

Convolution (1B)

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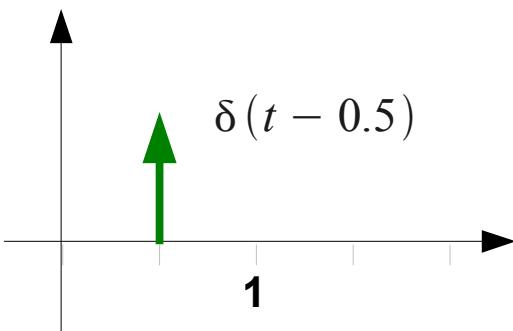
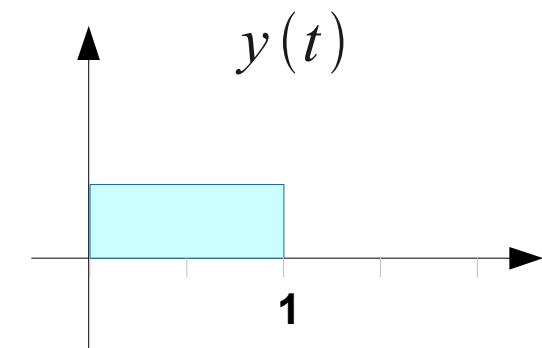
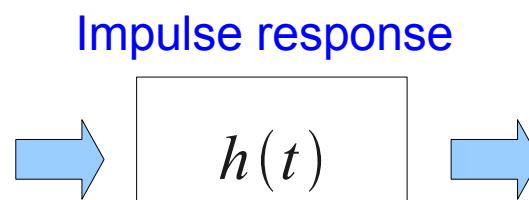
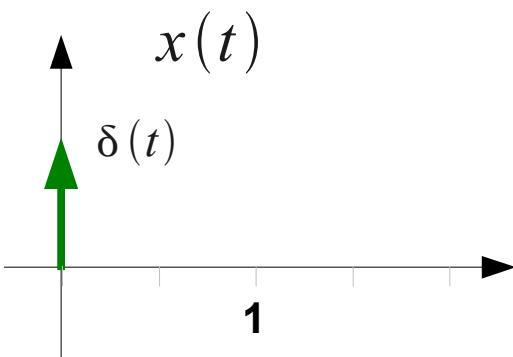
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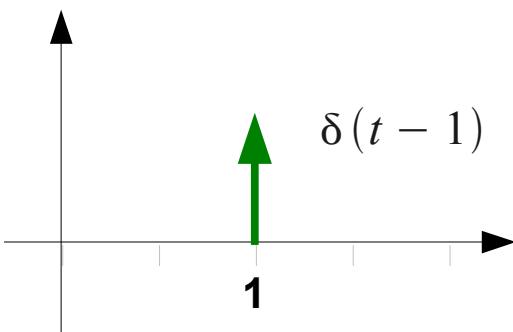
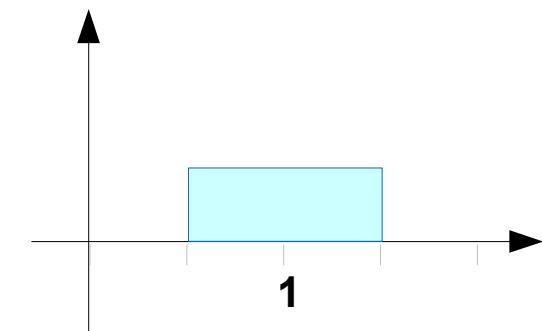
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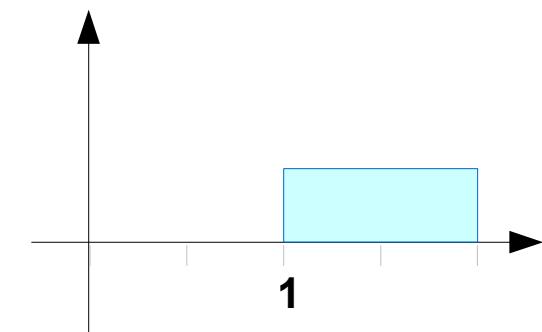
Impulse Response



