Triangles (2B)

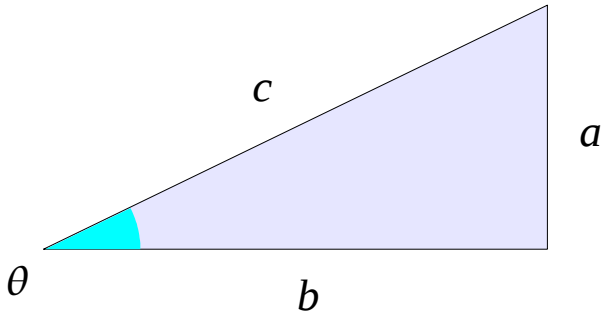
- Law of Sines - all acute angles
- Law of Cosines - all acute angles
- Law of Sines - one obtuse angle
- Law of Cosines - one obtuse angle

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice and Octave.

Young Won Lim
10/18/09

Right Triangles (1)



$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

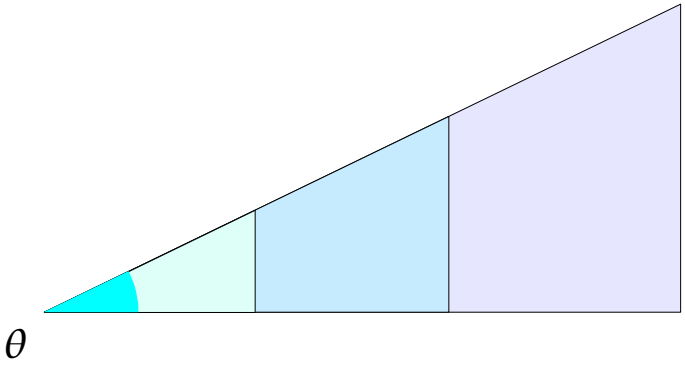
$$c^2 = a^2 + b^2$$

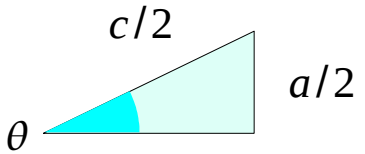
$$\sin\theta = \frac{a}{c}$$

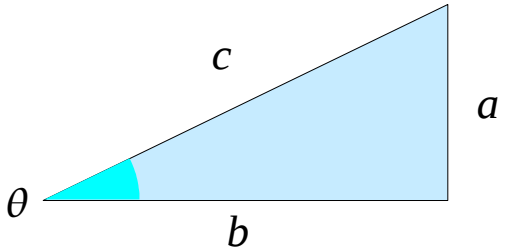
$$\cos\theta = \frac{b}{c}$$

$$\tan\theta = \frac{b}{a}$$

Right Triangles (2)



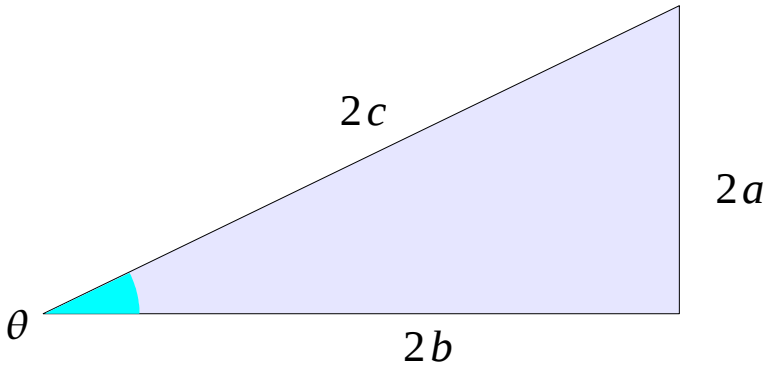




$$\sin\theta = \frac{a}{c}$$

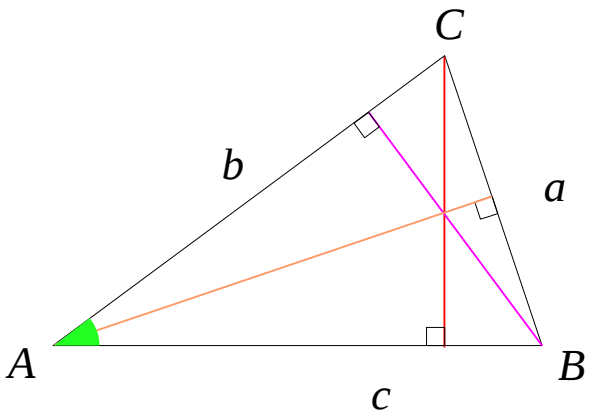
$$\cos\theta = \frac{b}{c}$$

$$\tan\theta = \frac{b}{a}$$



Oblique Triangles – (1) all acute angles

All Acute Angles



$$0^\circ < A < 90^\circ$$
$$0^\circ < B < 90^\circ$$
$$0^\circ < C < 90^\circ$$

The Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The Law of Cosines

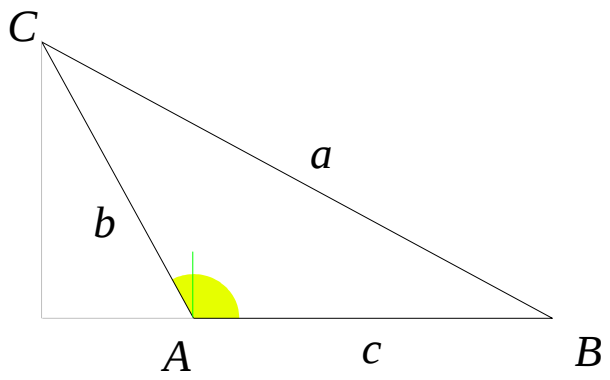
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = c^2 + a^2 - 2ca\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Oblique Triangles – (2) one obtuse angle

