# LMS Overview (1A)

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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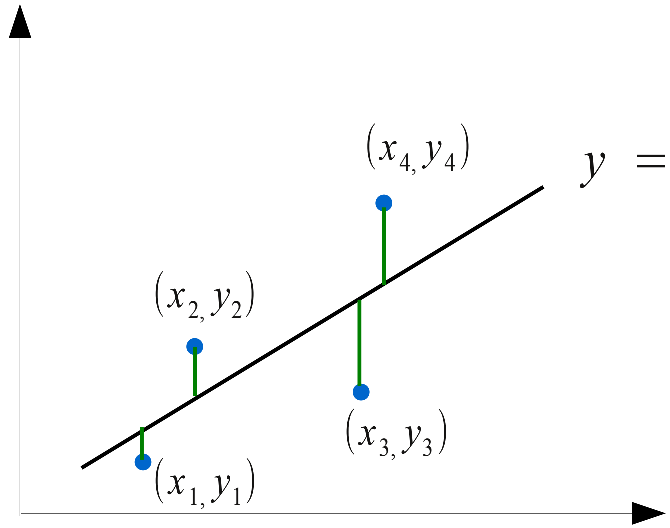



$$y = a_0 + a_1 x$$

$a_0,\ a_1$ *unknowns*

$(x_i,\ y_i)$ *measured data*

*random*

$$S_r = \sum_{i=1}^{n} \epsilon_i^2 = \sum_{i=1}^{n} \left(y_i - (a_0 + a_1 x_i)\right)^2$$

# Linear Regression

*Sum of the square of the residuals*

$$S_r = \sum_{i=1}^{n} \epsilon_i^2 = \sum_{i=1}^{n} (y_i - (a_0 + a_1 x_i))^2$$

$a_{0,}\ a_1$ *unknowns*

$(x_i,\ y_i)$ *measured data*

*random*

*Minimum Condition*

$$\frac{\partial S_r}{\partial a_0} = 2\sum_{i=1}^{n} (y_i - a_0 - a_1 x_i)(-1) = 0$$

$$\sum_{i=1}^{n} a_0 + \sum_{i=1}^{n} a_1 x_i = \sum_{i=1}^{n} y_i$$

$$\frac{\partial S_r}{\partial a_1} = 2\sum_{i=1}^{n} (y_i - a_0 - a_1 x_i)(-x_i) = 0$$

$$\sum_{i=1}^{n} a_0 x_i + \sum_{i=1}^{n} a_1 x_i^2 = \sum_{i=1}^{n} y_i x_i$$

# Linear Regression

$$\sum_{i=1}^{n} a_0 + \sum_{i=1}^{n} a_1 x_i = \sum_{i=1}^{n} y_i$$

$$n \cdot a_0 + \sum_{i=1}^{n} a_1 x_i = \sum_{i=1}^{n} y_i$$

$$a_0 = \frac{1}{n}\sum_{i=1}^{n} y_i - \frac{1}{n}\sum_{i=1}^{n} a_1 x_i$$

$$\sum_{i=1}^{n} a_0 x_i + \sum_{i=1}^{n} a_1 x_i^2 = \sum_{i=1}^{n} y_i x_i$$

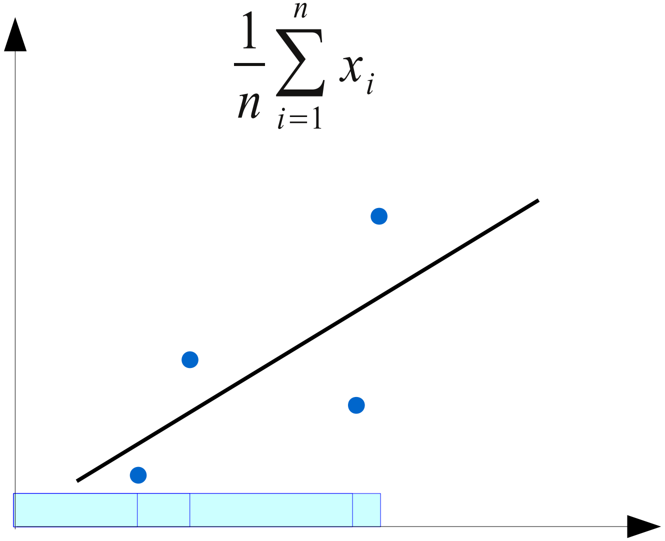
$$\left(\sum_{i=1}^{n} x_i\right)\left(\frac{1}{n}\sum_{i=1}^{n} y_i - \frac{1}{n}\sum_{i=1}^{n} a_1 x_i\right) + \sum_{i=1}^{n} a_1 x_i^2 = \sum_{i=1}^{n} y_i x_i$$

