General CORDIC Description (1A)

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CORDIC Background

1. CORDIC FAQ, G. R. Griffin, www.dspguru.com/info/faqs/cordic2.htm

Complex Multiplication

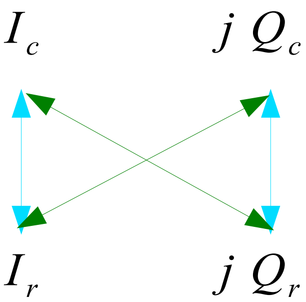
Given Complex Value $C = I_c + j\,Q_c$

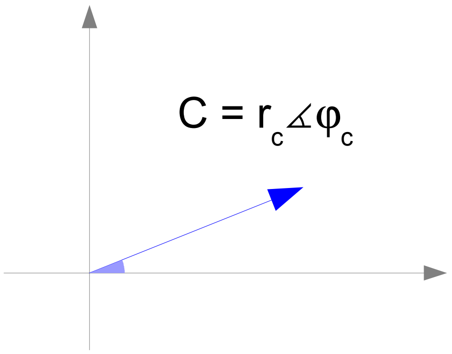
Rotation Value $R = I_r + j\,Q_r$

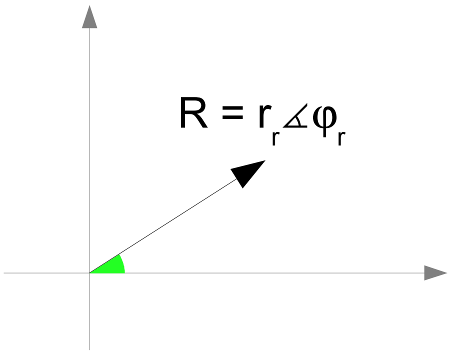
Rotated Complex Value $C' = I_c' + j\,Q_c'$

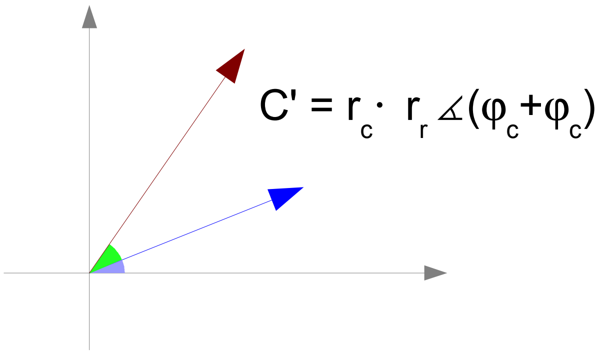
$$C' = C \cdot R$$

$$\begin{aligned} I_c' + j\,Q_c' &= (I_c + j\,Q_c)\cdot(I_r + j\,Q_r) \\ &= (I_c I_r - Q_c Q_r) + j\,(Q_c I_r + I_c Q_r) \end{aligned}$$





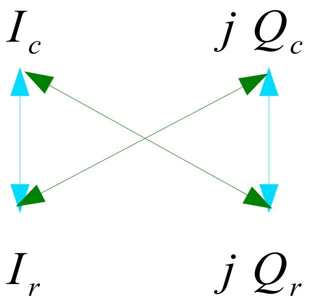




Adding / Subtracting Phase

To **add** R' phase to C

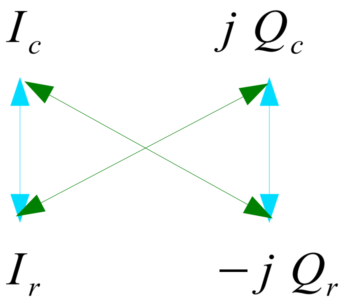
$$C' = C \cdot R$$



$$\begin{aligned} I_c' + j\,Q_c' &= (I_c + j\,Q_c) \cdot (I_r + j\,Q_r) \\ &= (I_c I_r - Q_c Q_r) + j\,(Q_c I_r + I_c Q_r) \end{aligned}$$

To **sub** R' phase to C

$$C' = C \cdot R^*$$

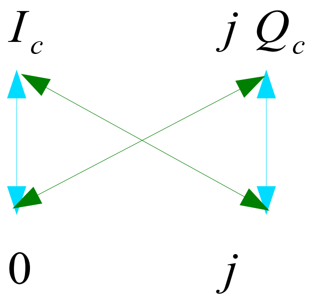


$$\begin{aligned} I_c' + j\,Q_c' &= (I_c + j\,Q_c) \cdot (I_r - j\,Q_r) \\ &= (I_c I_r + Q_c Q_r) + j\,(Q_c I_r - I_c Q_r) \end{aligned}$$

