Thu Nov 24, 2011 8:13 AM

For continuous cdf and pdf, the probability is

$$P(\hat{x}) = \lim_{x \to \hat{x}} |F_X(x) - F_X(\hat{x})| = 0 \qquad (1)$$

P(\hat x) := \lim_{xc \to \hat x } | F_X (x) - F_X (\hat x) | = 0

$$P(X < \hat{x}) = P(X \le \hat{x}) = \int_{-\infty}^{\hat{x}} f(X)dX =: F(\hat{x}) \qquad (2)$$

P (X < \hat x) = P (X \le \hat x) = \int_{-\infty}^{\hat x} f(X) dX =: F(\hat x)

Generalization of cdf to left or right continuity:

Take (1) as the starting point, and distinguish left and right limit of x going to x hat, i.e., there are two cases:

Case 1: x tends to x hat from below

Define the probability as follows:

$$P(\hat{x}) := \lim_{x \uparrow \hat{x}} |F_X(x) - F_X(\hat{x})| = 0 \qquad (3)$$

P(\hat x) := \lim_{x \ \uparrow \ \hat x } | F_X (x) - F_X (\hat x) | = 0

then $$F_X(\hat{x}^-) = F_X(\hat{x}) \qquad (4)$$

F_X (\hat x^-) = F_X (\hat x)

thus F is left continuous.

Note: For continuous cdf and pdf, start with the pdf to define the probability as in (2); then

$$P(x < X < \hat{x}) = P(x \le X \le \hat{x}) = F_X(\hat{x}) - F_X(x)$$

then

P (x < X < \hat x) = P (x \le X \le \hat x) = F_X (\hat x) - F_X (x) (5)

$$P(x = \hat{x}) = \lim_{x \to \hat{x}} |F_X(\hat{x}) - F_X(x)| = 0 \qquad (1)$$

P (x = \hat x) = \lim_{x \to \hat x} | F_X (\hat x) - F_X (x) | = 0 ///

Note: For Case 1, you would have in general

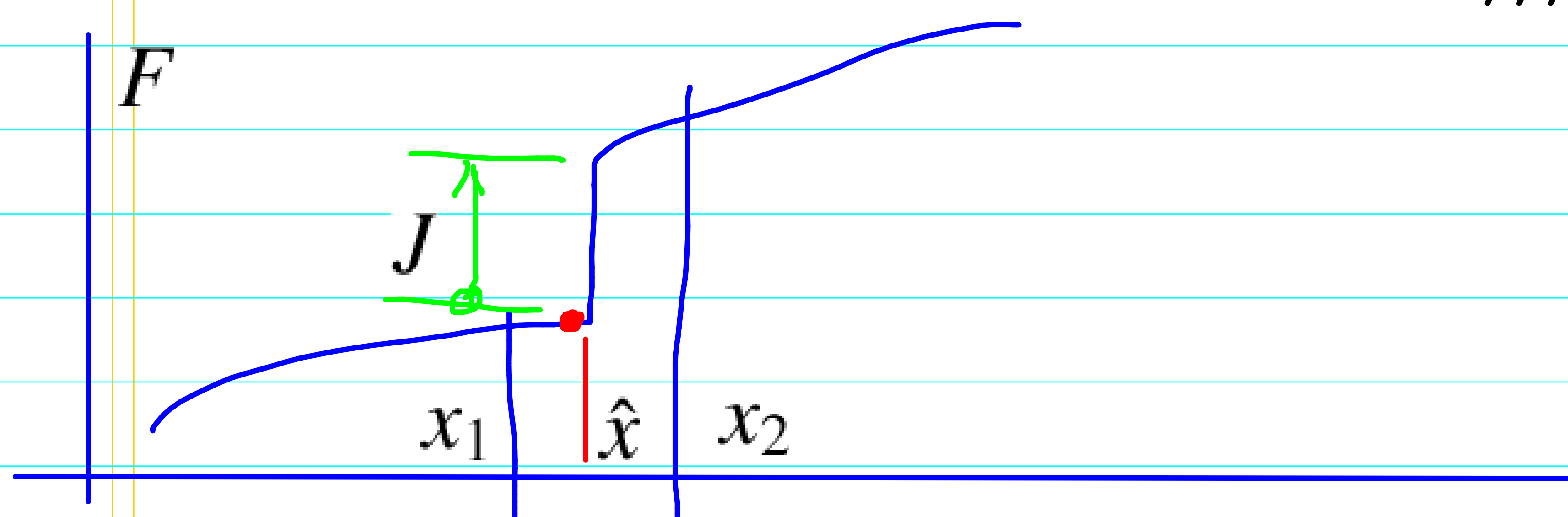
$$\lim_{x \downarrow \hat{x}} |F_X(x) - F_X(\hat{x})| \ne 0 \qquad (6)$$

\lim_{x \ \downarrow \ \hat x } | F_X (x) - F_X (\hat x) | \ne 0

thus

$$P(\hat{x}) \ne \lim_{x \downarrow \hat{x}} |F_X(x) - F_X(\hat{x})| \ne 0 \qquad (7)$$