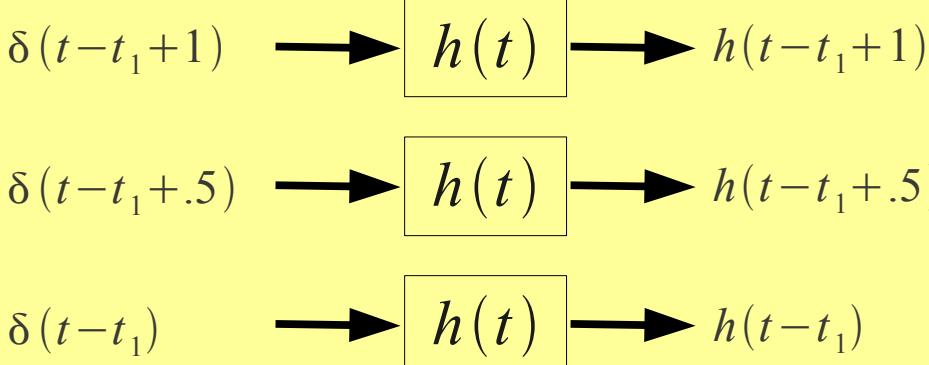
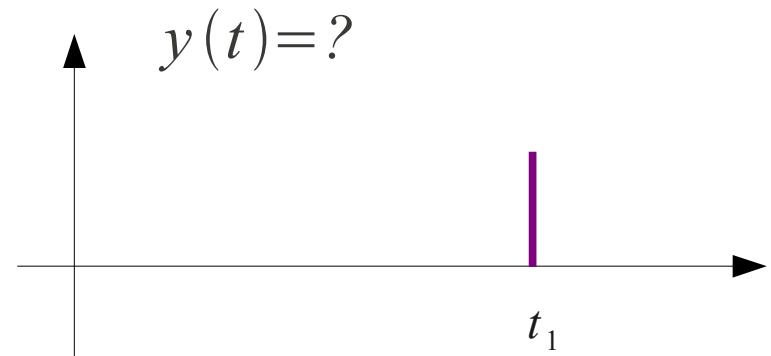
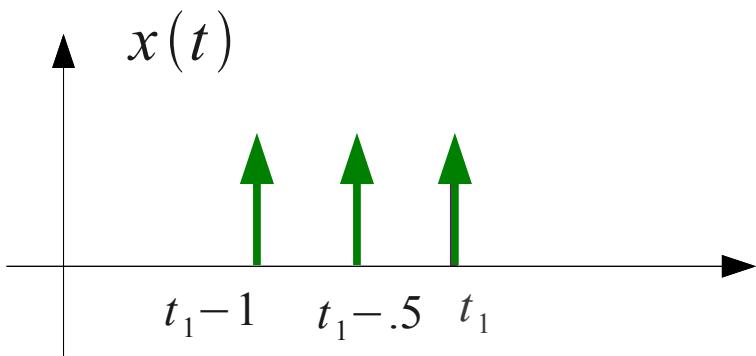
delayed response
by 0.5



delayed response
by 1



LTI System



A block diagram of the LTI system. An input signal enters a block labeled $h(t)$, which then produces an output signal $y(t)$.

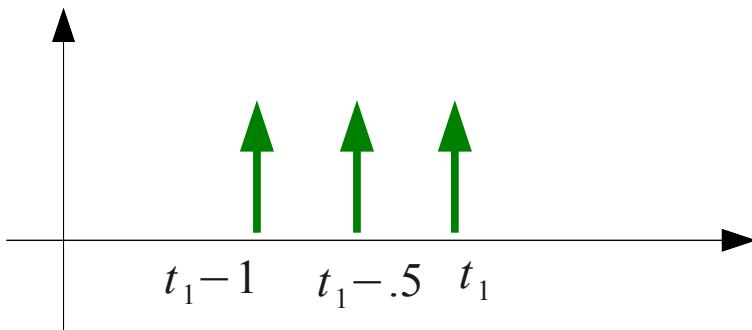
The input signal is defined as:

