

Reconstructor Spectra (9B)

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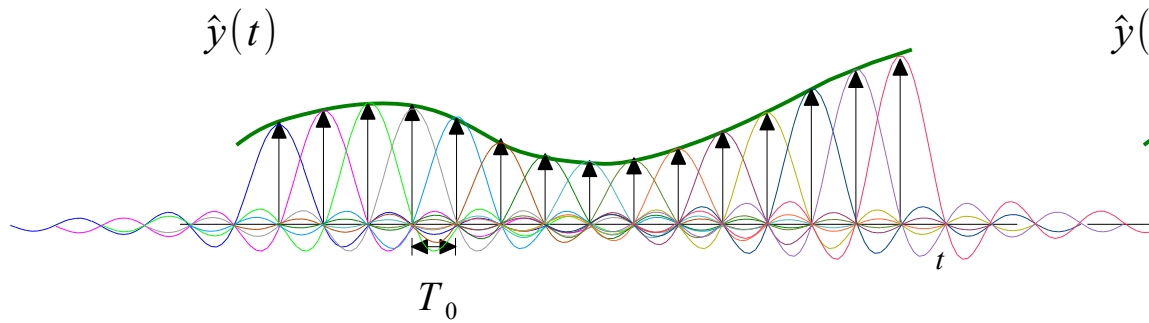
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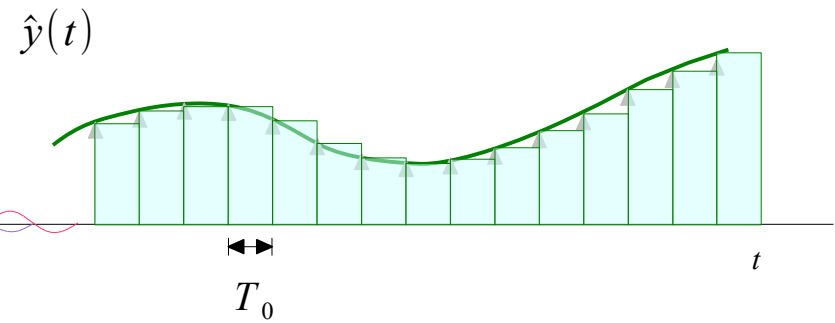
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Reconstructor

