Element viewpt: Element matrices

Instead of using (2) p.30-4, (1) p. 30-5, consider

$$\underline{\underline{\tilde{K}}} = [K_{ij}], \quad \underline{\underline{\tilde{K}}} = \sum_{e=1}^{nel} \underline{\underline{\tilde{K}}}^e, \quad \underline{\underline{\tilde{K}}}^e = [K_{ij}^e] \qquad (1)$$

(3) p.22-3 ; (1) p. 30-5 ; $(n+1)\times(n+1)$

Recall: "Tilde" (Mtg 22) means include all dofs (E + F) (on. + free) ///

$$\underset{(n+1)\times(n+1)}{\underline{\underline{\tilde{K}}}^e} = \underset{(n+1)\times n_e}{\underline{\underline{L}}^{eT}} \; \underset{n_e \times n_e}{\underline{\underline{k}}^e} \; \underset{n_e \times (n+1)}{\underline{\underline{L}}^e} \qquad (2)$$

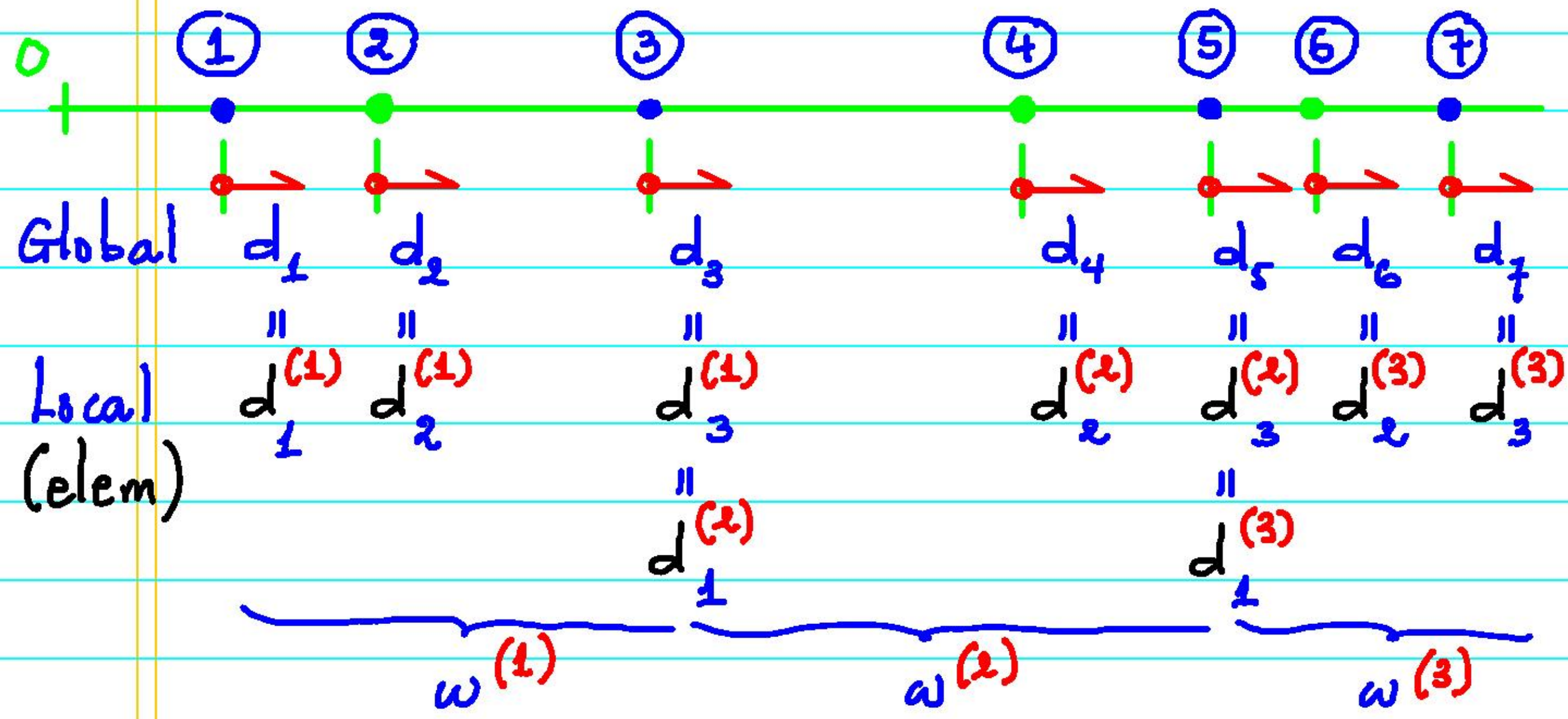
n_e = no. of dofs per elem

$\underline{\underline{L}}^e$ = "location" or "scatter" matrix

$$\underset{n_e \times 1}{\underline{d}^e} = \{ d_i^e \} = \underset{n_e \times (n+1)}{\underline{\underline{L}}^e} \; \underbrace{\underset{(n+1)\times 1}{\underline{\tilde{d}}}}_{} \qquad (3)$$

(5) p. 30-3 ; (4) p. 22-2

Ex: p. 30-2, 1D QLEBF $\Rightarrow n_e = 3$



$$\underline{d}^{(1)} = \begin{bmatrix} d_1^{(1)} \\ d_2^{(1)} \\ d_3^{(1)} \end{bmatrix}, \quad \underline{d}^{(2)} = \begin{bmatrix} d_1^{(2)} \\ d_2^{(2)} \\ d_3^{(2)} \end{bmatrix}, \quad \underline{d}^{(3)} = \begin{bmatrix} d_1^{(3)} \\ d_2^{(3)} \\ d_3^{(3)} \end{bmatrix} \qquad (1)$$

$$\underset{3\times 7}{\underline{L}^{(2)}} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad (2)$$

$\underline{k}^e$ = Element stiffness / Conductance matrix

$$\underline{k}^e = [k^e_{ij}], \quad k^e_{ij} = \int_{\omega^e} b_i^{e\prime}\, a_2\, b_j^{e\prime}\, dx \qquad (1)$$