One Obtuse Angles



$$90^\circ < A < 180^\circ$$

$$0^\circ < B < 90^\circ$$
$$0^\circ < C < 90^\circ$$

The Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$

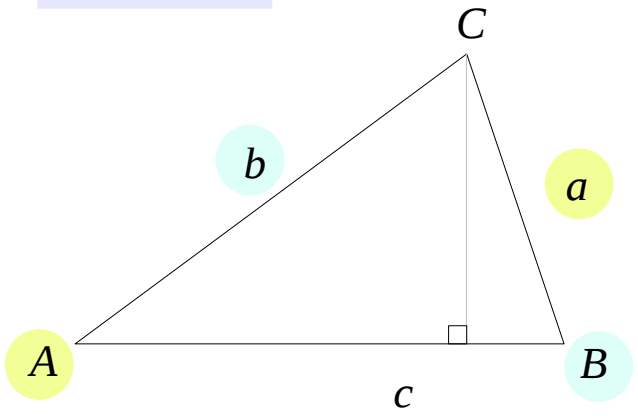
$$b^2 = c^2 + a^2 - 2ca\cos B$$

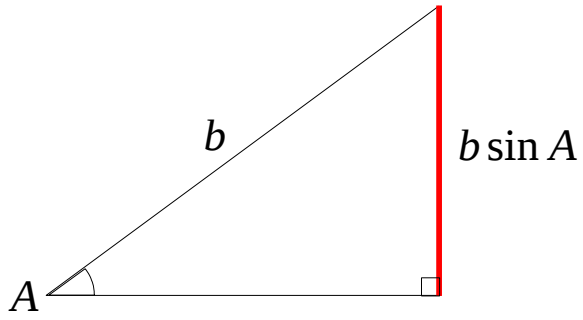
$$c^2 = a^2 + b^2 - 2ab\cos C$$

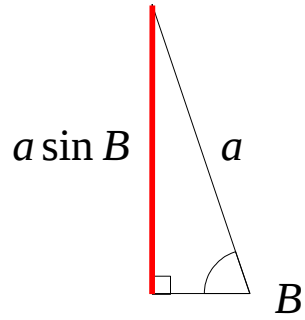
The Law of Sines	*- all acute angles*	*(1) ~ (4)*
The Law of Cosines	*- all acute angles*	*(1) ~ (4)*
The Law of Sines	*- one obtuse angle*	*(1) ~ (4)*
The Law of Cosines	*- one obtuse angle*	*(1) ~ (4)*

Law of Sines – all acute angles (1)

Case A:





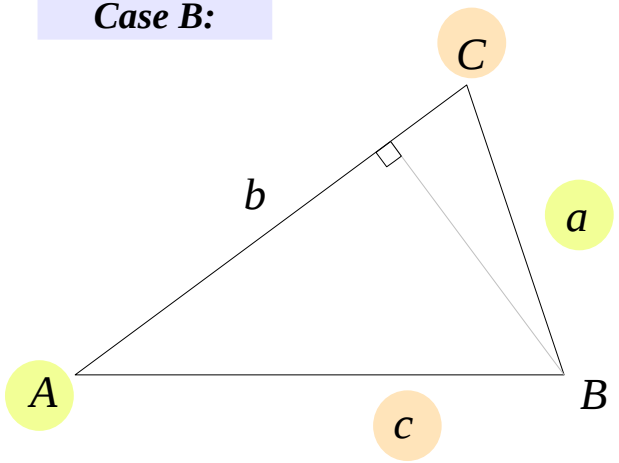


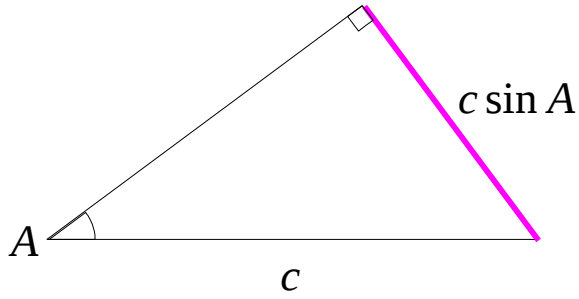
$$b \sin A = a \sin B$$

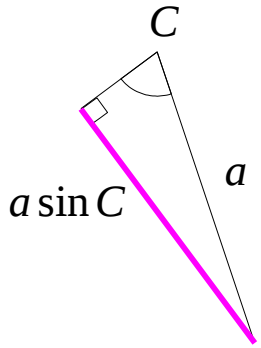
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Law of Sines – all acute angles (2)

Case B:



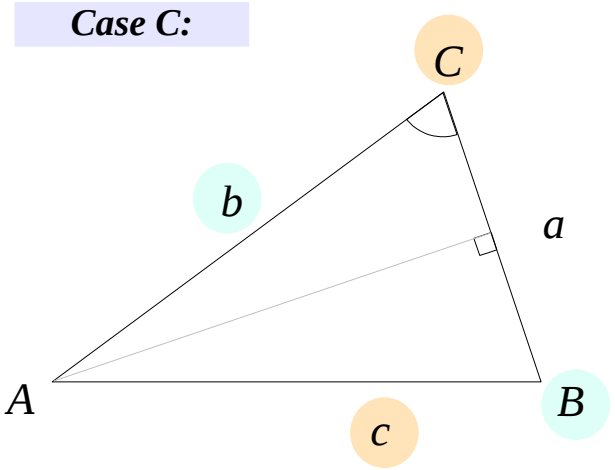


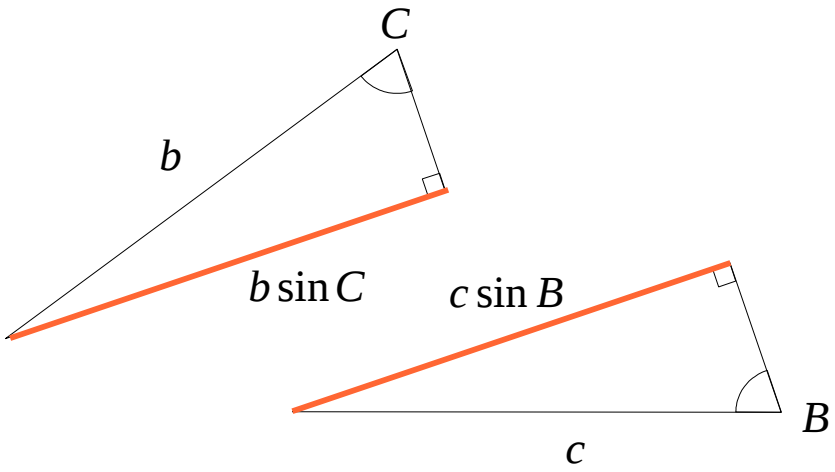


$$c \sin A = a \sin C$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

Law of Sines – all acute angles (3)



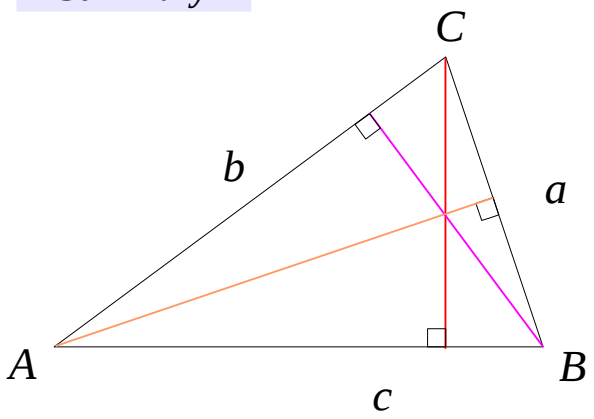


$$b \sin C = c \sin B$$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Sines – all acute angles (4)

Summary



The Law of Sines

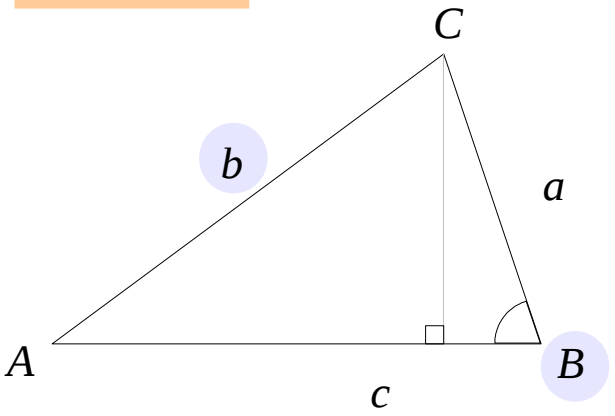
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

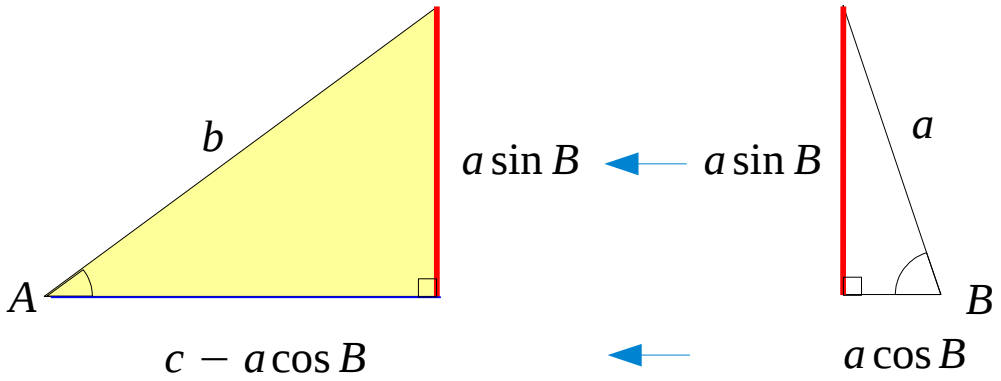
$$0^\circ < A < 90^\circ$$
$$0^\circ < B < 90^\circ$$
$$0^\circ < C < 90^\circ$$

