Reconstructor Spectra (9B)

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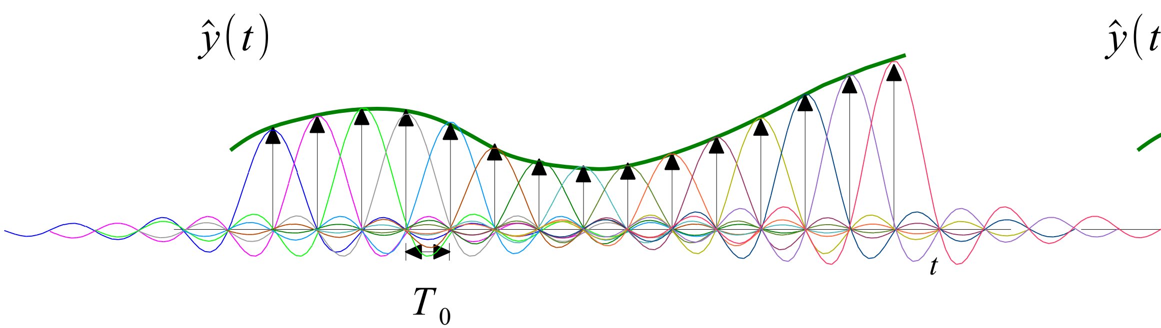
Please send corrections (or suggestions) to youngwlim@hotmail.com.

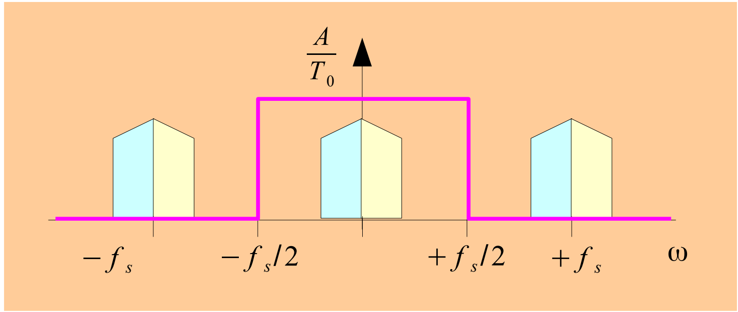
This document was produced by using OpenOffice and Octave.