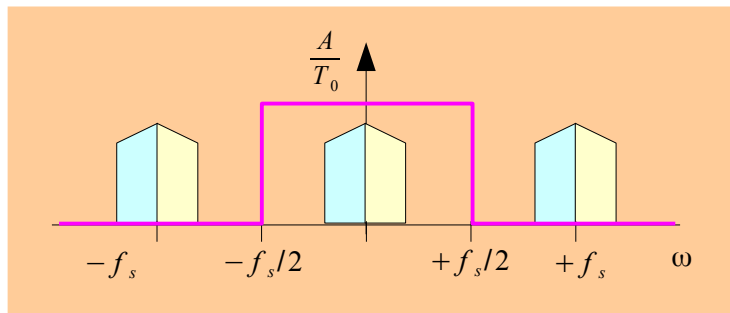
Ideal Reconstructor



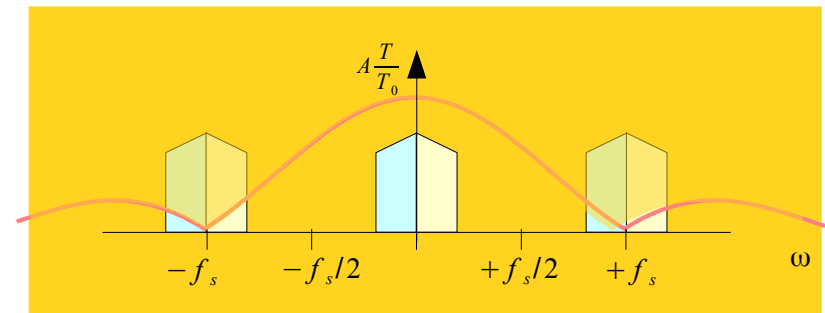
Practical Reconstructor



↓ CTFT

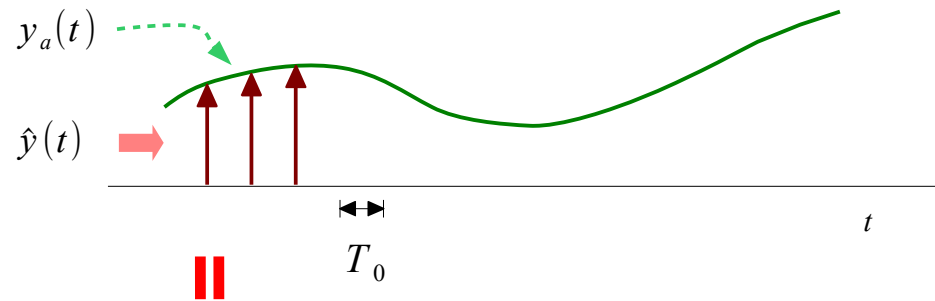


↓ CTFT

