

Complex Functions (1B)

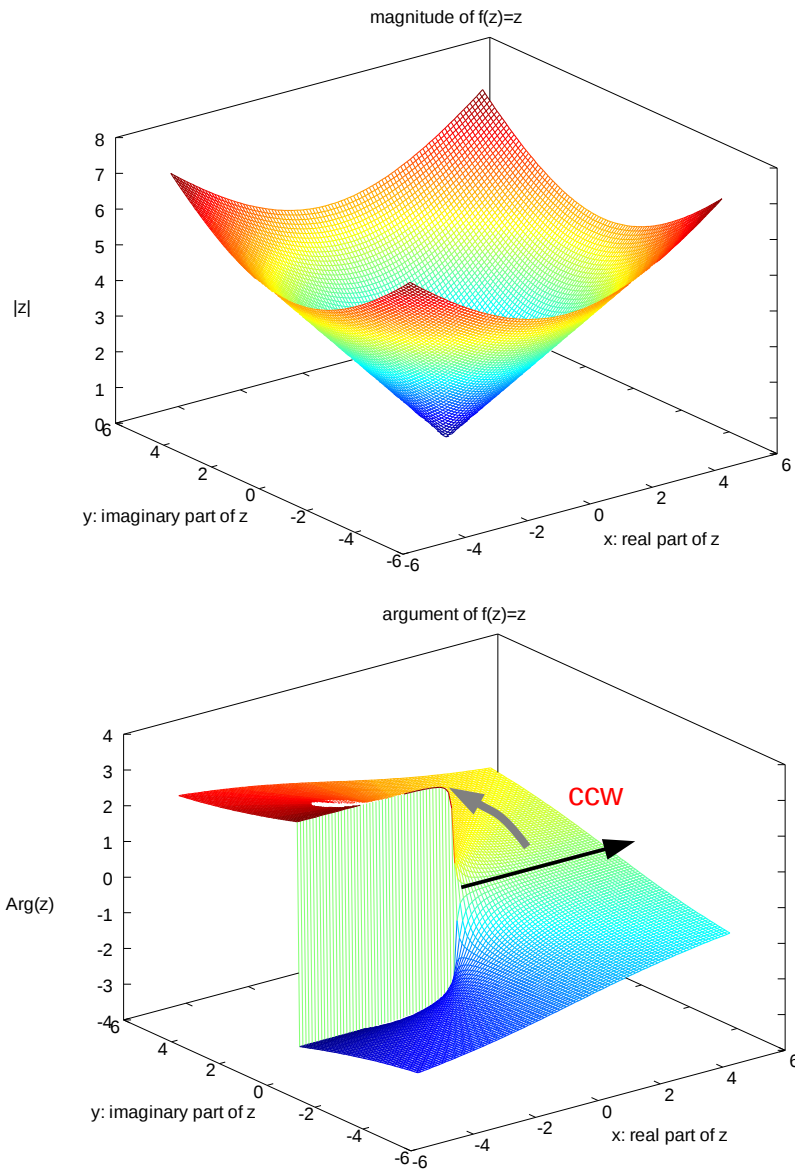
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$$f(z) = z$$



```

%-----
% Plot f(z) = z^2
% Licensing: This code is distributed under the GNU LGPL license.
% Modified: 2012.11.23
% Author: Young W. Lim
%-----

```

```

x = linspace(-5, +5, 100);
y = linspace(-5, +5, 100);
[xx yy] = meshgrid(x, y);

```

```

z = xx + i* yy;

```

```

mesh(xx, yy, abs(z))
title("magnitude of f(z)=z");
xlabel("x: real part of z");
ylabel("y: imaginary part of z");
zlabel("|z|");
print -demf z.mag.emf

