Bandpass Sampling (2B)

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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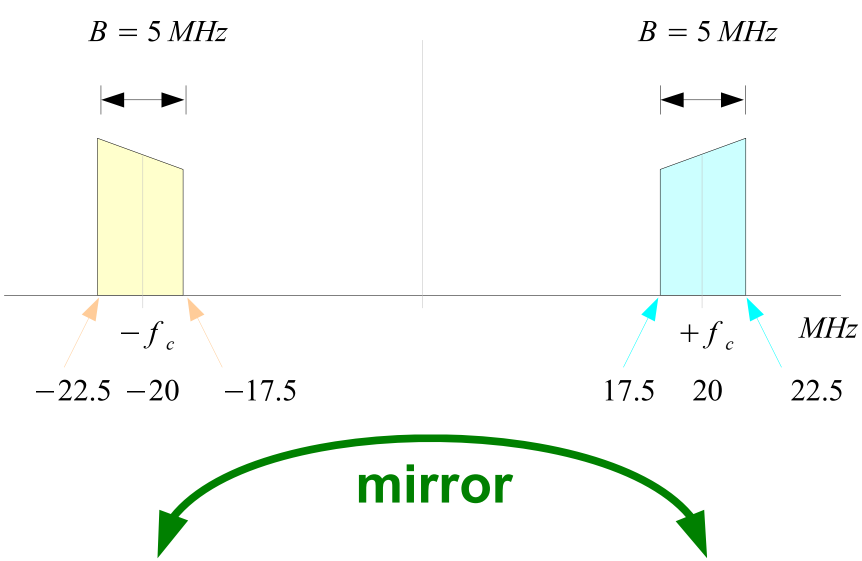
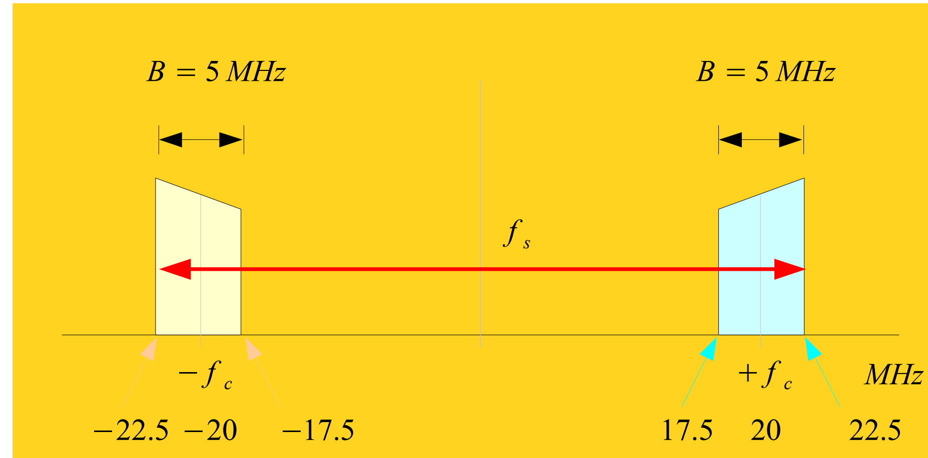
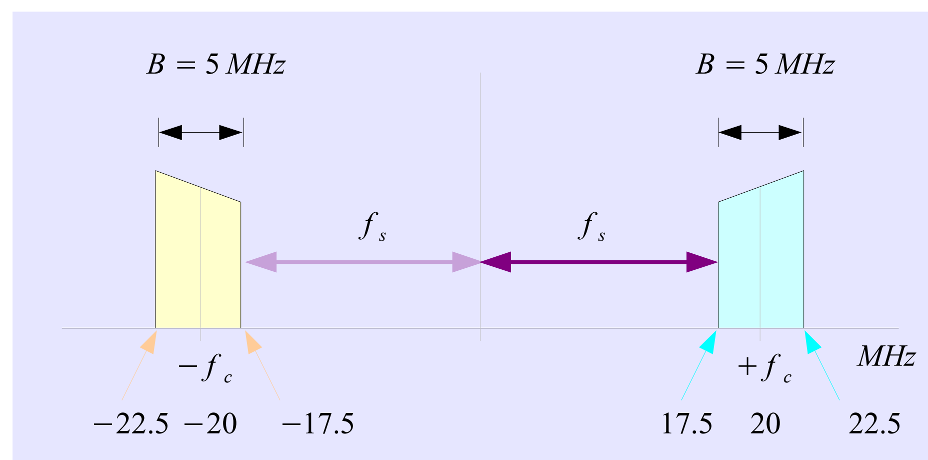
Band-limited Signal



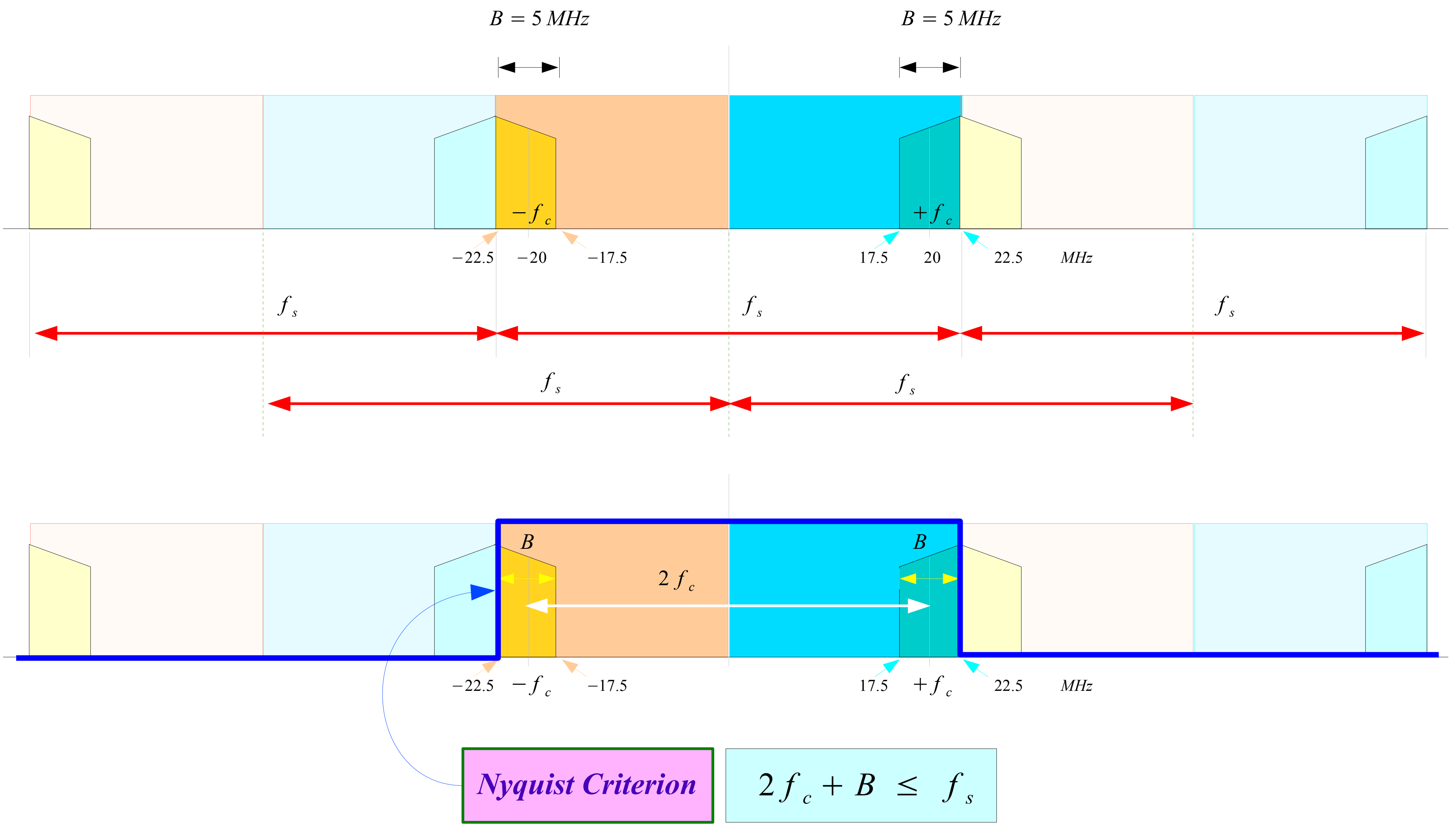


- **Lowpass Sampling**

- **Bandpass Sampling**
- **IF filtering**
- **Harmonic Sampling**
- **Sub-Nyquist Sampling**

