Convolution (1A)

•

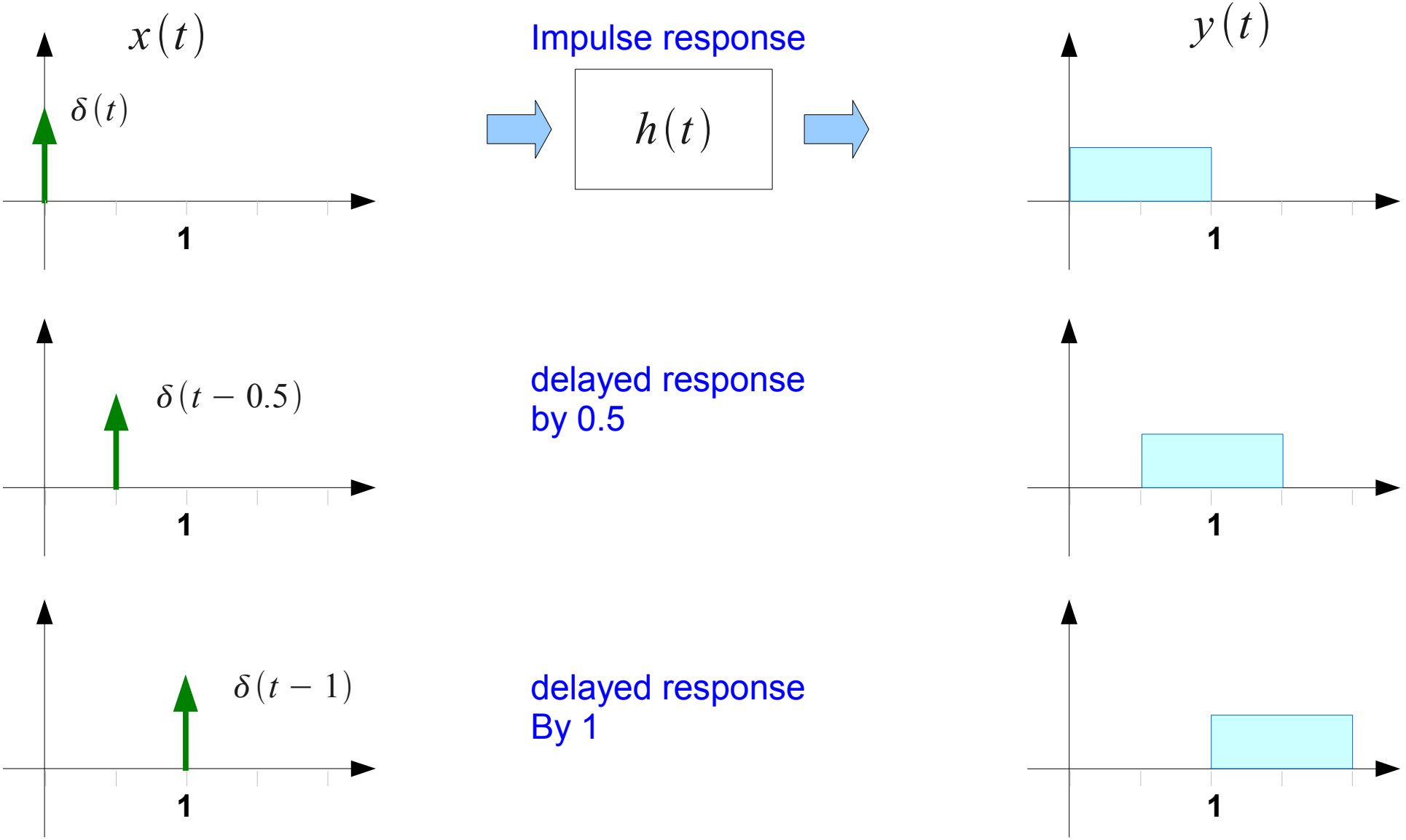
Young Won Lim
10년 11월 25일

Please send corrections (or suggestions) to youngwlim@hotmail.com.

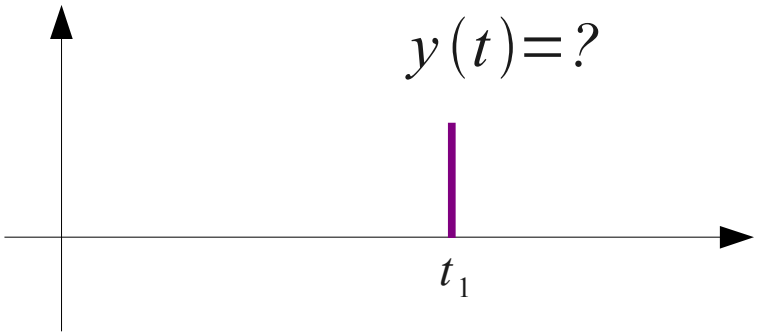
This document was produced by using OpenOffice and Octave.