Adding / Subtracting 90 Degrees

To add R' phase to C

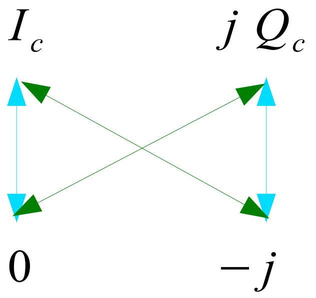
$$C' = C \cdot (+j)$$



$$I_c' + j\,Q_c' = (I_c + j\,Q_c) \cdot (0 + j)$$
$$= (-Q_c) + j\,(I_c)$$

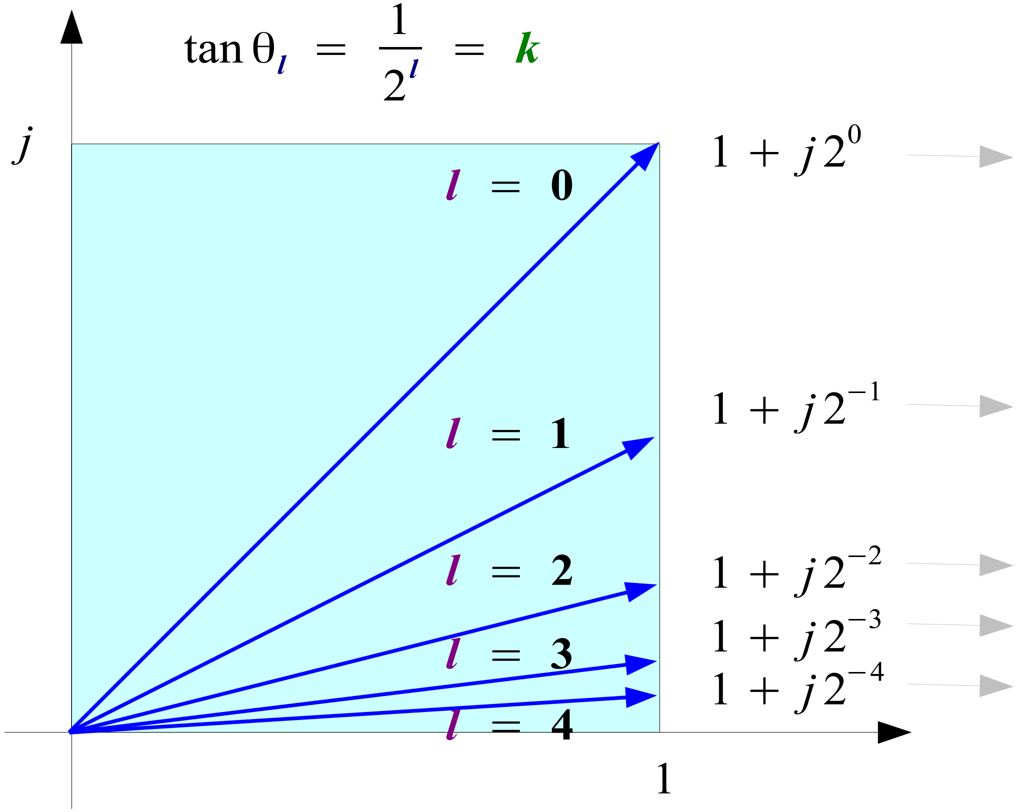
To sub R' phase to C

$$C' = C \cdot (-j)$$



$$I_c' + j\,Q_c' = (I_c + j\,Q_c) \cdot (0 - j)$$
$$= (Q_c) + j\,(-I_c)$$

Elementary Angle: $\tan^{-1}(k)$



$$\theta_l = \tan^{-1}(2^{-l}) = \tan^{-1}(k)$$

$$\theta_0 = \tan^{-1}(2^0) = 45.00000^\circ$$

$$\theta_1 = \tan^{-1}(2^{-1}) = 26.56505^\circ$$

$$\theta_2 = \tan^{-1}(2^{-2}) = 14.03624^\circ$$

$$\theta_3 = \tan^{-1}(2^{-3}) = 7.12502^\circ$$

$$\theta_4 = \tan^{-1}(2^{-4}) = 3.57633^\circ$$

Represent arbitrary angle θ

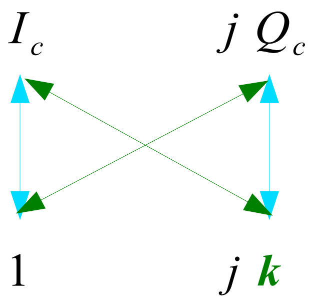
in terms of $\pm\theta_{0,}\ \pm\theta_{1,}\ \pm\theta_{2,}\ \pm\theta_{3,}\ \cdots,\ \pm\theta_l,\ \cdots$

$$\left(k = \tan\theta_i = \frac{1}{2^l},\quad l = 0, 1, 2, \cdots\right)$$

Adding / Subtracting $\tan^{-1}(k)$

To add R' phase to C

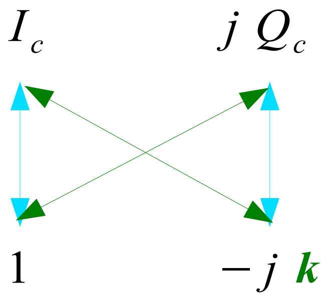
$$C' = C \cdot (1 + j k)$$



$$\begin{aligned} I_c' + j\,Q_c' &= (I_c + j\,Q_c) \cdot \boxed{(1 + j\,k)} \\ &= (I_c - k\,Q_c) + j\,(Q_c + k\,I_c) \\ &= (I_c - 2^{-l}Q_c) + j\,(Q_c + 2^{-l}I_c) \end{aligned}$$