The Law of Sines	*- all acute angles*	*(1) ~ (4)*
The Law of Cosines	*- all acute angles*	*(1) ~ (4)*
The Law of Sines	*- one obtuse angle*	*(1) ~ (4)*
The Law of Cosines	*- one obtuse angle*	*(1) ~ (4)*

Law of Cosines – all acute angles (1)

Case A:





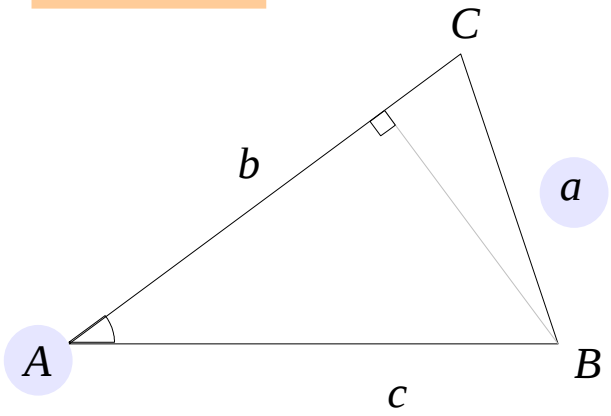
$$b^2 = (c - a\cos B)^2 + (a\sin B)^2$$

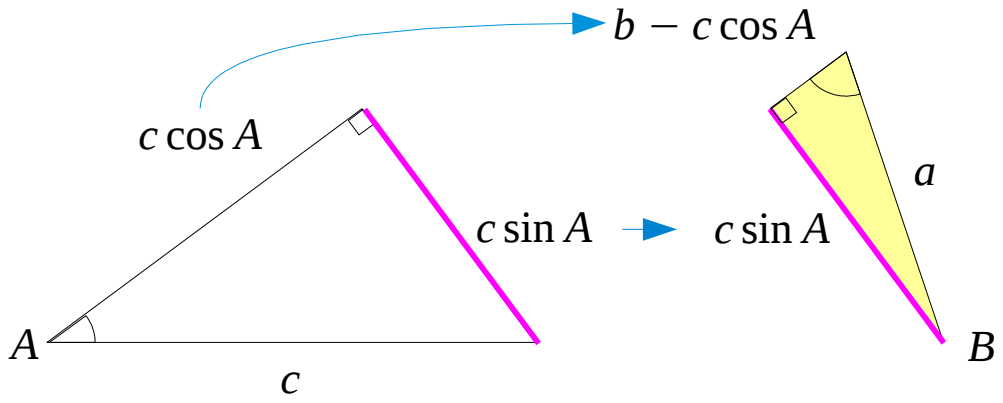
$$b^2 = c^2 - 2ca\cos B + a^2\cos^2 B + a^2\sin^2 B$$

$$b^2 = c^2 + a^2 - 2ca\cos B$$

Law of Cosines – all acute angles (2)

Case B:





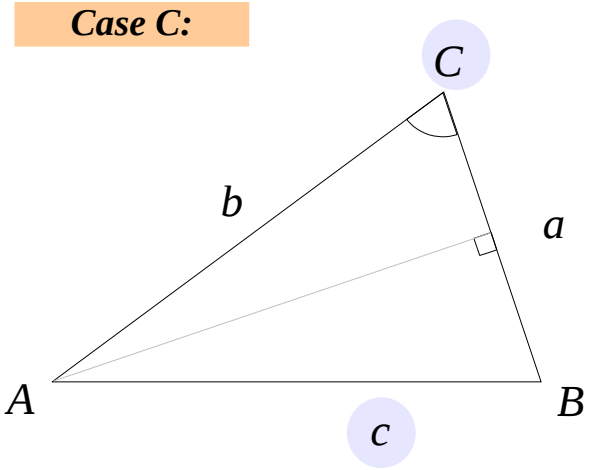
$$a^2 = (b - c\cos A)^2 + (c\sin A)^2$$

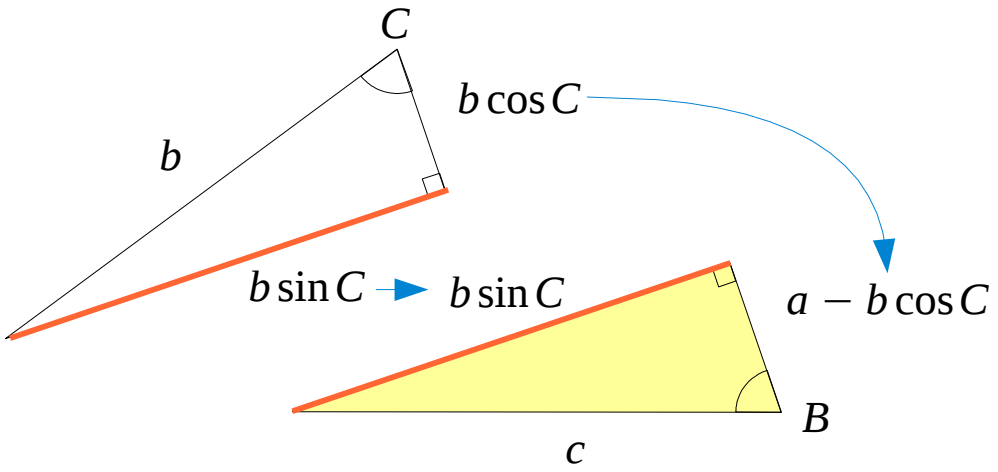
$$a^2 = b^2 - 2bc\cos A + c^2\cos^2 A + c^2\sin^2 A$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Law of Cosines – all acute angles (3)