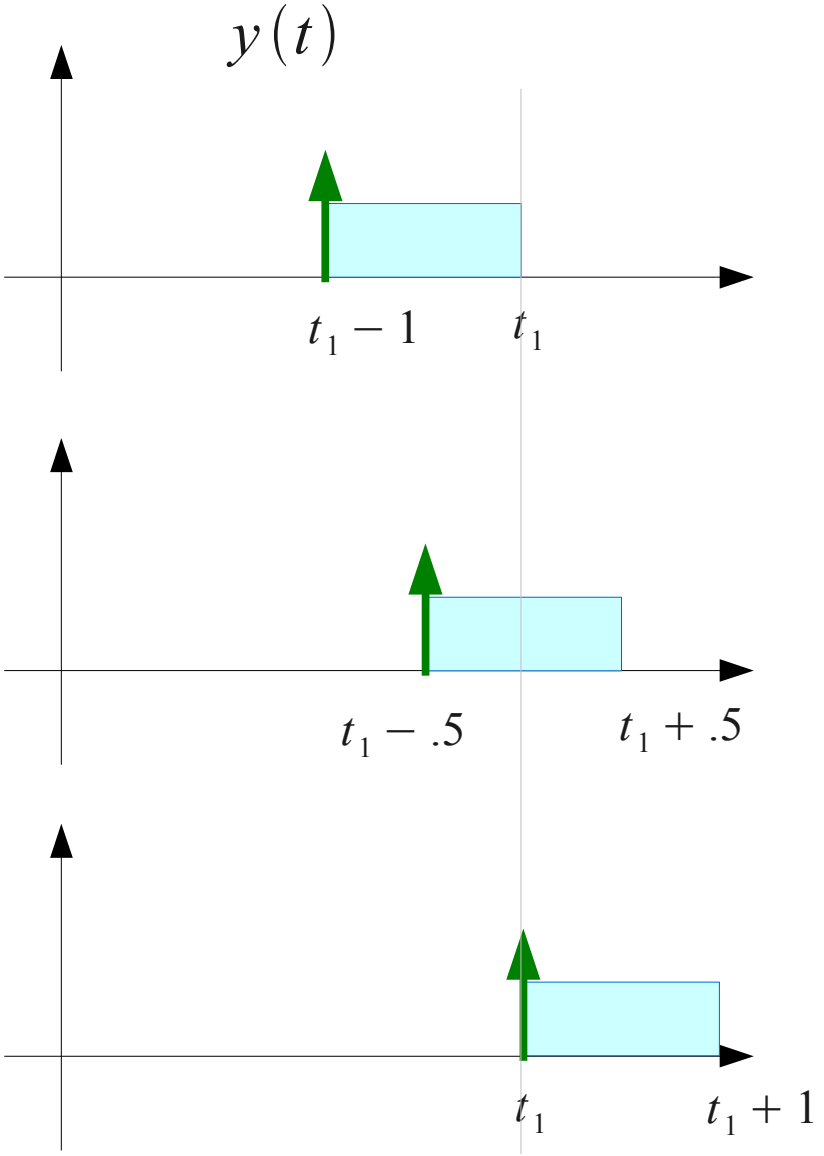
Young Won Lim
10년 11월 25일

Impulse Response

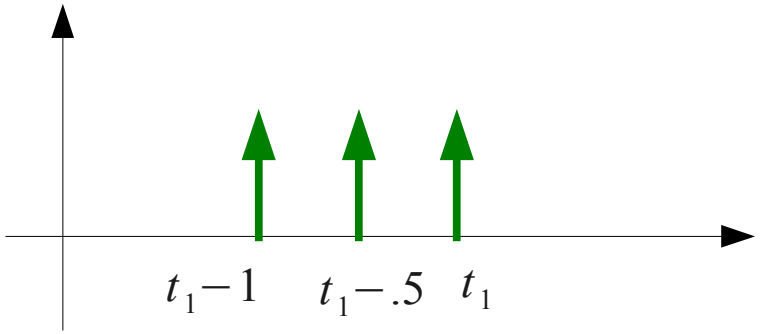


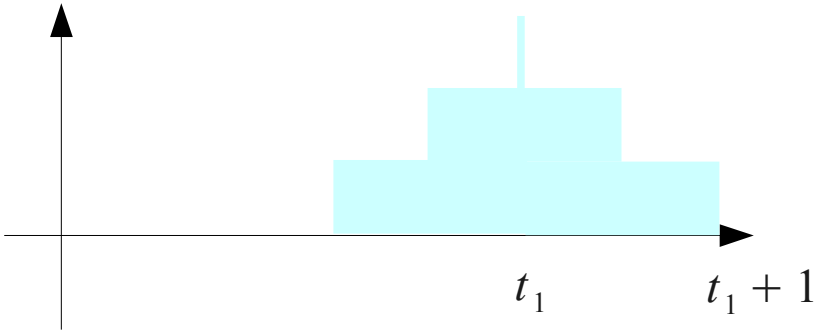
Output at t1





$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1) \qquad y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$



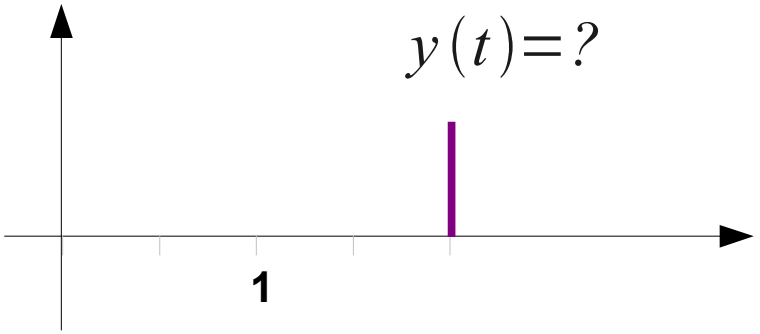


$$\begin{aligned} y(t) &= \int x(v)h(t-v)\,dv \\ &= \int \delta(v-t_1+1)h(t-v)\,dv \quad \rightarrow\ h(t-t_1+1) \\ &+ \int \delta(v-t_1+.5)h(t-v)\,dv \quad \rightarrow\ h(t-t_1+.5) \\ &+ \int \delta(v-t_1)h(t-v)\,dv \quad \rightarrow\ h(t-t_1) \end{aligned}$$

$$\begin{aligned} y(t_1) &= h(t_1-t_1+1) + h(t_1-t_1+.5) + h(t_1-t_1) \\ &= h(1) + h(.5) + h(0) \end{aligned}$$

N=8 DFT

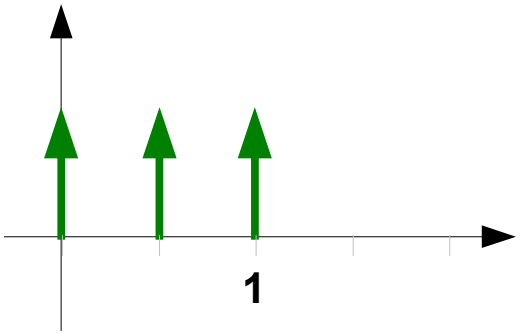
$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

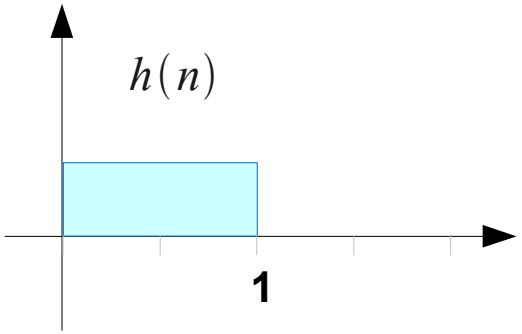


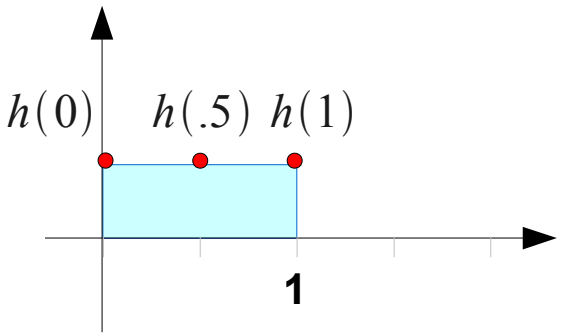
$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

$$y(t_1) = h(1) + h(.5) + h(0)$$

$$\delta(t) + \delta(t-.5) + \delta(t-1)$$







N=8 DFT

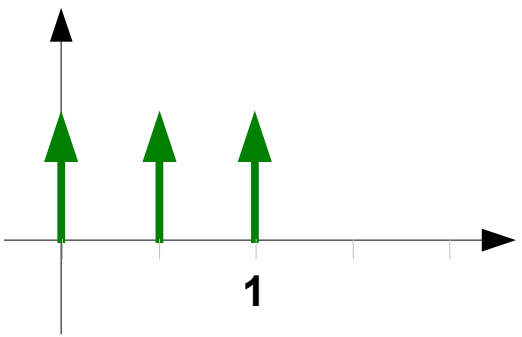
$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

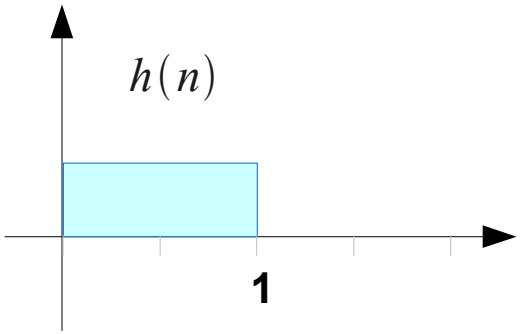
$$x(v) = \delta(v-t_1+1) + \delta(v-t_1+.5) + \delta(v-t_1)$$

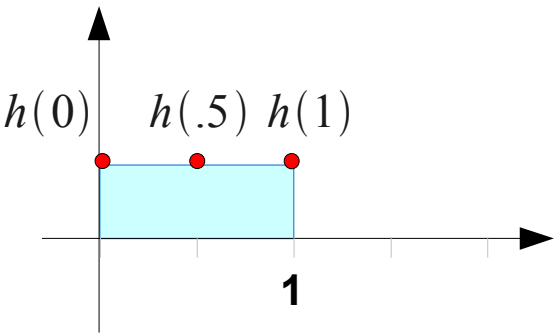
$$x(t-v) = \delta(t-v-t_1+1) + \delta(t-v-t_1+.5) + \delta(t-v-t_1)$$

$$\begin{aligned} y(t) &= \int x(t-v)h(v)\,dv \\ &= \int \delta(t-v-t_1+1)h(v)\,dv \quad \rightarrow \; h(t-t_1+1) \\ &+ \int \delta(t-v-t_1+.5)h(v)\,dv \quad \rightarrow \; h(t-t_1+.5) \\ &+ \int \delta(t-v-t_1)h(v)\,dv \quad \rightarrow \; h(t-t_1) \end{aligned}$$