To sub R' phase to C

$$C' = C \cdot (1 - j k)$$



$$\begin{aligned} I_c' + j\,Q_c' &= (I_c + j\,Q_c) \cdot \boxed{(1 - j\,k)} \\ &= (I_c + k\,Q_c) + j\,(Q_c - k\,I_c) \\ &= (I_c + 2^{-l}Q_c) + j\,(Q_c - 2^{-l}I_c) \end{aligned}$$

$$\theta_l = \tan^{-1}(k) = \tan^{-1}(2^{-l})$$

$$k = \frac{1}{2^l}, \quad l = 0, 1, 2, \cdots$$

Phase and Magnitude of 1 + jk (1)

Cumulative Magnitude

L	$k = \frac{1}{2^l}$	$R = 1 + jk$	*Phase of R*	*Magnitude of R*	***CORDIC Gain***
0	1.0	1 + j1.0	45°	1.41421356	1.414213562
1	0.5	1 + j0.5	26.56505°	1.11803399	1.581138830
2	0.25	1 + j0.25	14.03624°	1.03077641	1.629800601
3	0.125	1 + j0.125	7.12502°	1.00778222	1.642484066
4	0.0625	1 + j0.0625	3.57633°	1.00195122	1.645688916
5	0.03125	1 + j0.03125	1.78991°	1.00048816	1.646492279
6	0.015625	1 + j0.015625	0.89517°	1.00012206	1.646693254
7	0.007813	1 + j0.007813	0.44761°	1.00003052	1.646743507
•••	•••	•••	•••	•••	•••
					1.647 ⟵

$$R = 1 + jk \xrightarrow{k = 1/2^l \qquad l = 0, 1, 2, \cdots} |R| = \sqrt{1^2 + k^2}$$

> 1.0

Phase and Magnitude of 1 + j*k* (2)

Represent arbitrary angle θ

in terms of $\pm\theta_0, \ \pm\theta_1, \ \pm\theta_2, \ \pm\theta_3, \ \cdots, \ \pm\theta_l, \ \cdots$ $\left(k = \tan\theta_i = \frac{1}{2^l}, \quad l = 0, 1, 2, \cdots\right)$

Binary Search → Shift and Add → No multiplier

Phase of R

$\theta_0 = \tan^{-1}(2^0) =$ 45°

$\theta_1 = \tan^{-1}(2^{-1}) =$ 26.56505°

$\theta_2 = \tan^{-1}(2^{-2}) =$ 14.03624°

$\theta_3 = \tan^{-1}(2^{-3}) =$ 7.12502°

$\theta_4 = \tan^{-1}(2^{-4}) =$ 3.57633°

$\theta_5 = \tan^{-1}(2^{-5}) =$ 1.78991°

$\theta_6 = \tan^{-1}(2^{-6}) =$ 0.89517°

$\theta_7 = \tan^{-1}(2^{-7}) =$ 0.44761°

•••

$\approx\frac{1}{2}$ (between each successive pair of values)

> 92°

$\theta \in$ [-180, +180]

↓ $R = 0 \pm j$

$\theta \in$ [-90, +90]

$R = 1 \pm j\,k$

4 or more rotations

45° + 26.56505° + 14.03624° + 7.12502° > 92°

Phase and Magnitude of 1 + j*k* (3)

