Bandpass Sampling (2B)

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2/2/12

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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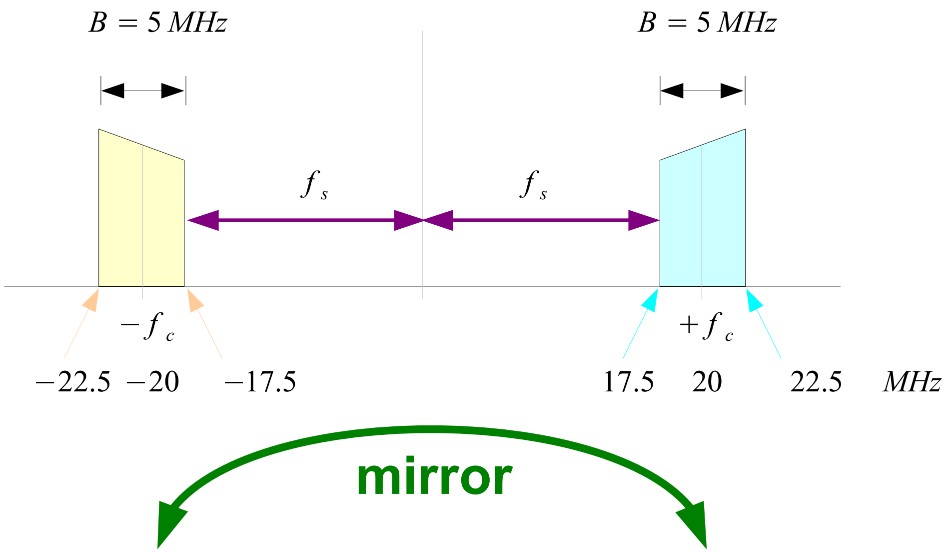
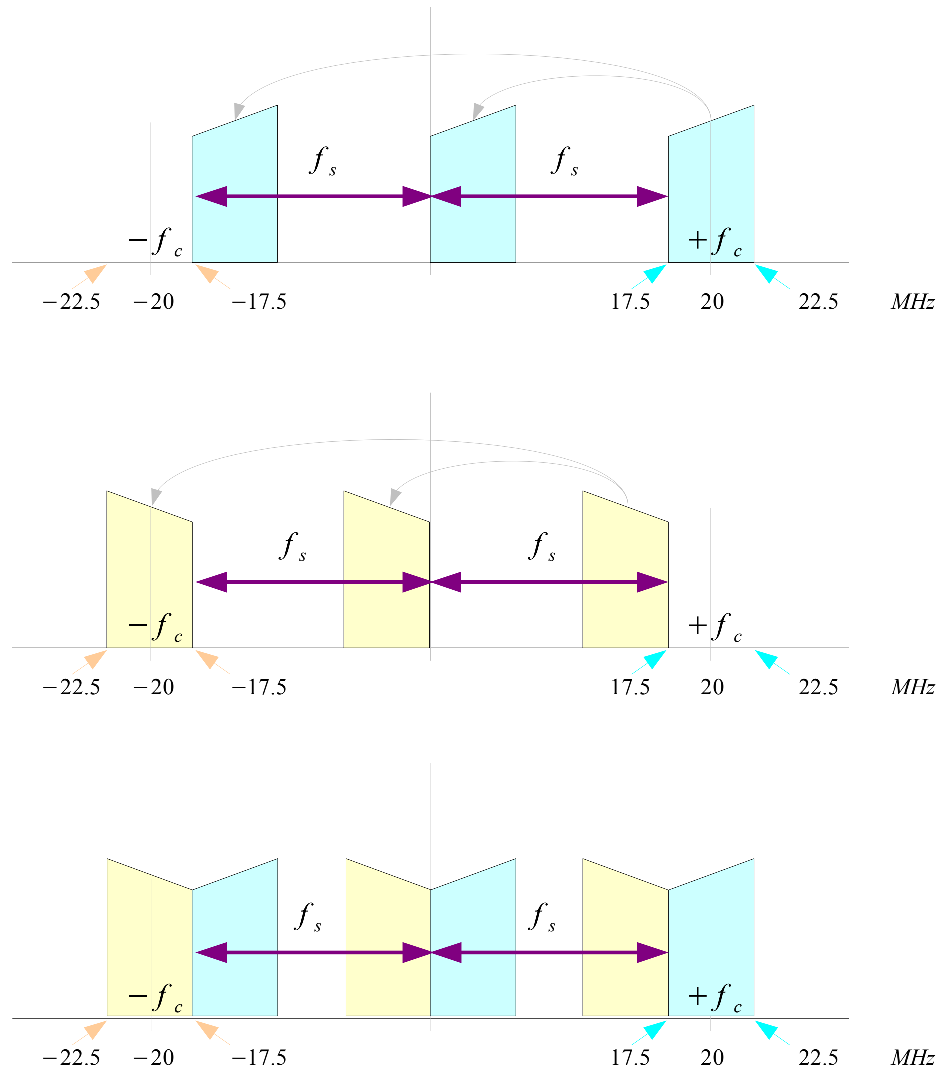
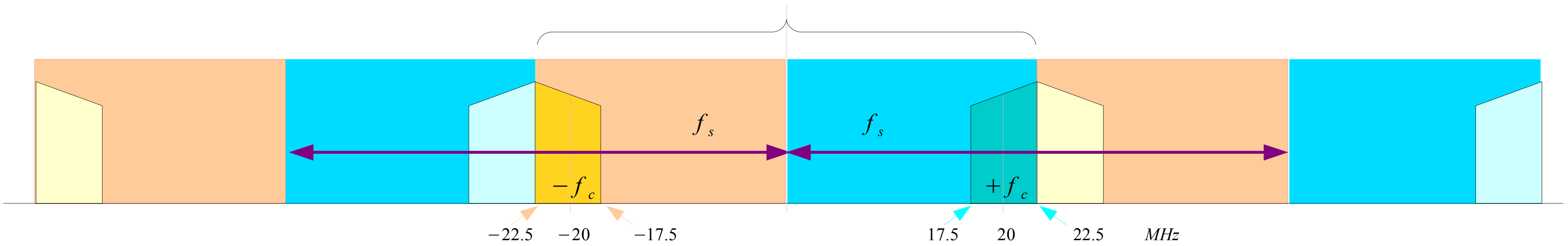
Band-limited Signal