$$i, j = 1, 2, \ldots, n_e$$

In general, $n = n_F$ = no. of free dofs

n_E = no. of prescribed dofs on Γ_g (ess. b.c.)

$$\tilde{n} = n_E + n_F = \text{total no. of dofs} \qquad (2)$$

$$\underline{\tilde{K}} = [K_{ij}], \quad \underline{\tilde{K}} = \sum_{e=1}^{nel} \underline{\tilde{K}}^e, \quad \underline{\tilde{K}}^e = [K^e_{ij}] \qquad (3)$$

$\underline{\tilde{K}}$: $\tilde{n} \times \tilde{n}$; $\underline{\tilde{K}}^e$: $\tilde{n} \times \tilde{n}$

$$\underline{\tilde{K}}^e = \underline{L}^{eT}\, \underline{k}^e\, \underline{L}^e \qquad (4)$$

$\underline{\tilde{K}}^e$: $\tilde{n} \times \tilde{n}$; $\underline{L}^{eT}$: $\tilde{n} \times n_e$; $\underline{k}^e$: $n_e \times n_e$; $\underline{L}^e$: $n_e \times \tilde{n}$

HW 6.1: Similar to HW5.{1,3,7}, but using QLEBF with uniform discretization (equidistant elem nodes)

nel = 2, 4, 6, 8, ...

1) For nel = 2, Comp. $\underline{\underline{\tilde{K}}} = \sum_{e=1}^{2} \underline{\underline{\tilde{K}}}^{e}$, with $\underline{\underline{\tilde{K}}}^{e}$ by (1) p.30-5, display $\underline{\underline{\tilde{K}}}^{e}$, e = 1,2.

2) Comp. $\underline{\underline{k}}^{e}$, $\underline{\underline{L}}^{e}$, for e = 1,2.

3) Comp. $\underline{\underline{\tilde{K}}}^{e} = \underline{\underline{L}}^{eT}\, \underline{\underline{k}}^{e}\, \underline{\underline{L}}^{e}$, for e = 1,2. Compare to 1).

4) Plot all QLEBF for nel = 3.

5) Plot $u^{h}_{\tilde{n}}$ vs. u, $\left[u^{h}_{\tilde{n}}(0.5) - u(0.5) \right]$ vs. $\tilde{n}$.

HW 6.2: Similar to HW5.{2,4,8}, but using QLEBF with uniform discretization (equidistant elem nodes)

nel = 2, 4, 6, 8, ... Same tasks as in HW 6.1.