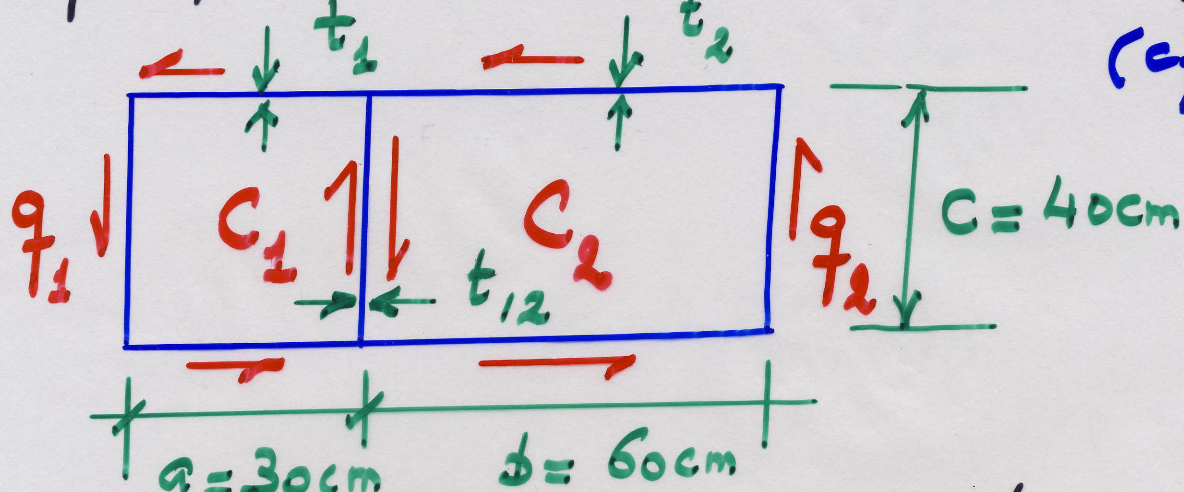
Multicell airfoil: (Sec 3.6)

Specific example first (generalization later)
(cf. p. 94)



$t_1 = 0.3cm$, $t_2 = 0.5cm$, $t_{12} = 0.4\,cm$

p. 17-1: Find θ as function of T, and J (torsional const.)

$$(1) \quad \boxed{T = T_1 + T_2 = 2q_1\bar{A}_1 + 2q_2\bar{A}_2}$$

$\bar{A}_1 = ac$, $\bar{A}_2 = bc$

$$\theta_1 = \frac{1}{2G\bar{A}_1}\oint \frac{q_1}{t_1}\,ds \quad (1)$$

$$(3) \quad = \frac{1}{2G\bar{A}_1}\left[\frac{2q_1 a}{t_1} + \frac{q_1 c}{t_1} + \frac{(q_1 - q_2)c}{t_{12}}\right]$$

$$\theta_2 = \frac{1}{2G\bar{A}_2} \oint \frac{q_2}{t_2(s)} ds$$

$$\overset{(4)}{=} \frac{1}{2G\bar{A}_2}\left[\frac{2q_2 b}{t_2} + \frac{q_2 c}{t_2} + \frac{(q_2 - q_1)c}{t_{12}}\right]$$

C_1 and C_2 have same rate of twist angle: $\boxed{\theta_1 = \theta_2}$ (2) constraint eq. on θ_i's.

Think of (q_1, q_2) as 2 unknowns. Eqs (1) & (2) are 2 eqs. for the 2 unknowns (q_1, q_2) w/ T being a variable. $\Rightarrow$ use (1) & (2) to find expr. for (q_1, q_2) in terms of T:

$$\begin{cases} q_1 = \beta_1 T \\ q_2 = \beta_2 T \end{cases}$$

(β_1, β_2) are actually numbers.

<u>HW</u>.

Next, use the expr. (3) or (4) to find the expr. betw. θ and T:

$$\theta = \theta_1 = \theta_2 = \frac{T}{\cancel{2}GJ}$$

and deduce J.

Recall: $\boxed{T = GJ\theta}$

Once θ is found as a func. of T, use $\theta = \frac{T}{GJ}$ or $J = \frac{T}{G\theta}$

to find J.

HW4: 3-cell NACA 2415 air foil.

p. 16-2: 2 partition walls

1st wall at $1/4$ c from leading edge

2nd " " $3/4$ c " "

3 unknowns q_1, q_2, q_3

$$T = 2\sum_{i=1}^{3} q_i \bar{A}_i \qquad (1)$$

$$\theta_1 = \theta_2 \qquad (2)$$

$$\theta_2 = \theta_3 \qquad (3)$$

Find J.