Iteration

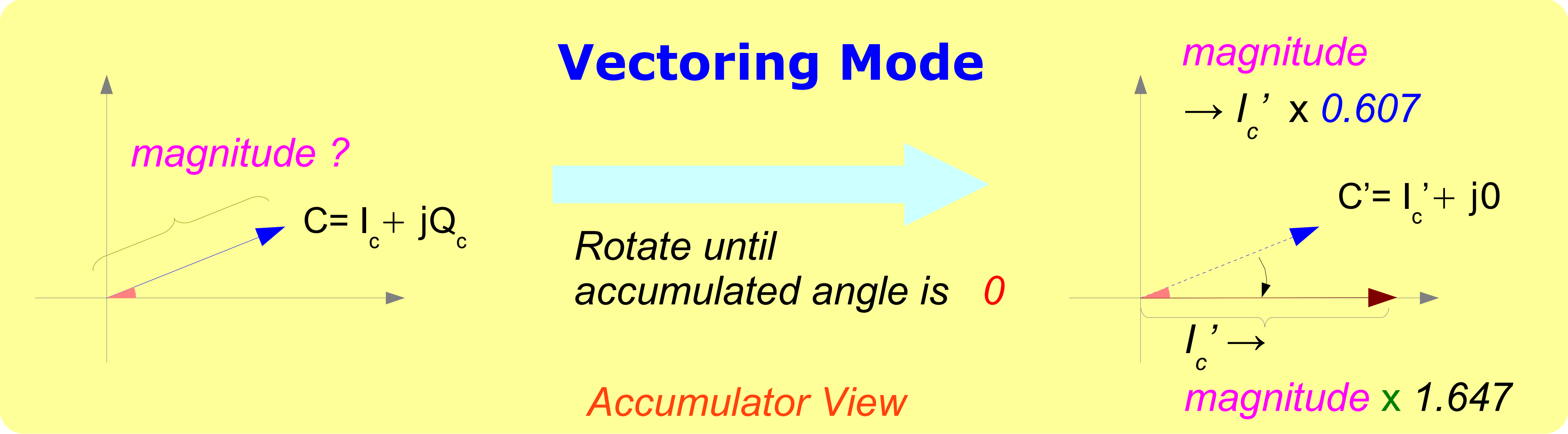
$R = 1 \pm j\,k$	*Magnitude of R*	$\sqrt{1^2 + k^2}$ > 1.0	*Cumulative Magnitude* *CORDIC Gain*
$R_0 = 1 \pm j\,(1/2^0)$	$\sqrt{1^2 + 1^2} =$	1.41421356	1.414213562
$R_1 = 1 \pm j\,(1/2^1)$	$\sqrt{1^2 + (1/2)^2} =$	1.11803399	1.581138830
$R_2 = 1 \pm j\,(1/2^2)$	$\sqrt{1^2 + (1/2^2)^2} =$	1.03077641	1.629800601
$R_3 = 1 \pm j\,(1/2^3)$	$\sqrt{1^2 + (1/2^3)^2} =$	1.00778222	1.642484066
$R_4 = 1 \pm j\,(1/2^4)$	$\sqrt{1^2 + (1/2^4)^2} =$	1.00195122	1.645688916
$R_5 = 1 \pm j\,(1/2^5)$	$\sqrt{1^2 + (1/2^5)^2} =$	1.00048816	1.646492279
$R_6 = 1 \pm j\,(1/2^6)$	$\sqrt{1^2 + (1/2^6)^2} =$	1.00012206	1.646693254
$R_7 = 1 \pm j\,(1/2^7)$	$\sqrt{1^2 + (1/2^7)^2} =$	1.00003052	1.646743507
		•••	••• 1.647

The actual CORDIC Gain
depends on the number of iterations

For each iteration, the corresponding θ_i
is always used (either $+\theta_i$ *or* $-\theta_i$ *)*

The magnitude is growing →
for correction, multiply by 0.6073 = 1 / 1.647

Calculating Magnitude (1)



Vectoring Mode

magnitude ?

$C = I_c + jQ_c$

Rotate until accumulated angle is 0

Accumulator View

magnitude

$\rightarrow I_c' \times 0.607$

$C' = I_c' + j0$

$I_c' \rightarrow$

magnitude x 1.647

Each iteration, the magnitude is increased by $\sqrt{1^2 + k^2}$

CORDIC Gain (cumulative gain)