Reconstructor

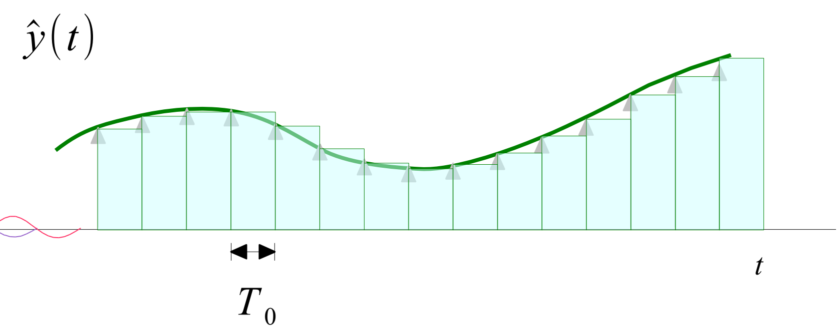
Ideal Reconstructor



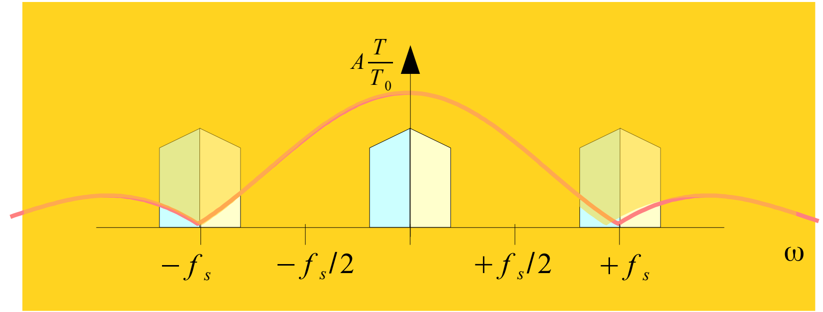
CTFT



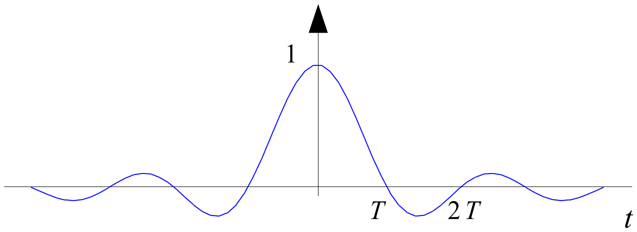
Practical Reconstructor



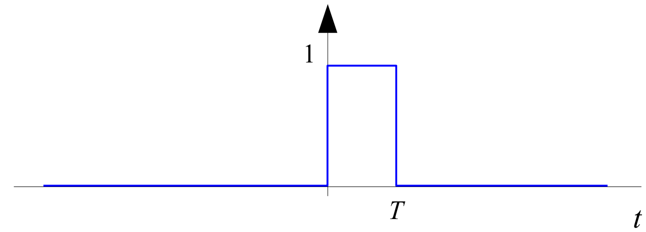
CTFT



CTFT of Reconstructors



$$\frac{1}{T} \equiv f_s$$

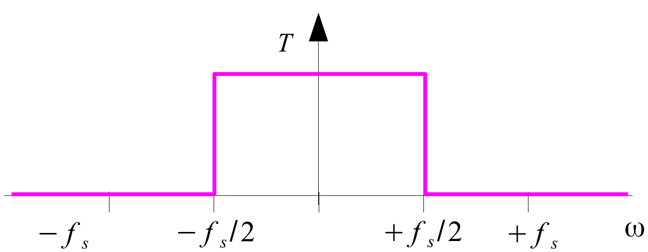


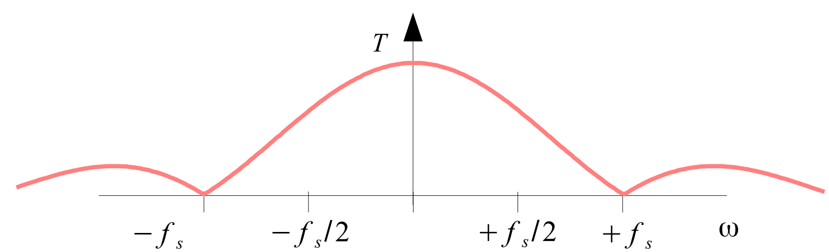
$$h(t) = \frac{\sin(\pi t/T)}{\pi t/T} = \frac{\sin(\pi f_s t)}{\pi f_s t}$$

$$h(t) = u(t) - u(t-T) = \begin{cases} 1, & 0 \le t \le T \\ 0, & otherwise \end{cases}$$