Note: need thickness t, but G cancels out.

Now theory (deriv.)

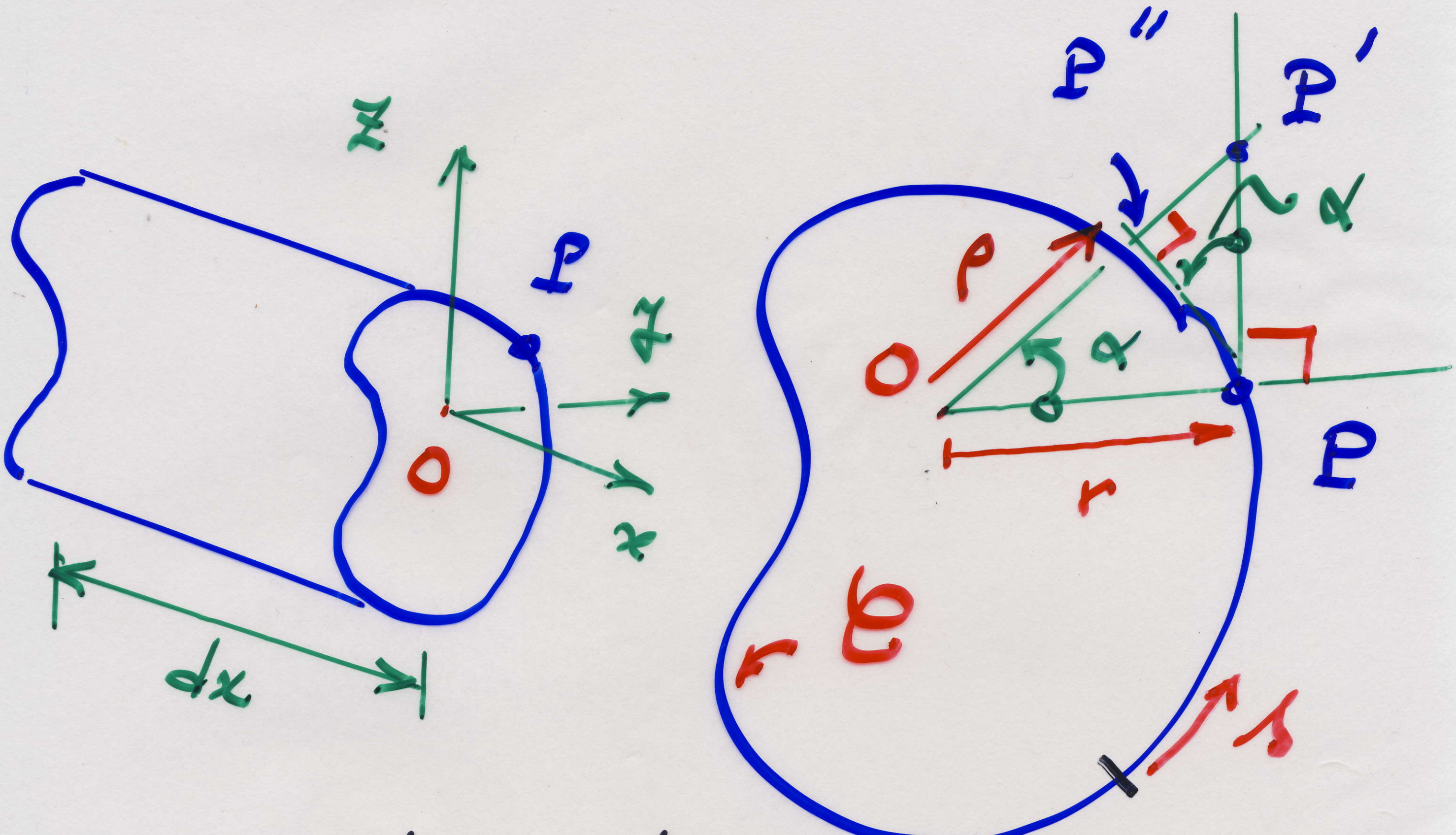
$T = GJ\theta$ (done)

$T = 2q\bar{A}$ (done)

Engineering (ad-hoc) deriv. of Eq. -2

$$\boxed{\theta = \frac{1}{2G\bar{A}} \oint \frac{q}{t}\, ds}$$

uniform bar w/ non-circular cross section subject. twist:



Disp PP' due to α:

$$\frac{PP'}{OP} = \tan\alpha \cong \alpha \quad \text{(for } \alpha \text{ small)}$$

Proj. disp. PP' on the dir. $\perp$ OP':

$$PP'' = PP' \cos\alpha$$

$$PP'' = (OP \tan\alpha)\cos\alpha$$
$$= \underbrace{(OP \cdot \cos\alpha)}_{OP''}\tan\alpha$$

Recall: $\begin{cases} OP = r & \text{(radial coord.)} \\ OP'' = \rho & \text{(p. 12-5)} \end{cases}$

$$\underbrace{PP''}_{\downarrow} = \underbrace{(r\cos\alpha)}_{\rho}\underbrace{\tan\alpha}_{\approx \alpha}$$

disp. of P in dir. "tangent" to lateral surf. of bar.