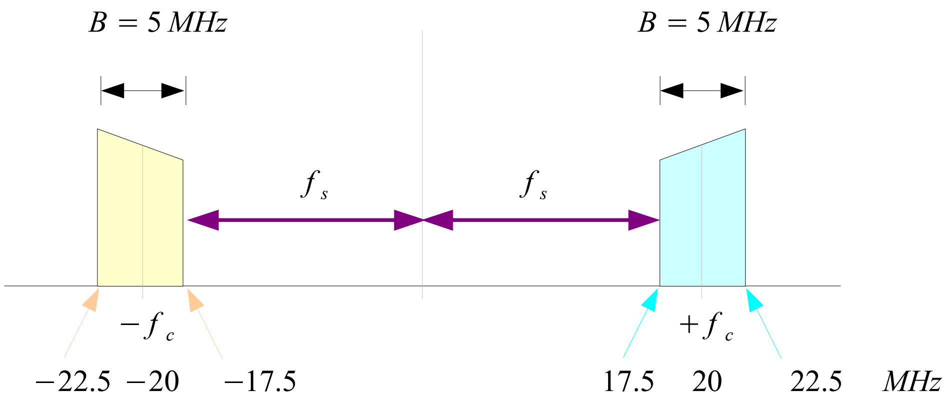


Low-pass Signal Sampling



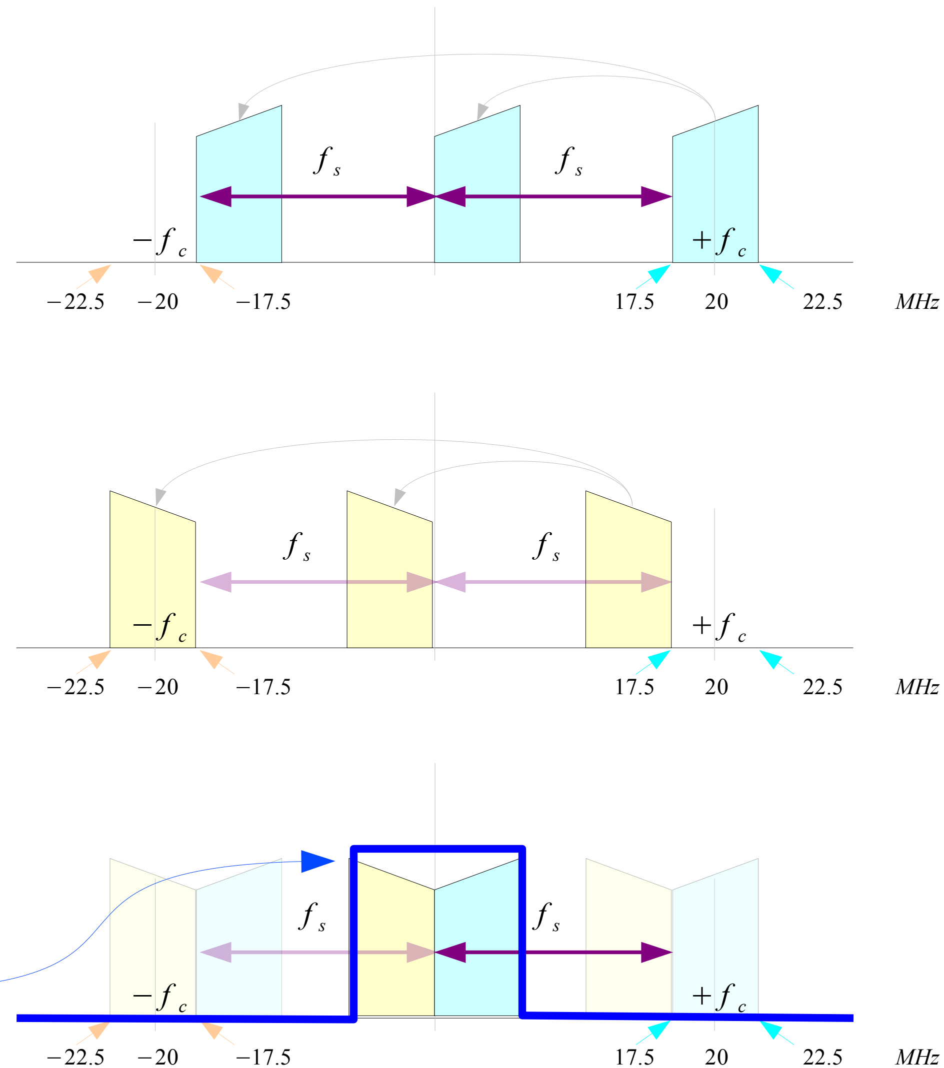
$$2f_c + B \le f_s$$

Band-pass Signal Sampling

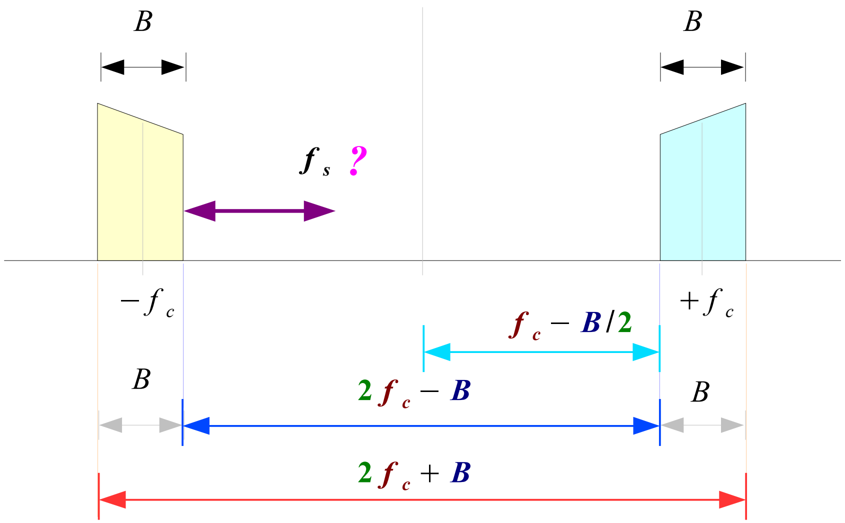


- **Bandpass Sampling**
- **IF filtering**
- **Harmonic Sampling**
- **Sub-Nyquist Sampling**

Nyquist Criterion $2B \leq f_s$



Sampling Frequency f_s (1)



- **Bandpass Sampling**
- **IF filtering**
- **Harmonic Sampling**
- **Sub-Nyquist Sampling**