```

```

pause

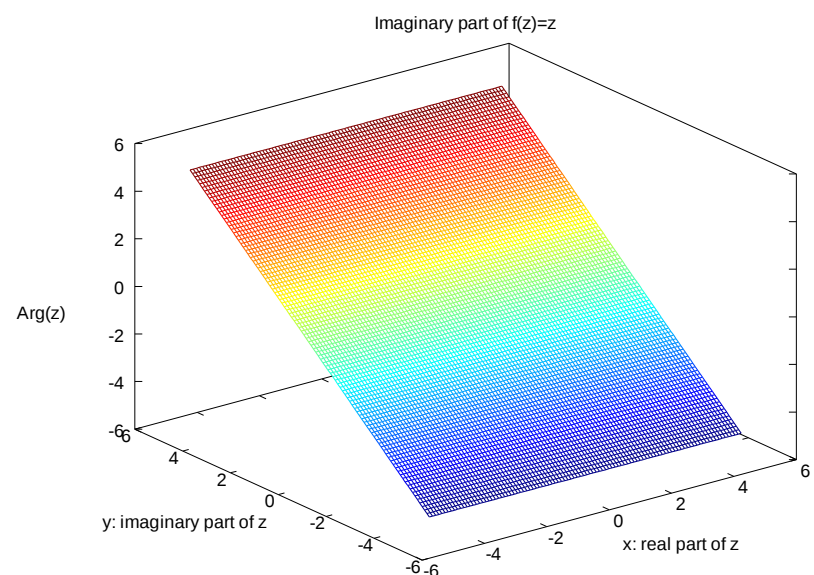
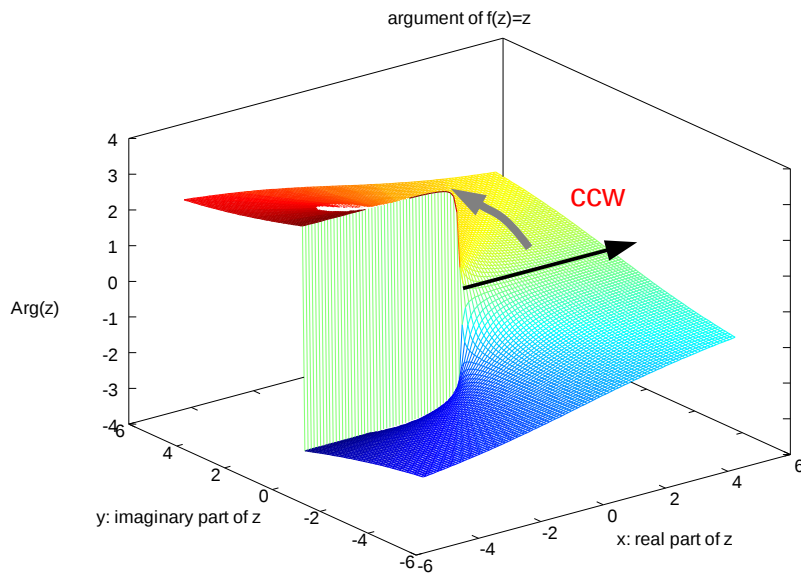
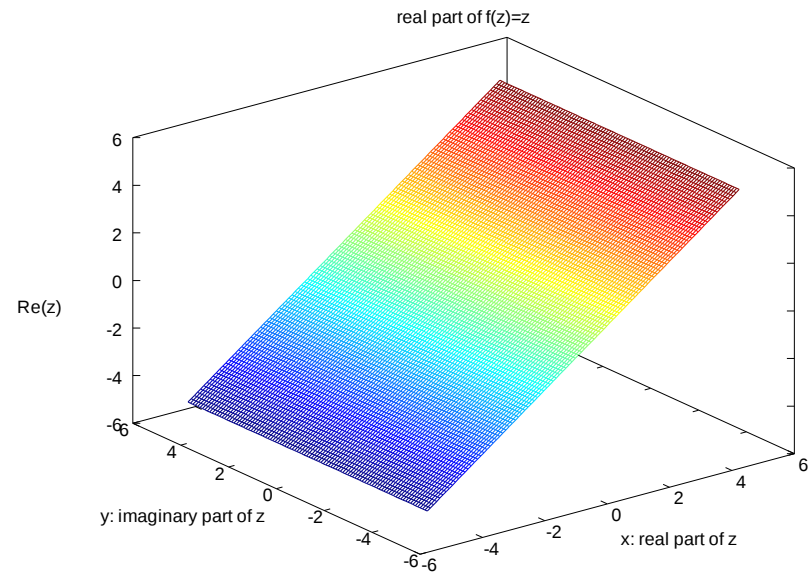
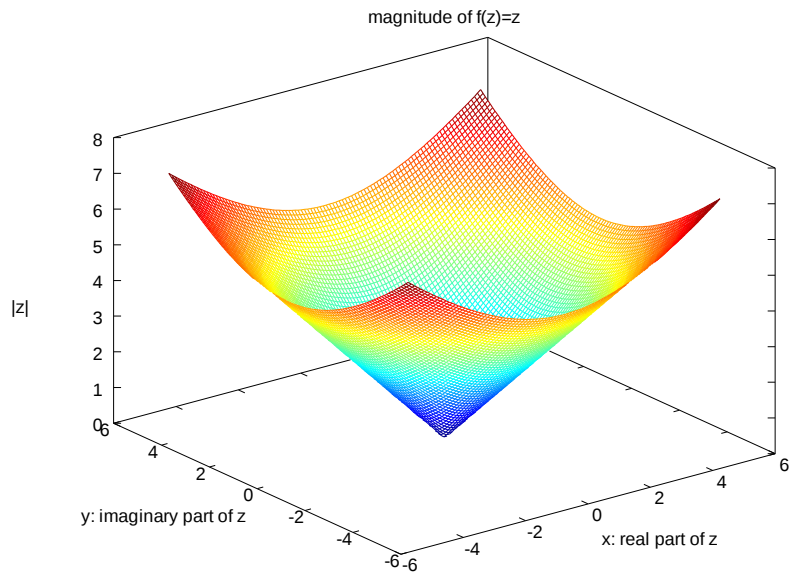
```

```

mesh(xx, yy, arg(z))
title("argument of f(z)=z");
xlabel("x: real part of z");
ylabel("y: imaginary part of z");
zlabel("Arg(z)");
print -demf z.arg.emf

```

$$f(z) = z$$



$$f(z) = z^2$$

$$z = x + iy \quad \Rightarrow \quad f(z) = u(x, y) + i v(x, y)$$

$$\begin{aligned} f(z) &= z^2 = (x + iy)^2 = (x^2 + i2xy - y^2) \\ &= (x^2 - y^2) + i(2xy) \end{aligned}$$

$$u(x, y) = (x^2 - y^2) \quad v(x, y) = (2xy)$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