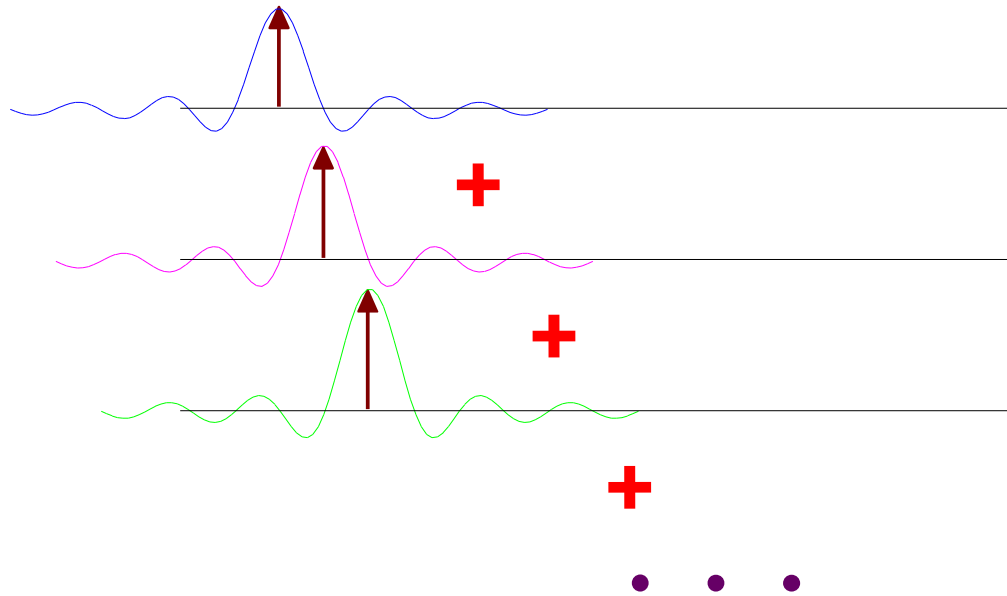


Reconstructor

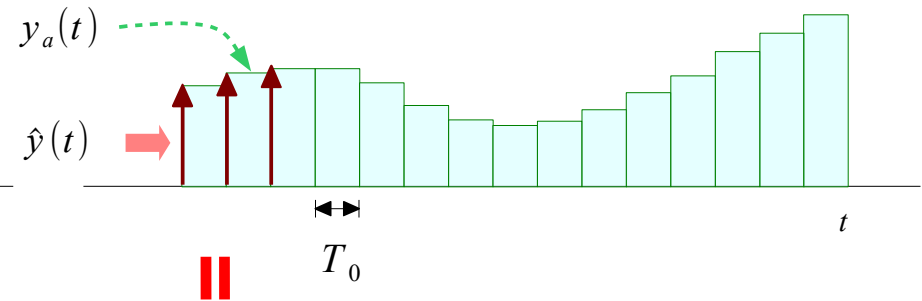
Ideal Reconstructor



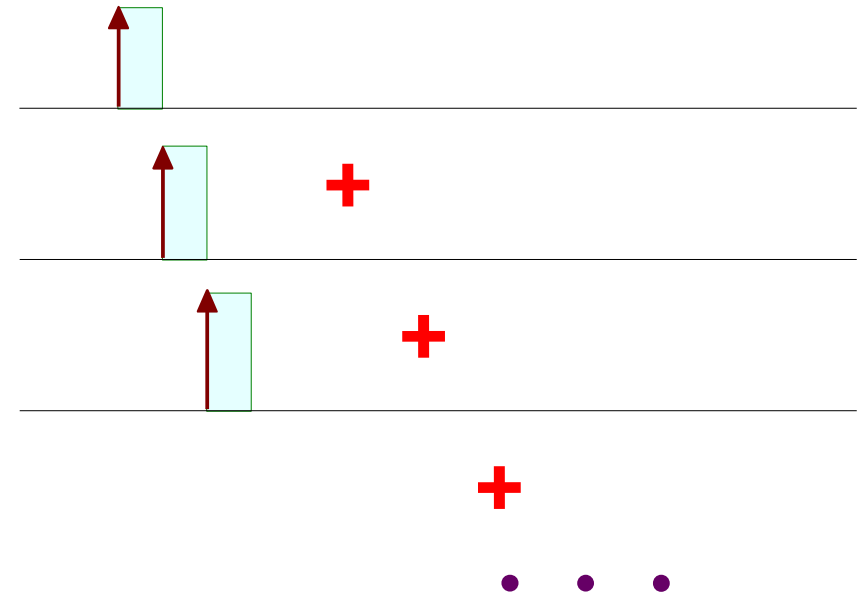
$$y_a(t) = \sum_{n=-\infty}^{\infty} y(nT_0) h(t-nT_0)$$