Assume there are m multiples of f_s

Given an integer m

$$2f_c - B = m \cdot f_s$$

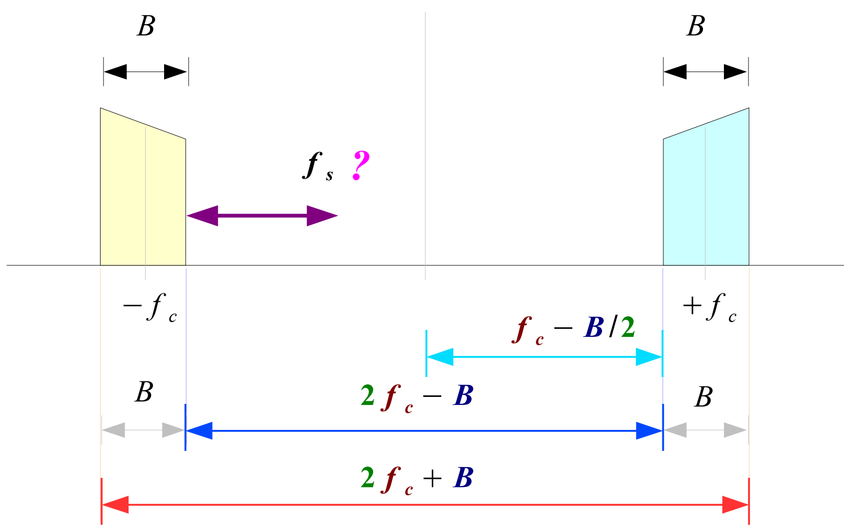
Max f_s condition

f_s can be decreased according to the following condition without introducing aliasing problems

$$2f_c + B = (m+1) \cdot f_s$$

Min f_s condition

Sampling Frequency f_s (2)



When $m = 6$

$$\text{max } f_s = \frac{2f_c - B}{6}$$

$$\text{min } f_s = \frac{2f_c + B}{7}$$

$$\text{min } f_s \quad \frac{2f_c + B}{7} \le f_s \le \frac{2f_c - B}{6} \quad \text{max } f_s$$

Assume there are m multiples of f_s

Given an integer m

$$2f_c - B = m \cdot f_s$$

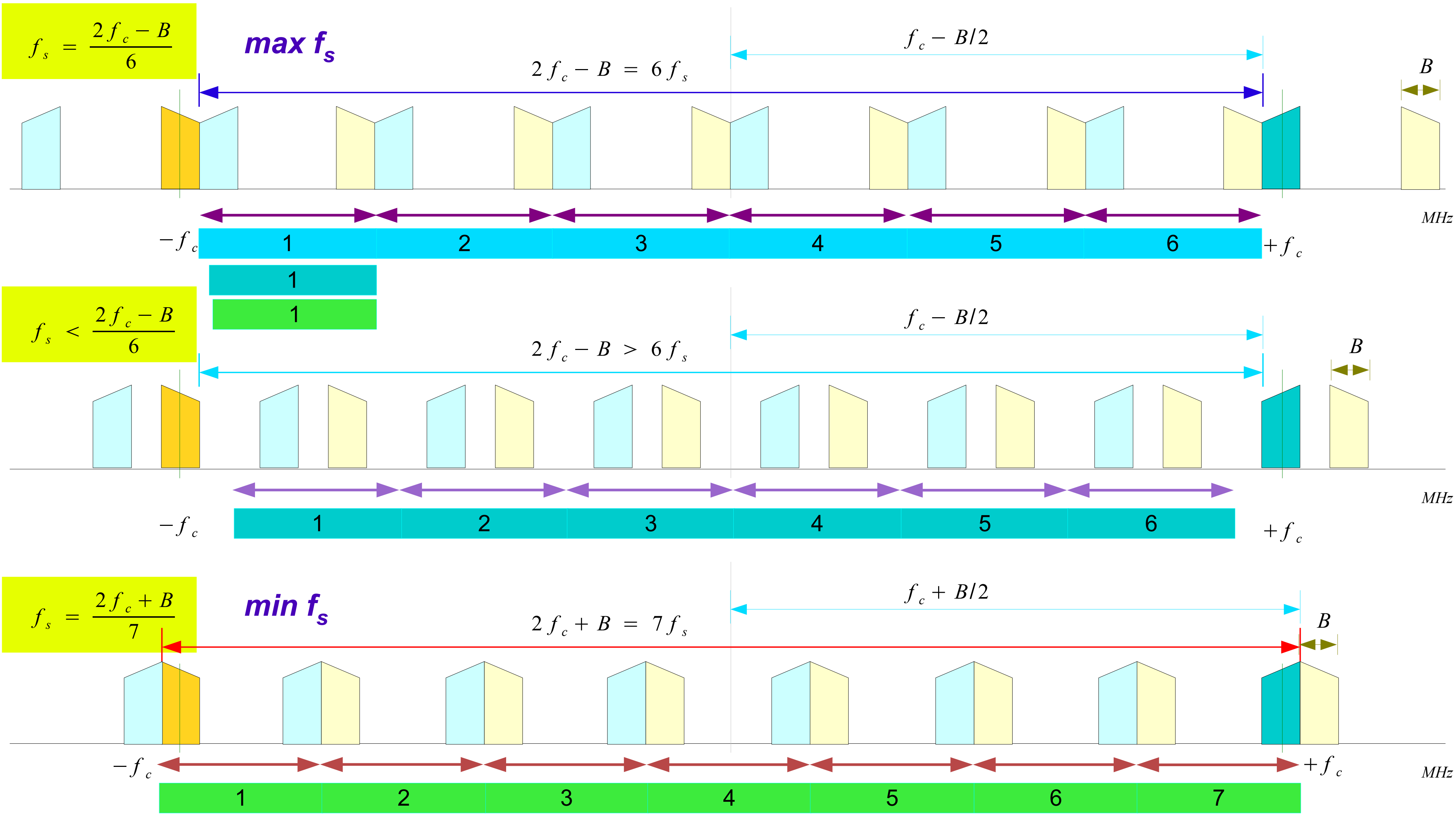
Max f_s condition

f_s can be decreased according to the following condition without introducing aliasing problems