Strain: $\gamma = \dfrac{PP''}{dx} = \dfrac{\rho\,\alpha}{dx} = \rho\,\theta$

with $\theta = \dfrac{\alpha}{dx}$ rate of twist

(α very small) can be denoted by $d\alpha$

Rafael.

$$\theta = \frac{d\alpha}{dx}$$

Hooke's law: $\tau = G\gamma = G\rho\theta$

$$\tau(s) = G\rho(s)\,\theta(x)$$

Int. along contour $\mathcal{C}$:

$$\oint_{\mathcal{C}} \underbrace{\tau(s)}_{\frac{q(s)}{t(s)}} ds = G\theta(x) \underbrace{\oint_{\mathcal{C}} \rho(s)\, ds}_{2\bar{A}}$$

Hence expr. for θ p. 20-2.

Q: What is ad-hoc about the above deriv. of θ expr. on p. 20-2 and the deriv. of $T = 2q\bar{A}$?

1) Strain γ must be obtained using the disp. of P in the dir. tangent to $\mathcal{C}$ at P, but PP''

on p. 20-2 is not nec. tangent to $\mathscr{C}$ (but actually close).

2) $\tau = \dfrac{q}{t}$ obtained from ad hoc assump. that τ was uniform across wall thickness.

Now formal justification (deriv.) by elasticity theory:

Roadmap p. 16-2

A. Kinematic assump. p. 16-1.

$$u_x(y,z) = \theta \, \psi(y,z)$$

considered const. wrt x: uniform bar

$$u_y(x,z) = -\theta x z$$

$$u_z(x,y) = \theta x y$$

To transf. eqs. in Sun [2006] to those using our unified notation, use cyclic permutation

$$x \to y \to z \to x$$

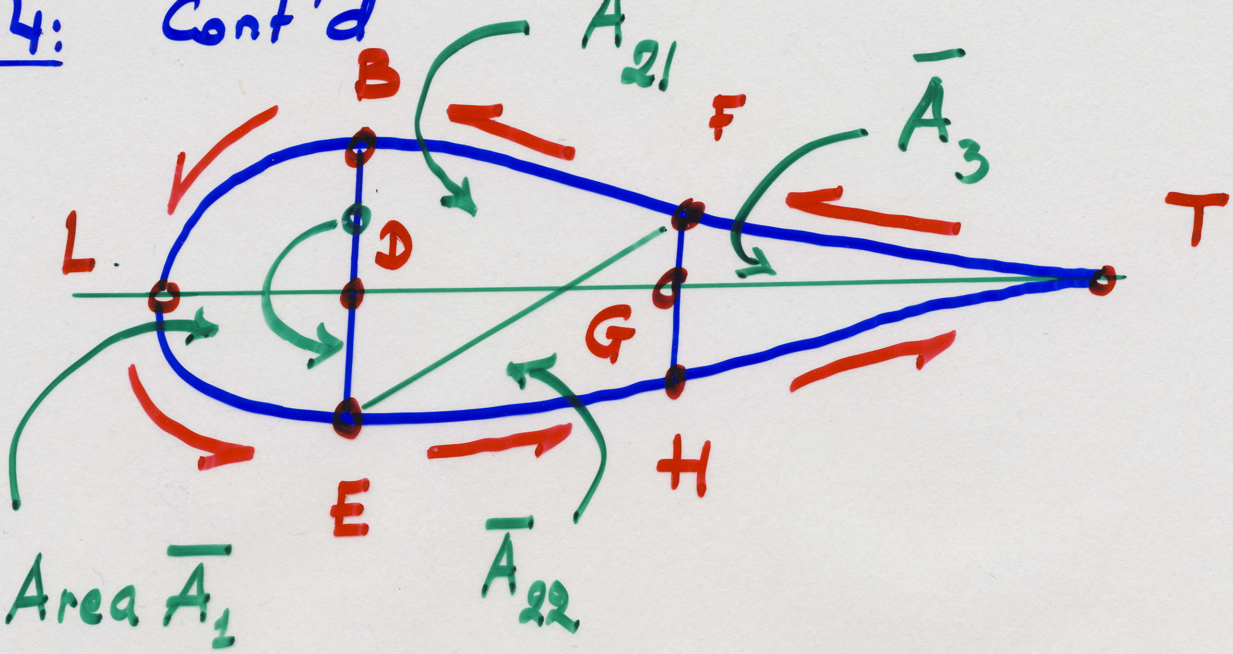
$$\text{(1)} \quad \boxed{\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} = \gamma_{yz} = 0}$$