$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

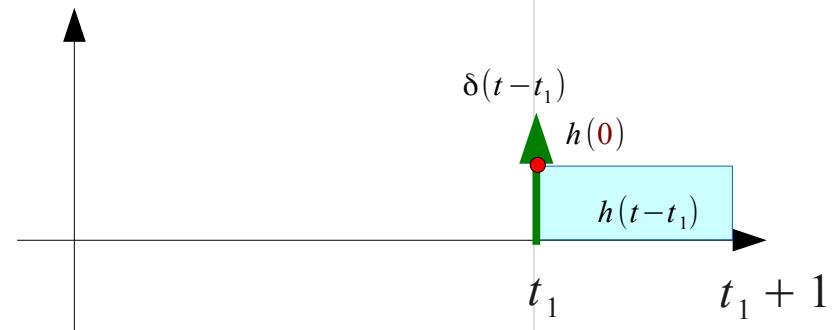
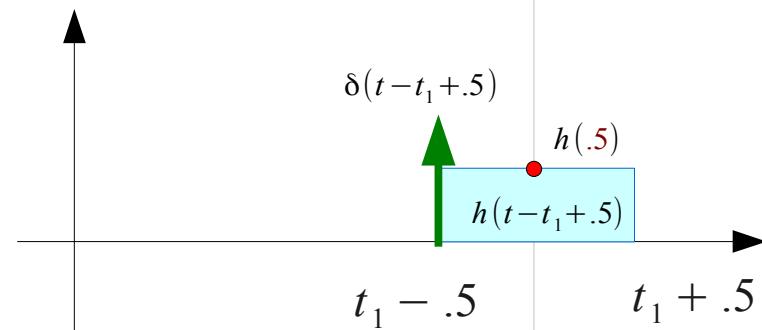
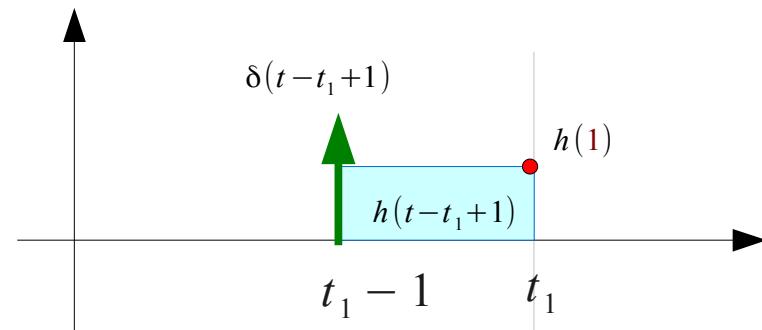
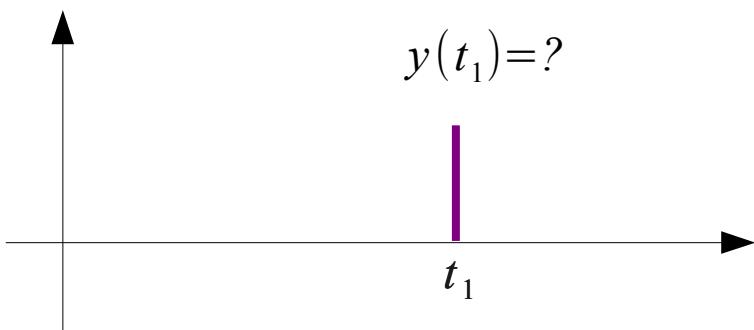
The output signal is defined as:

$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

Computing $y(t_1)$: Output at $t = t_1$

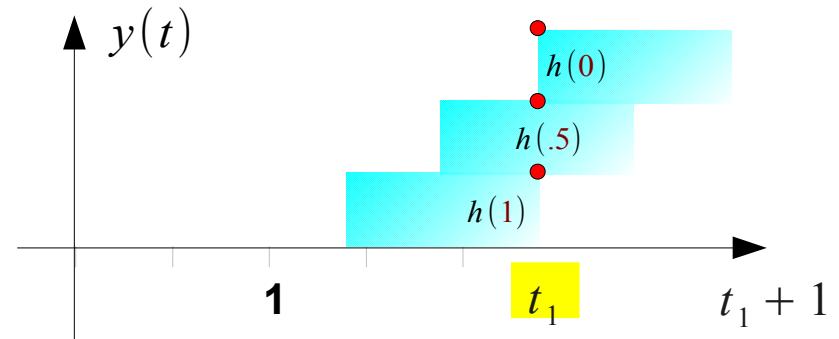
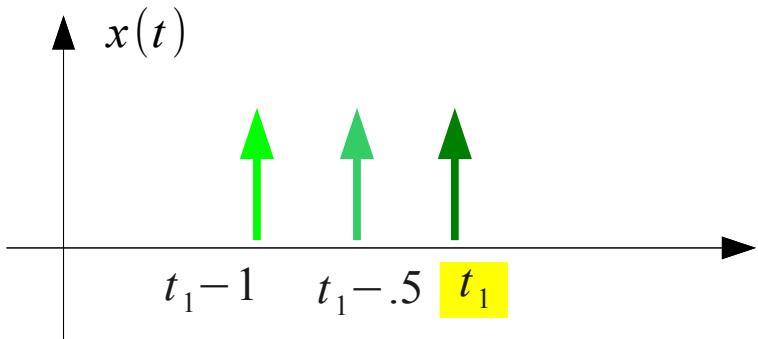