$\delta(t) + \delta(t-.5) + \delta(t-1)$







N=8 DFT

$$y(t) = \int x(v)h(t-v)\,dv$$

$$= \int \delta(v-t_1+1)h(t-v)\,dv \quad \rightarrow \quad h(t-t_1+1)$$

$$+ \int \delta(v-t_1+.5)h(t-v)\,dv \quad \rightarrow \quad h(t-t_1+.5)$$

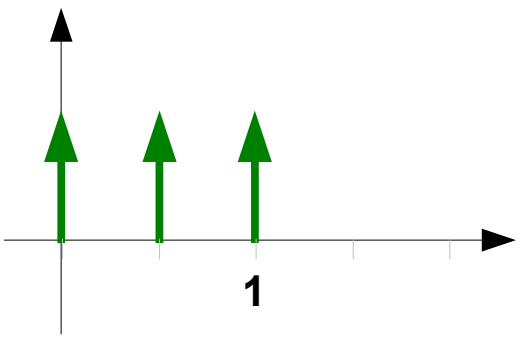
$$+ \int \delta(v-t_1)h(t-v)\,dv \quad \rightarrow \quad h(t-t_1)$$

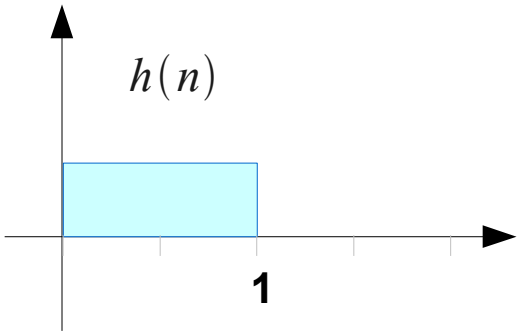
$$y(t) = \int x(t-v)h(v)\,dv$$

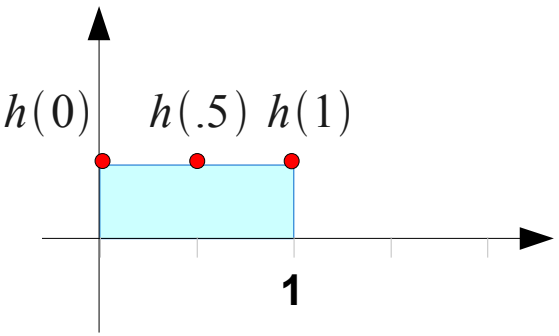
$$= \int \delta(t-v-t_1+1)h(v)\,dv \quad \rightarrow \quad h(t-t_1+1)$$

$$+ \int \delta(t-v-t_1+.5)h(v)\,dv \quad \rightarrow \quad h(t-t_1+.5)$$

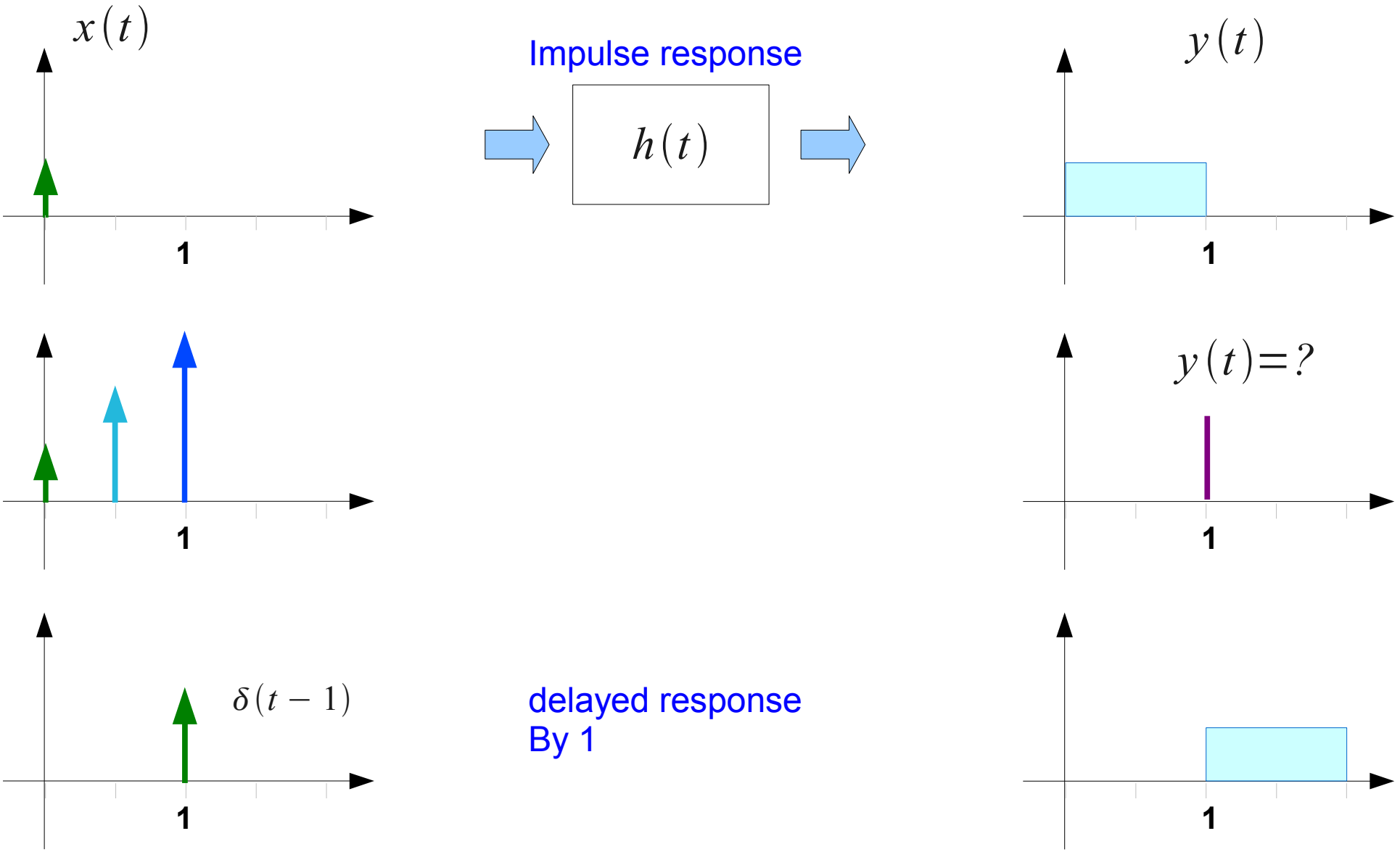
$$+ \int \delta(t-v-t_1)h(v)\,dv \quad \rightarrow \quad h(t-t_1)$$