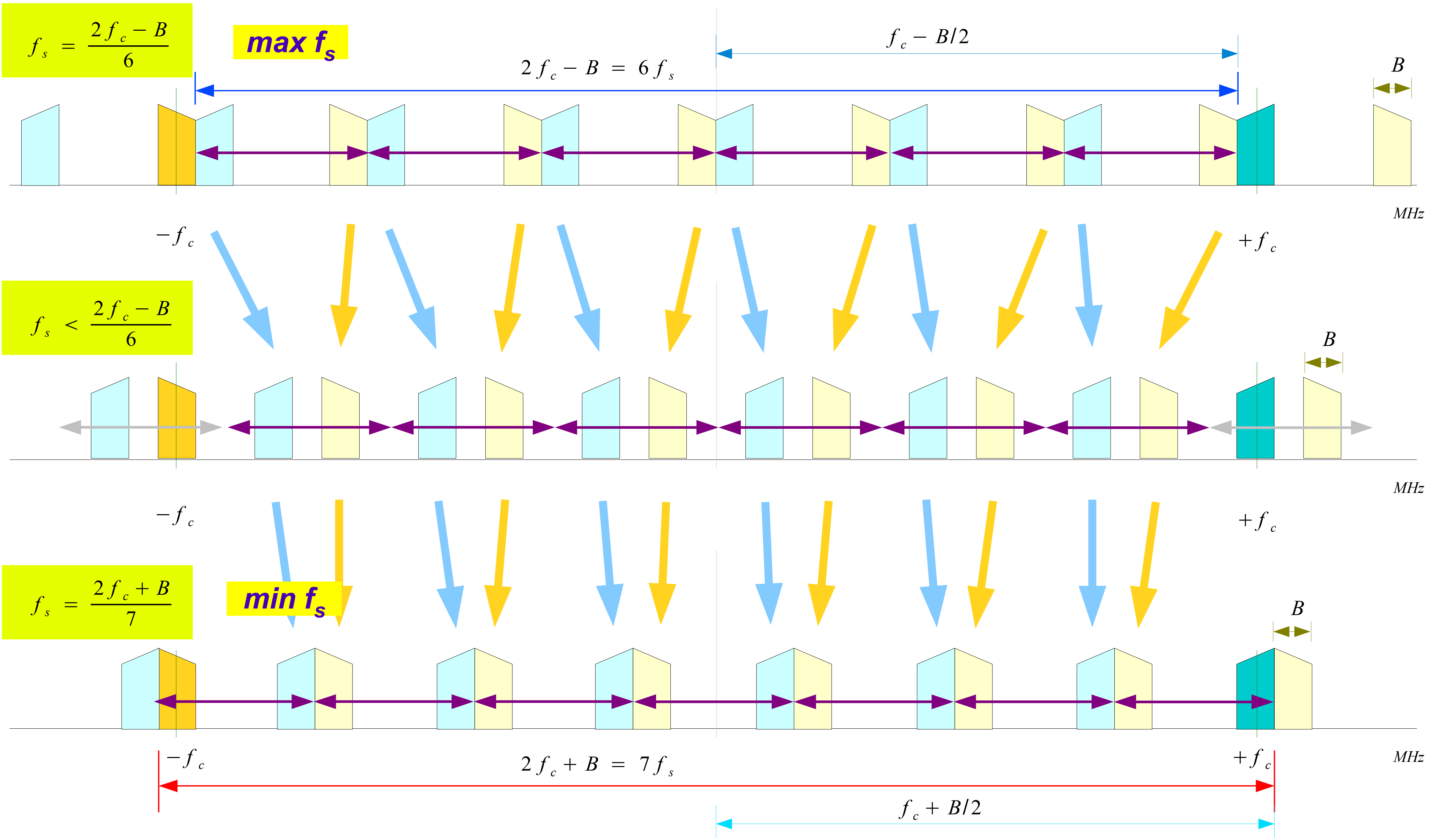
$$2f_c + B = (m+1) \cdot f_s$$

Min f_s condition

Sampling Frequency f_s (3)



Sampling Frequency f_s (4)



Range of f_s (1)

For a given m	$\dfrac{2f_c + B}{m+1} \le f_s \le \dfrac{2f_c - B}{m}$	*Nyquist Criterion*	$2B \le f_s$

$f_c = 20\ MHz$

$B = 5\ MHz$

min f_s ← $\dfrac{2f_c + B}{m+1}$

max f_s ← $\dfrac{2f_c - B}{m}$

Optimum Sampling Frequency

$m = 1$ ➡ $\dfrac{2\cdot 20 + 5}{1+1} = 22.5 \le f_s \le \dfrac{2\cdot 20 - 5}{1} = 35$ ➡ $f_s = 22.5\ MHz \quad (10 \le f_s)$

$m = 2$ ➡ $\dfrac{2\cdot 20 + 5}{2+1} = 15 \le f_s \le \dfrac{2\cdot 20 - 5}{2} = 17.5$ ➡ $f_s = 17.5\ MHz \quad (10 \le f_s)$

$m = 3$ ➡ $\dfrac{2\cdot 20 + 5}{3+1} = 11.25 \le f_s \le \dfrac{2\cdot 20 - 5}{3} = 11.67$ ➡ $f_s = 11.25\ MHz \quad (10 \le f_s)$

$m = 4$ ➡ $\dfrac{2\cdot 20 + 5}{4+1} = 9 \ge \dfrac{2\cdot 20 - 5}{4} = 8.75$ ➡ X

$m = 5$ ➡ $\dfrac{2\cdot 20 + 5}{5+1} = 7.5 \ge \dfrac{2\cdot 20 - 5}{5} = 7.0$ ➡ X

Range of f_s (2)

$$\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}$$

$$\frac{f_c + B/2}{B} = \boldsymbol{R}$$

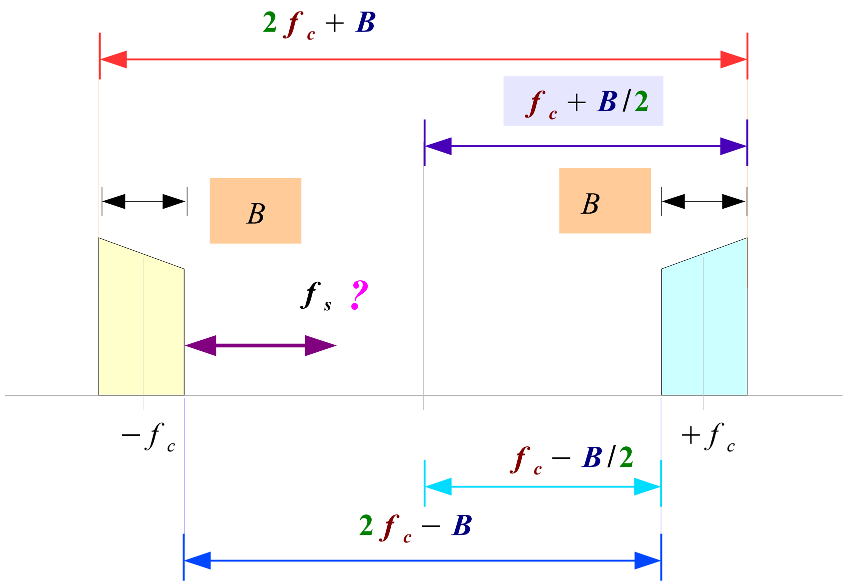
$\Rightarrow$ $\dfrac{\text{highest signal frequency}}{\text{bandwidth B}}$

$$\frac{2f_c + B}{(m+1)B} = \frac{f_{s,min}}{B} = g(m,R)$$

$\Rightarrow$ $\dfrac{\text{minimum sampling rate}}{\text{bandwidth B}}$

$$\frac{2(f_c + B/2)}{(m+1)B} = \frac{2R}{m+1} = g(m,R)$$

$m=1$	$g(1,R) = R$	$m=5$	$g(5,R) = \frac{1}{3}R$
$m=2$	$g(2,R) = \frac{2}{3}R$	$m=6$	$g(6,R) = \frac{2}{7}R$
$m=3$	$g(3,R) = \frac{1}{2}R$	$m=7$	$g(7,R) = \frac{1}{4}R$
$m=4$	$g(4,R) = \frac{2}{5}R$	$m=8$	$g(8,R) = \frac{2}{9}R$