$$\simeq 1.647 \; = \; 0.607^{-1}$$

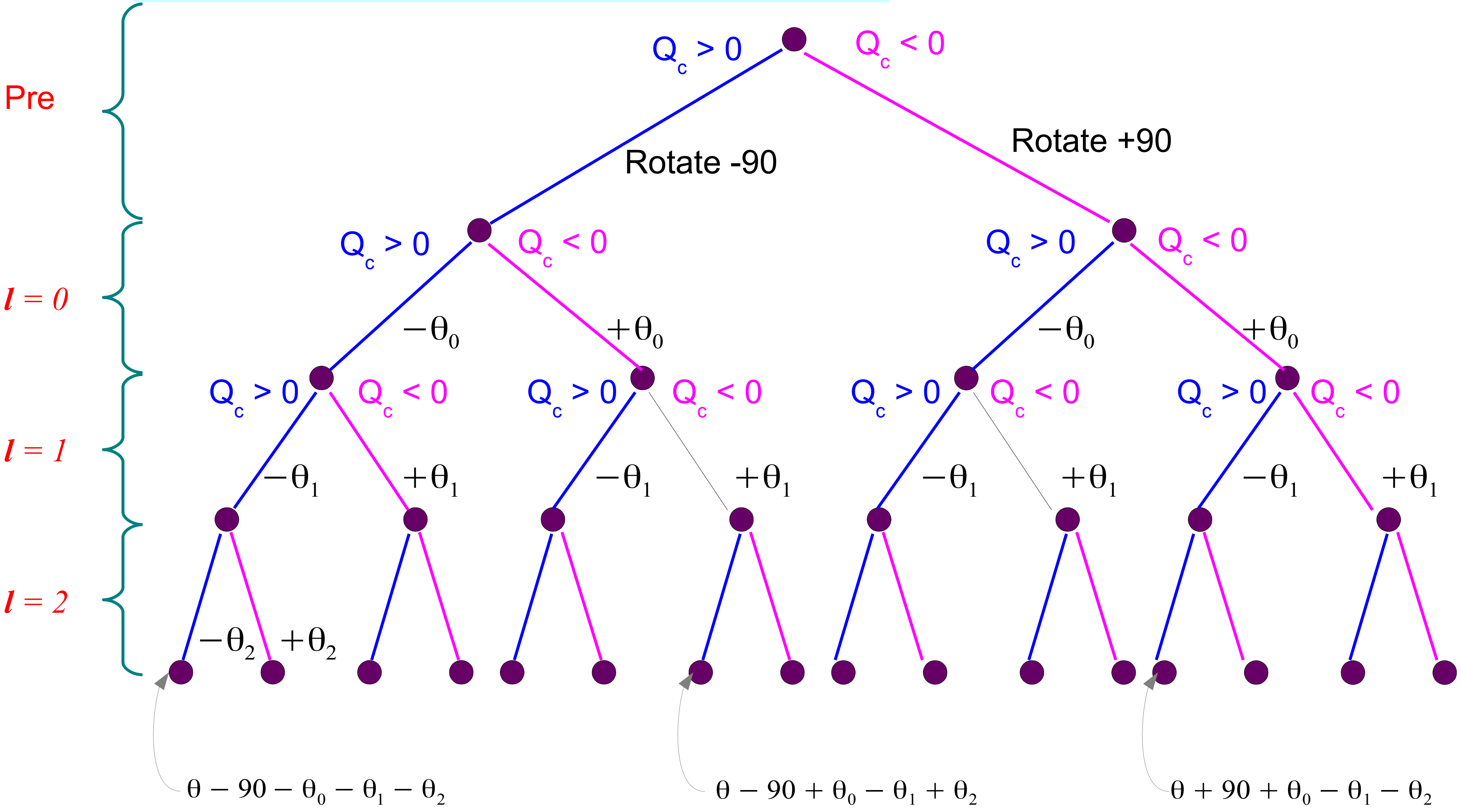
To compensate, multiply by 0.607

Can't perform this gain adjustment by simple shift and add

- *This adjustment can be done in other part of a system when relative magnitude is enough*

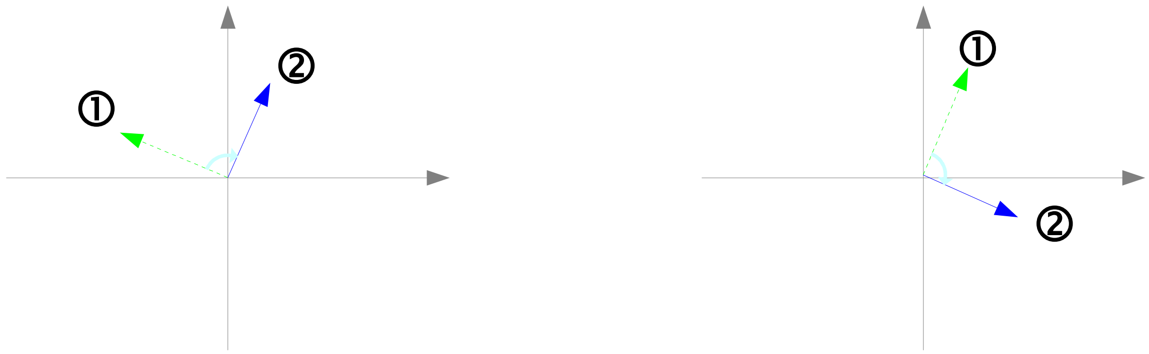
Calculating Magnitude (2)

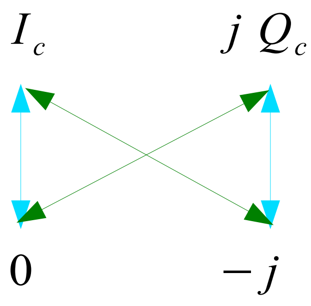
$$I_c' + j\,Q_c' = (I_c + j\,Q_c) \cdot \boxed{(1 + j\,k)}$$