$$\frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right) - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2 a_1 + \left(\sum_{i=1}^{n} x_i^2\right) a_1 = \left(\sum_{i=1}^{n} y_i x_i\right)$$

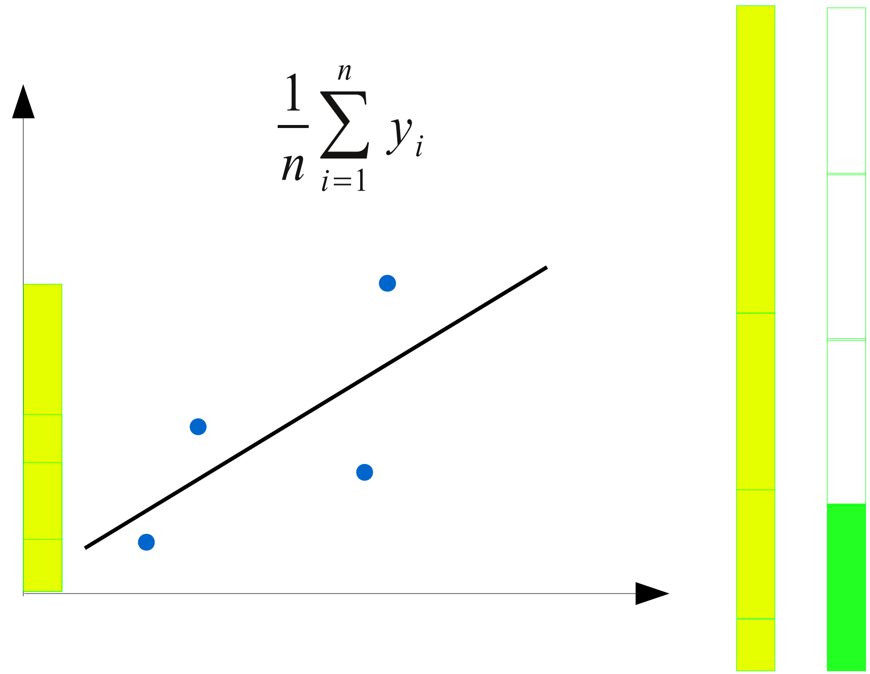
$$n\left(\sum_{i=1}^{n} x_i^2\right) a_1 - \left(\sum_{i=1}^{n} x_i\right)^2 a_1 = n\left(\sum_{i=1}^{n} y_i x_i\right) - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)$$

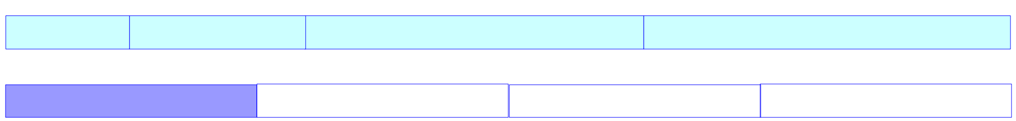
$$a_1 = \frac{n\left(\sum_{i=1}^{n} y_i x_i\right) - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n\left(\sum_{i=1}^{n} x_i^2\right) - \left(\sum_{i=1}^{n} x_i\right)^2}$$