Case C:





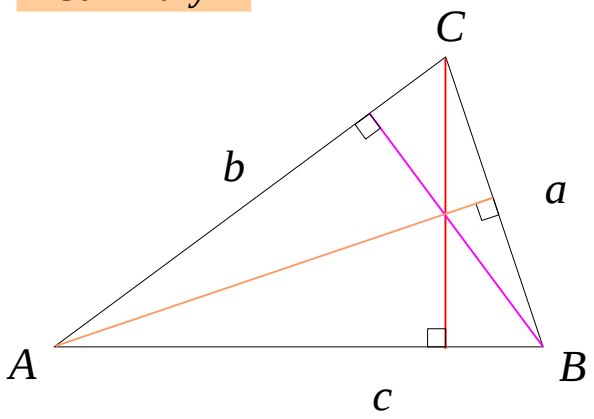
$$c^2 = (a - b\cos C)^2 + (b\sin C)^2$$

$$c^2 = a^2 - 2ab\cos C + b^2\cos^2 C + b^2\sin^2 C$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Law of Cosines – all acute angles (4)

Summary



$$0^\circ < A < 90^\circ$$
$$0^\circ < B < 90^\circ$$
$$0^\circ < C < 90^\circ$$

The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = c^2 + a^2 - 2ca\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

The Law of Sines - all acute angles (1) ~ (4)

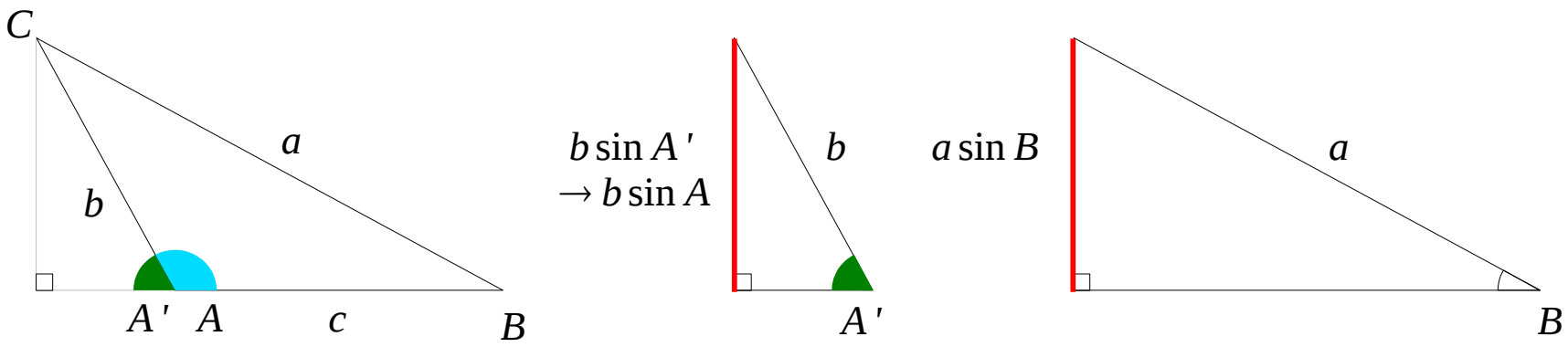
The Law of Cosines - all acute angles (1) ~ (4)

The Law of Sines - one obtuse angle (1) ~ (4)

The Law of Cosines - one obtuse angle (1) ~ (4)

Law of Sines – one obtuse angle (1)

Case A:



Supposing

$$\sin A' = \sin A,$$

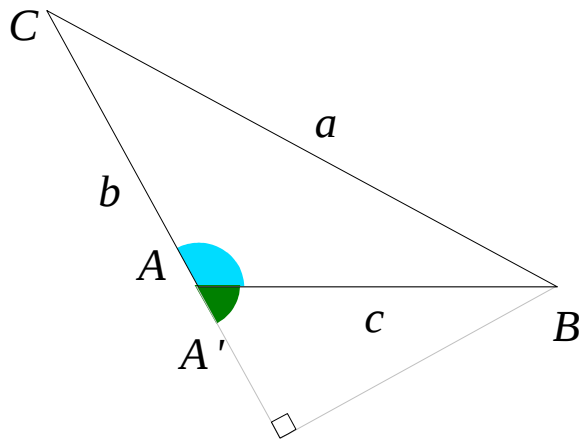
$$\sin(\pi - A) = \sin A$$

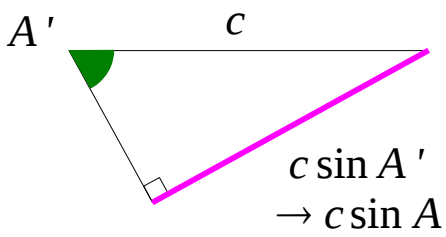
$$b \sin A = a \sin B$$

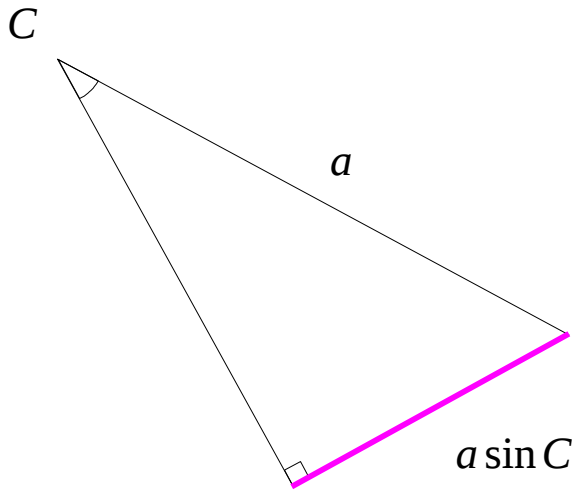
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Law of Sines – one obtuse angle (2)