$$\varepsilon_{xx} = \frac{\partial u_x(y,z)}{\partial x} = 0$$

$$\gamma_{yz} = \underbrace{\frac{\partial u_y^{(x,z)}}{\partial z}}_{-\theta x} + \underbrace{\frac{\partial u_z^{(x,y)}}{\partial y}}_{+\theta x} = 0$$

HW: Do ε_{yy}, ε_{zz}.

HW4: Cont'd



1) Set $P_0 = D$, sweep area $B \to L \to E$

Area $\bar{A}_{21}$

2) Set $P_0 = E$, sweep area $F \to B$

2.1

2.2) Area $\bar{A}_{22}$: Set $P_0 = F$, sweep area $E \to H$

3) Area $\bar{A}_3$: Set $P_0 = G$, sweep area $H \to T \to F$

$$\bar{A} = \bar{A}_1 + \underbrace{\bar{A}_{21} + \bar{A}_{22}}_{\bar{A}_2} + \bar{A}_3$$

within 1% of previous results

p.21-2 cont'd : 3rd "ad-hoc" pt in engineering deriv.

3) Inconsistency in assump. on size of α: To get line PP', assume α small; to get PP'', assume α finite ($\cos\alpha$) and $\rho = r\cos\alpha$; reintroduce small α after that.

p.20-2

Back to formal deriv. (p.21-3)

How many strain comp. in 3-D: 6

$$\underline{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix}_{3\times 3}$$

9 coeff (Diego)

$$\left.\begin{aligned} x &\longleftrightarrow 1 \\ y &\longleftrightarrow 2 \\ z &\longleftrightarrow 3 \end{aligned}\right\} \text{indicial notation}$$

$$\underline{\varepsilon} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} = \left[\varepsilon_{ij} \right]$$

$i, j = 1, 2, 3$

tensorial

ε_{ij}

row col.

Sym. of $\underline{\varepsilon}$: $\boxed{\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}$

Coord. $x \longleftrightarrow x_1$

$y \longleftrightarrow x_2$

$z \longleftrightarrow x_3$

$$\varepsilon_{11} = \varepsilon_{xx} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_1}{\partial x_1}\right)$$

$$= \frac{\partial u_1}{\partial x_1} = \frac{\partial u_x}{\partial x}$$

Sym : $\varepsilon_{ij} = \varepsilon_{ji}$ (HW)

$$\varepsilon_{12} = \varepsilon_{21} \Longleftrightarrow \varepsilon_{xy} = \varepsilon_{yx}$$

Hence: only 6 indep. comp. of $\underline{\varepsilon}$ in 3D

Similarly for stress tensor $\underline{\sigma}$

$$\underline{\sigma} = \left[\sigma_{ij} \right]_{3\times 3}$$

Also only 6 indep. comp. of $\underline{\sigma}$ in 3-D

Q: (Jeff) Does sym. of $\underline{\varepsilon}$ related to isotropy of materials?

No: Isotropic elasticity is related to material behavior ($\underline{\sigma}$ - $\underline{\varepsilon}$ rel.)

Following Eq. (1) on p. 21-3,

$$\boxed{\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \tau_{yz} = 0}$$

(cf. p. 70)

due to σ-ε rel. Why?

Consider isotropic elastic mat'l:

$\hookrightarrow$ mat'l behavior

Read: MIT OCW Prof. Lagace Unit 4, 12.

How to relate strains to E, ν ?

Young's modulus "nu" Poisson's ratio

Steve: Hooke's law.

Normal strains:

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu\,\sigma_{yy}}{E} - \frac{\nu\,\sigma_{zz}}{E}$$

$\varepsilon_{yy} =$ HW

$\varepsilon_{zz} =$ HW

Shear strains:

$$\gamma_{xy} = 2\varepsilon_{xy} = \frac{\tau_{xy}}{G}$$

$\tau_{xy} = \sigma_{xy}$ etc.