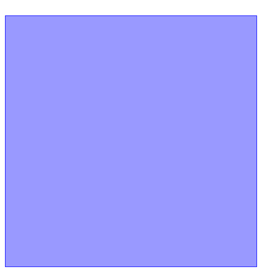
$$\frac{1}{n}\sum_{i=1}^{n} x_i$$



$$\frac{1}{n}\sum_{i=1}^{n} y_i$$

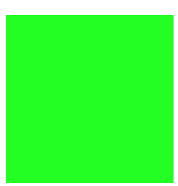


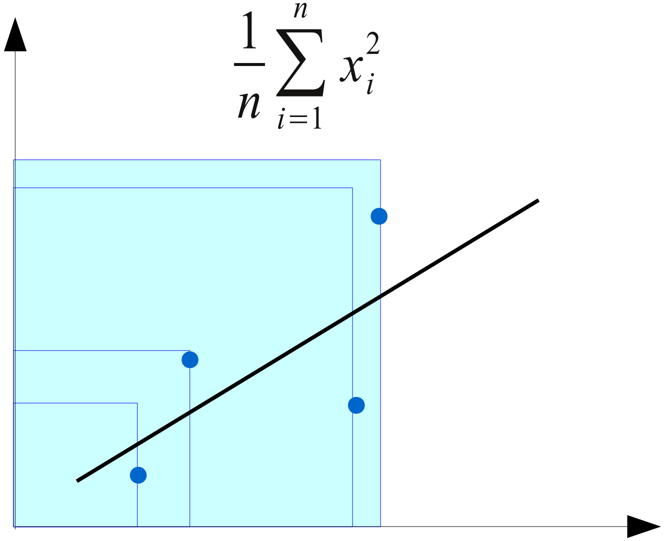




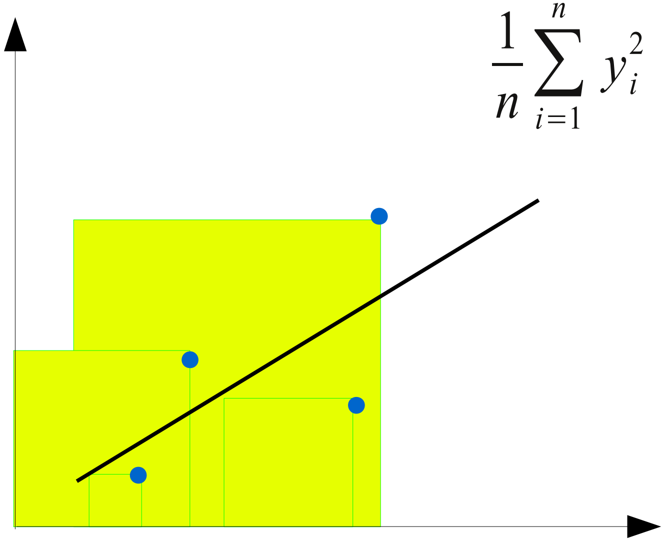
$$\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2$$

$$\left(\frac{1}{n}\sum_{i=1}^{n} y_i\right)^2$$

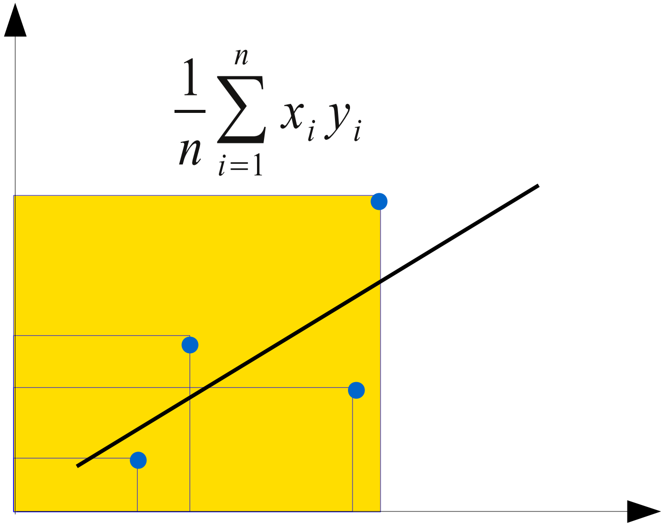




$$\frac{1}{n}\sum_{i=1}^{n} x_i^2$$



$$\frac{1}{n}\sum_{i=1}^{n} y_i^2$$



$$\frac{1}{n}\sum_{i=1}^{n} x_i y_i$$

# Non-Linear Regression

*Sum of the square of the residuals*

$$S_r = \sum_{i=1}^{n} \epsilon_i^2 = \sum_{i=1}^{n} \left(y_i - \left(a_0 + a_1 x_i + a_2 x_i^2\right)\right)^2$$