Practical Reconstructor

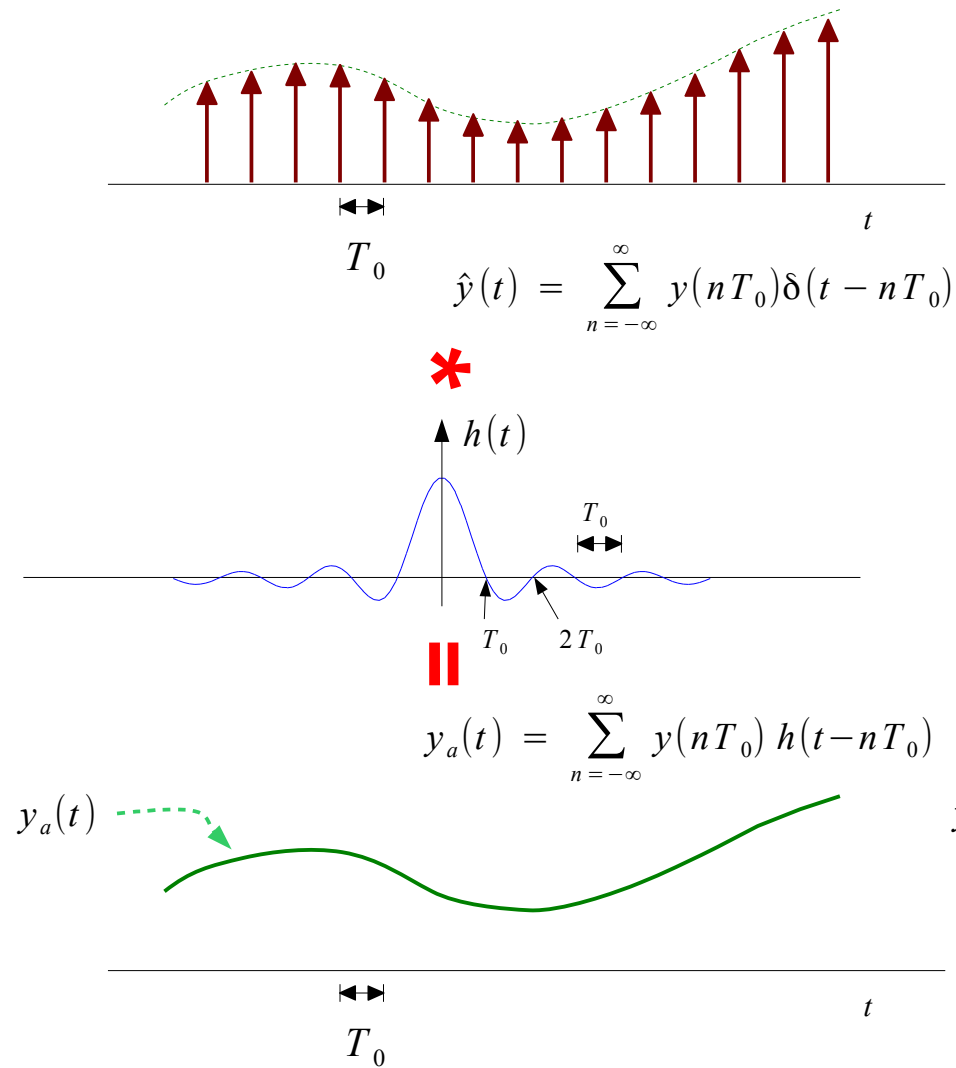


$$y_a(t) = \sum_{n=-\infty}^{\infty} y(nT_0) h(t-nT_0)$$

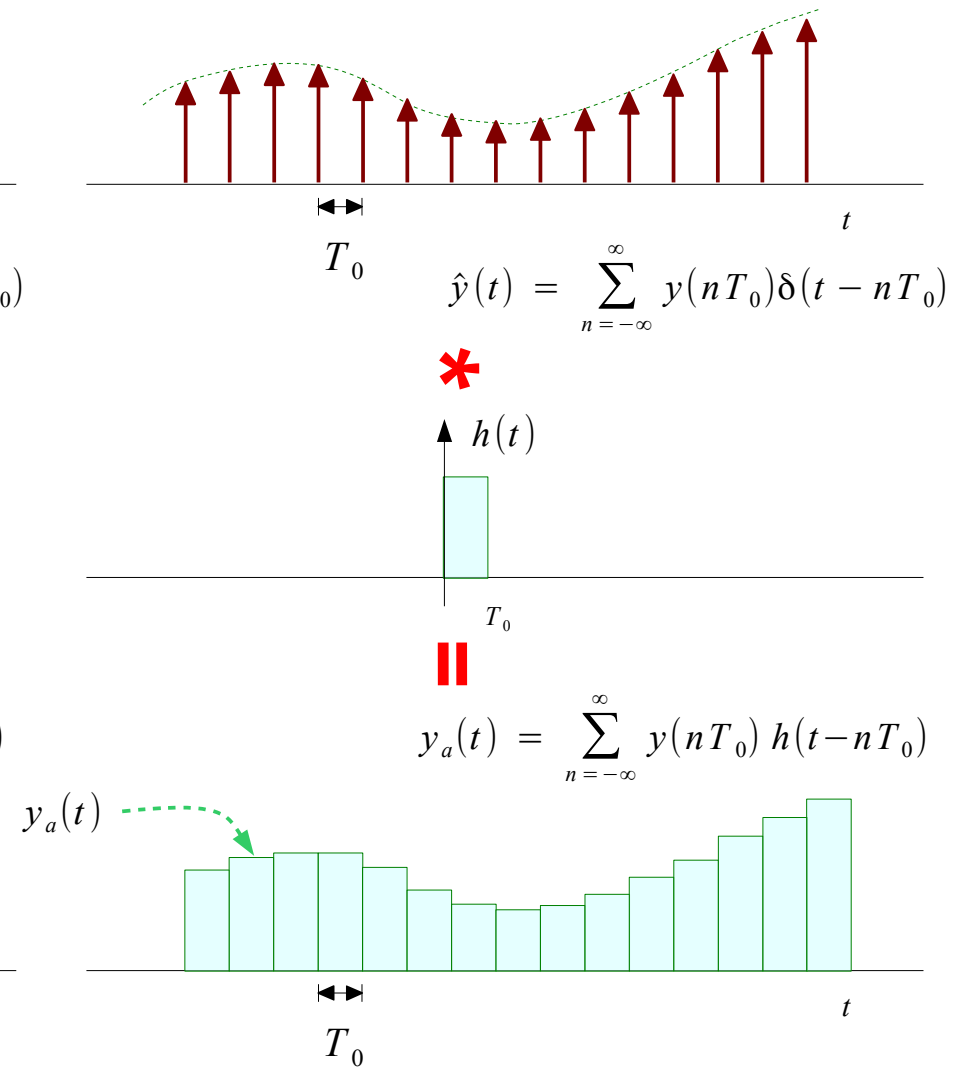


Reconstructor

Ideal Reconstructor



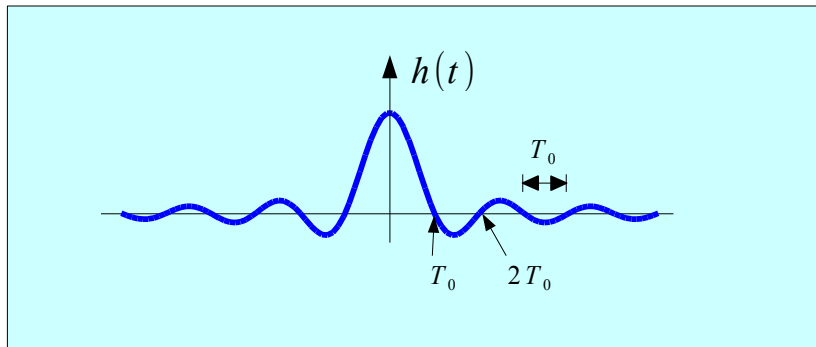
Practical Reconstructor



Reconstructor

Ideal Reconstructor

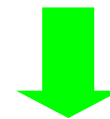
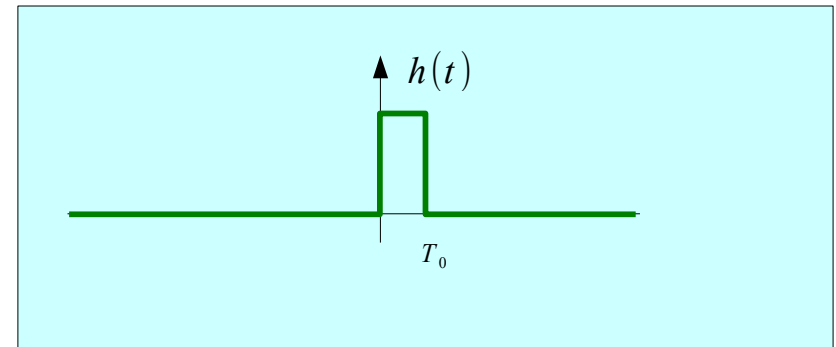
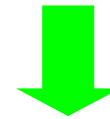
$$\hat{y}(t) = \sum_{n=-\infty}^{\infty} y(nT_0) \delta(t - nT_0)$$



