Group Delay and Phase Delay (1A)

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Beat Signal

Very similar frequency signals

1.1 Hz $\cos(2\pi * 1.1 * t)$

0.9 Hz $\cos(2\pi * 0.9 * t)$

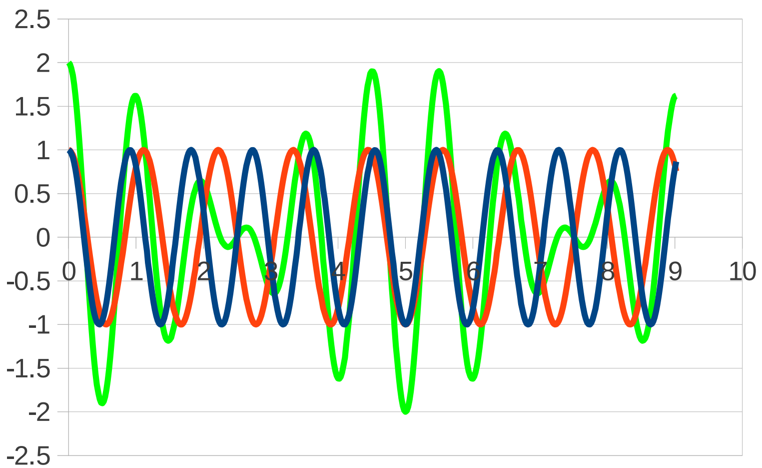
$$\cos(2\pi * 1.1 * t) + \cos(2\pi * 0.9 * t)$$

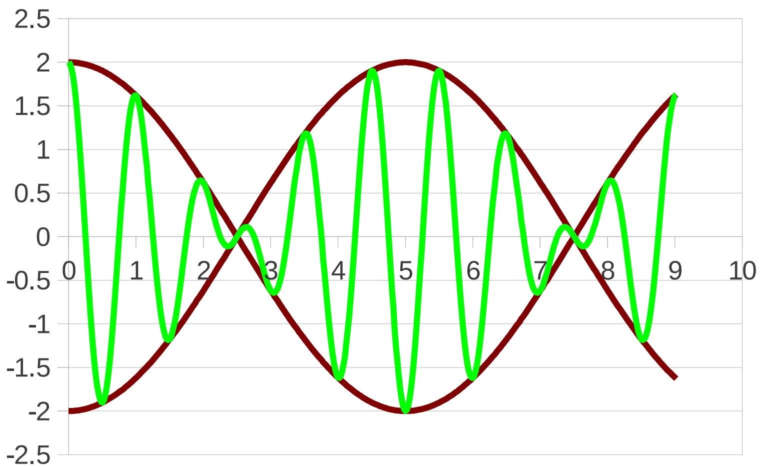
$$= \cos\left(2\pi * \frac{(1.1 - 0.9)}{2} * t\right) \cdot \cos\left(2\pi * \frac{(1.1 + 0.9)}{2} * t\right)$$