Multiplication of ±j

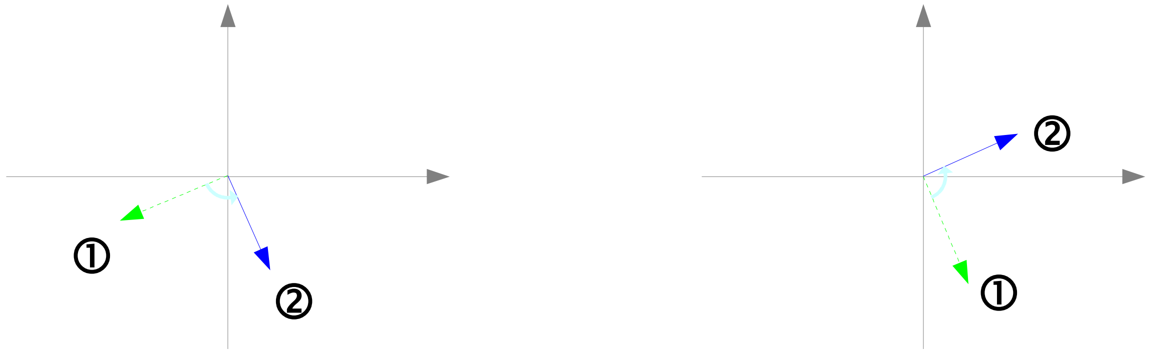
Positive Phase ($Q_c > 0$) → *Rotate by –90 degrees*

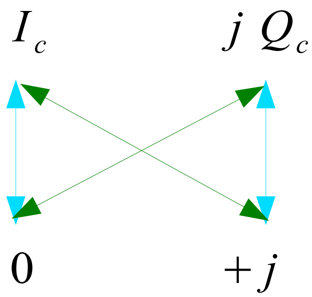




$$(Q_c) \; + \; j\,(-I_c)$$

Negative Phase ($Q_c < 0$) → *Rotate by +90 degrees*



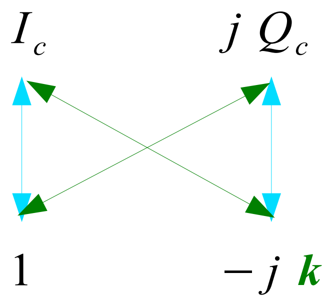


$$(-Q_c) \; + \; j\,(I_c)$$

Resulting Phase → [-90, +90]

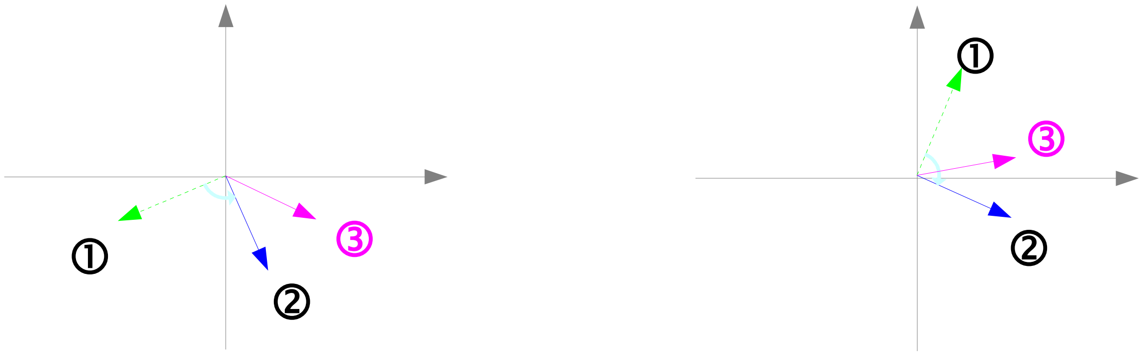
Multiplication of $1 \pm jk$

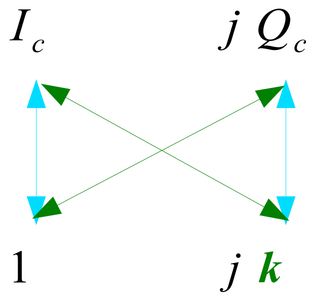
Positive Phase ($Q_c > 0$) ➡ *Rotate by* $1 - j\,k$