$$\frac{\partial u}{\partial x} = 2x$$

$$\frac{\partial v}{\partial x} = 2y$$

$$\frac{\partial u}{\partial y} = -2y$$

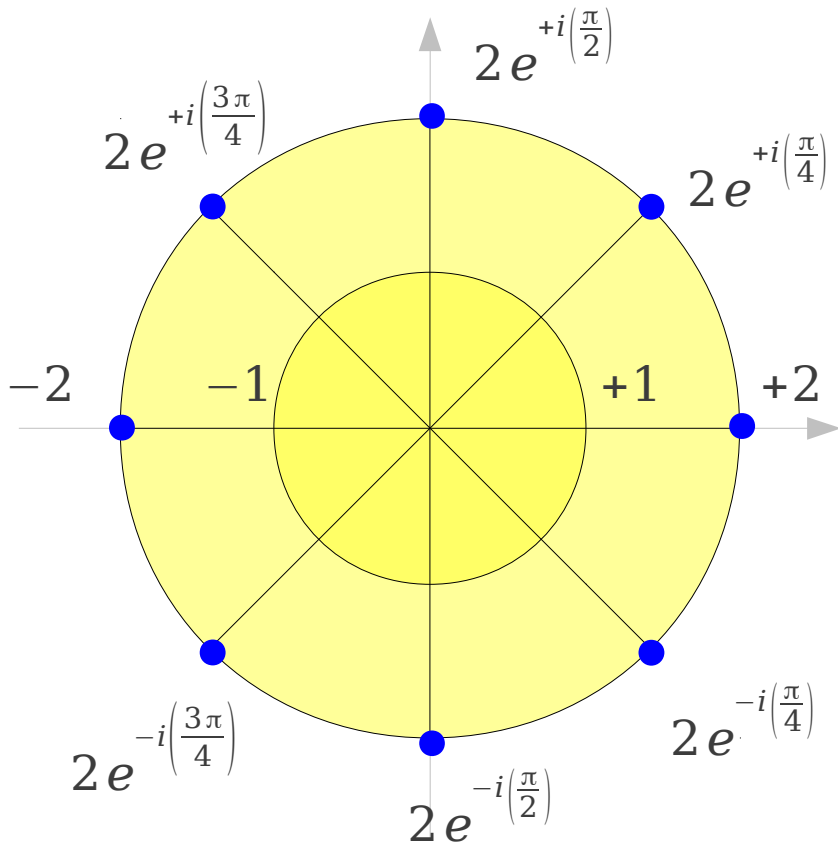
$$\frac{\partial v}{\partial y} = 2x$$

$$f'(z) = \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x} = 2x + i2y$$

$$f'(z) = -i \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} = i2y + 2x$$

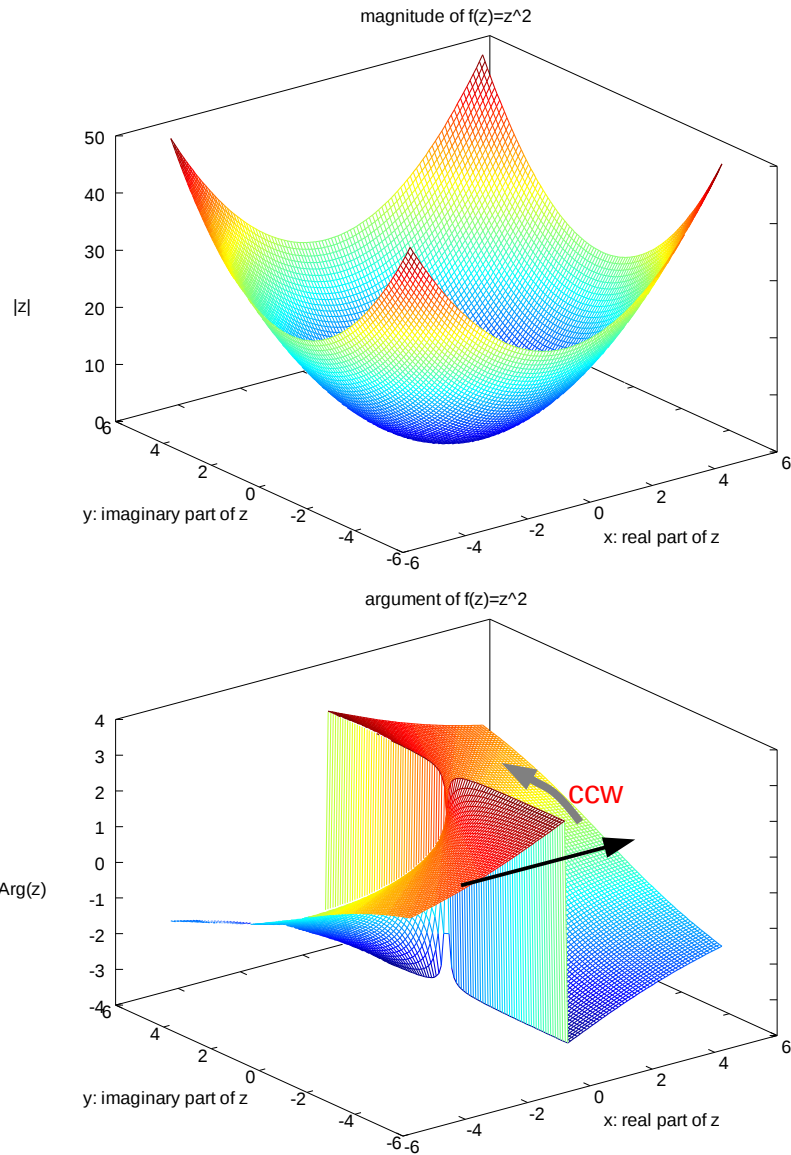
$$f'(z) = 2z = 2(x + iy)$$

Complex Numbers



polar	rectangular
+2	+2
$2e^{+i(\pi/4)}$	$\sqrt{2}(+1+i)$
$2e^{+i(\pi/2)}$	$2i$
$2e^{+i(3\pi/4)}$	$\sqrt{2}(-1+i)$
-2	-2
$2e^{-i(3\pi/4)}$	$\sqrt{2}(-1-i)$
$2e^{-i(\pi/2)}$	$-2i$
$2e^{-i(\pi/4)}$	$\sqrt{2}(+1-i)$

