LMS Overview (1A)

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Linear Regression

Sum of the square of the residuals

$$S_r = \sum_{i=1}^{n} \epsilon_i^2 = \sum_{i=1}^{n} (y_i - (a_0 + a_1 x_i))^2$$

$a_0, \; a_1$ *unknowns*

$(x_i, \; y_i)$ measured data

random

Minimum Condition

$$\frac{\partial S_r}{\partial a_0} = 2 \sum_{i=1}^{n} (y_i - a_0 - a_1 x_i)(-1) = 0$$

$$\Rightarrow \quad \sum_{i=1}^{n} a_0 + \sum_{i=1}^{n} a_1 x_i = \sum_{i=1}^{n} y_i$$

$$\frac{\partial S_r}{\partial a_1} = 2 \sum_{i=1}^{n} (y_i - a_0 - a_1 x_i)(-x_i) = 0$$

$$\Rightarrow \quad \sum_{i=1}^{n} a_0 x_i + \sum_{i=1}^{n} a_1 x_i^2 = \sum_{i=1}^{n} y_i x_i$$

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Linear Regression

$$\sum_{i=1}^{n} a_0 + \sum_{i=1}^{n} a_1 x_i = \sum_{i=1}^{n} y_i$$

$$\sum_{i=1}^{n} a_0 x_i + \sum_{i=1}^{n} a_1 x_i^2 = \sum_{i=1}^{n} y_i x_i$$

$$n \cdot a_0 + \sum_{i=1}^{n} a_1 x_i = \sum_{i=1}^{n} y_i$$

$$\left(\sum_{i=1}^{n} x_i\right)\left(\frac{1}{n}\sum_{i=1}^{n} y_i - \frac{1}{n}\sum_{i=1}^{n} a_1 x_i\right) + \sum_{i=1}^{n} a_1 x_i^2 = \sum_{i=1}^{n} y_i x_i$$

$$a_0 = \frac{1}{n}\sum_{i=1}^{n} y_i - \frac{1}{n}\sum_{i=1}^{n} a_1 x_i$$

$$\frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right) - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2 a_1 + \left(\sum_{i=1}^{n} x_i^2\right) a_1 = \left(\sum_{i=1}^{n} y_i x_i\right)$$

$$n\left(\sum_{i=1}^{n} x_i^2\right) a_1 - \left(\sum_{i=1}^{n} x_i\right)^2 a_1 = n\left(\sum_{i=1}^{n} y_i x_i\right) - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)$$

$$a_1 = \frac{n\left(\sum_{i=1}^{n} y_i x_i\right) - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n\left(\sum_{i=1}^{n} x_i^2\right) - \left(\sum_{i=1}^{n} x_i\right)^2}$$

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References

[1] http://en.wikipedia.org/
[2] http://numericalmethods.eng.usf.edu/
[3]