$$y_a(t) = \sum_{n=-\infty}^{\infty} y(nT_0) h(t - nT_0)$$

Practical Reconstructor

$$\hat{y}(t) = \sum_{n=-\infty}^{\infty} y(nT_0) \delta(t - nT_0)$$

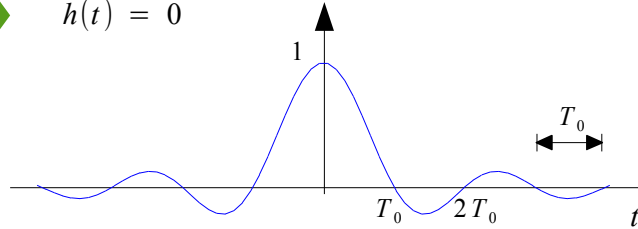


$$y_a(t) = \sum_{n=-\infty}^{\infty} y(nT_0) h(t - nT_0)$$

CTFT of Reconstructors

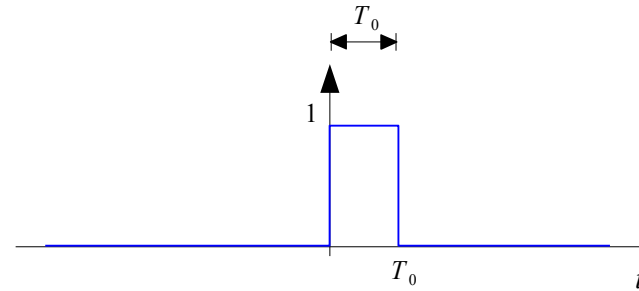
$$t = \pm T_0, \pm 2T_0, \pm 3T_0, \dots$$

$$\rightarrow h(t) = 0$$



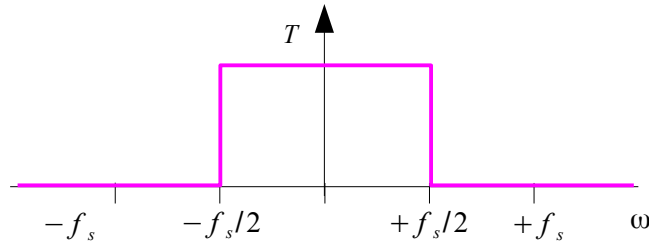
$$h(t) = \frac{\sin(\pi t/T)}{\pi t/T} = \frac{\sin(\pi f_s t)}{\pi f_s t}$$