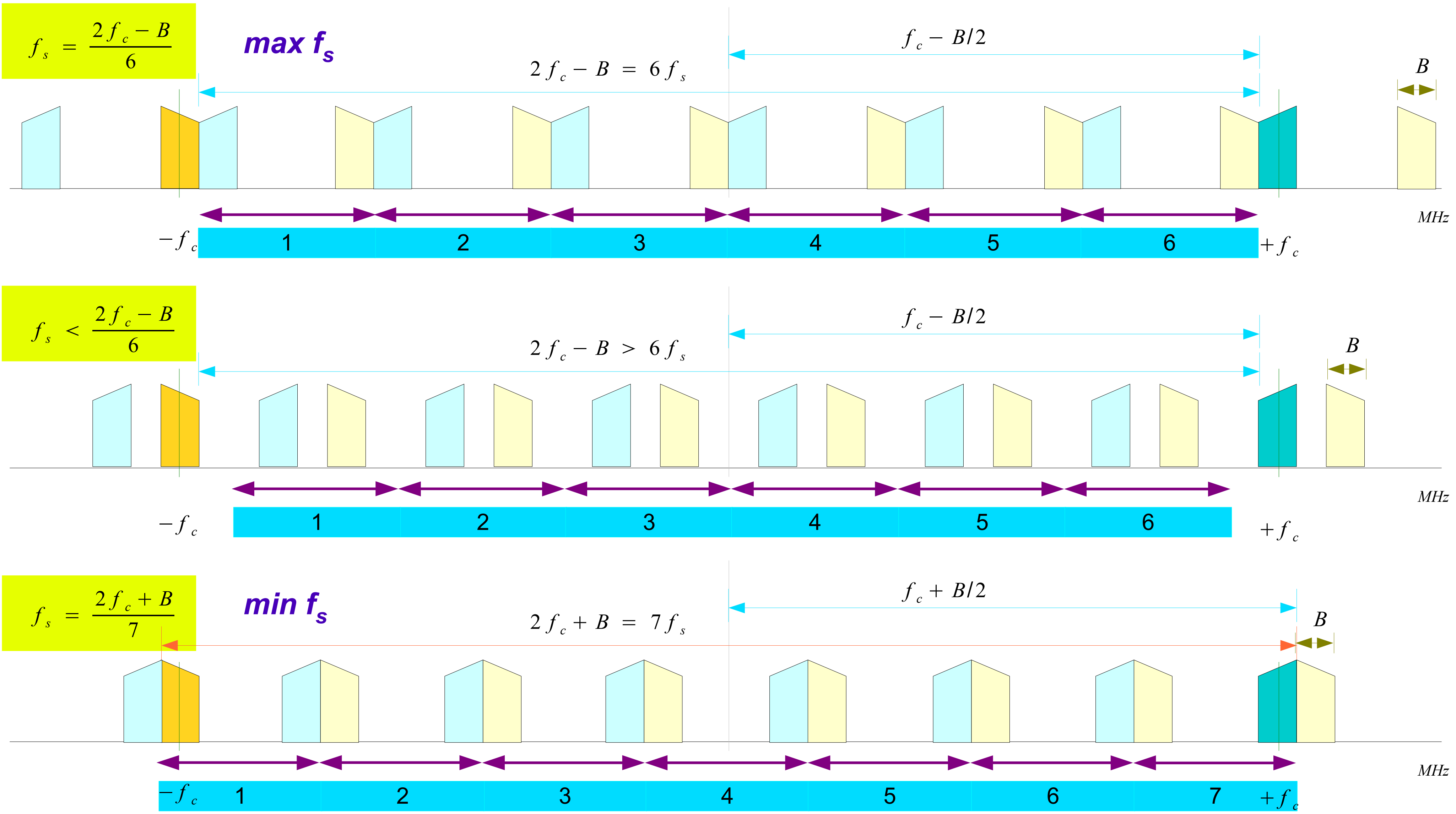


- **Bandpass Sampling**
- **IF filtering**
- **Harmonic Sampling**
- **Sub-Nyquist Sampling**