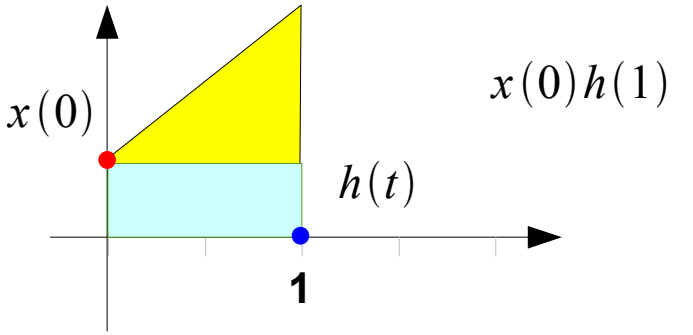




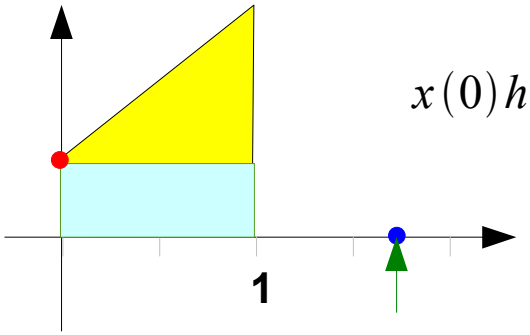
Impulse Response

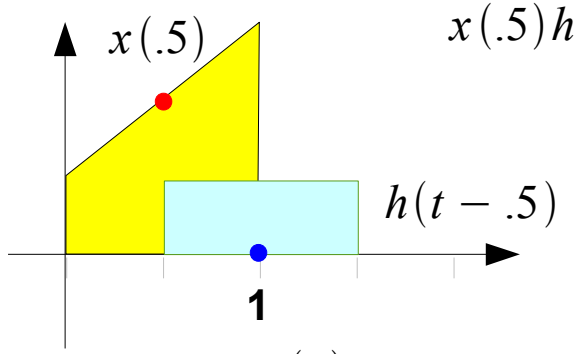


Impulse Response

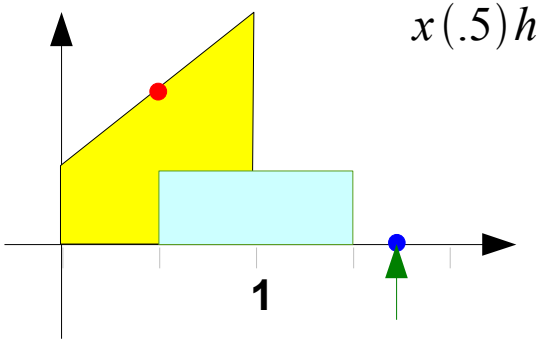


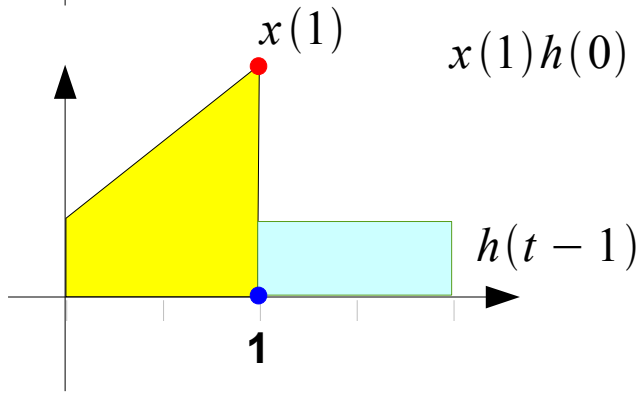
$x(0)h(1)$



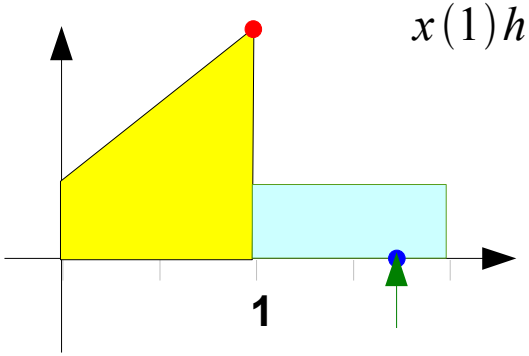


$x(.5)h(.5)$

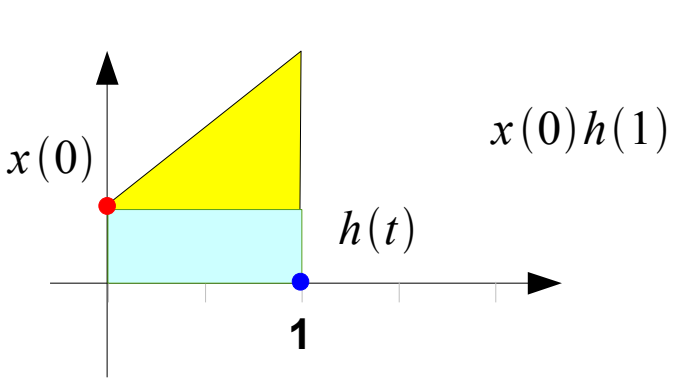




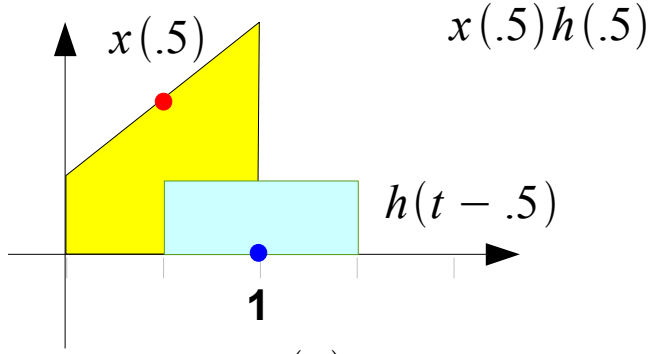
$x(1)h(0)$



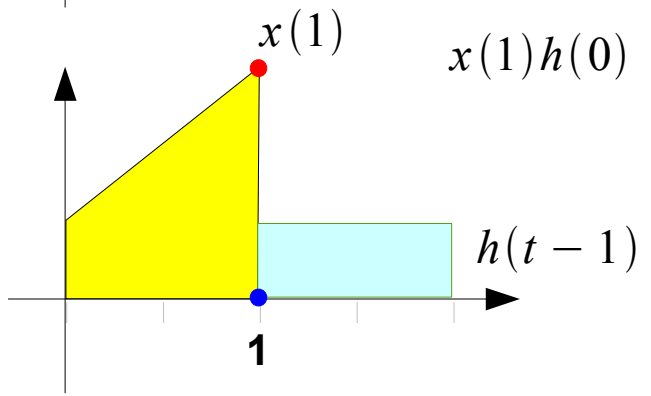
Impulse Response



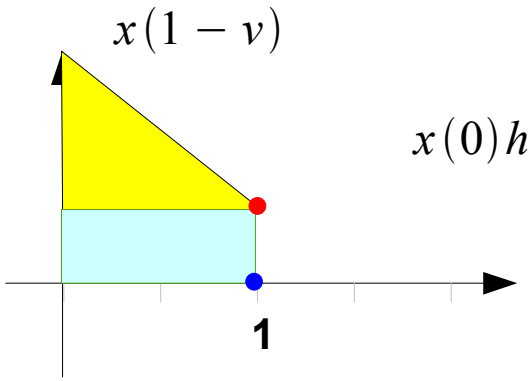