Range of f_s (3)

$$\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}$$

$$\frac{f_c + B/2}{B} = R \quad \Rightarrow \quad \frac{\text{highest signal frequency}}{\text{bandwidth B}}$$

$$f_H = f_c + B/2$$

$$R = f_H / B$$

$$\frac{2f_c + B}{(m+1)B} = g(m, R) \quad \Rightarrow \quad \frac{\text{minimum sampling rate}}{\text{bandwidth B}}$$

$$f_{s,min} = \frac{2f_c + B}{m+1} = \frac{2f_H}{k}$$

$$\frac{2f_c + B}{(m+1)B} = g(m, R) = \frac{2R}{m+1} = \frac{2R}{k}$$

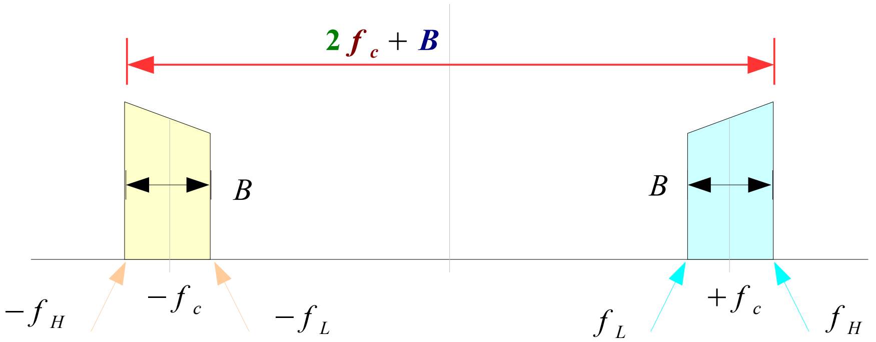
$$m + 1 = k$$

$$g(m, R) = \frac{2f_H}{kB} = \frac{2R}{k}$$

***k* represents how many f_s are in $2f_c + B$ in**

Min f_s condition

$$2f_c + B = (m+1)\cdot f_s = k \cdot f_s$$



Range of f_s (4)

$$\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}$$

$$\frac{2f_c + B}{k} \le f_s \le \frac{2f_c - B}{k-1}$$

$$\frac{2(f_c + B/2)}{k} \le f_s \le \frac{2(f_c + B/2) - 2B}{k-1}$$

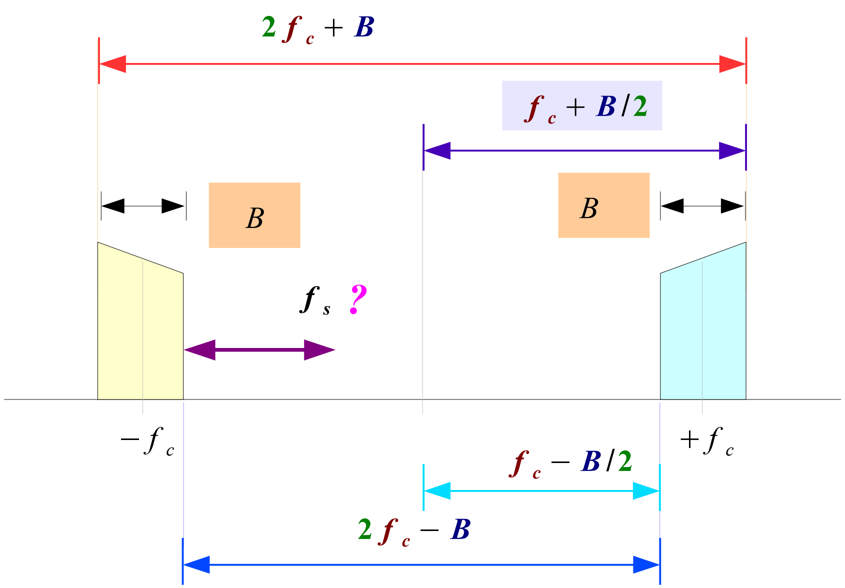
$$\frac{2f_H}{k} \qquad \frac{2(f_H - B)}{k-1}$$

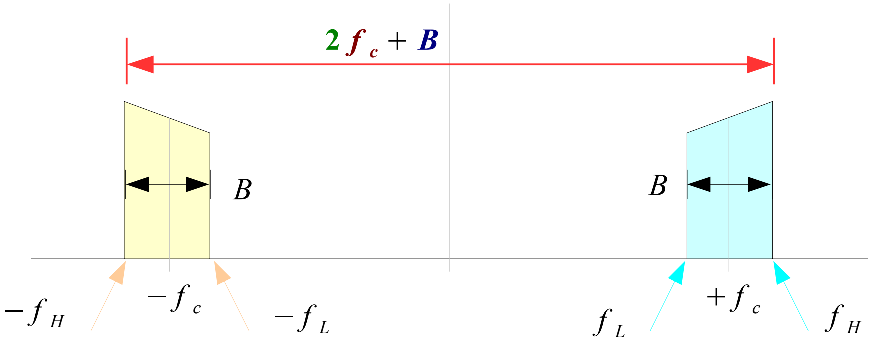
$$\frac{2f_H}{k} \le f_s \le \frac{2(f_H - B)}{k-1}$$