$$= \cos(2\pi * 0.1 * t) \cdot \cos(2\pi * 1.0 * t)$$

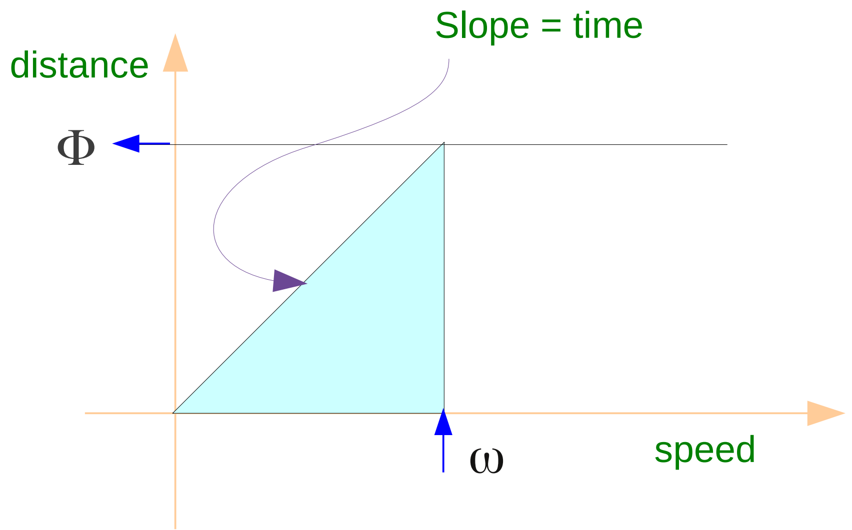
Slow moving envelop

Fast moving carrier



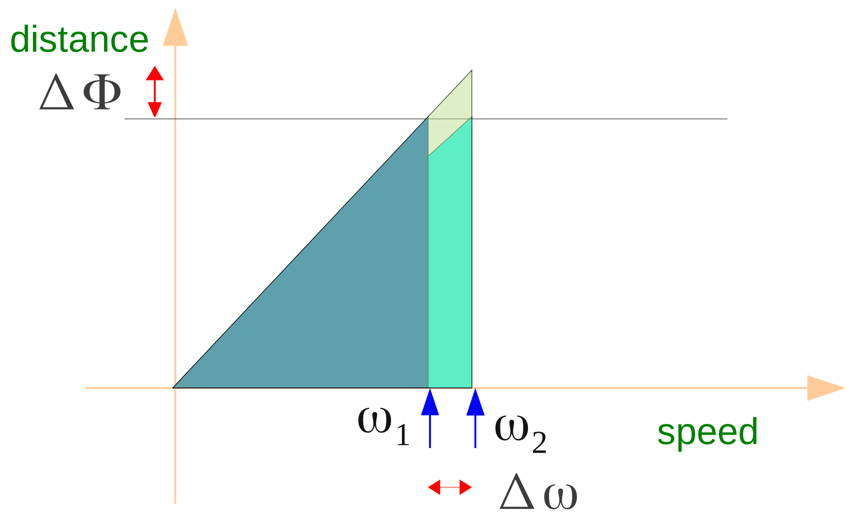


Angle and Angular Speed



$$\Phi = \omega \cdot t$$

$$t = \frac{\Phi}{\omega}$$



$$\Delta\Phi = \Delta\omega \cdot \Delta t$$

$$\Delta t = \frac{\Delta\Phi}{\Delta\omega}$$

Phase Shift and Time Shift

measure phase shift not in second
But in portions of a cosine wave cycle

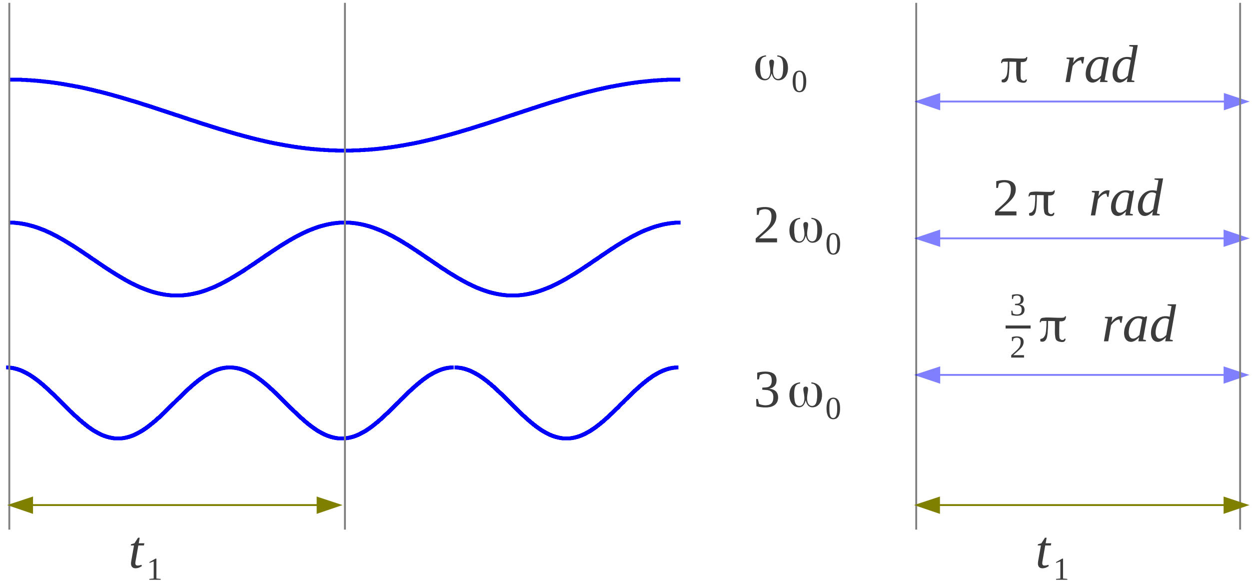
within phase change in one cycle

Phase Shift → in radians, degrees
Delay → in seconds (time)

Given time shift (delay)

$$\Phi = 2\pi f \cdot t$$

The actual phase shift is different
According to the frequency



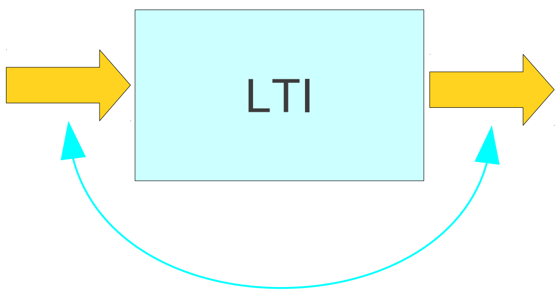
Uniform Time Delay

The same delay applied to all frequencies → The different phase shift to the different frequency

Linear Phase System

Phase Shift $\propto$ Frequency

Frequency Response
: uniform magnitude & delay



The waveform shape is preserved.

Uniform Time Delay

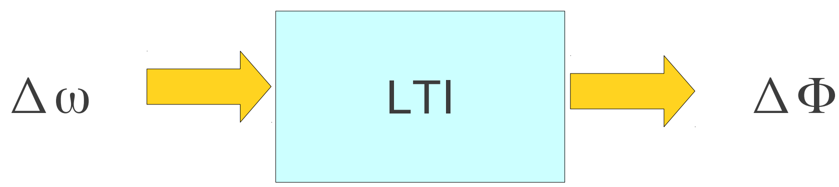
→ ***Could remove delay from the <u>phase response</u> to achieve a horizontal line at zero degree (No delay)***

$$2\pi f \cdot t$$

Group Delay

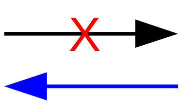
Consider the cosine components at *closely spaced frequencies* and *their phase shifts* in relation to each other

Group Delay:
The phase shift changes for small changes in frequency



A uniform, waveform preserving phase response → linear

Constant Group Delay



Uniform Time Delay

Group Delay

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] http://www.libinst.com/tpfd.htm