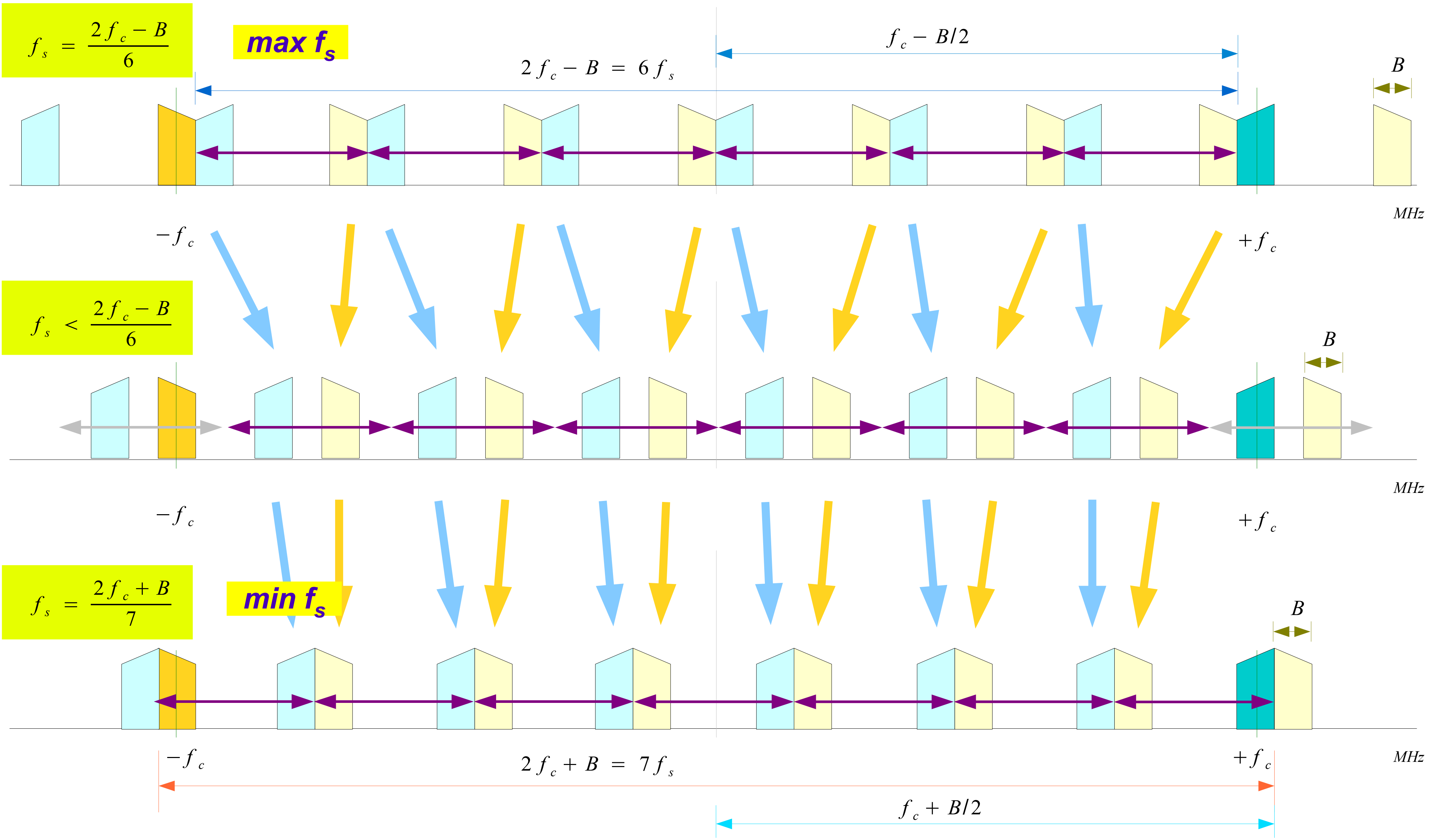
Low-pass Signal Sampling



Band-pass Signal Sampling



Band-pass Signal Sampling



$$\boxed{\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}}$$

$$f_c = 20\,MHz$$
$$B = 5\,MHz$$

$$\boxed{2B \le f_s}$$

$$\frac{2\cdot 20 + 5}{1+1} = 22.5 \le f_s \le \frac{2\cdot 20 - 5}{1} = 35 \Rightarrow f_s = 22.5\,MHz \quad (10 \le f_s)$$

$$\frac{2\cdot 20 + 5}{2+1} = 15 \le f_s \le \frac{2\cdot 20 - 5}{2} = 17.5 \Rightarrow f_s = 17.5\,MHz \quad (10 \le f_s)$$

$$\frac{2\cdot 20 + 5}{3+1} = 11.25 \le f_s \le \frac{2\cdot 20 - 5}{3} = 11.67 \Rightarrow f_s = 11.25\,MHz \quad (10 \le f_s)$$

$$\frac{2\cdot 20 + 5}{4+1} = 9 \ge \frac{2\cdot 20 - 5}{4} = 8.75 \Rightarrow$$ X

$$\frac{2\cdot 20 + 5}{5+1} = 7.5 \ge \frac{2\cdot 20 - 5}{5} = 7.0 \Rightarrow$$ X