$k = 2 \qquad f_H \le f_s \le 2f_H - 2B$

$k = 3 \qquad \frac{2f_H}{3} \le f_s \le f_H - B$

$k = 4 \qquad \frac{f_H}{2} \le f_s \le \frac{2f_H}{3} - \frac{3B}{3}$



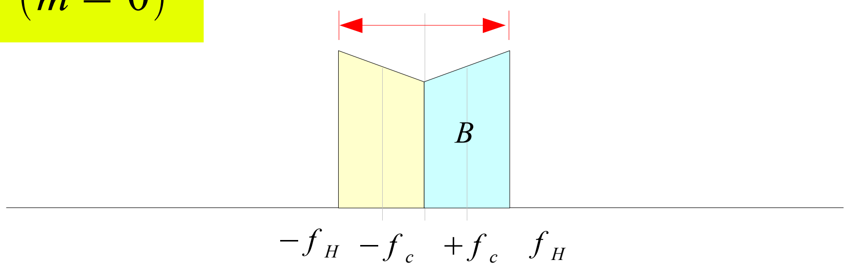


Young Won Lim
3/20/12

Example k=1

$$k = 1 \quad (m = 0)$$

$$f_H = f_c + B/2 = 1B$$



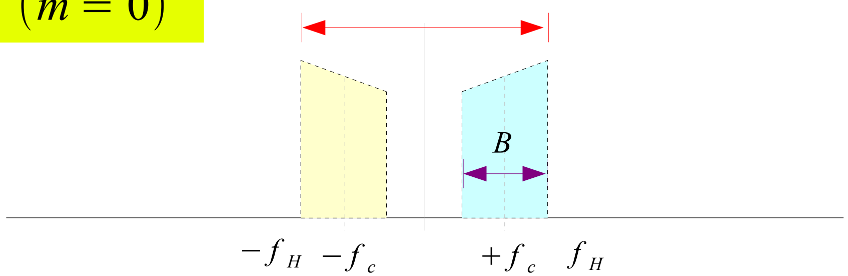
$$R = f_H / B = 1$$

$$\frac{f_{s,min}}{B} = \frac{2 f_H}{k B} = \mathbf{2}$$

$$\frac{f_{s,max}}{B} = \frac{2(f_H - B)}{(k-1) B} = +\infty$$

$$k = 1 \quad (m = 0)$$

$$f_H = f_c + B/2 = 1.5B$$



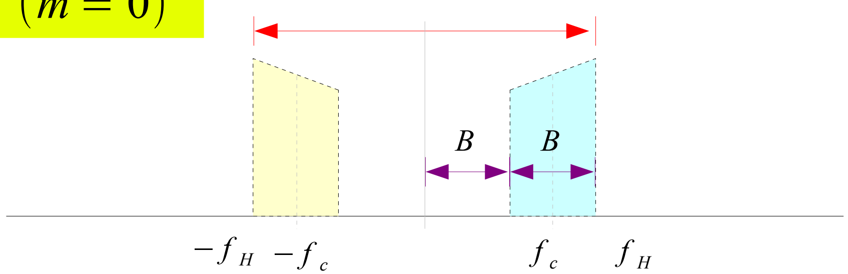
$$R = f_H / B = 1.5$$

$$\frac{f_{s,min}}{B} = \frac{2 f_H}{k B} = \mathbf{3}$$

$$\frac{f_{s,max}}{B} = \frac{2(f_H - B)}{(k-1) B} = +\infty$$

$$k = 1 \quad (m = 0)$$

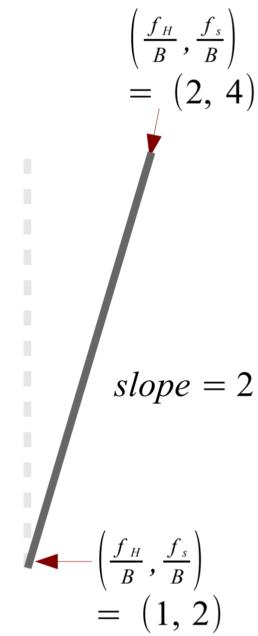
$$f_H = f_c + B/2 = 2B$$



$$R = f_H / B = 2$$

$$\frac{f_{s,min}}{B} = \frac{2 f_H}{k B} = \mathbf{4}$$

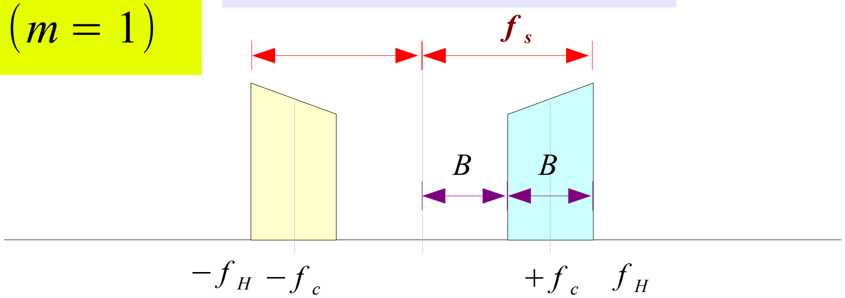
$$\frac{f_{s,max}}{B} = \frac{2(f_H - B)}{(k-1) B} = +\infty$$



Example k=2

$$k = 2 \quad (m = 1)$$

$$f_H = f_c + B/2 = 2B$$



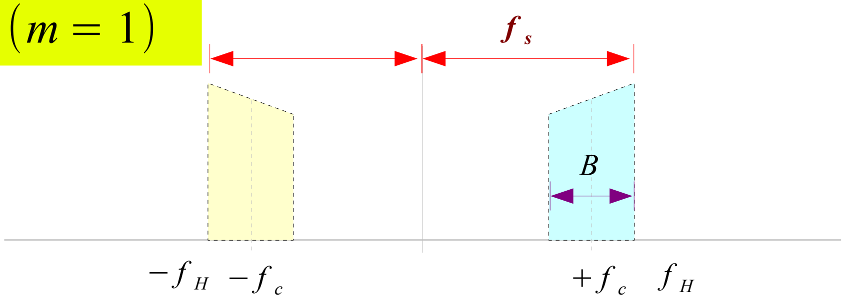
$$R = f_H / B = 2$$

$$\frac{f_{s,min}}{B} = \frac{2 f_H}{k B} = 2$$

$$\frac{f_{s,max}}{B} = \frac{2(f_H - B)}{(k-1)B} = 2$$

$$k = 2 \quad (m = 1)$$

$$f_H = f_c + B/2 = 2.5B$$



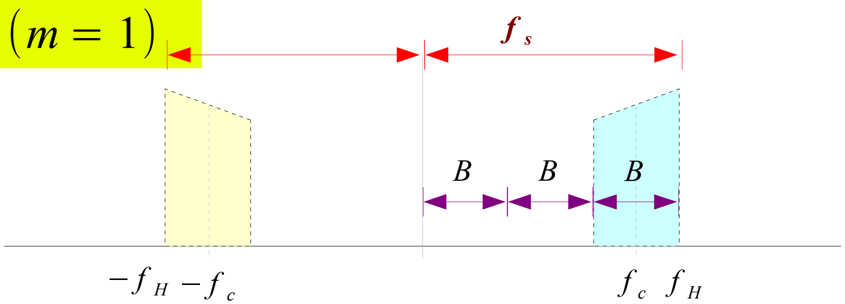
$$R = f_H / B = 2.5$$

$$\frac{f_{s,min}}{B} = \frac{2 f_H}{k B} = 2.5$$

$$\frac{f_{s,max}}{B} = \frac{2(f_H - B)}{(k-1)B} = 3$$

$$k = 2 \quad (m = 1)$$

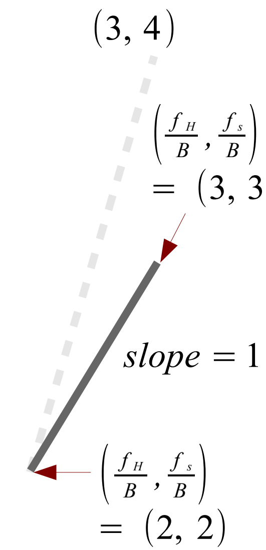
$$f_H = f_c + B/2 = 3B$$



$$R = f_H / B = 3$$

$$\frac{f_{s,min}}{B} = \frac{2 f_H}{k B} = 3$$

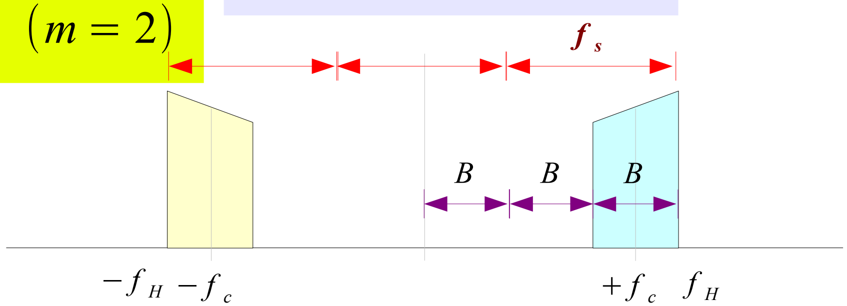
$$\frac{f_{s,max}}{B} = \frac{2(f_H - B)}{(k-1)B} = 4$$