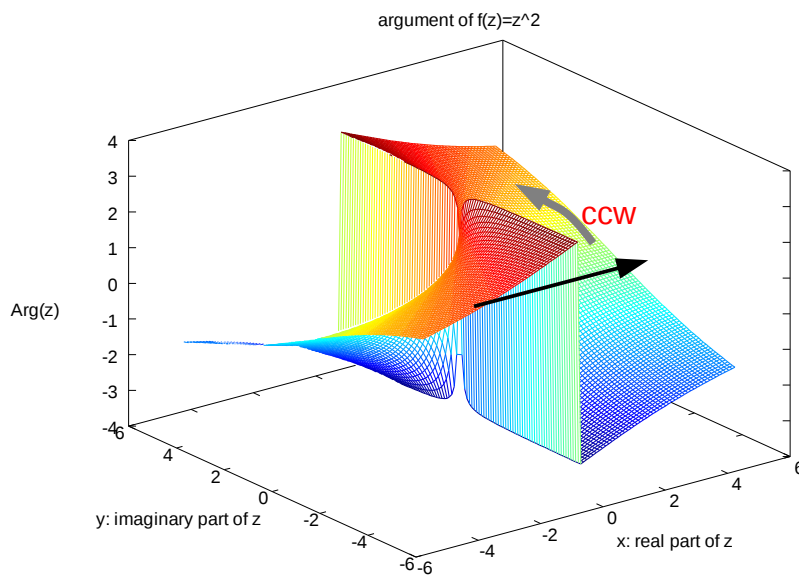
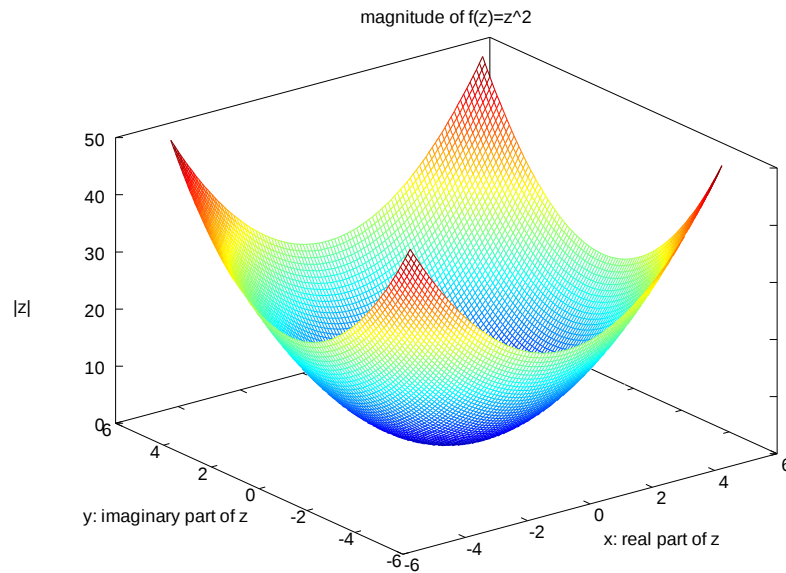
Complex Functions



$$f(z) = z^2$$

$f(+2)$	4
$f\left(2e^{+i\left(\frac{\pi}{4}\right)}\right)$	$4e^{+i\left(\frac{\pi}{2}\right)}$
$f\left(2e^{+i\left(\frac{\pi}{2}\right)}\right)$	$4e^{+i(\pi)}$
$f\left(2e^{+i\left(\frac{3\pi}{4}\right)}\right)$	$4e^{-i\left(\frac{\pi}{2}\right)}$
$f(-2)$	4
$f\left(2e^{-i\left(\frac{3\pi}{4}\right)}\right)$	$4e^{+i\left(\frac{\pi}{2}\right)}$
$f\left(2e^{-i\left(\frac{\pi}{2}\right)}\right)$	$4e^{-i(\pi)}$
$f\left(2e^{-i\left(\frac{\pi}{4}\right)}\right)$	$4e^{-i\left(\frac{\pi}{2}\right)}$

$$f(z) = z^2$$



```

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% Plot f(z) = z^2
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y = linspace(-5, +5, 100);
[xx yy] = meshgrid(x, y);

```

```

z = xx + i* yy;

```

```

mesh(xx, yy, abs(z))
title("magnitude of f(z)=z^2");
xlabel("x: real part of z");
ylabel("y: imaginary part of z");
zlabel("|z|");
print -demf z.mag.emf