$\gamma_{yz} =$ HW

$\gamma_{zx} =$ HW

Voigt notation:

Due sym. of $\underset{\sim}{\varepsilon} = [\varepsilon_{ij}]_{3\times3}$, $\underset{\sim}{\sigma} = [\sigma_{ij}]_{3\times3}$

arrange ε_{ij}'s and σ_{ij}'s in col. mat.

$$\{\varepsilon_{ij}\}_{6\times 1} = \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{31} \\ \varepsilon_{12} \end{Bmatrix}_{6\times 1} \qquad \varepsilon_{31} = \varepsilon_{13}$$

$$\{\sigma_{ij}\}_{6\times 1} = \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{Bmatrix}_{6\times 1} \qquad \sigma_{31} = \sigma_{13}$$

Hooke's law for <u>isotropic</u> <u>elasticity</u>:

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \vdots \\ \gamma_{\cancel{x}yz} \\ \gamma_{31} \\ \varepsilon_{12} \to \gamma \end{Bmatrix} = \left[\begin{array}{ccc|ccc} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ \text{HW} & & & & & \\ \text{HW} & & & & & \\ \hline 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ \text{HW} & & & & \cdot & \\ \text{HW} & & & & & \cdot \end{array}\right] \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \vdots \\ \vdots \\ \vdots \\ \sigma_{12} \end{Bmatrix}$$

$\gamma_{yz} = \gamma_{23}$ (Jared, Greg)

<u>HW:</u> Fill out all rows (Chris)

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{31} \\ \varepsilon_{12} \end{Bmatrix} = \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & \frac{1}{2G} & & \\ & & & & \cdot & \\ & & & & & \cdot \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{Bmatrix}$$

Note: steel, $\nu = 0.3$

cork, $\nu = 0$

rubber, $\nu = 0.5$ (incompressible)

regular mat'l: $0 \leq \nu \leq 0.5$

Artificial $\underbrace{\text{neg. Poisson's ratio}}_{\nu < 0}$ mat'l

Mtg 24: Mon, 20 Oct 08. EAS 4200c

p. 21.2: kinematic assump. (")

p. 21-3: 4 zero strain comp. (cf. p. 70)

p. 22-4: 4 zero stress comp. (")

p. 23-2: $\varepsilon - \sigma$ rel.

(2 forms: tensorial or engineering)

p. 23-2 p 28-1b

p. 23.2: Rewrite $\varepsilon - \sigma$ rel:

full (not diag.)

$$\{\varepsilon_{ij}\}_{6\times 1} = \underbrace{\begin{bmatrix} \underline{A}_{3\times 3} & \underline{0}_{3\times 3} \\ \underline{0}_{3\times 3} & \underline{B}_{3\times 3} \end{bmatrix}_{6\times 6}}_{\underline{C}} \{\sigma_{ij}\}_{6\times 1}$$

diag

$\sigma - \varepsilon$ rel:

full

$$\{\sigma_{ij}\}_{6\times 1} = \underbrace{\begin{bmatrix} \underline{A}^{-1}_{3\times 3} & \underline{0}_{3\times 3} \\ \underline{0}_{3\times 3} & \underline{B}^{-1}_{3\times 3} \end{bmatrix}_{6\times 6}}_{\underline{C}^{-1}} \{\varepsilon_{ij}\}_{6\times 1}$$

diag

Id. mat.

Verification: (Adam) $\underline{C}^{-1}_{6\times 6}\,\underline{C}_{6\times 6} = \underline{I}_{6\times 6}$

$$\underline{C}^{-1}\,\underline{C} = \begin{bmatrix} \underline{A}^{-1}\underline{A} & \underline{0} \\ \underline{0} & \underline{B}^{-1}\underline{B} \end{bmatrix} = \underline{I}$$

p. 21-3:

$$\{\sigma_{ij}\} = \begin{bmatrix} \underline{A}^{-1} & \underline{0} \\ \underline{0} & \underline{B}^{-1} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \hline 0 \\ \varepsilon_{31} \\ \varepsilon_{12} \end{Bmatrix}$$

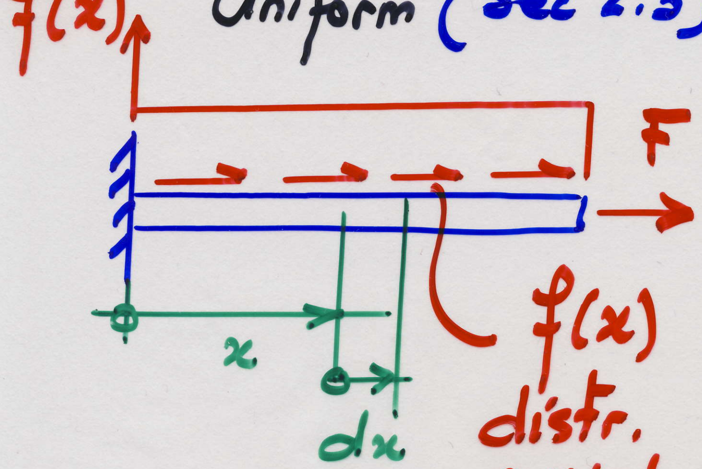
Diag.