$$\boxed{\frac{2f_c + B}{m+1}} \le f_s \le \frac{2f_c - B}{m}$$

$$f_c = 20\,MHz$$
$$B = 5\,MHz$$

$$2B \le f_s$$

$$\frac{f_c + B/2}{B} = R$$

highest signal frequency / bandwidth

$$\frac{2f_c + B}{(m+1)B} = f(m, R)$$

minimum sampling rate / bandwidth

$$\frac{2(f_c + B/2)}{(m+1)B} = \frac{2R}{m+1} = f(m, R)$$

$m = 1 \quad f(1, R) = R$

$m = 2 \quad f(2, R) = \frac{2}{3}R$

$m = 3 \quad f(3, R) = \frac{1}{2}R$

$m = 4 \quad f(4, R) = \frac{2}{5}R$

$m = 5 \quad f(5, R) = \frac{1}{3}R$

$m = 6 \quad f(6, R) = \frac{2}{7}R$

$m = 7 \quad f(7, R) = \frac{1}{4}R$

$m = 8 \quad f(8, R) = \frac{2}{9}R$

$$\boxed{\frac{2f_c + B}{m+1}} \le f_s \le \frac{2f_c - B}{m}$$

$$f_c = 20\,MHz$$
$$B = 5\,MHz$$

$$2B \le f_s$$

$$\frac{f_c + B/2}{B} = R$$

$$\frac{\textbf{highest signal frequency}}{\textbf{bandwidth}}$$

$$f_H = f_c + B/2$$

$$R = f_H / B$$

$$\frac{2f_c + B}{m+1} \cdot \frac{1}{B} = f(m, R)$$

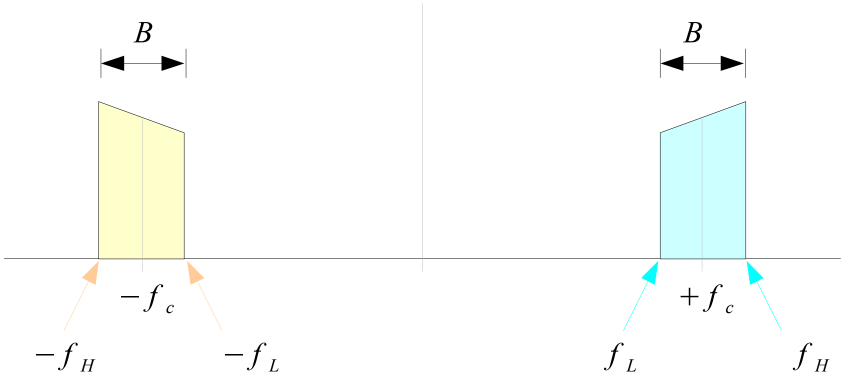
$$\frac{\textbf{minimum sampling rate}}{\textbf{bandwidth}}$$

$$f_{s,min} = \frac{2f_c + B}{m+1} = \frac{2f_H}{k}$$

$$\frac{2(f_c + B/2)}{(m+1)B} = \frac{2R}{m+1} = f(m, R)$$

$$f(m, R) = \frac{2f_H}{kB} = \frac{2R}{k}$$

$$m + 1 = k$$



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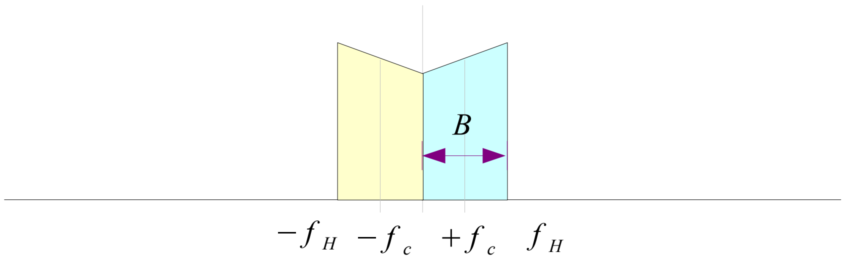
$k = 1 \quad (m = 0)$