```

```

pause

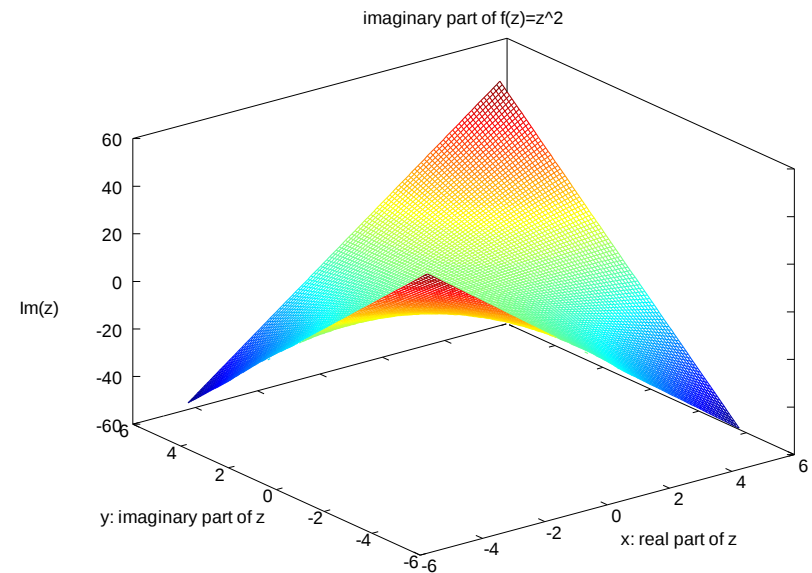
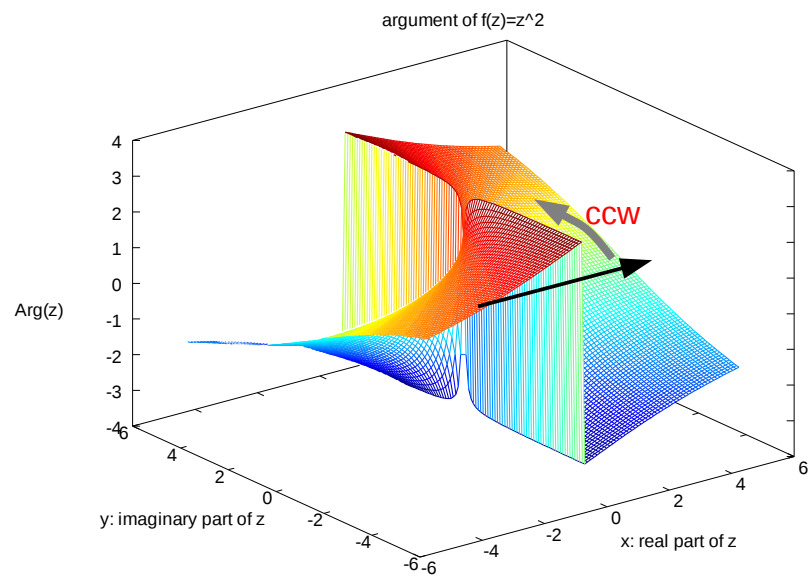
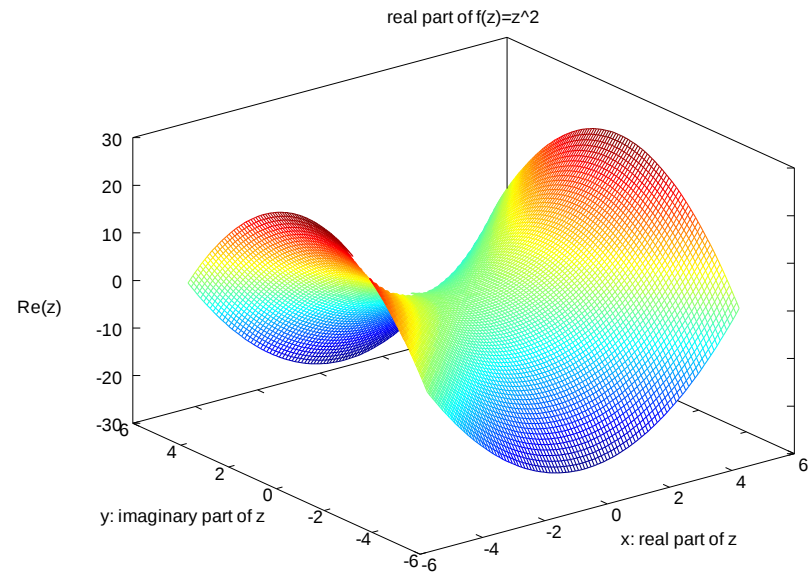
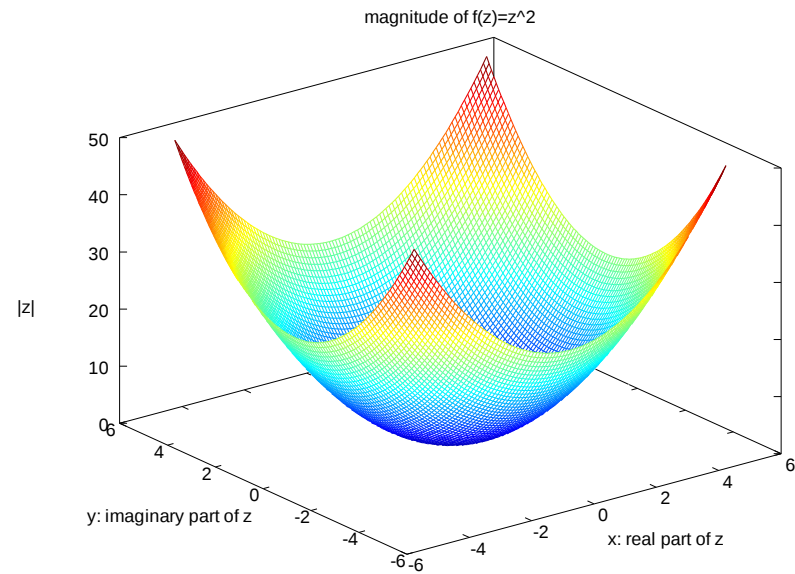
```

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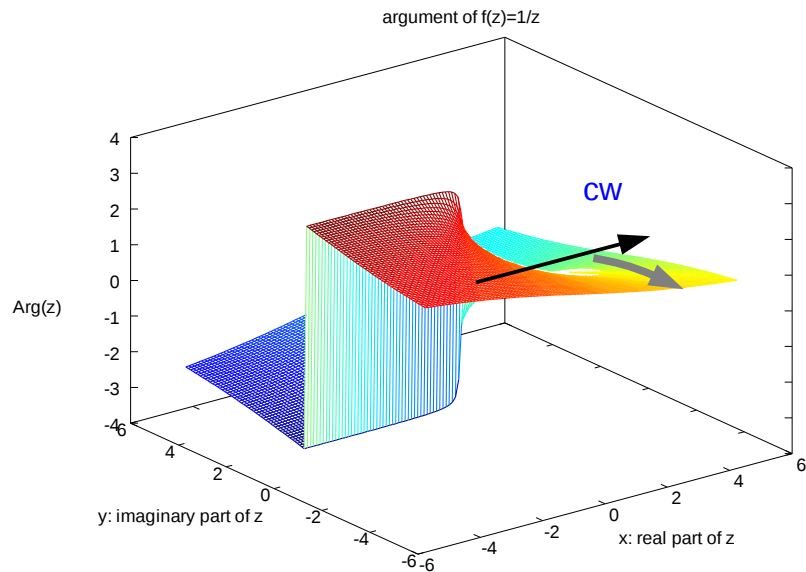
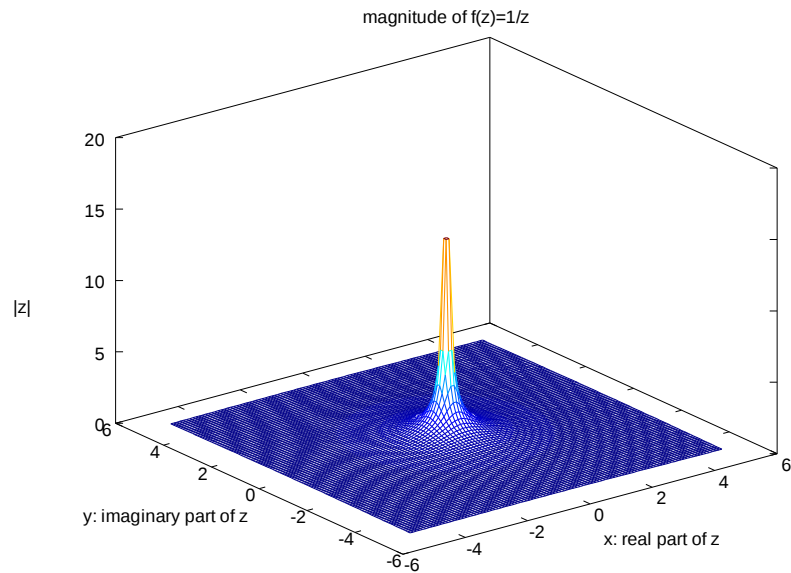
mesh(xx, yy, arg(z))
title("argument of f(z)=z^2");
xlabel("x: real part of z");
ylabel("y: imaginary part of z");
zlabel("Arg(z)");
print -demf z.arg.emf

```


$f(z) = z^2$



$f(z) = 1/z$