P(\hat x) \ne \lim_{x \ \downarrow \ \hat x } | F_X (x) - F_X (\hat x) | \ne 0 ///



$$P(X < x_1) < P(X < x_2) \tag{8}$$

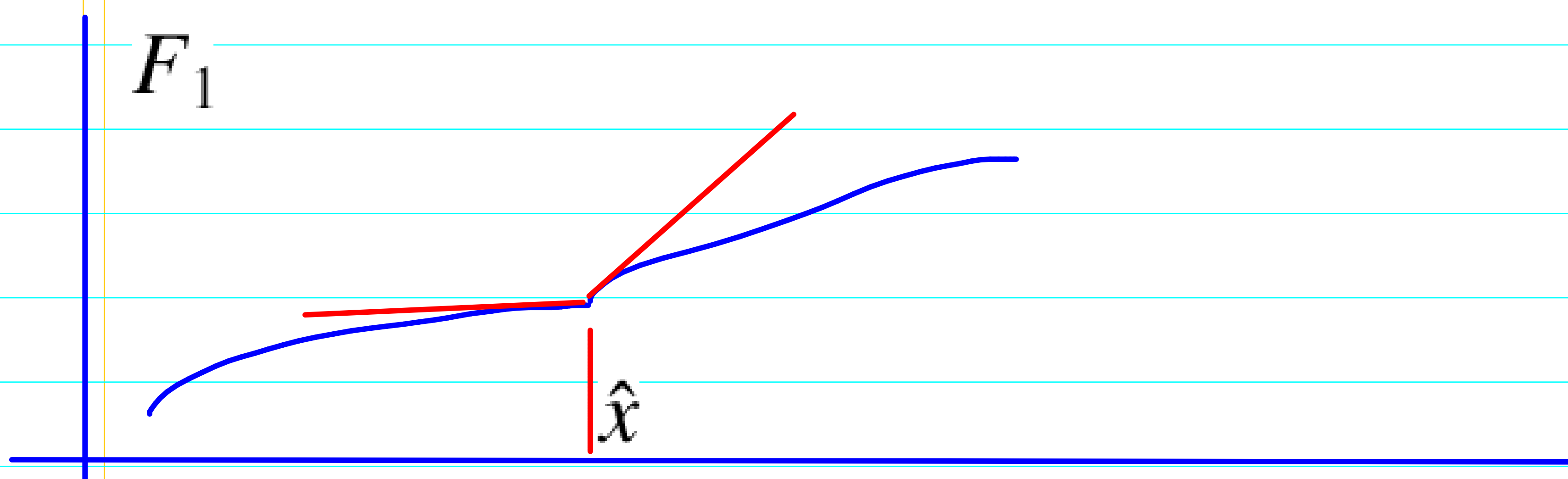
P (X < x_1) < P (X < x_2)

$$P(X < \hat{x}^+) - P(X < \hat{x}) = J \tag{9}$$

P (X < \hat x^+) - P (X < \hat x) = J

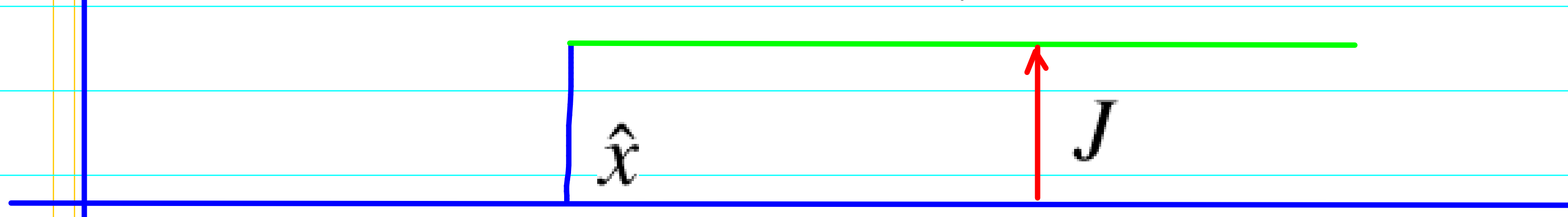
$$f(x) = \frac{dF(x)}{dx} = \left. F'(x) \right|_{x \ne \hat{x}} + \bar{J}\delta(x - \hat{x}) \tag{10}$$

f(x) = \frac{d F (x)}{dx} = \left. F' (x) \right|_{x \ne \hat x} + \bar J \delta (x - \hat x)



$F_2(x) \equiv H(x)$

Heaviside function = step function



$$\bar F = F_1 + H \tag{11}$$

\bar F = F_1 + H

$$\bar J = J_1 + J \tag{12}$$

\bar J = J_1 + J

$$F' = F_1' + H' = \left. F'(x) \right|_{x \ne \hat x} + \bar J \delta(x - \hat x) \tag{13}$$