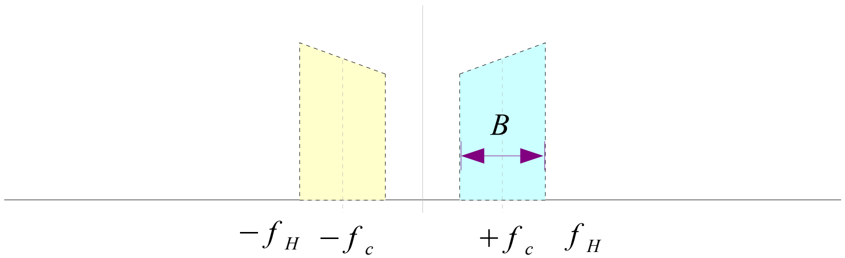


$$f_H = f_c + B/2 = 1\,B$$

$$f_H / B = R = 1$$

$$f_{s,min} = \frac{2 f_H}{k} = \frac{2 f_H}{m+1} = 2\,B$$

$k = 1 \quad (m = 0)$

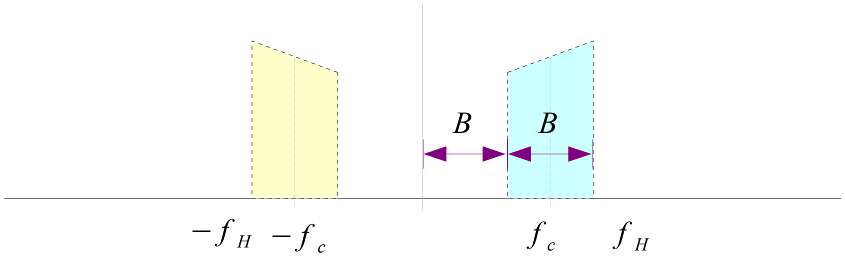


$$f_H = f_c + B/2 = 1.5\,B$$

$$f_H / B = R = 1.5$$

$$f_{s,min} = \frac{2 f_H}{k} = \frac{2 f_H}{m+1} = 3\,B$$

$k = 1 \quad (m = 0)$

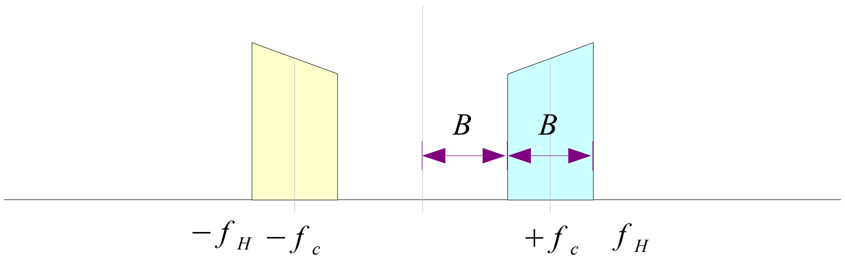


$$f_H = f_c + B/2 = 2\,B$$

$$f_H / B = R = 2$$

$$f_{s,min} = \frac{2 f_H}{k} = \frac{2 f_H}{m+1} = 4\,B$$

$k = 2 \quad (m = 1)$

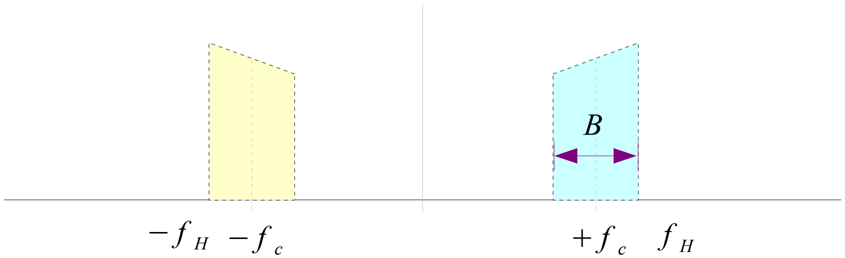


$$f_H = f_c + B/2 = 2B$$