$$\frac{1}{T_0} \equiv f_s$$



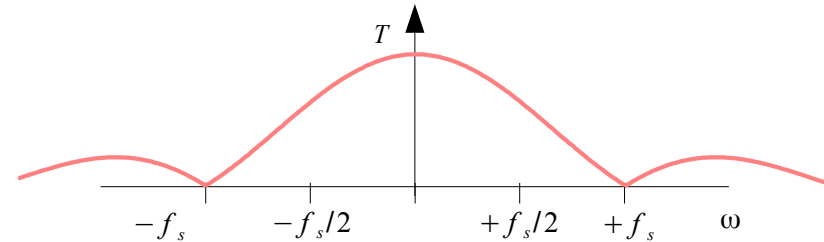
$$h(t) = u(t) - u(t - T_0) = \begin{cases} 1, & 0 \leq t \leq T_0 \\ 0, & \text{otherwise} \end{cases}$$

CTFT



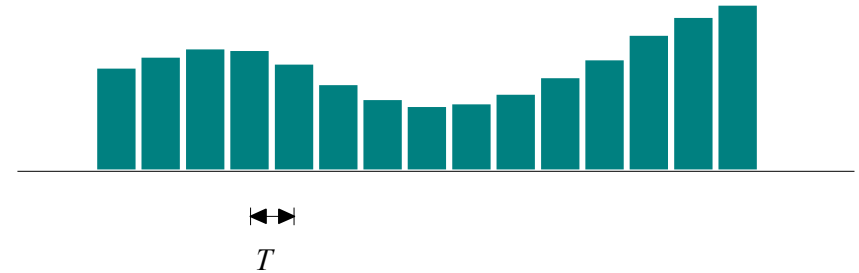
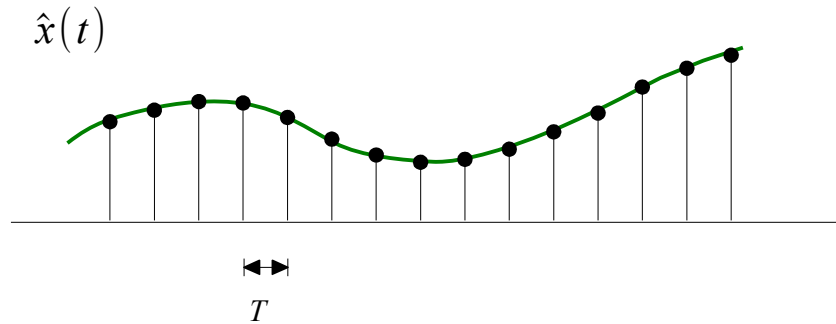
$$H(f) = \begin{cases} T, & |f| \leq f_s/2 \\ 0, & \text{otherwise} \end{cases}$$