```

%-----
% Plot  $f(z) = 1/z$ 
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% Modified: 2012.11.23
% Author: Young W. Lim
%-----

```

```

x = linspace(-5, +5, 100);
y = linspace(-5, +5, 100);
[xx yy] = meshgrid(x, y);

```

```

z1 = xx + i* yy;
z = 1 ./ z1;

```

```

mesh(xx, yy, abs(z))
title("magnitude of  $f(z)=1/z$ ");
xlabel("x: real part of  $z$ ");
ylabel("y: imaginary part of  $z$ ");
zlabel("|z|");
print -demf 1_z.mag.emf

```

```

pause

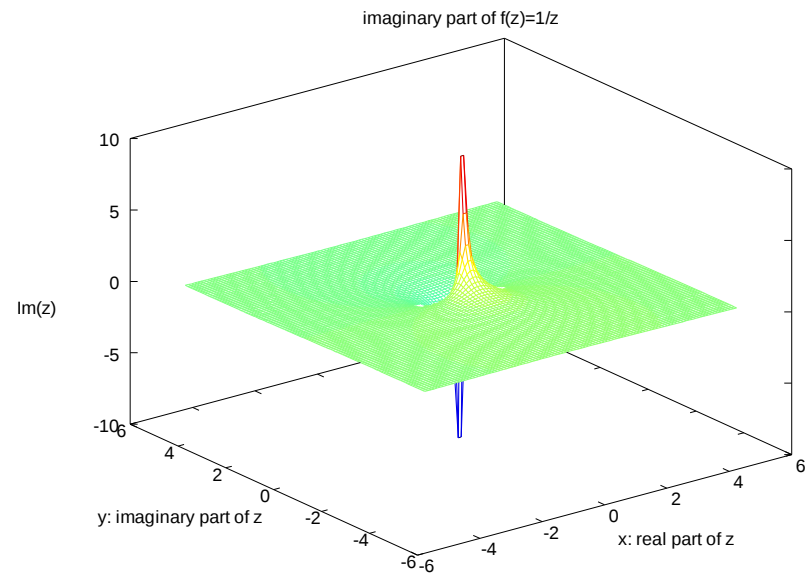
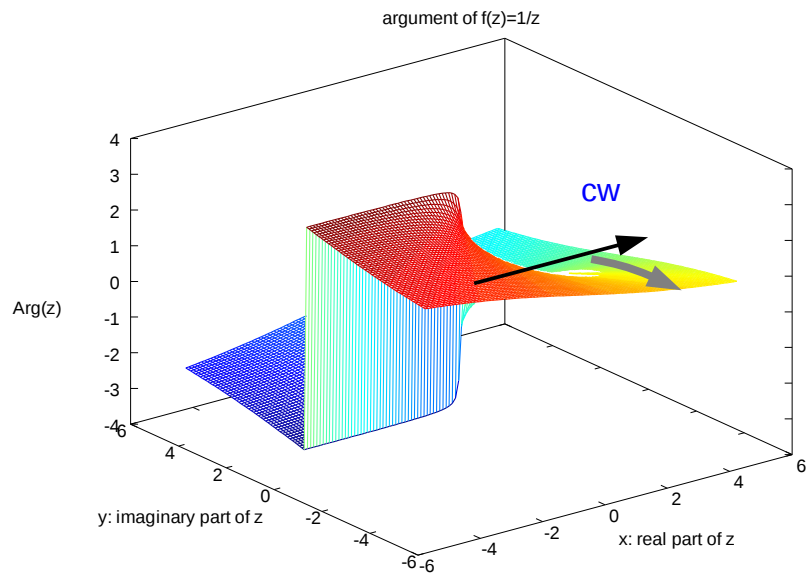
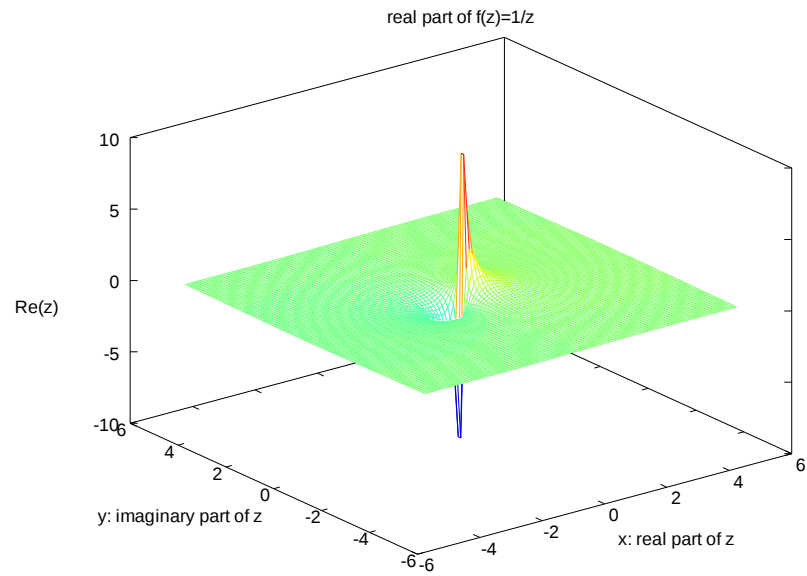
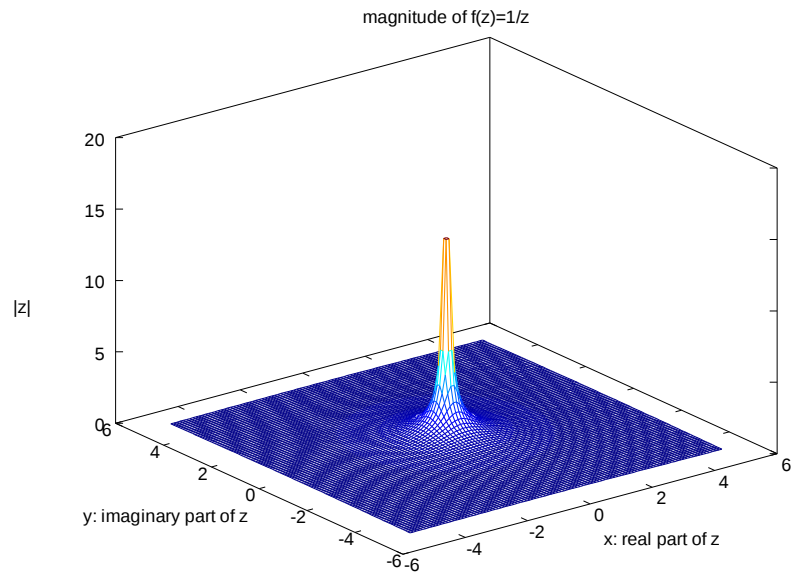
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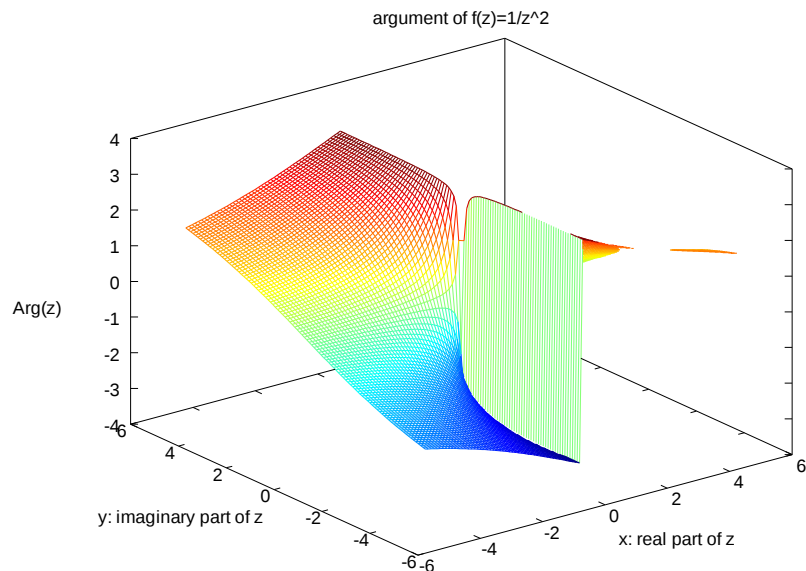