F' = F_1' + H' = \left. F' (x) \right|_{x \ne \hat x} + \bar J \delta (x - \hat x)

$$F_1' = \left. F'(x) \right|_{x \ne \hat x} + J_1 \delta(x - \hat x) \tag{14}$$

F_1' = \left. F' (x) \right|_{x \ne \hat x} + J_1 \delta (x - \hat x)

$$H' = J \delta(x - \hat x) \tag{15}$$

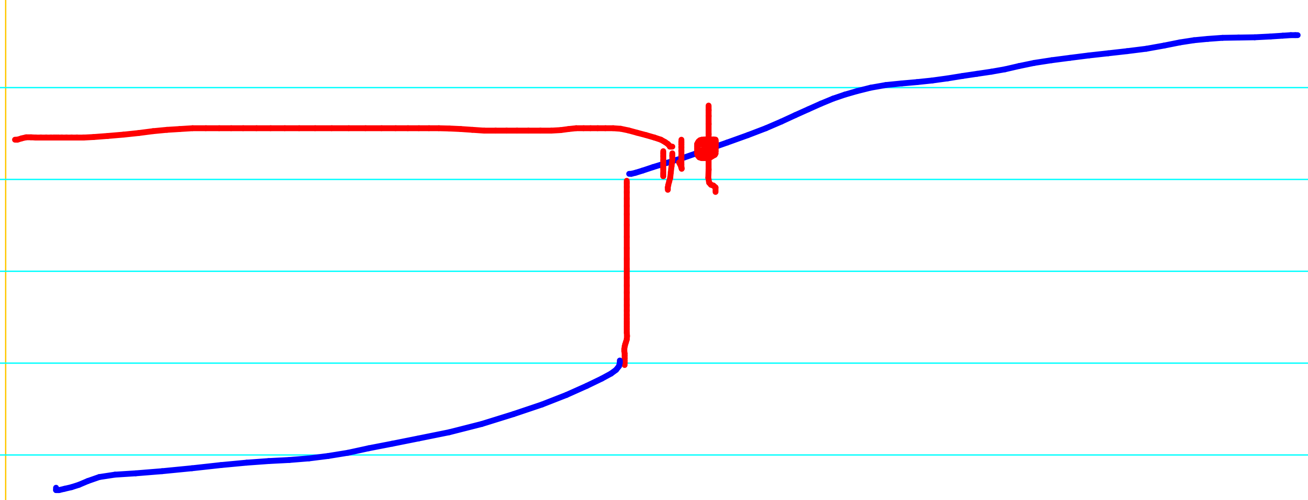
H' = J \delta (x - \hat x)

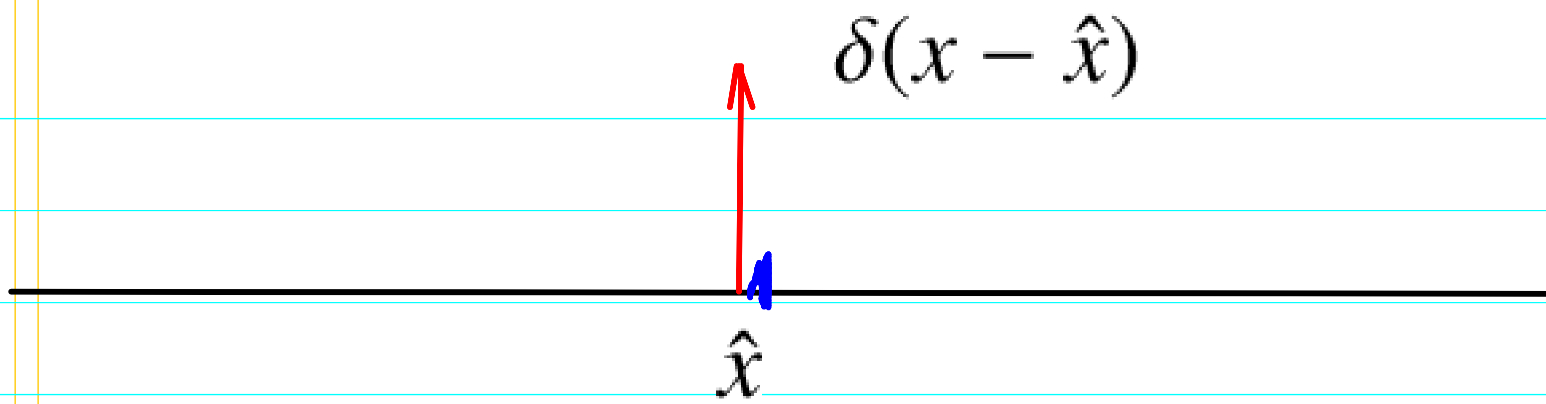
$$f = F' \tag{16}$$

f = F'

$$P(X < \hat x^+) - P(X < \hat x) = F(\hat x^+) - F(\hat x) = J \tag{9}$$

P (X < \hat x^+) - P (X < \hat x) = F(\hat x^+) - F(\hat x) = J





$$\int_{-\infty}^{+\infty} \delta(x - \hat{x}) dx = 1$$

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$$\delta(x - \hat{x}) = 0 \text{ for } x \neq \hat{x}$$

\delta(x - \hat x) = 0 \text{ for } x \ne \hat x