Example k=3

$k = 3$ $(m = 2)$

$f_H = f_c + B/2 = 3B$



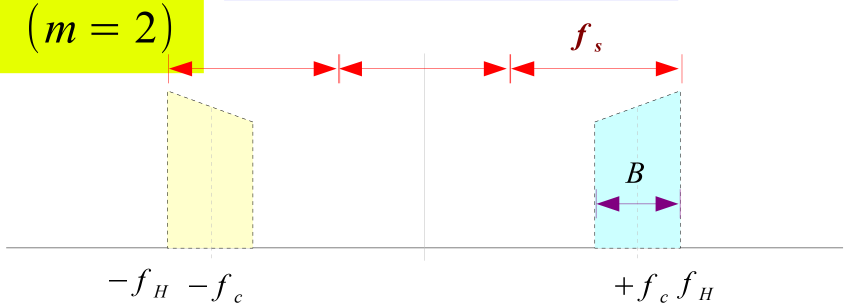
$$R = f_H / B = 3$$

$$\frac{f_{s,min}}{B} = \frac{2 f_H}{k B} = 2$$

$$\frac{f_{s,max}}{B} = \frac{2(f_H - B)}{(k-1) B} = 2$$

$k = 3$ $(m = 2)$

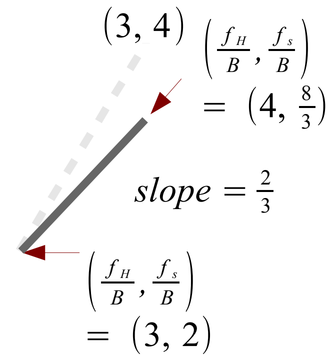
$f_H = f_c + B/2 = 3.5B$



$$R = f_H / B = 3.5$$

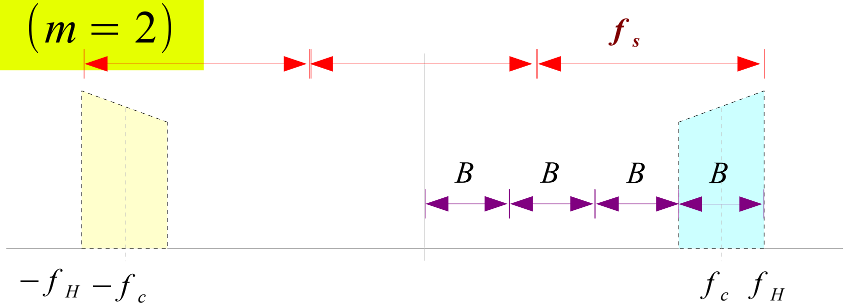
$$\frac{f_{s,min}}{B} = \frac{2 f_H}{k B} = \frac{7}{3}$$