$$(I_c + 2^{-l} Q_c) + j\,(Q_c - 2^{-l} I_c)$$

Negative Phase ($Q_c < 0$) ➡ *Rotate by* $1 + j\,k$

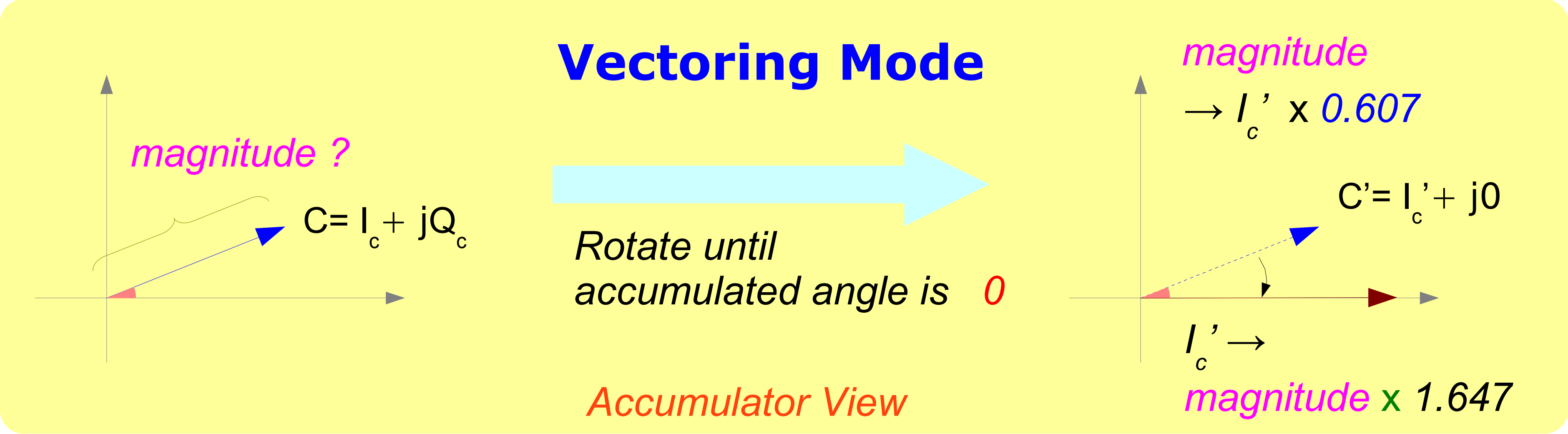




$$(I_c - 2^{-l} Q_c) + j(Q_c + 2^{-l} I_c)$$

After iterations, the result ➡ $I_c + j\,0$

Calculating Phase



Vectoring Mode

Rotate until accumulated angle is 0

Accumulator View

Phase of R

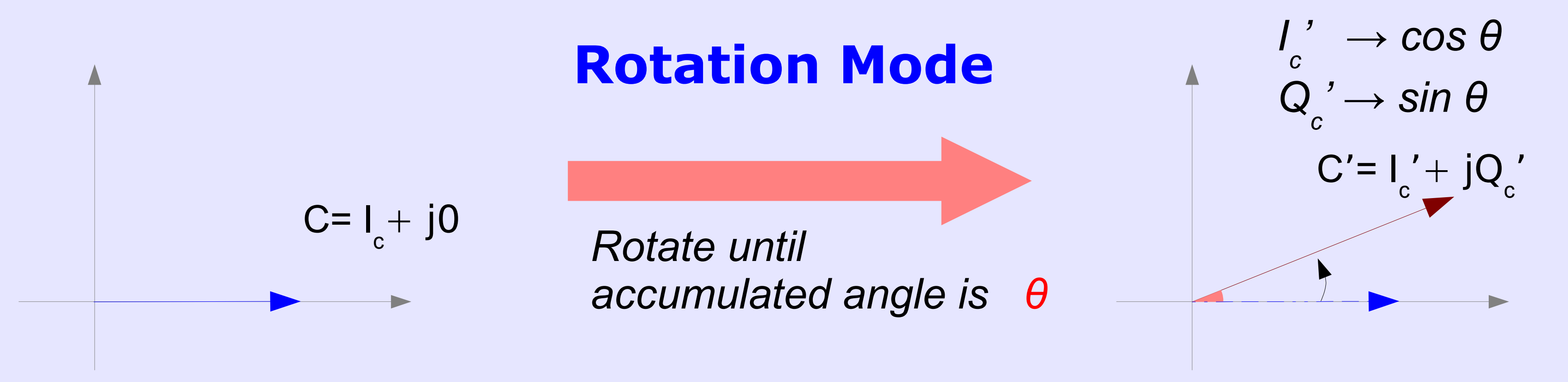
$$\theta_0 = \tan^{-1}(2^0) = 45°$$
$$\theta_1 = \tan^{-1}(2^{-1}) = 26.56505°$$
$$\theta_2 = \tan^{-1}(2^{-2}) = 14.03624°$$
$$\theta_3 = \tan^{-1}(2^{-3}) = 7.12502°$$
$$\theta_4 = \tan^{-1}(2^{-4}) = 3.57633°$$
$$\theta_5 = \tan^{-1}(2^{-5}) = 1.78991°$$

•••

$$\theta \quad \pm\theta_0 \pm\theta_1 \pm\theta_2 \pm\theta_3 \cdots = 0$$

Accumulate each rotating angles then negate the result

Calculating Sine and Cosine (1)



