Time Domain Techniques (3B)

for Noisy Signals

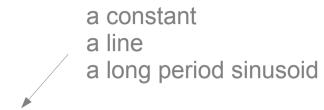
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Signal Detrending / Spike Removal

Remove low frequency noise



least square fitting a low frequency function tr[i]

subtracting the trend from the measurements y[i] = x[i] - tr[i]

example)

removing the dc component

$$y[i] = x[i] - tr[i] = x[i] - \frac{1}{N} \sum_{i=0}^{N-1} x[i-1]$$

removing spikes

$$y[i] = x[i]$$
 if $|x[i] - y[i-1]| < threshold$

$$y[i] = \frac{x[i-1] + x[i+1]}{2}$$
 else

Stacking

Remove background noise

Assume noise has a zero mean

Measuring multiple times (ensemble)

Averaging across the ensemble

Reduces the noise level in the averaged signal Increases the SNR of the correlated component (signal)

$$E[X[i]] \rightarrow x[i]$$

$$E[N[i]] \rightarrow 0$$

$$E[N^2[i]] \rightarrow \frac{\sigma_n}{\sqrt{M}}$$

$$SNR \propto \sqrt{M}$$

Moving Average Filter

L-point Running Averager

$$y[n] = \frac{1}{L} \sum_{k=0}^{L-1} x[n-k] = \frac{1}{L} \{x[n] + x[n-1] + \dots + x[n-L+1] \}$$

$$Y(e^{j\hat{\omega}}) = \frac{1}{L} \sum_{k=0}^{L-1} e^{-j\hat{\omega}k} X(e^{j\hat{\omega}}) = \frac{1}{L} \{X(e^{j\hat{\omega}}) + X(e^{j\hat{\omega}})e^{-j\hat{\omega}\mathbf{1}} + \dots + X(e^{j\hat{\omega}})e^{-j\hat{\omega}(L-1)}\}$$

$$H(e^{j\hat{\omega}}) = \frac{1}{L} \sum_{k=0}^{L-1} e^{j\hat{\omega}k} = \frac{1}{L} \{1 + e^{-j\hat{\omega}1} + \dots + e^{-j\hat{\omega}(L-1)}\} = \frac{1}{L} \left(\frac{1 - e^{-j\hat{\omega}L}}{1 - e^{-j\hat{\omega}}}\right)$$

$$= \frac{1}{L} \left(\frac{e^{-j\hat{\omega}L/2}}{e^{-j\hat{\omega}/2}} \right) \left(\frac{e^{+j\hat{\omega}L/2} - e^{-j\hat{\omega}L/2}}{e^{+j\hat{\omega}/2} - e^{-j\hat{\omega}/2}} \right) = \frac{1}{L} \left(\frac{e^{+j\hat{\omega}L/2} - e^{-j\hat{\omega}L/2}}{e^{+j\hat{\omega}/2} - e^{-j\hat{\omega}/2}} \right) e^{-j\hat{\omega}(L-1)/2}$$

$$= \left(\frac{\sin(\hat{\omega}L/2)}{L\sin(\hat{\omega}/2)}\right)e^{-j\hat{\omega}(L-1)/2}$$

Dirichlet Function

$$H(e^{j\hat{\omega}}) = D_L(e^{j\hat{\omega}}) e^{-j\hat{\omega}(L-1)/2}$$

$$D_{\underline{L}}(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}\underline{L}/2)}{\underline{L}\sin(\hat{\omega}/2)}$$

Moving Average Filter

Frequency Response

$H(e^{j\hat{\omega}}) = D_{I}(e^{j\hat{\omega}}) e^{-j\hat{\omega}(L-1)/2}$

Dirichlet Function

$$D_{L}(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}L/2)}{L\sin(\hat{\omega}/2)}$$

$$D_{L}(e^{j(\hat{\omega}+2\pi)}) = \frac{\sin((\hat{\omega}+2\pi)L/2)}{L\sin((\hat{\omega}+2\pi)/2)}$$
$$= \frac{\sin(\hat{\omega}L/2+L\pi)}{L\sin(\hat{\omega}/2+\pi)}$$

$$\begin{cases} +D_L(e^{j\hat{\omega}}) & \text{for an odd } L \text{ (period: } 2\pi) \\ -D_L(e^{j\hat{\omega}}) & \text{for an even } L \end{cases}$$

$$0 \le \hat{\omega} \le +\pi$$

$$0 \le \hat{\omega}/2 \le +\frac{\pi}{2} \qquad \qquad 0 \le \hat{\omega} L/2 \le +L\frac{\pi}{2}$$

$$0 \le \sin(\hat{\omega}/2) \le +1$$
 $-1 \le \sin(\hat{\omega}/2) \le +1$

Envelope:
$$\frac{1}{\sin(\hat{\omega}/2)}$$
 Zeros: $\hat{\omega} = \frac{2\pi}{L}k$

$$\sin(\hat{\omega} L/2) = 0$$

$$D_{L}(e^{-j\hat{\omega}}) = \frac{\sin(-\hat{\omega}L/2)}{L\sin(-\hat{\omega}/2)} = D_{L}(e^{j\hat{\omega}})$$

an even function

$$\lim_{\hat{\omega} \to 0} D_{L}(e^{j\hat{\omega}}) = \lim_{\hat{\omega} \to 0} \frac{L/2\cos(\hat{\omega}L/2)}{L/2\cos(\hat{\omega}/2)} = 1$$

$$\max \text{ value } D_{L}(e^{j\hat{\omega}}) = 1 \text{ when } \hat{\omega} = 0$$

Correlation: Identifying Similarities

References

- [1] http://en.wikipedia.org/
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] J.C. Santamarina, D Fratta, "Discrete Signals and Inverse Problems: An Introduction for Engineers and Scientists", 2005