$$\frac{f_{s,max}}{B} = \frac{2(f_H - B)}{(k-1) B} = 3$$



$k = 3$ $(m = 2)$

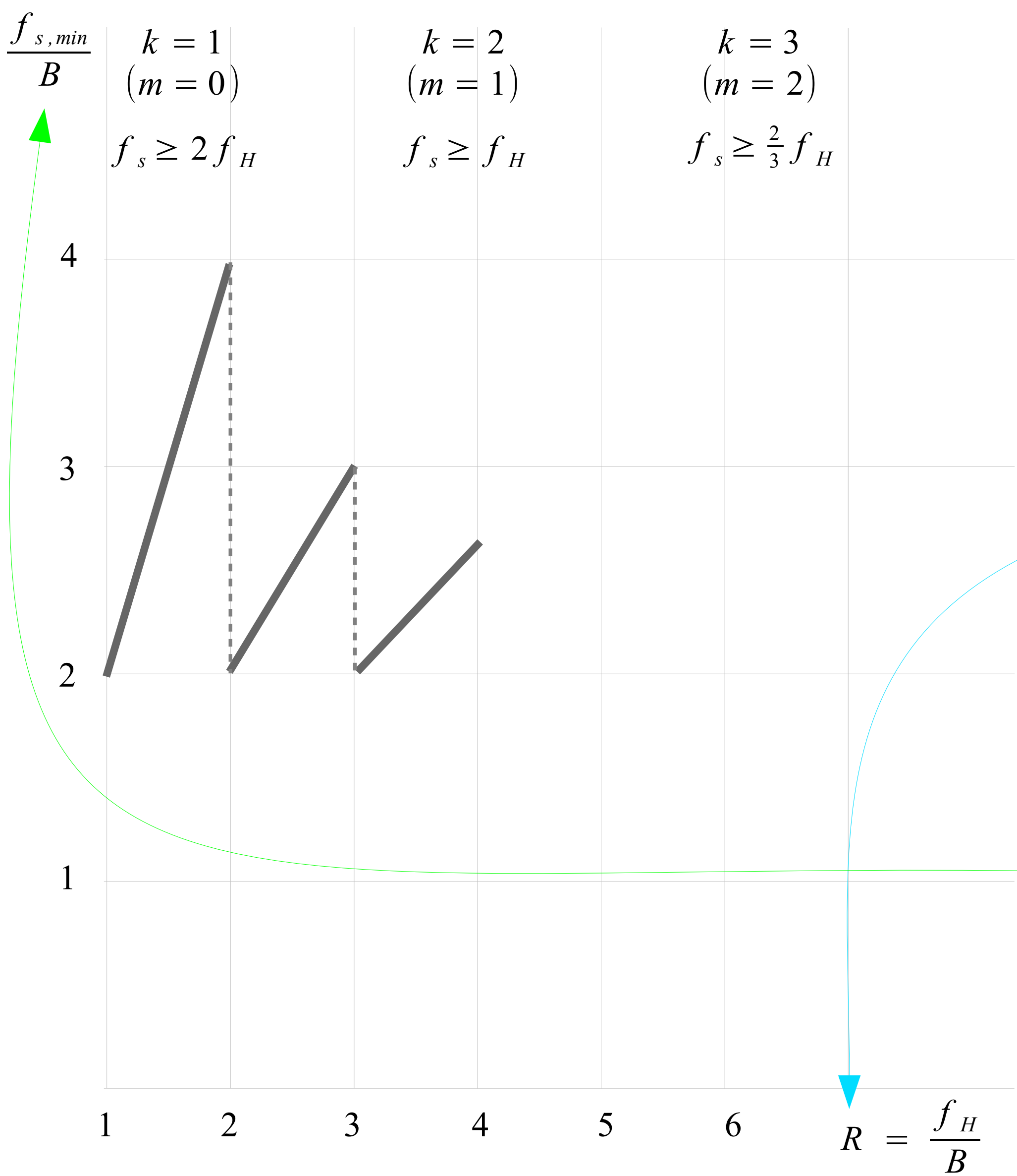
$f_H = f_c + B/2 = 4B$



$$R = f_H / B = 4$$

$$\frac{f_{s,min}}{B} = \frac{2 f_H}{k B} = \frac{8}{3}$$

$$\frac{f_{s,max}}{B} = \frac{2(f_H - B)}{(k-1) B} = 4$$



$$\frac{2 f_c + B}{m+1} \le f_s \le \frac{2 f_c - B}{m}$$

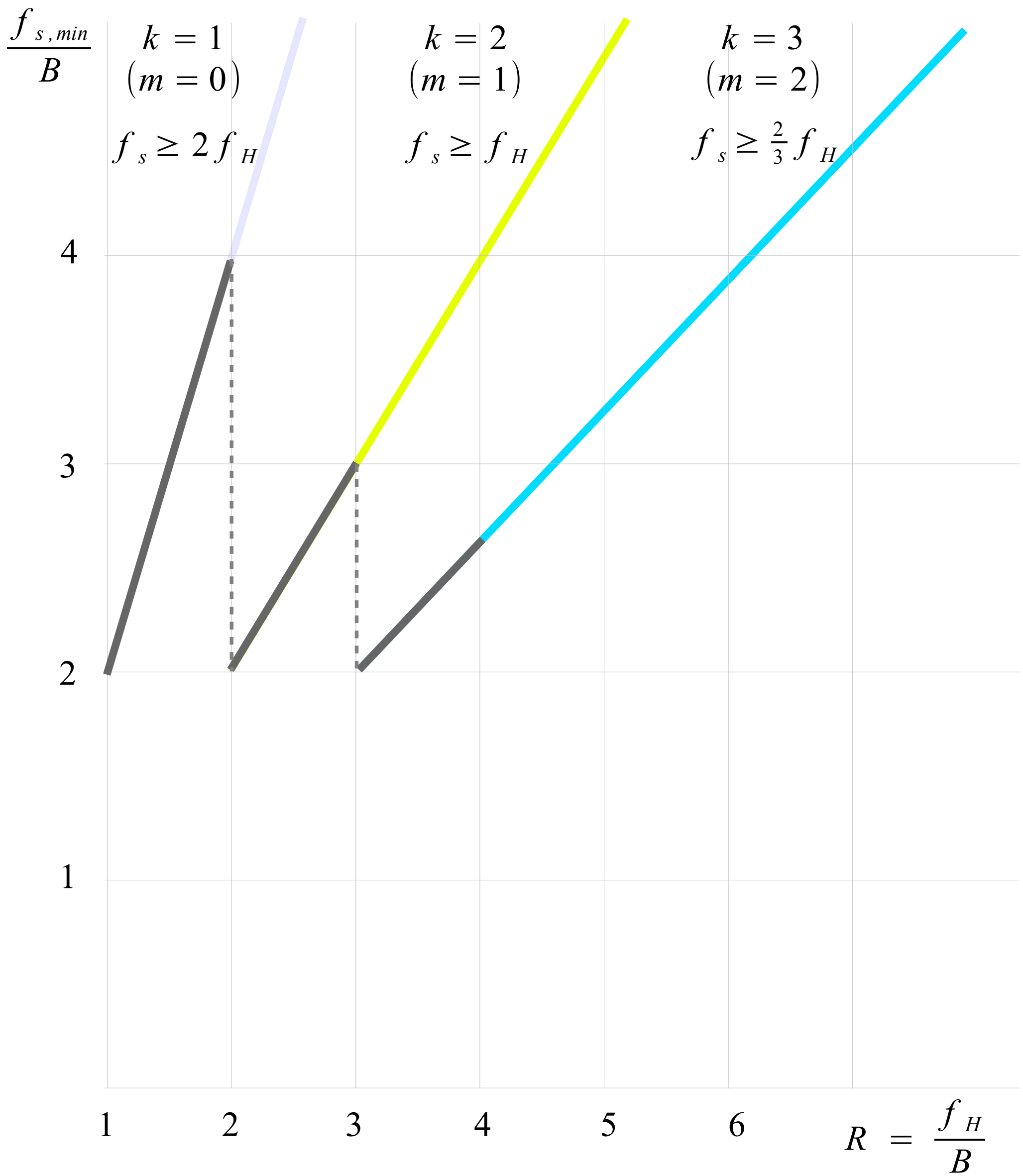
$$\frac{2 f_c + B}{\boldsymbol{k}} \le f_s \le \frac{2 f_c - B}{\boldsymbol{k-1}}$$

$$\frac{f_c + B/2}{\boldsymbol{B}} = \boldsymbol{R}$$

➡ $\dfrac{\textit{highest signal frequency}}{\textit{bandwidth B}}$

$$\frac{2 f_c + B}{(m+1)\boldsymbol{B}} = \frac{f_{s,min}}{B} = g(m, R)$$

➡ $\dfrac{\textit{minimum sampling rate}}{\textit{bandwidth B}}$



$$\frac{2f_c + B}{m+1} \leq f_s \leq \frac{2f_c - B}{m}$$

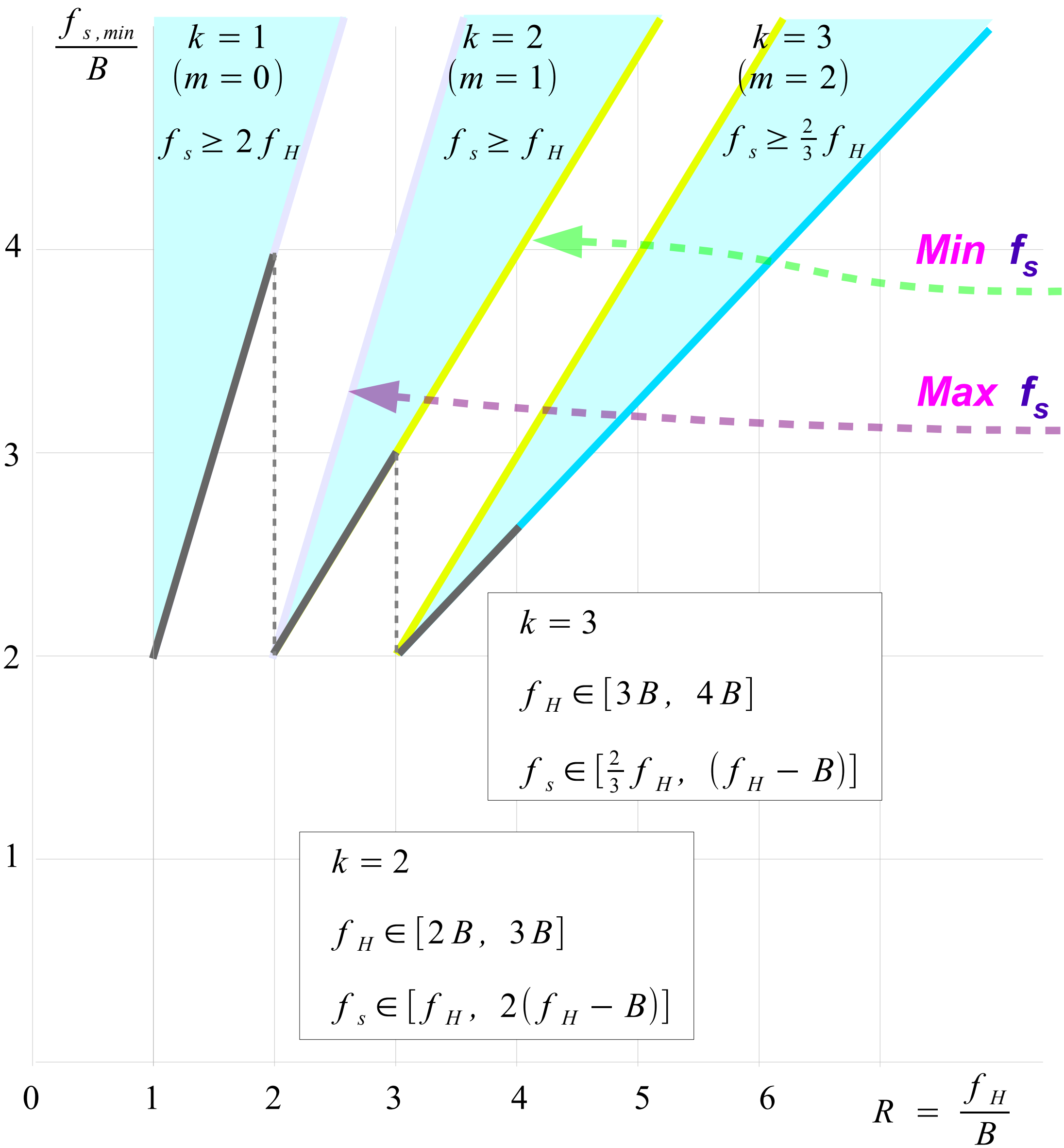
$$\frac{2f_c + B}{k} \leq f_s \leq \frac{2f_c - B}{k-1}$$

$$\frac{f_c + B/2}{B} = R$$

➡ $\dfrac{\text{highest signal frequency}}{\text{bandwidth B}}$

$$\frac{2f_c + B}{(m+1)B} = \frac{f_{s,min}}{B} = g(m, R)$$

➡ $\dfrac{\text{minimum sampling rate}}{\text{bandwidth B}}$



$$\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}$$

$$\frac{2f_c + B}{k} \le f_s \le \frac{2f_c - B}{k-1}$$

	Max f_s	Min f_s
	$y = 2(x-2)+2$	$y = 1(x-2)+2$
$k = 2$	$y = 2x-2$	$y = x$
	$y = 1(x-3)+2$	$y = \frac{2}{3}(x-3)+2$
$k = 2$	$y = x-1$	$y = \frac{2}{3}x$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A “graphical interpretation” of the DFT and FFT, by Steve Mann
[4] R. G. Lyons, Understanding Digital Signal Processing, 1997