CTFT

CTFT

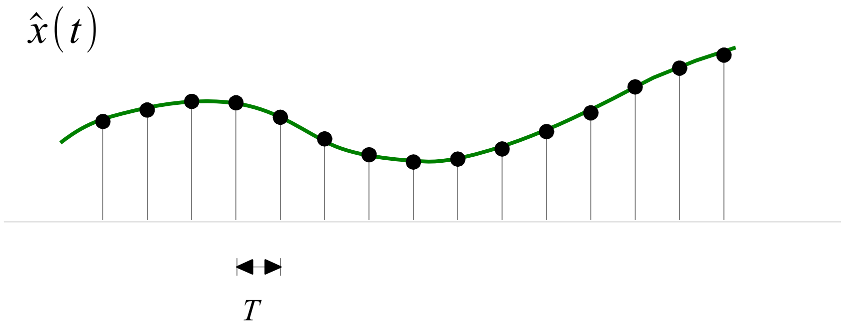


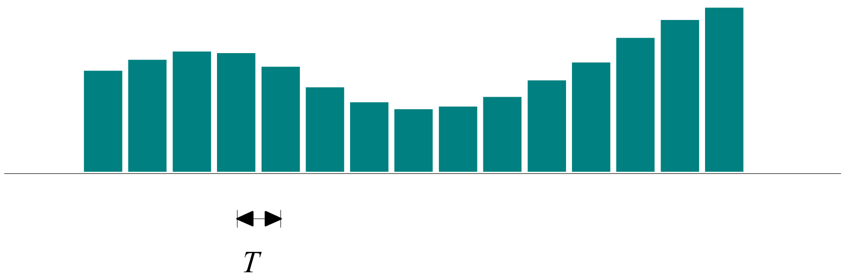


$$H(f) = \begin{cases} T, & |f| \le f_s/2 \\ 0, & otherwise \end{cases}$$

$$H(f) = T \cdot \frac{\sin(\pi f T)}{\pi f T} e^{-j\pi f T}$$

Analog Reconstructor





$$\hat{y}(t) = \sum_{n=-\infty}^{+\infty} y(nT)\,\delta(t-nT)$$

$$y_a(t) = \int_{-\infty}^{+\infty} h(t-t')\,\hat{y}(t')\,dt'$$

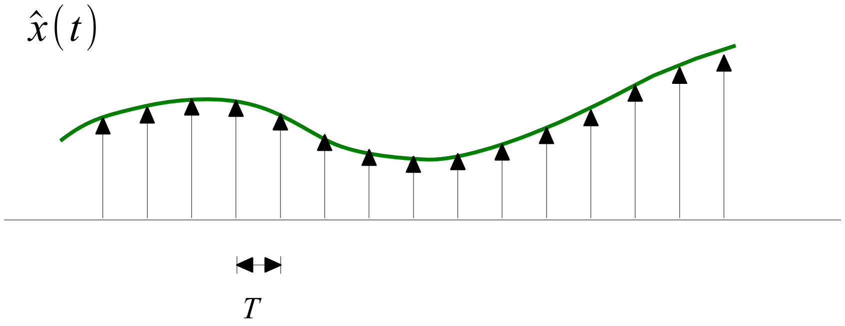
$$y_a(t) = \sum_{n=-\infty}^{+\infty} y(nT)\,h(t-nT)$$