$x(0)h(1)$



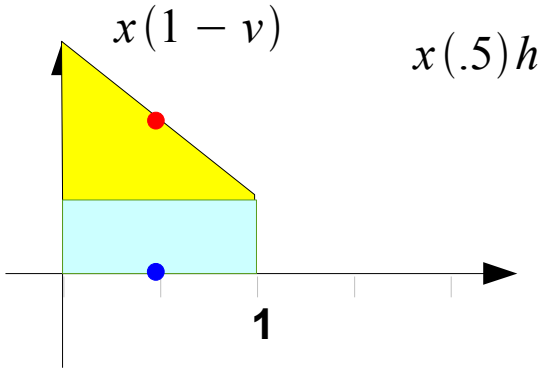
$x(.5)h(.5)$



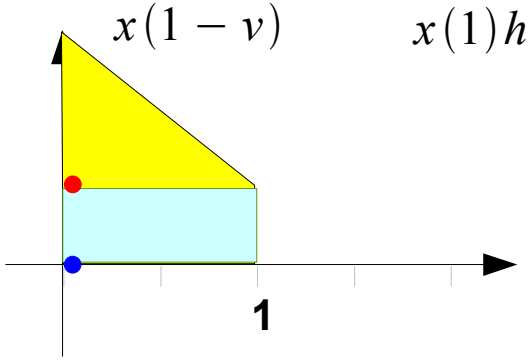
$x(1)h(0)$



$x(0)h(1)$



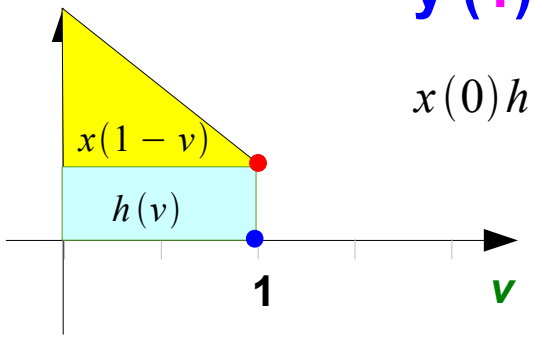
$x(.5)h(.5)$



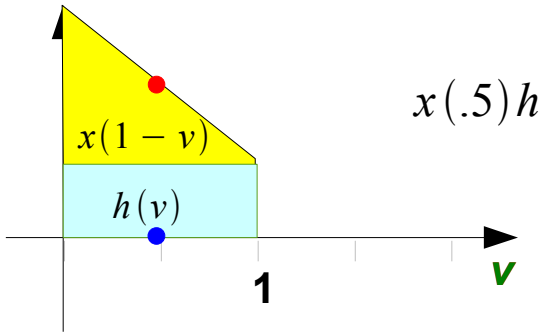
$x(1)h(0)$

Impulse Response

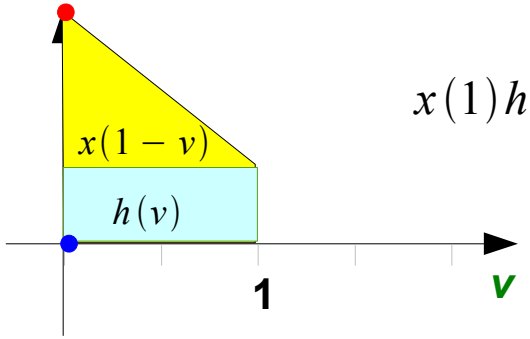
y (1)



$$x(0)h(1)$$

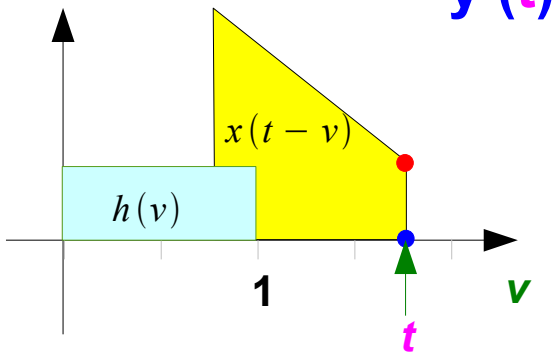


$$x(.5)h(.5)$$

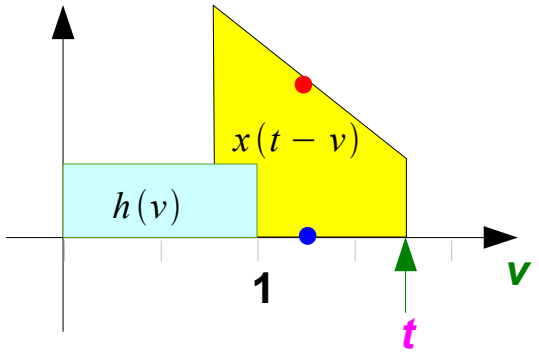


$$x(1)h(0)$$

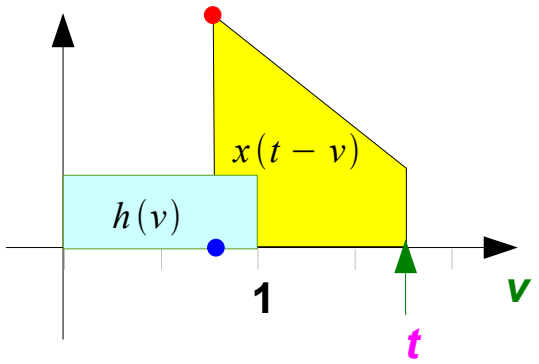
y (t)