CTFT



$$H(f) = T \cdot \frac{\sin(\pi f T)}{\pi f T} e^{-j\pi f T}$$

Analog Reconstructor



$$\hat{y}(t) = \sum_{n=-\infty}^{+\infty} y(nT) \delta(t-nT)$$

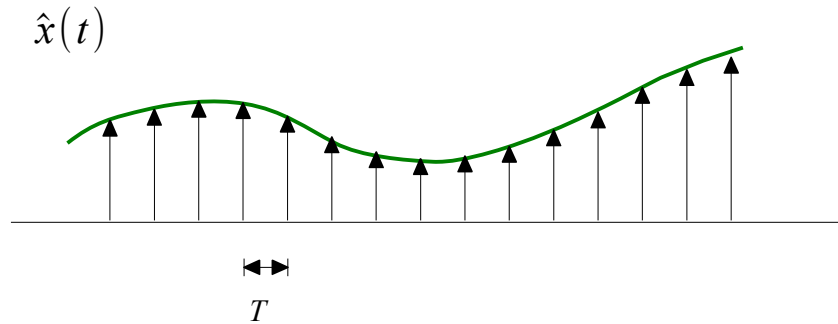
$$Y_a(f) = H(f) \hat{Y}(f)$$

$$y_a(t) = \int_{-\infty}^{+\infty} h(t-t') \hat{y}(t') dt'$$

$$\hat{Y}_a(f) = \frac{1}{T} \sum_{m=-\infty}^{+\infty} Y(f - m f_s)$$

$$y_a(t) = \sum_{n=-\infty}^{+\infty} y(nT) h(t-nT)$$

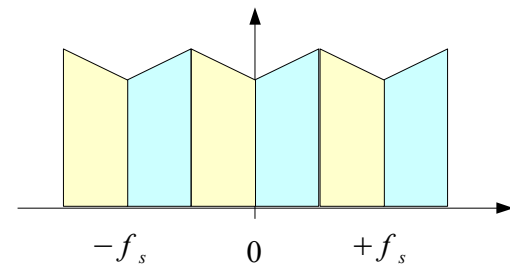
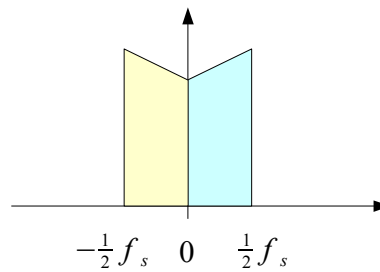
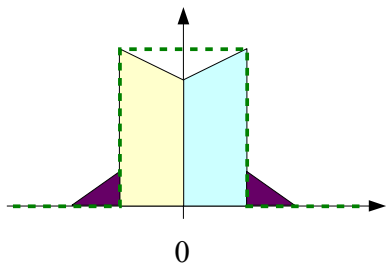
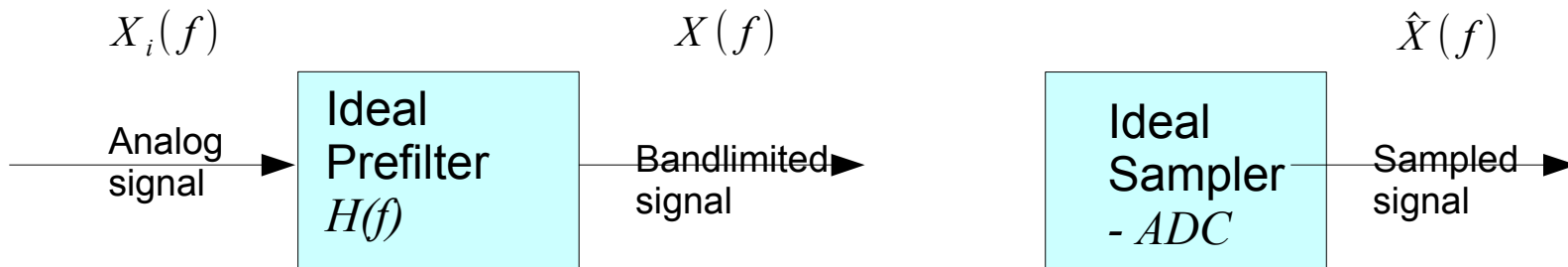
Impulse Response of Ideal Reconstructor



$$\hat{Y}(f) = \frac{1}{T} Y(f) \quad -\frac{f_s}{2} \leq f \leq +\frac{f_s}{2}$$

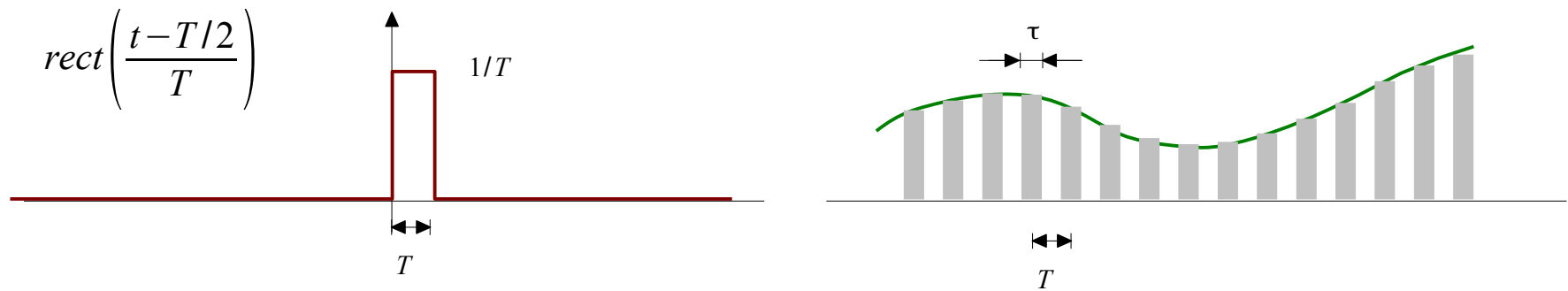
$$y(t) = \sum_{n=-\infty}^{+\infty} y(nT) h(t-nT)$$