$$f_H / B = R = 2$$

$$f_{s,min} = \frac{2 f_H}{k} = \frac{2 f_H}{m+1} = 2B$$

$k = 2 \quad (m = 1)$

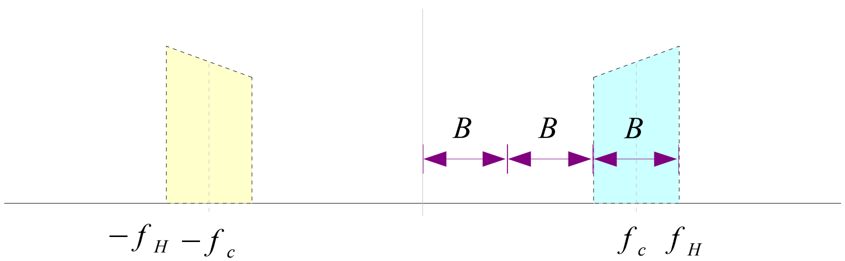


$$f_H = f_c + B/2 = 2.5B$$

$$f_H / B = R = 2.5$$

$$f_{s,min} = \frac{2 f_H}{k} = \frac{2 f_H}{m+1} = 2.5B$$

$k = 2 \quad (m = 3)$

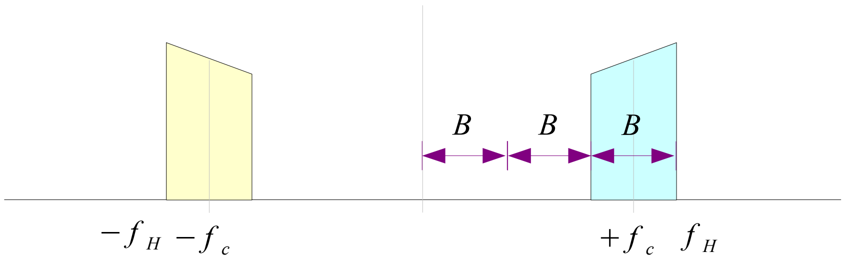


$$f_H = f_c + B/2 = 3B$$

$$f_H / B = R = 3$$

$$f_{s,min} = \frac{2 f_H}{k} = \frac{2 f_H}{m+1} = 3B$$

$k = 3 \quad (m = 2)$

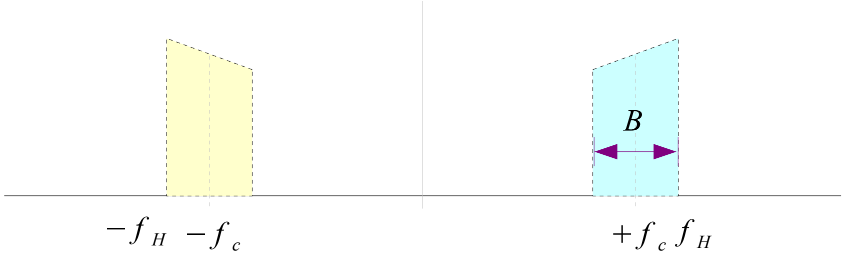


$$f_H = f_c + B/2 = 3B$$

$$f_H / B = R = 3$$

$$f_{s,min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = 2B$$