$$x(t-t)h(t) = x(0)h(t)$$

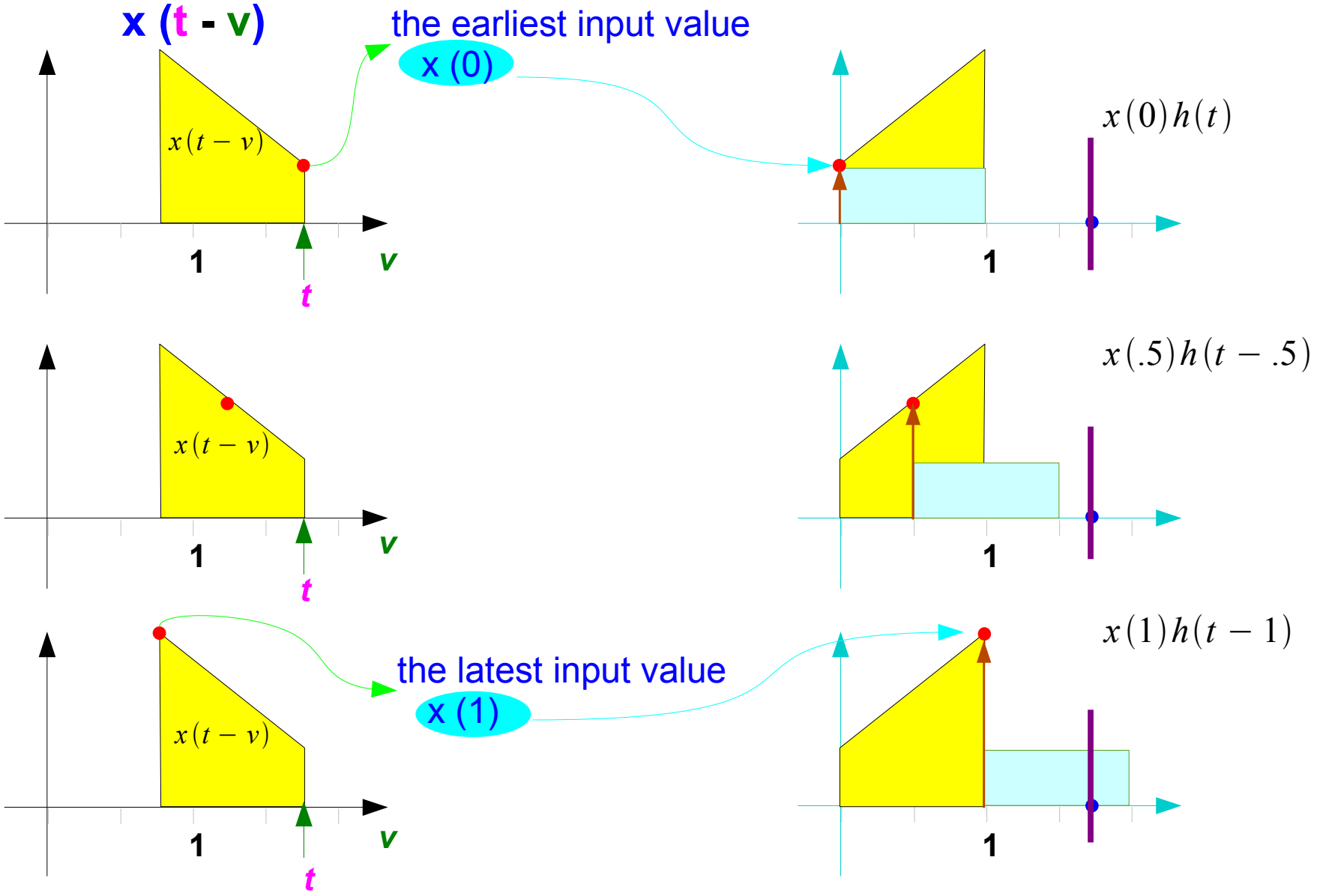


$$x(t-t+.5)h(t-.5) = x(.5)h(t-.5)$$

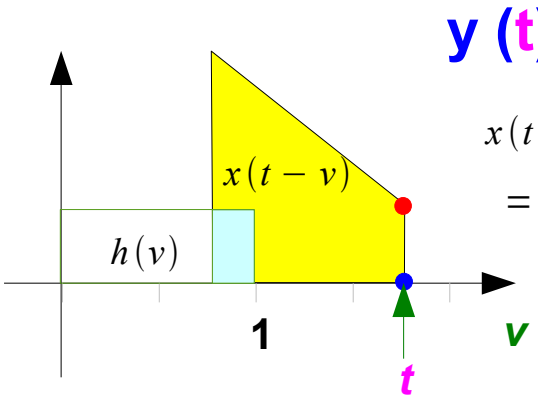


$$x(t-t+1)h(t-1)$$
$$x(1)h(t-1)$$

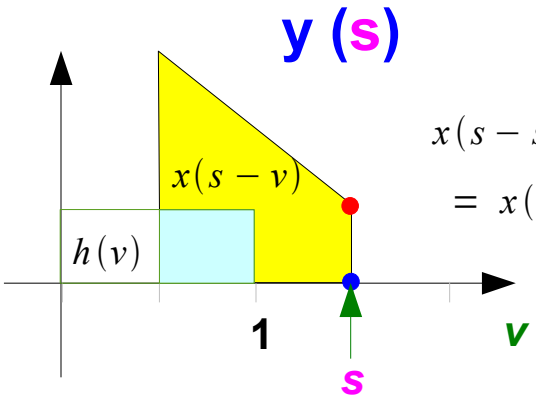
Impulse Response



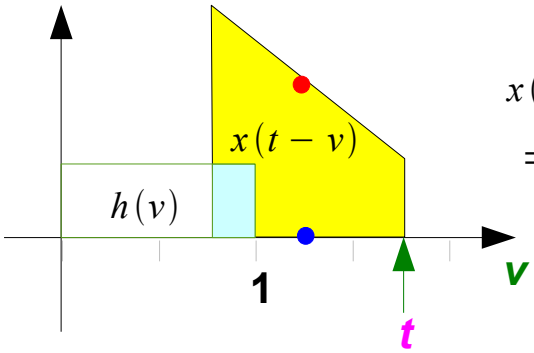
Impulse Response



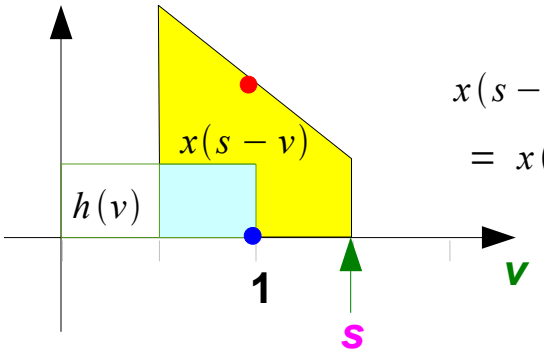
$$x(t-t)h(t)$$
$$= x(0)h(t)$$



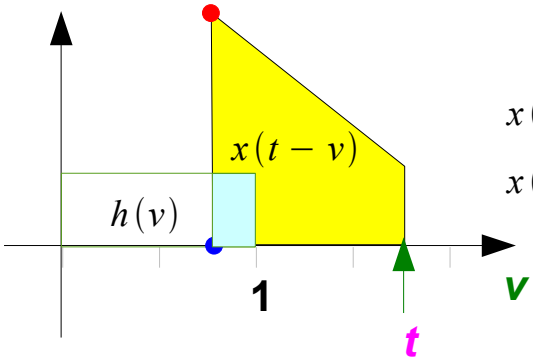
$$x(s-s)h(s)$$
$$= x(0)h(s)$$



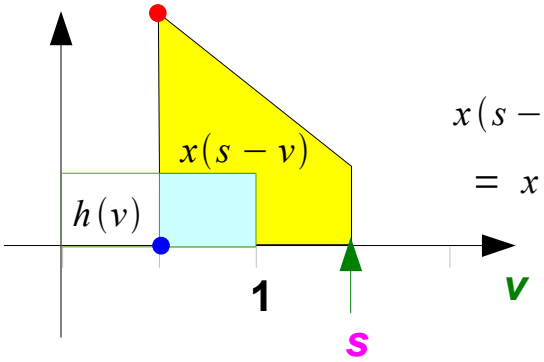
$$x(t-t+.5)h(t-.5)$$
$$= x(.5)h(t-.5)$$



$$x(s-s+.5)h(s-.5)$$
$$= x(.5)h(s-.5)$$



$$x(t-t+1)h(t-1)$$
$$x(1)h(t-1)$$

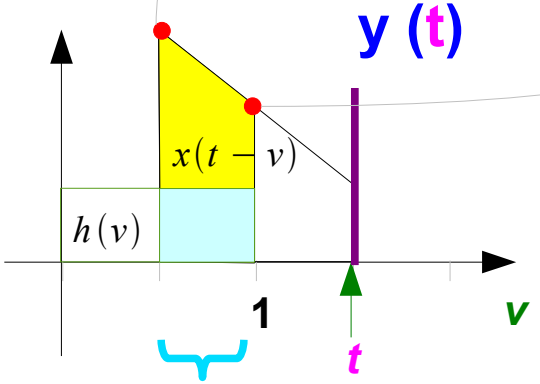


$$x(s-s+1)h(s-1)$$
$$= x(1)h(s-1)$$

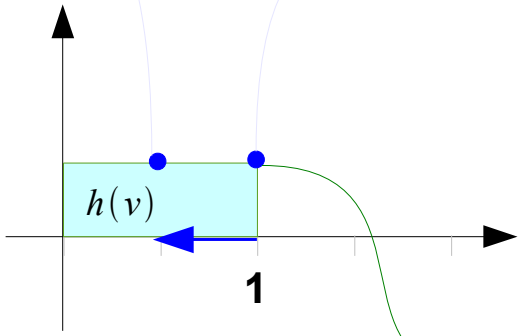
Impulse Response

the latest input value
that affects the output value y(t)

the earliest input value
that affects the output value y(t)