Rotation Mode

$C = I_c + j0$

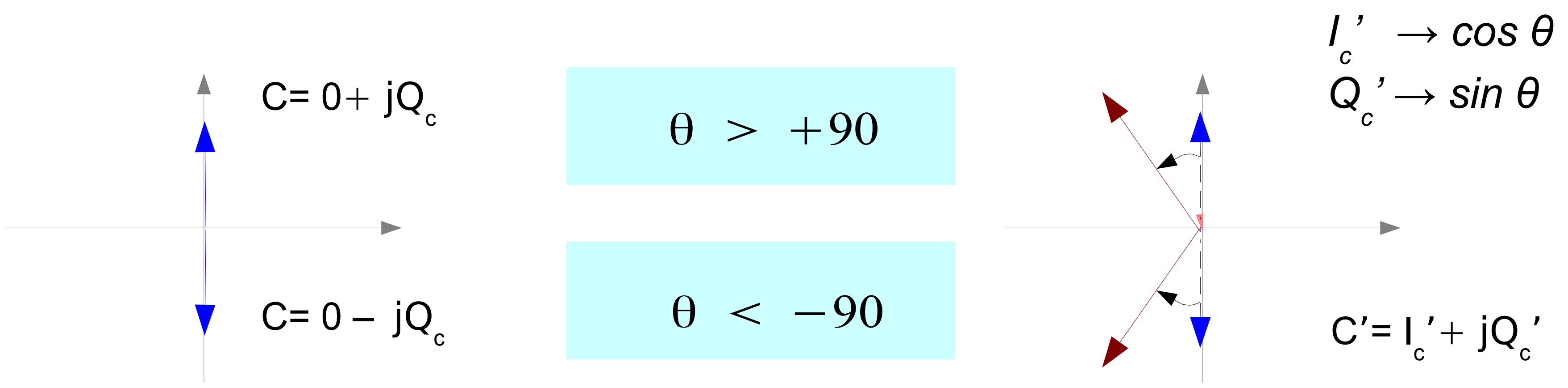
Rotate until accumulated angle is θ

$I_c' \rightarrow \cos\theta$

$Q_c' \rightarrow \sin\theta$

$C' = I_c' + jQ_c'$

Accumulator View



$C = 0 + jQ_c$

$C = 0 - jQ_c$

$\theta > +90$

$\theta < -90$

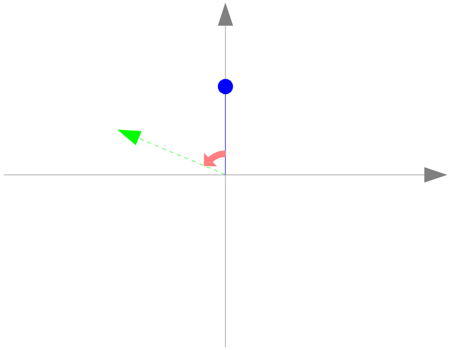
$I_c' \rightarrow \cos\theta$

$Q_c' \rightarrow \sin\theta$

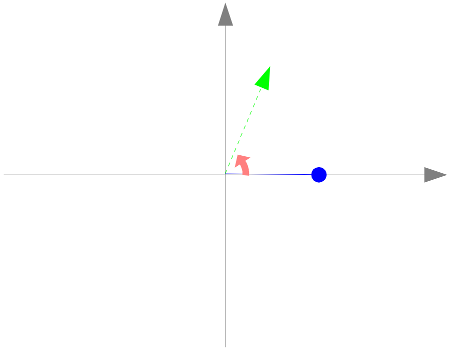
$C' = I_c' + jQ_c'$

Calculating Sine and Cosine (3)

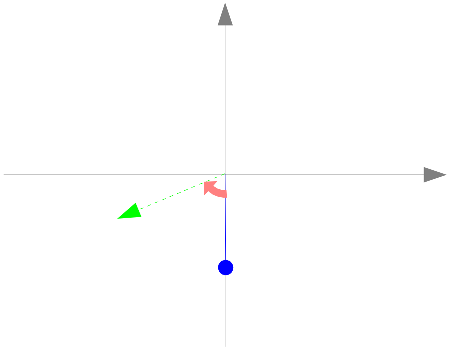
$\Theta > +90$ ➡ *starting from* $0 + j1$



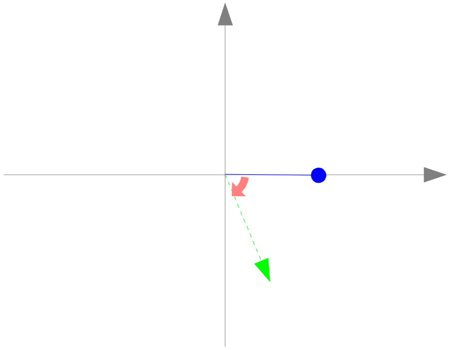
$\Theta < +90$ ➡ *starting from* $1 + j0$



$\Theta < -90$ ➡ *starting from* $0 - j1$



$\Theta > -90$ ➡ *starting from* $1 - j0$



Initialize the accumulate rotation ➡ -90, +90, 0

Calculating Sine and Cosine (3)

In each iteration

(Θ – *the accumulated rotation*) < 0

➡ *then add the next angle*

(Θ – *the accumulated rotation*) > 0

➡ *then subtract the next angle*

The final

I_c ➡ *cosine*

Q_c ➡ *sine*

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A “graphical interpretation” of the DFT and FFT, by Steve Mann
[4] CORDIC FAQ, www.dspguru.com