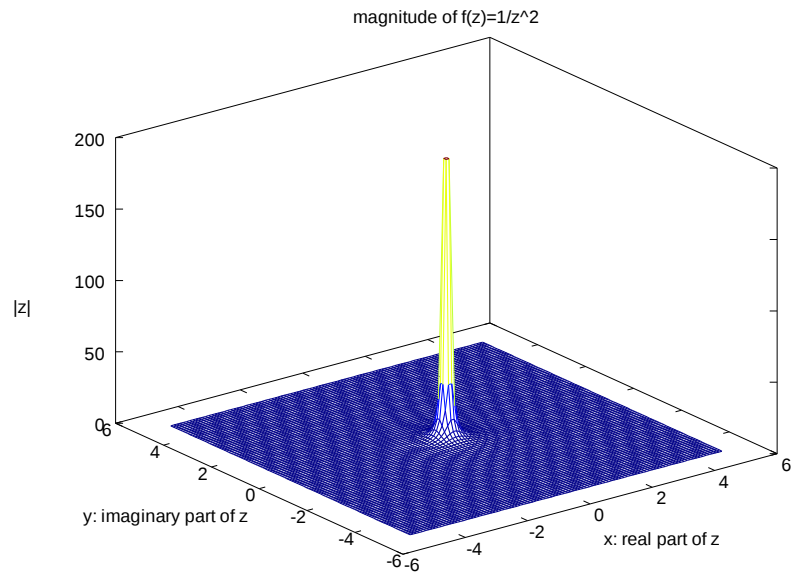
mesh(xx, yy, arg(z))
title("argument of  $f(z)=1/z$ ");
xlabel("x: real part of  $z$ ");
ylabel("y: imaginary part of  $z$ ");
zlabel("Arg( $z$ )");
print -demf 1_z.arg.emf

```

$$f(z) = 1/z$$



$$f(z) = 1/z^2$$



```

%-----
% Plot f(z) = 1/z^2
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% Author: Young W. Lim
%-----

```

```

x = linspace(-5, +5, 100);
y = linspace(-5, +5, 100);
[xx yy] = meshgrid(x, y);

```

```

z1 = xx + i* yy;
z2 = z1 .* z1;
z = 1 ./ z2;

```

```

mesh(xx, yy, abs(z))
title("magnitude of f(z)=1/z^2");
xlabel("x: real part of z");
ylabel("y: imaginary part of z");
zlabel("|z|");
print -demf 1_z2.mag.emf

```