$k = 3 \quad (m = 2)$

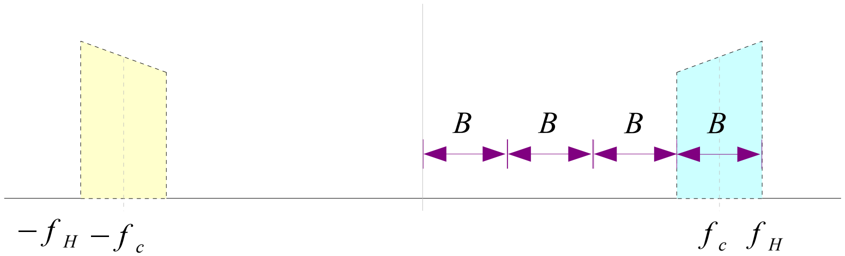


$$f_H = f_c + B/2 = 3.5B$$

$$f_H / B = R = 3.5$$

$$f_{s,min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = \frac{7}{3}B$$

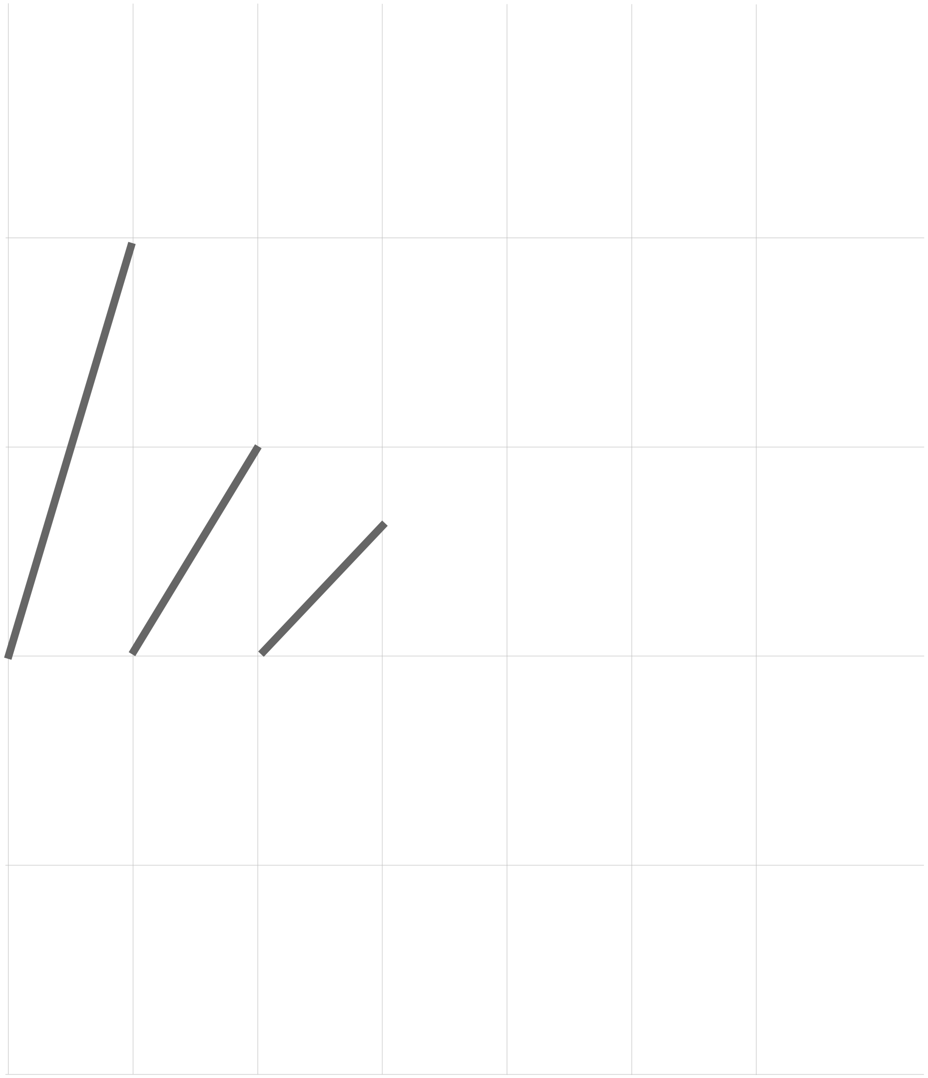
$k = 3 \quad (m = 2)$



$$f_H = f_c + B/2 = 4B$$

$$f_H / B = R = 4$$

$$f_{s,min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = \frac{8}{3}B$$



References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A "graphical interpretation" of the DFT and FFT, by Steve Mann
[4] R. G. Lyons, Understanding Digital Signal Processing, 1997