$$\sigma_{11} = 0$$
$$\sigma_{22} = 0$$
$$\sigma_{33} = 0$$
$$\sigma_{23} = 2G\,\cancel{\varepsilon_{23}} \qquad 0 = \frac{1}{2}\gamma_{23} - \frac{1}{2}\gamma_{yz}$$
$$\sigma_{31} = 2G\,\varepsilon_{31}$$
$$\sigma_{12} = \text{HW}$$

Roadmap (p. 16-2): C. Equil. Eq. for stresses in a nonuniform stress field (Sec. 2.4)

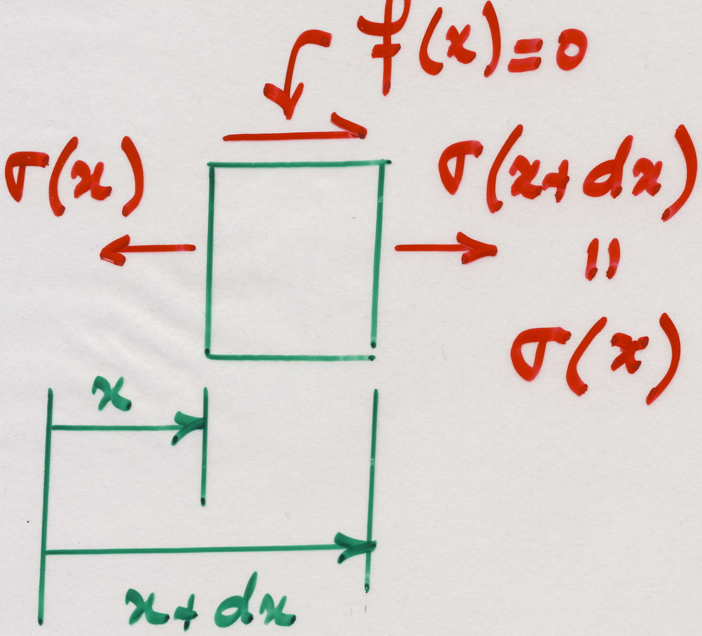
Consider 1-D case as model.

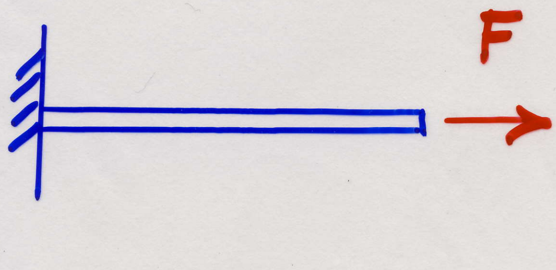
Uniform (Sec 2.3)



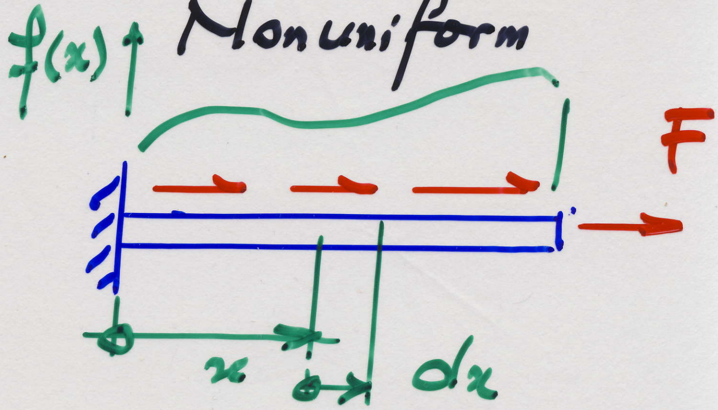
$f(x) = \text{const}$

$= 0$

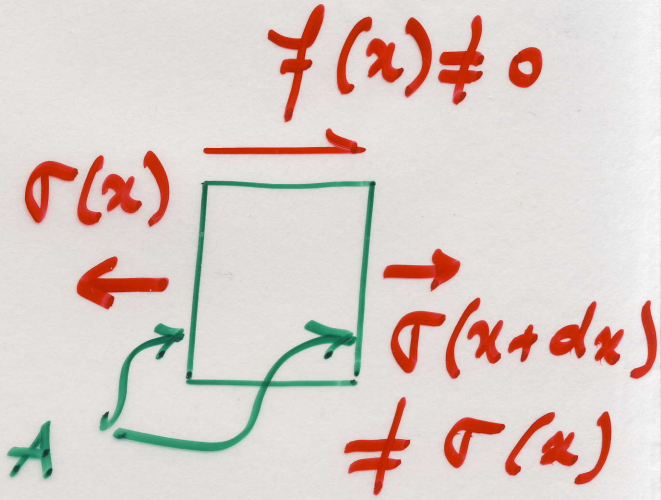




Nonuniform



$f(x) = \text{non-const.}$



Case includes

$f(x) = \text{const} \neq 0$

(26-0

Mtg 26 : Mon, 27 Oct 08

(Mtg 25 : Exam 2 on Wed, 22 Oct 08)