$$Y_a(f) = H(f)\,\hat{Y}(f)$$

$$\hat{Y}_a(f) = \frac{1}{T}\sum_{m=-\infty}^{+\infty} Y(f-mf_s)$$

Young Won Lim
6/23/12

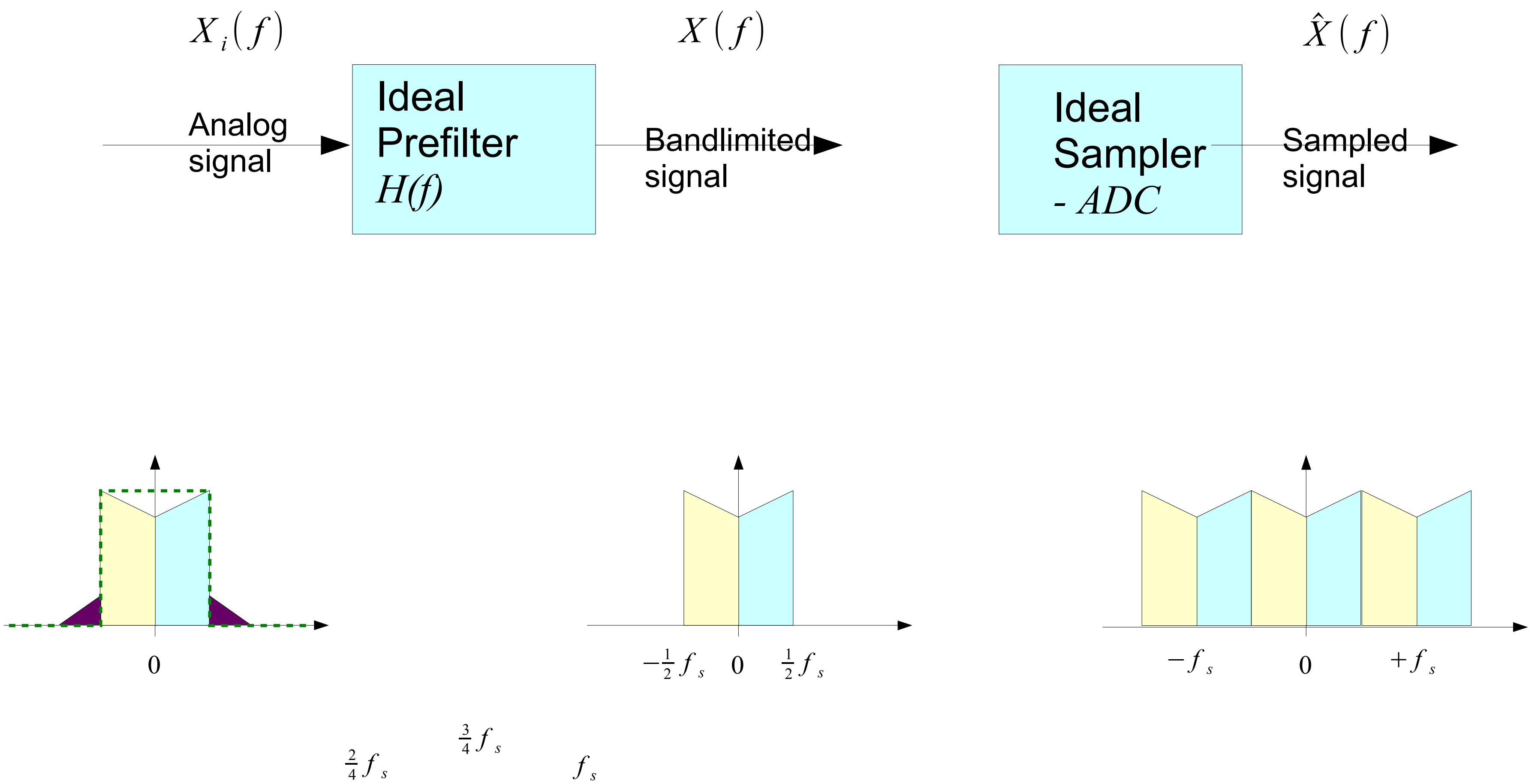
Impulse Response of Ideal Reconstructor



$$\hat{Y}(f) = \frac{1}{T} Y(f) \qquad -\frac{f_s}{2} \le f \le +\frac{f_s}{2}$$

$$y(t) = \sum_{n=-\infty}^{+\infty} y(nT) h(t-nT)$$

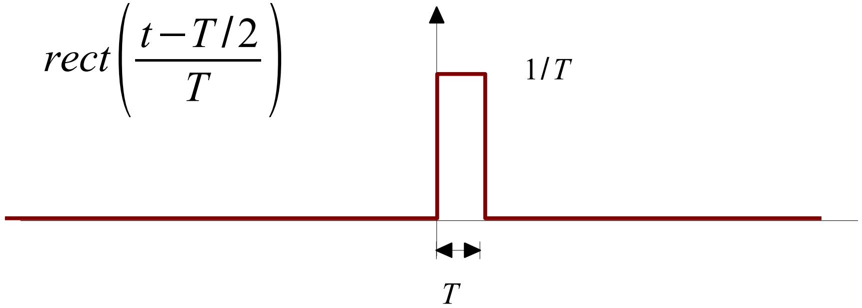
$$h(t) = \frac{\sin(\pi t/T)}{\pi t/T} = \frac{\sin(\pi f_s t)}{\pi f_s t}$$

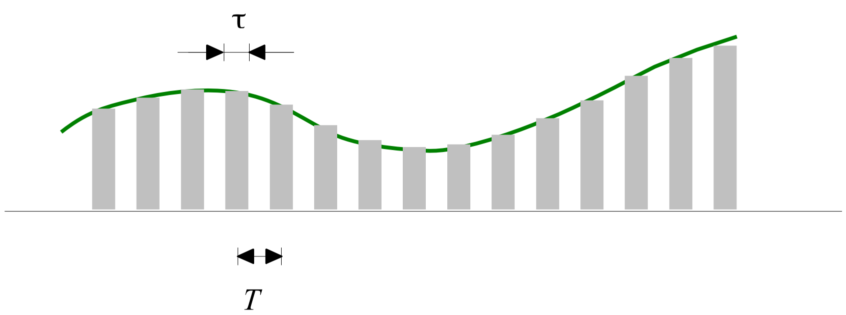


$X_i(f)$
Analog signal
Ideal Prefilter $H(f)$
$X(f)$
Bandlimited signal
Ideal Sampler - ADC
$\hat{X}(f)$
Sampled signal
0
$-\frac{1}{2}f_s$
0
$\frac{1}{2}f_s$
$-f_s$
0
$+f_s$
$\frac{2}{4}f_s$
$\frac{3}{4}f_s$
f_s