```

pause

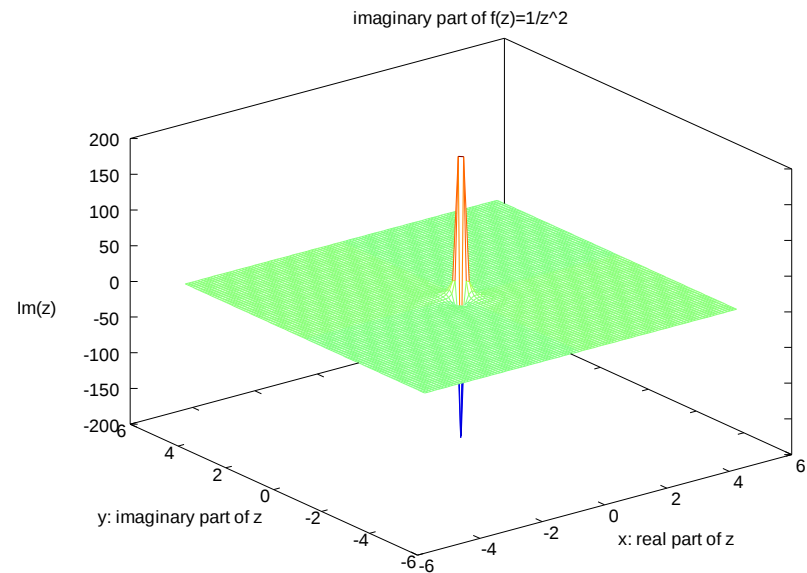
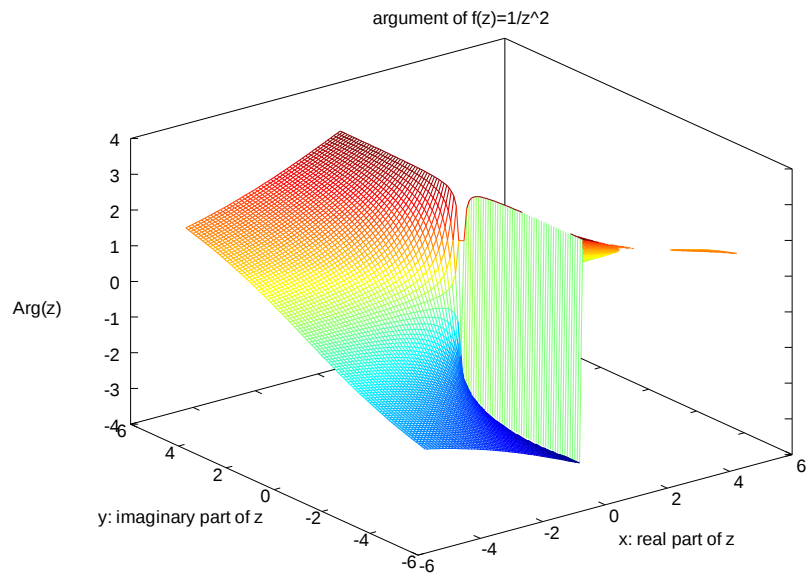
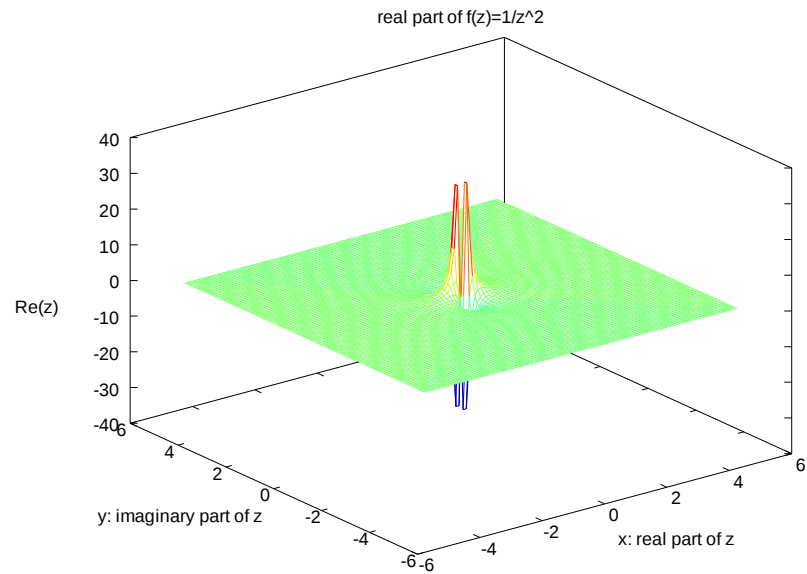
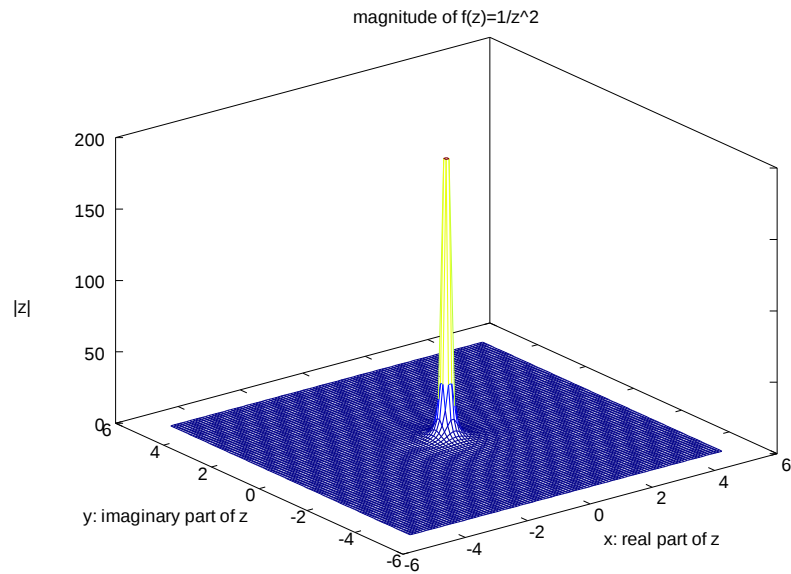
```

```

mesh(xx, yy, arg(z))
title("argument of f(z)=1/z^2");
xlabel("x: real part of z");
ylabel("y: imaginary part of z");
zlabel("Arg(z)");
print -demf 1_z2.arg.emf

```

$$f(z) = 1/z^2$$



Right Hand Rule

References

- [1] <http://en.wikipedia.org/>
- [2] <http://planetmath.org/>
- [3] M.L. Boas, “Mathematical Methods in the Physical Sciences”
- [4] D.G. Zill, “Advanced Engineering Mathematics”