Case B:







Supposing

$\sin A' = \sin A$,

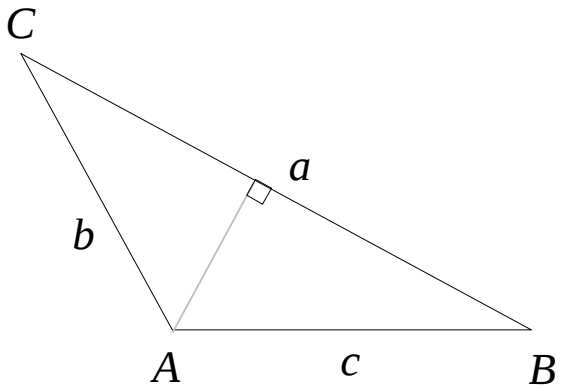
$\sin(\pi - A) = \sin A$

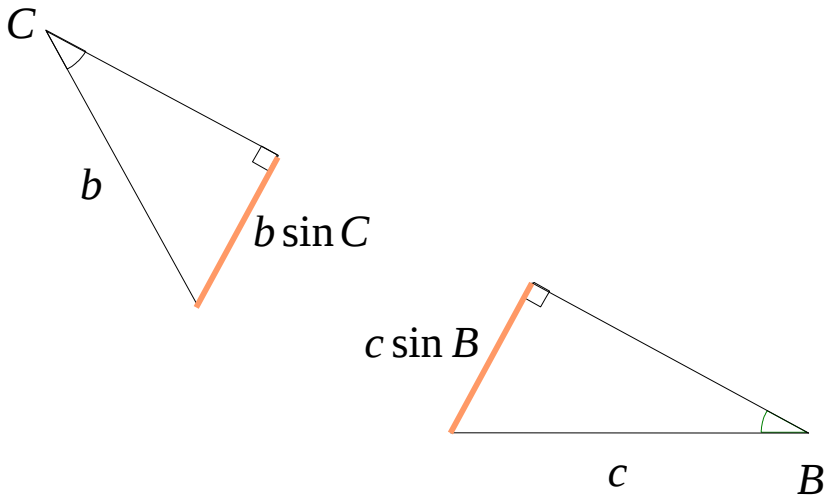
$$c \sin A = a \sin C$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

Law of Sines – one obtuse angle (3)

Case C:



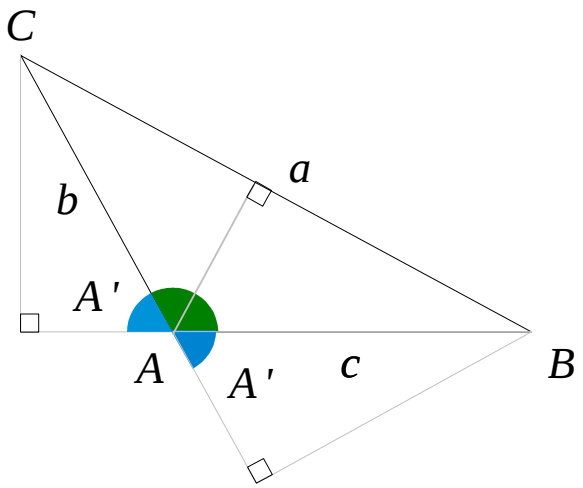


$$b \sin C = c \sin B$$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Sines – one obtuse angle (4)

Summary



Supposing

$$\sin A' = \sin A,$$

$$\sin(\pi - A) = \sin A$$

$$90^\circ < A < 180^\circ$$

$$0^\circ < B < 90^\circ$$

$$0^\circ < C < 90^\circ$$

The Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The Law of Sines - all acute angles (1) ~ (4)