$$h(t) = \frac{\sin(\pi t/T)}{\pi t/T} = \frac{\sin(\pi f_s t)}{\pi f_s t}$$

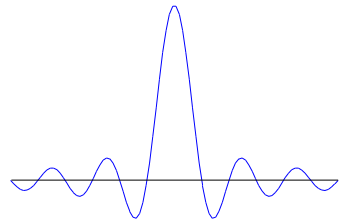


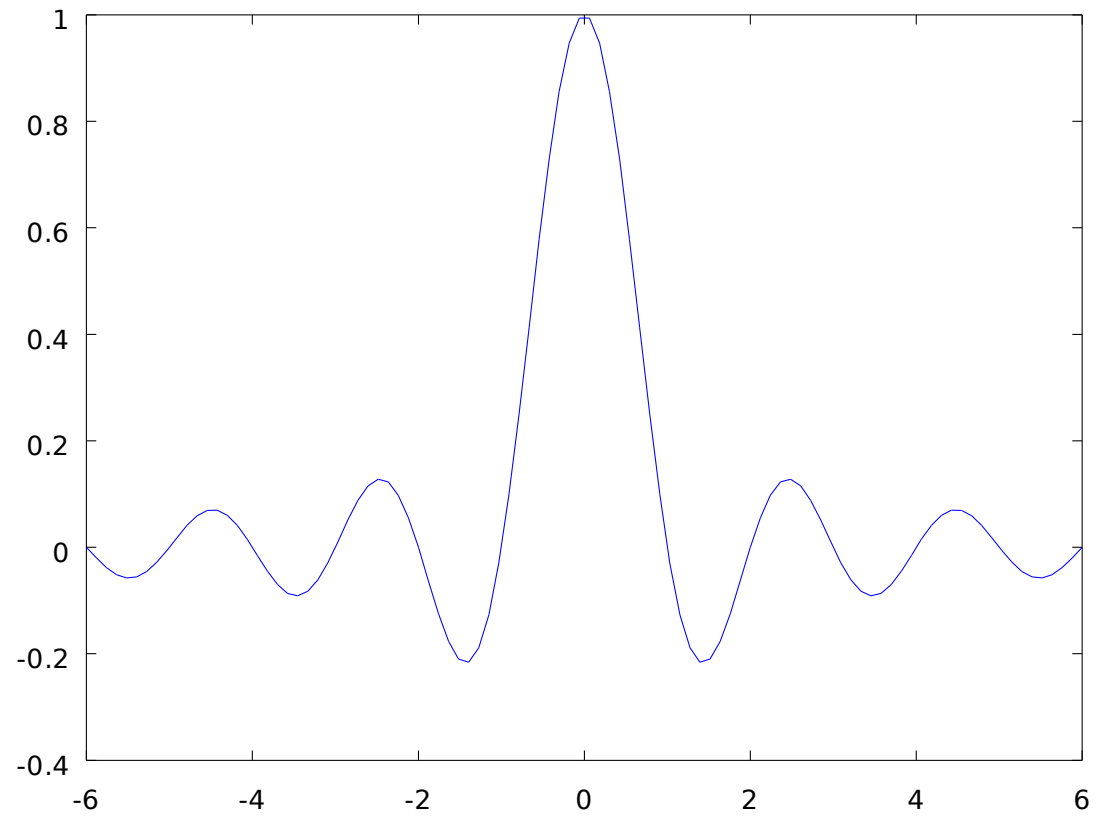
$\frac{2}{4}f_s$ $\frac{3}{4}f_s$ f_s

Zero Order Hold (ZOH)



$$x_{ZOH}(t) = \sum_{n=-\infty}^{+\infty} x[n] \cdot rect\left(\frac{t-T/2-nT}{T}\right)$$





References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] A “graphical interpretation” of the DFT and FFT, by Steve Mann
- [4] R. G. Lyons, Understanding Digital Signal Processing, 1997
- [5] AVR121: Enhancing ADC resolution by oversampling
- [6] S.J. Orfanidis, Introduction to Signal Processing
www.ece.rutgers.edu/~orfanidi/intro2sp