$a_{0,}\ \ a_1$ *unknowns*

$(x_i,\ y_i)$ *measured data*

*random*

*Minimum Condition*

$$\frac{\partial S_r}{\partial a_0} = 2 \sum_{i=1}^{n} \left(y_i - a_0 - a_1 x_i - a_2 x_i^2\right)(-1) = 0$$

$$\frac{\partial S_r}{\partial a_1} = 2 \sum_{i=1}^{n} \left(y_i - a_0 - a_1 x_i - a_2 x_i^2\right)(-x_i) = 0$$

$$\frac{\partial S_r}{\partial a_2} = 2 \sum_{i=1}^{n} \left(y_i - a_0 - a_1 x_i - a_2 x_i^2\right)(-x_i^2) = 0$$

# Non-Linear Regression

$$\left(\sum_{i=1}^{n} 1\right) \cdot a_0 + \left(\sum_{i=1}^{n} x_i\right) \cdot a_1 + \left(\sum_{i=1}^{n} x_i^2\right) \cdot a_2 = \left(\sum_{i=1}^{n} y_i\right)$$

$$\left(\sum_{i=1}^{n} x_i\right) \cdot a_0 + \left(\sum_{i=1}^{n} x_i^2\right) \cdot a_1 + \left(\sum_{i=1}^{n} x_i^3\right) \cdot a_2 = \left(\sum_{i=1}^{n} x_i y_i\right)$$

$$\left(\sum_{i=1}^{n} x_i^2\right) \cdot a_0 + \left(\sum_{i=1}^{n} x_i^3\right) \cdot a_1 + \left(\sum_{i=1}^{n} x_i^4\right) \cdot a_2 = \left(\sum_{i=1}^{n} x_i^2 y_i\right)$$

# Multivariate Regression

*Sum of the square of the residuals*

$$S_r = \sum_{i=1}^{n} \epsilon_i^2 = \sum_{i=1}^{n} \left(y_i - \left(a_0 + a_1 x_{\mathbf{1},i} + a_2 x_{\mathbf{2},i}\right)\right)^2$$

$a_{0,}\ a_1$ unknowns

$(x_i,\ y_i)$ measured data

random

*Minimum Condition*

$$\frac{\partial S_r}{\partial a_0} = 2 \sum_{i=1}^{n} \left(y_i - a_0 - a_1 x_{\mathbf{1},i} - a_2 x_{\mathbf{2},i}\right)(-1) = 0$$

$$\frac{\partial S_r}{\partial a_1} = 2 \sum_{i=1}^{n} \left(y_i - a_0 - a_1 x_{\mathbf{1},i} - a_2 x_{\mathbf{2},i}\right)(-x_{\mathbf{1},i}) = 0$$

$$\frac{\partial S_r}{\partial a_2} = 2 \sum_{i=1}^{n} \left(y_i - a_0 - a_1 x_{\mathbf{1},i} - a_2 x_{\mathbf{2},i}\right)(-x_{\mathbf{2},i}) = 0$$

# Multivariate Regression

$$\left(\sum_{i=1}^{n} 1\right) \cdot a_0 + \left(\sum_{i=1}^{n} x_{\mathbf{1},i}\right) \cdot a_1 + \left(\sum_{i=1}^{n} x_{\mathbf{2},i}\right) \cdot a_2 = \left(\sum_{i=1}^{n} y_i\right)$$

$$\left(\sum_{i=1}^{n} x_{\mathbf{1},i}\right) \cdot a_0 + \left(\sum_{i=1}^{n} x_{\mathbf{1},i}^2\right) \cdot a_1 + \left(\sum_{i=1}^{n} x_{\mathbf{1},i} x_{\mathbf{2},i}\right) \cdot a_2 = \left(\sum_{i=1}^{n} x_{\mathbf{1},i} y_i\right)$$

$$\left(\sum_{i=1}^{n} x_{\mathbf{2},i}\right) \cdot a_0 + \left(\sum_{i=1}^{n} x_{\mathbf{1},i} x_{\mathbf{2},i}\right) \cdot a_1 + \left(\sum_{i=1}^{n} x_{\mathbf{2},i}^2\right) \cdot a_2 = \left(\sum_{i=1}^{n} x_{\mathbf{2},i} y_i\right)$$

## References

[1] http://en.wikipedia.org/
[2] http://numericalmethods.eng.usf.edu/
[3] S.C. Chapra, Applied Numerical Methods W/ml Engineering And Science