- Biaxial ~~bending~~ (Sec. 4.2)
  Directional

- Example 4.1, p. 124

- HW5 : Will give areas of stringers A_B, A_E, A_H, A_F in e-mail.

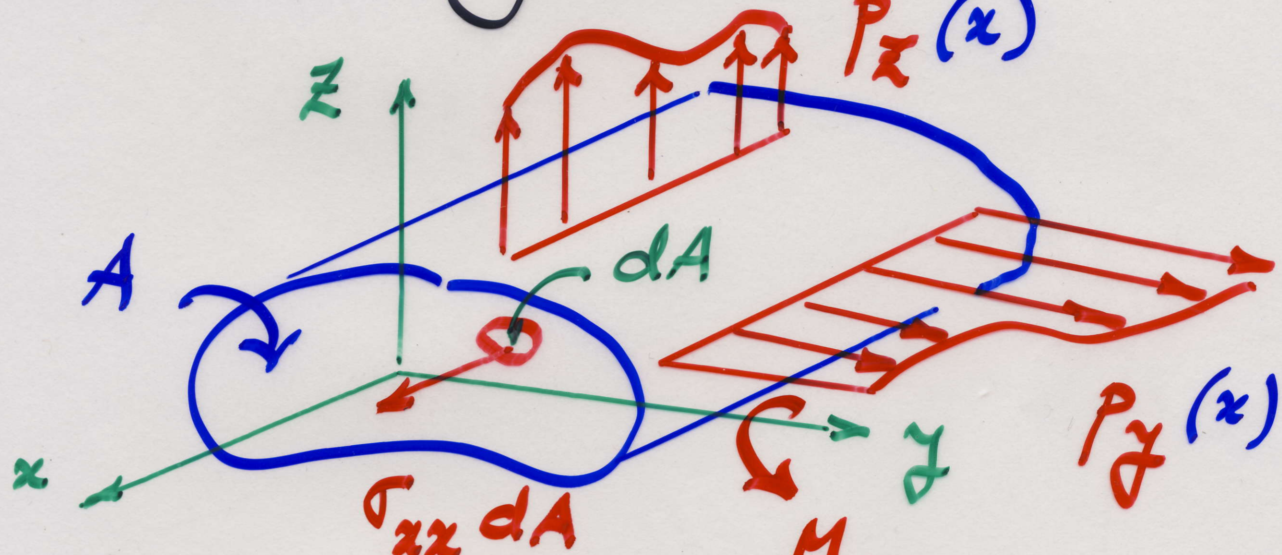
- Continue Road map for torsional analysis
  * Eq. of equil. for stresses.

(Mtg 25: Exam 2, Wed, 22 Oct 08)

- Bidirectional bending (Sec. 4.2), Read 4.1.
  * Example 4.1, p. 124

HW5: Stringers @ pts B, E, H, F (p. 22-1)
skin and spar web do not participate in bending; only stringers.
Need areas of stringers: A_B, A_E, A_H, A_F

Bidir. bending recipe:



$$M_y = \int_A z\, \sigma_{xx} \underbrace{dA}_{dy\,dz} \qquad (4.26)$$

Similarly for M_z (4.27)

Moment of inertia tensor: I_y, I_z, I_{yz}

$$I_y = I_{yy} = I_{22}, \quad I_z = I_{zz} = I_{33}, \quad I_{yz} = I_{23}$$

$$I_y = \int_A z^2 \, dA \qquad (4.28a)$$

$$I_z = \qquad (4.28b)$$

$$I_{yz} = \int_A y z \, dA \qquad (4.28c)$$

$$\sigma_{xx} = E \, \varepsilon_{xx} = \underbrace{\frac{I_y M_z - I_{yz} M_y}{\underbrace{\left[I_y I_z - (I_{yz})^2 \right]}_{D}}}_{m_y} y + \underbrace{\frac{I_z M_y - I_{yz} M_z}{\underbrace{\left[I_y I_z - (I_{yz})^2 \right]}_{D}}}_{m_z} z \qquad (4.29)$$