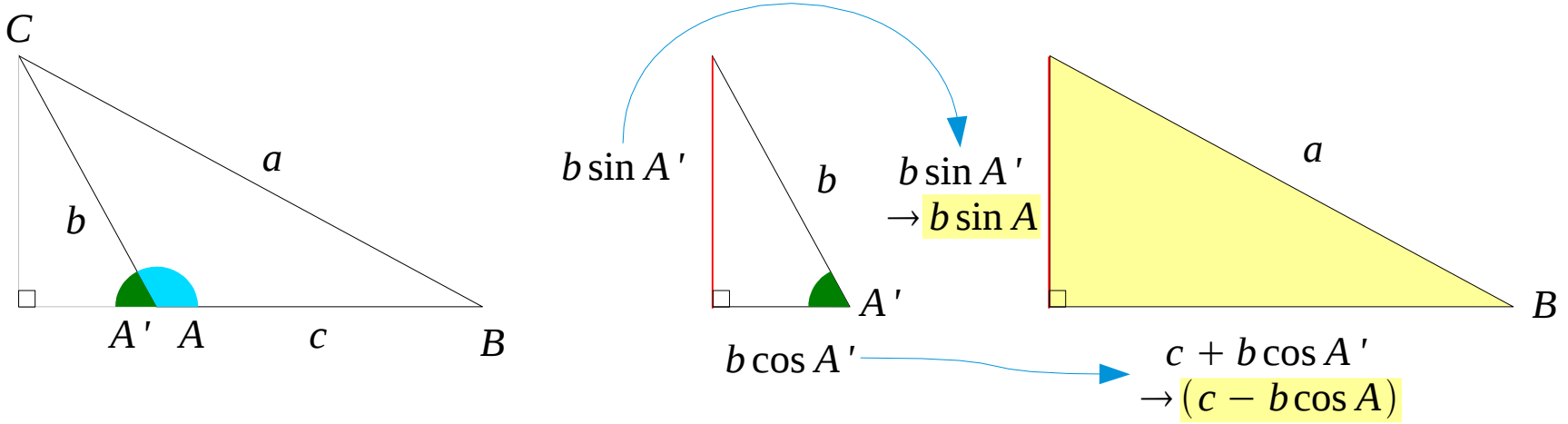
The Law of Cosines - all acute angles (1) ~ (4)

The Law of Sines - one obtuse angle (1) ~ (4)

The Law of Cosines - one obtuse angle (1) ~ (4)

Law of Cosines – one obtuse angle (1)

Case A:



Supposing

$$\sin A' = \sin A$$
$$\cos A' = -\cos A$$

$$\sin(\pi - A) = \sin A$$
$$\cos(\pi - A) = -\cos A$$

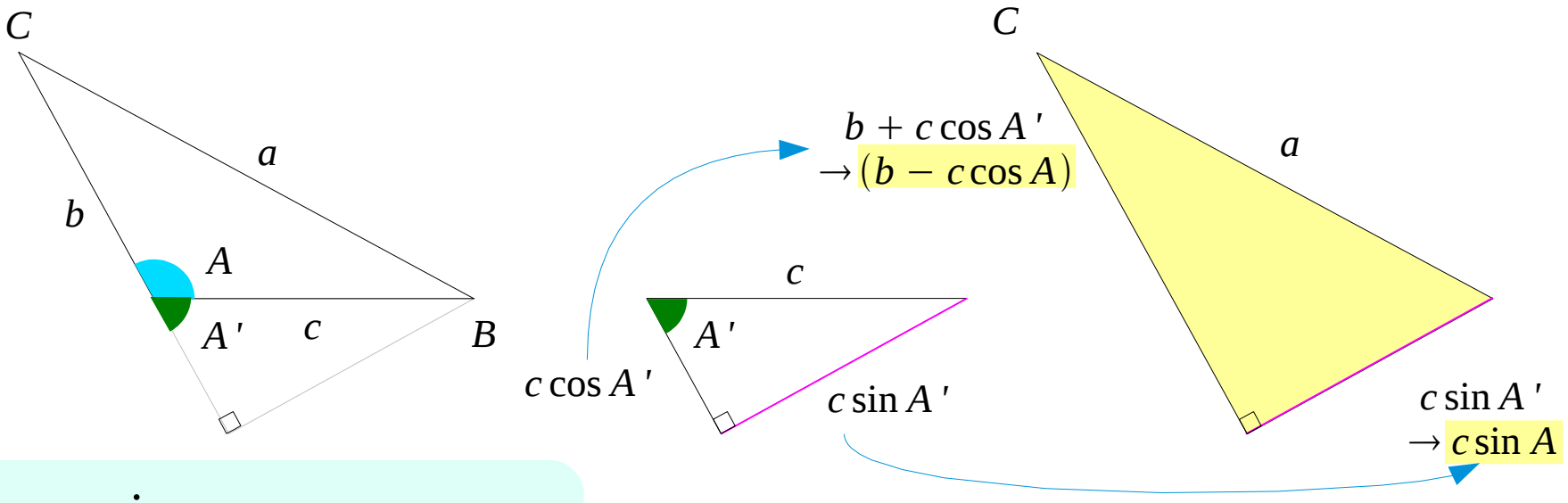
$$a^2 = (b\sin A)^2 + (c - b\cos A)^2$$

$$a^2 = b^2\sin^2 A + c^2 - 2bc\cos A + b^2\cos^2 A$$

$$a^2 = b^2 + c^2 - 2bc\cos C$$

Law of Cosines – one obtuse angle (2)

Case B:



Supposing

$$\sin A' = \sin A$$
$$\cos A' = -\cos A$$

$$\sin(\pi - A) = \sin A$$
$$\cos(\pi - A) = -\cos A$$

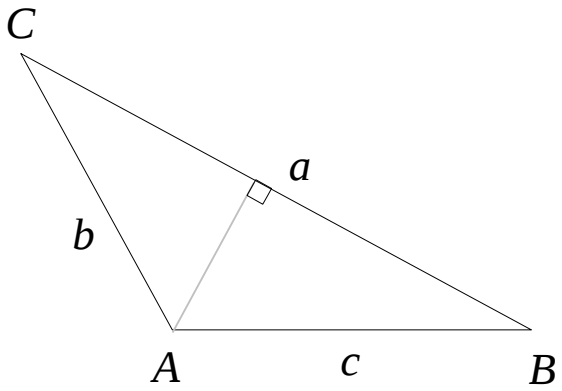
$$a^2 = (c\sin A)^2 + (b - c\cos A)^2$$

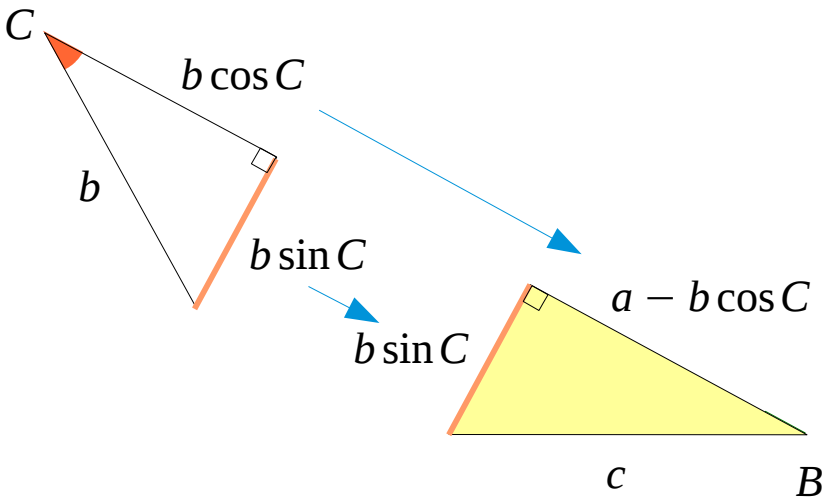
$$a^2 = c^2\sin^2 A + b^2 - 2bc\cos A + c^2\cos^2 A$$

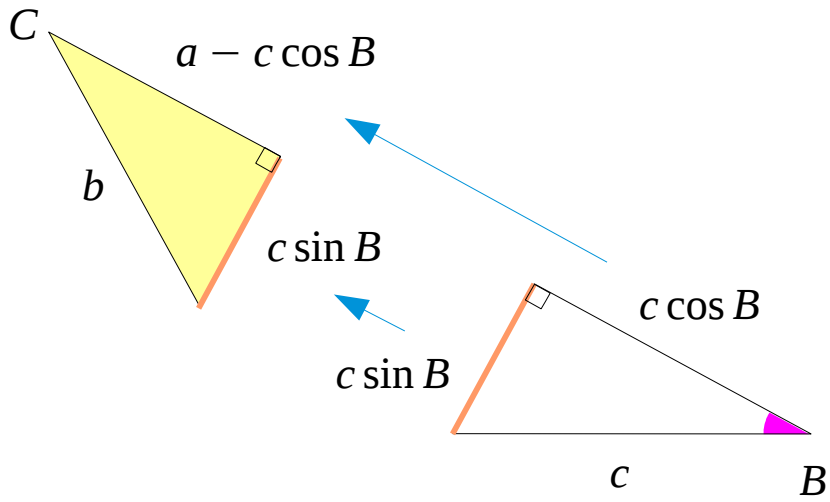
$$a^2 = b^2 + c^2 - 2bc\cos C$$

Law of Sines – one obtuse angle (3)

Case C:







$$b^2 = (c\sin B)^2 + (a - c\cos B)^2$$
$$b^2 = c^2\sin^2 B + a^2 - 2ac\cos B + c^2\cos^2 B$$

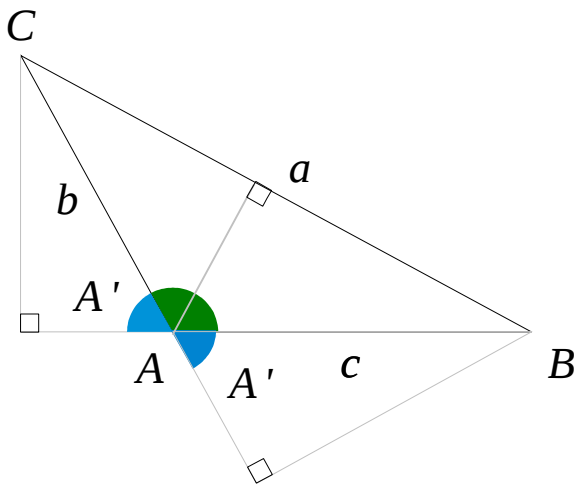
$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = (b\sin C)^2 + (a - b\cos C)^2$$
$$c^2 = b^2\sin^2 C + a^2 - 2ab\cos C + b^2\cos^2 C$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Law of Cosines – one obtuse angle (4)

Summary



$$90^\circ < A < 180^\circ$$

$$0^\circ < B < 90^\circ$$
$$0^\circ < C < 90^\circ$$

Supposing

$$\sin A' = \sin(\pi - A) = \sin A$$

$$\cos A' = \cos(\pi - A) = -\cos A$$

The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = c^2 + a^2 - 2ca\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

References

[1] http://en.wikipedia.org/
[2] http://planetmath.org/
[3] Blitzer, R. “Algebra & Trigonometry.” 3rd ed, Prentice Hall
[4] Smith, R. T., Minton, R. B. “Calculus: Concepts & Connections,” Mc Graw Hill
[5] 홍성대, “기본/실력 수학의 정석,”성지출판