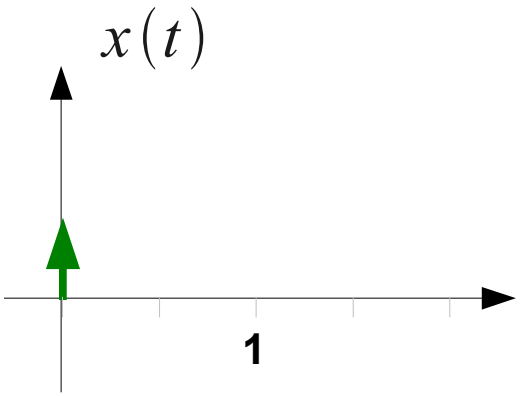


$$y(t) = \int x(t-v)h(v)\,dv$$

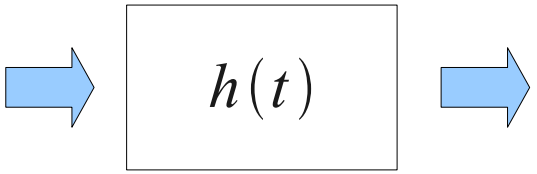


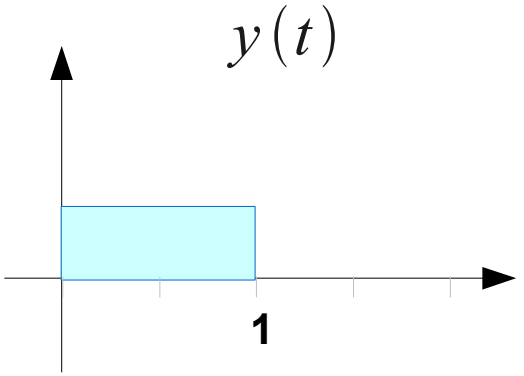
this value of h(t)
is used first

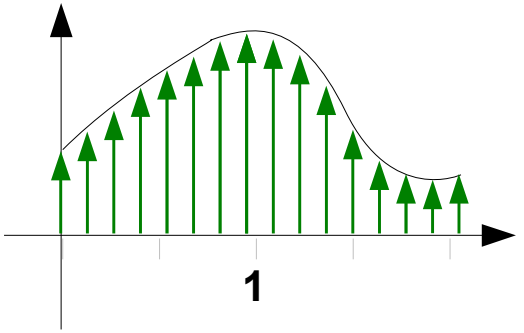
Impulse Response

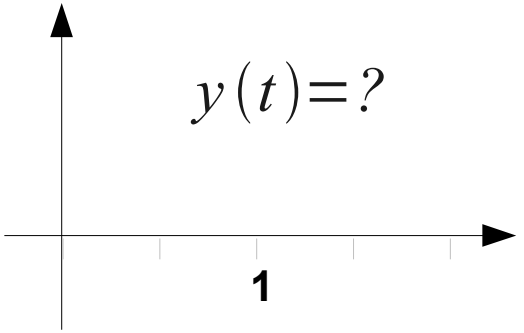


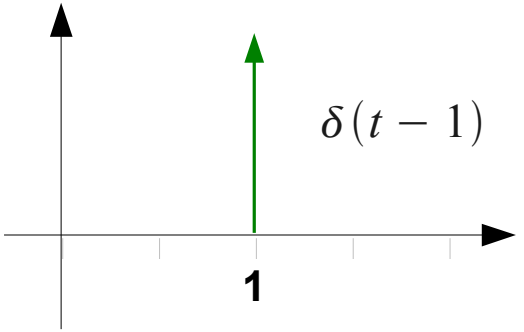
Impulse response



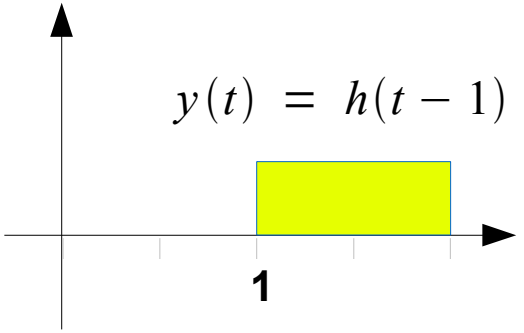




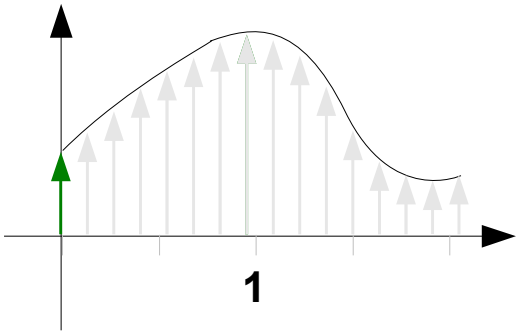


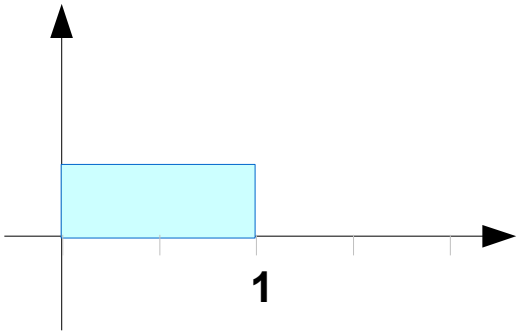


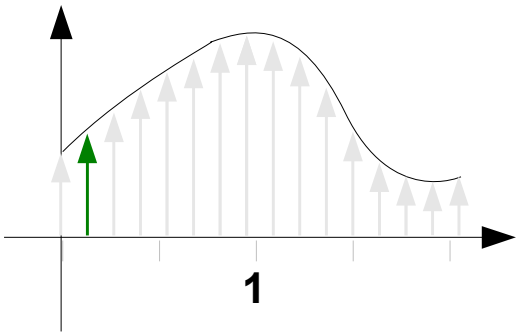
delayed response
By 1

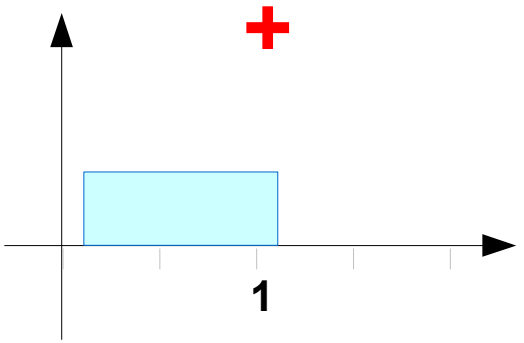


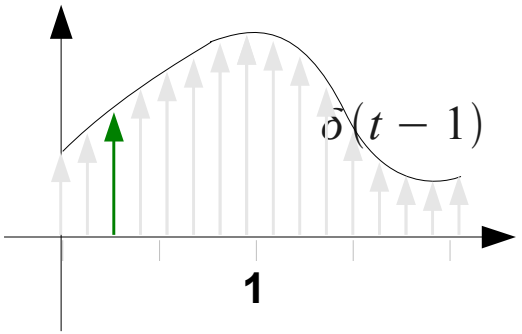