Recall:

$$\underline{I} = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ & I_{22} & I_{23} \\ \text{sym.} & & I_{33} \end{bmatrix}$$

Note: $D := I_y I_z - (I_{yz})^2$

$= I_{22} I_{33} - (I_{23})^2$

is the determinant of $\begin{bmatrix} I_{22} & I_{23} \\ I_{32} & I_{33} \end{bmatrix}$, $I_{32} = I_{23}$

(Jared)

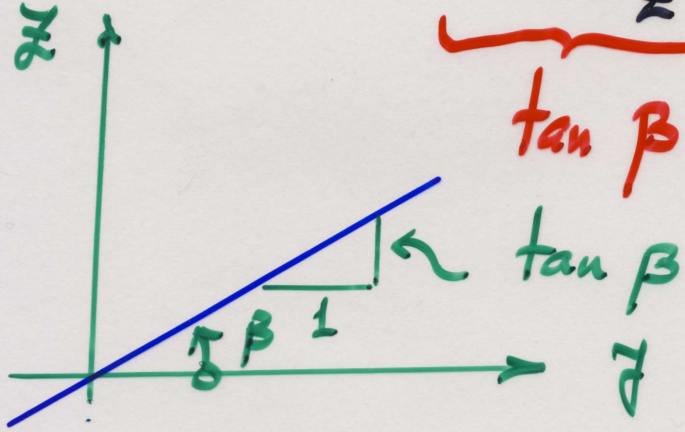
$$I_{32} = I_{zy} \overset{def}{:=} \int_A \underbrace{zy}_{= yz} \, dA = I_{yz} = I_{23}$$

<u>Neutral axis</u>: where $\sigma_{xx} = 0$

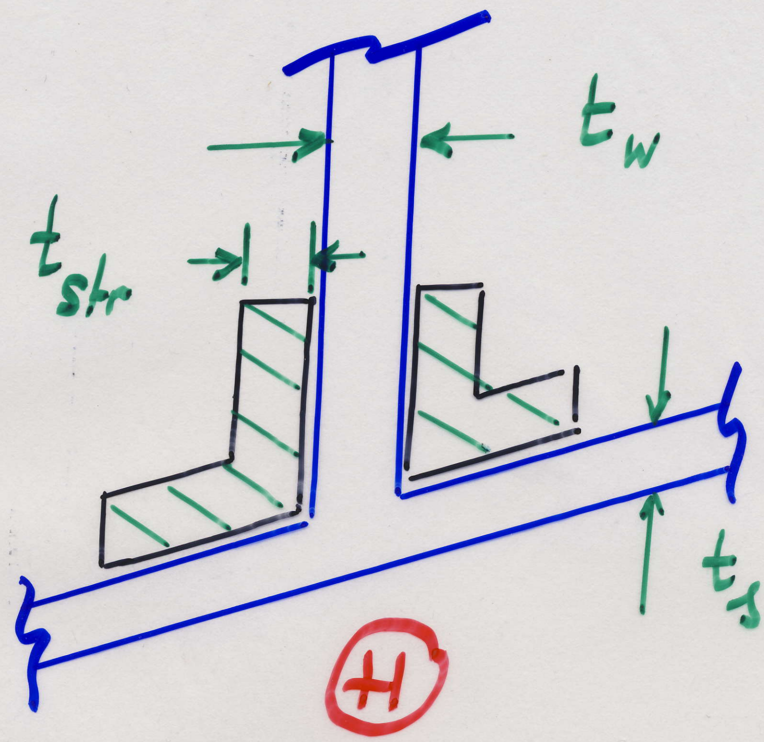
(4.29) $\Rightarrow \sigma_{xx} = m_y y + m_z z = 0$

$$\Rightarrow z = \underbrace{\left(- \frac{m_y}{m_z} \right)}_{\tan\beta} y = (\tan\beta) \, y$$

$\tan\beta = -\tan\alpha$ (4.30)



Mtg 27 HW5 Cont'd



Cont. from p. 24-3 : Eq. of equil. in terms of σ

Goal: $$\boxed{\frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} = 0} \quad \text{cf. (3.14)} \quad (1)$$

Let's use indicial notation

$$\boxed{\frac{\partial \sigma_{21}}{\partial x_2} + \frac{\partial \sigma_{31}}{\partial x_3} = 0} \quad (2)$$

Recall: (Sec. 2.4)

(3) $$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} = 0$$ cf. (2.21)

Similarly, see (2.22) & (2.23) HW5

In indicial notation, (2.21), (2.22), (2.23):

(4) $$\boxed{\sum_{i=1}^{3} \frac{\partial \sigma_{ij}}{\partial x_i} = 0 \quad \text{for } j = 1, 2, 3}$$