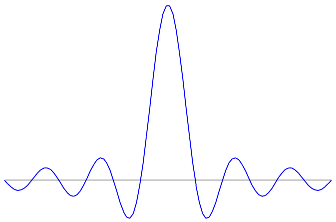
Zero Order Hold (ZOH)

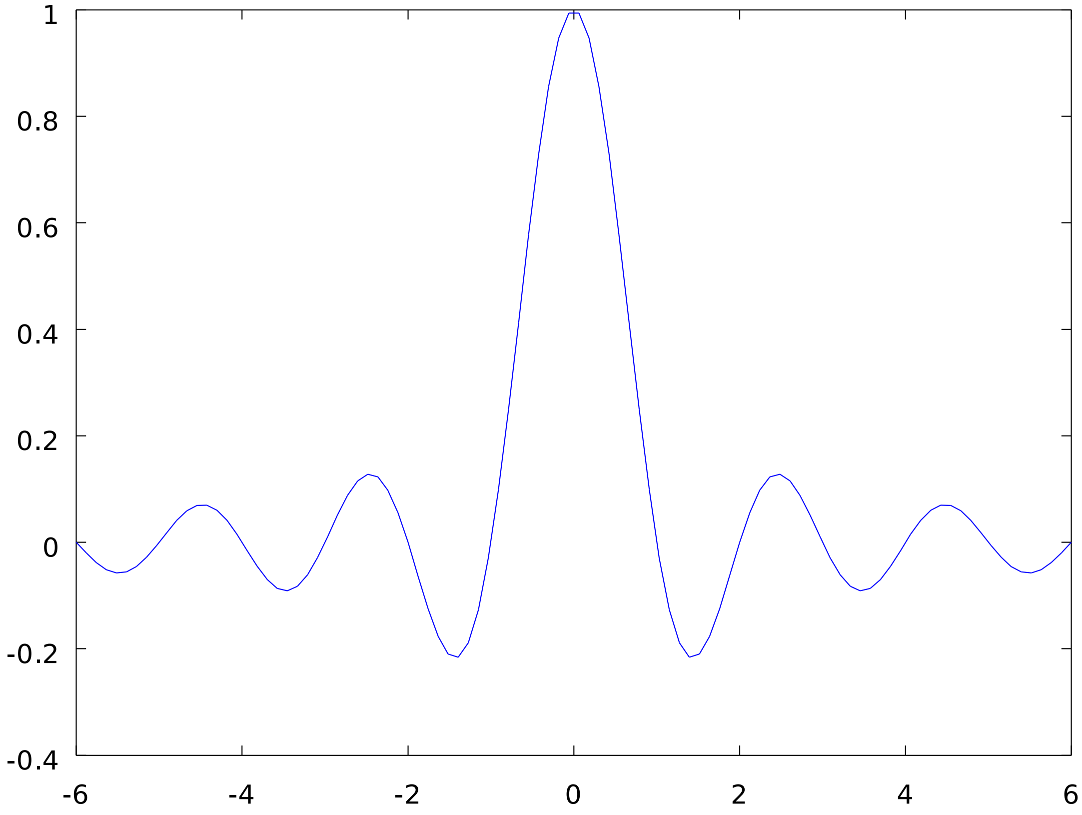
$$rect\left(\frac{t-T/2}{T}\right)$$





$$x_{ZOH}(t) = \sum_{n=-\infty}^{+\infty} x[n] \cdot rect\left(\frac{t-T/2-nT}{T}\right)$$




1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-6
-4
-2
0
2
4
6

References

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[3] A "graphical interpretation" of the DFT and FFT, by Steve Mann
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www.ece.rutgers.edu/~orfanidi/intro2sp