Impulse Response

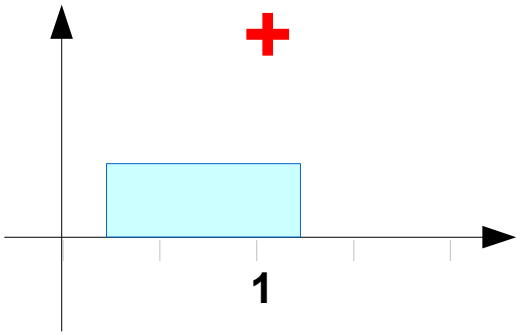




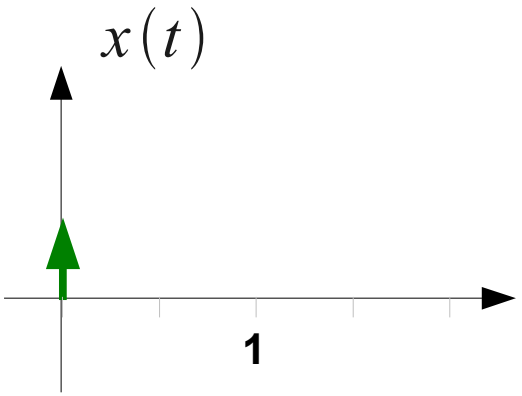




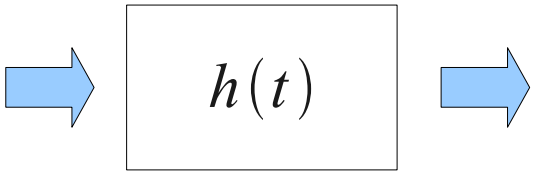


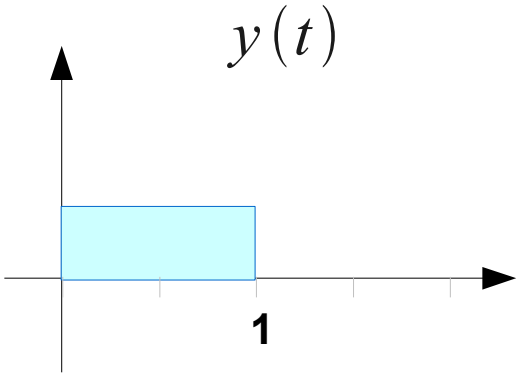


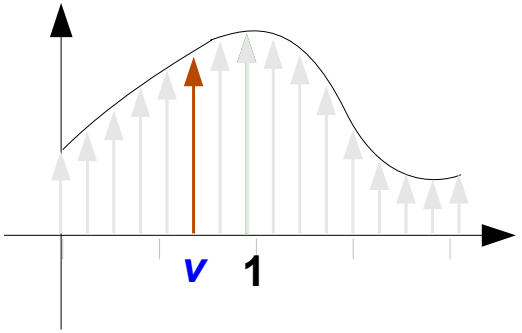
Impulse Response



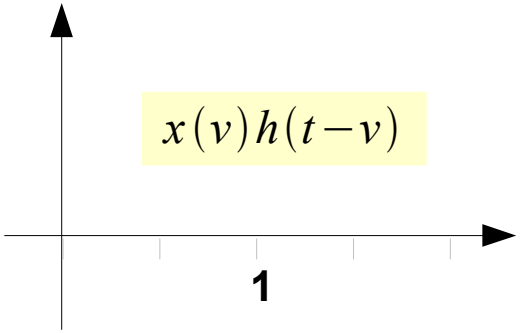
Impulse response

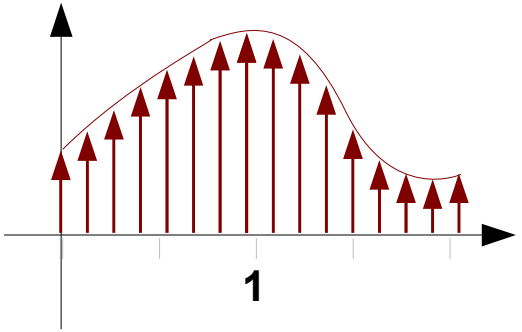


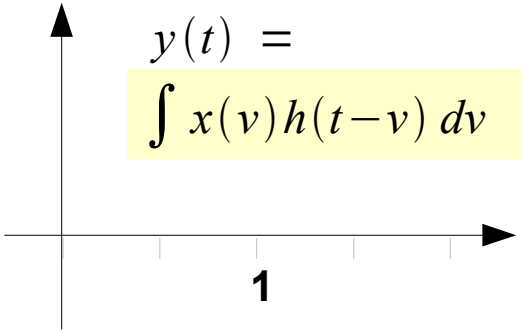




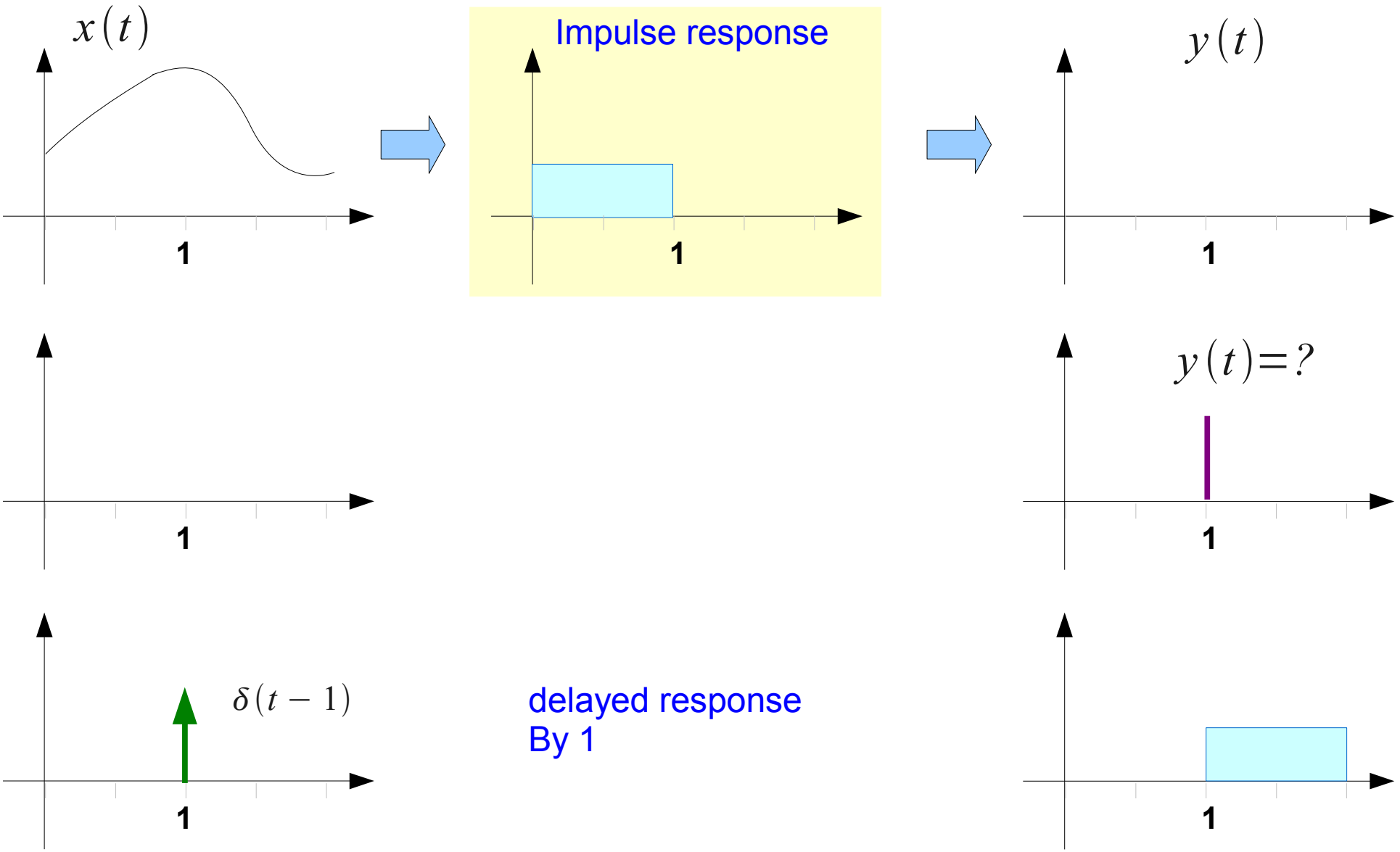
input value at time v
→ x (v)
delayed impulse response
→ h (t - v)







Impulse Response



References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003