$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$



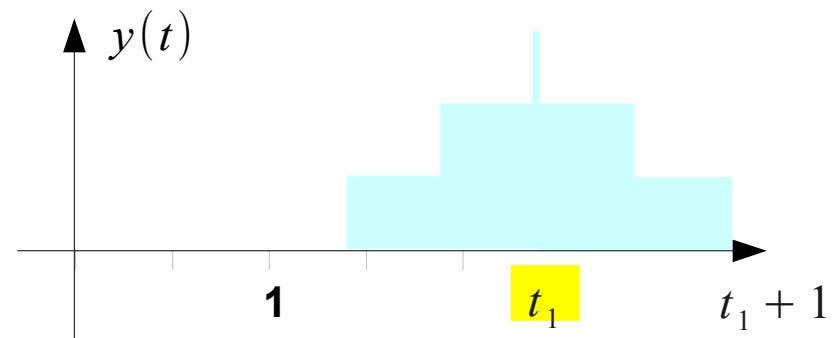
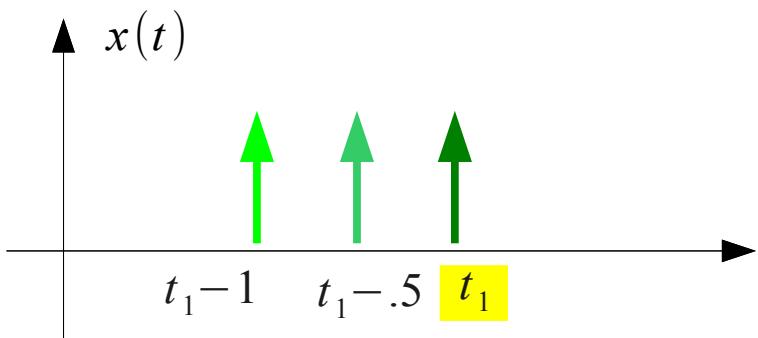
Computing $y(t_1)$: delayed impulse response $h(t)$

delayed impulse response – $h(t)$



$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

$$y(t_1) = h(1) + h(.5) + h(0)$$



$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

Computing $y(t_1)$: flip and shift $x(t)$

$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

↓ Change of variables $t \rightarrow v$

$$x(v) = \delta(v-t_1+1) + \delta(v-t_1+.5) + \delta(v-t_1)$$

↓ Flip around y axis and then shift to the right by t $v \rightarrow t-v$

$$x(t-v) = \delta(t-v-t_1+1) + \delta(t-v-t_1+.5) + \delta(t-v-t_1)$$

$$y(t) = \int x(t-v) h(v) dv$$

$$= \int \underline{\delta(t-v-t_1+1)} h(v) dv \rightarrow h(t-t_1+1) \text{ impulse response delayed by } t_1-1$$

$$+ \int \underline{\delta(t-v-t_1+.5)} h(v) dv \rightarrow h(t-t_1+.5) \text{ impulse response delayed by } t_1-.5$$

$$+ \int \underline{\delta(t-v-t_1)} h(v) dv \rightarrow h(t-t_1) \text{ impulse response delayed by } t_1$$

$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

→ $y(t_1) = h(1) + h(.5) + h(0)$

Computing $y(t_1)$: flip and shift $h(t)$

$h(t)$



Change of variables

$$t \rightarrow v$$

$h(v)$



Flip around y axis and then shift to the right by t

$$v \rightarrow t - v$$

$h(t-v)$

$$y(t) = \int x(v) h(t-v) dv$$

$$= \int \delta(v-t_1+1) \underline{h(t-v)} dv \rightarrow h(t-t_1+1) \text{ impulse response delayed by } t_1-1$$

$$+ \int \delta(v-t_1+.5) \underline{h(t-v)} dv \rightarrow h(t-t_1+.5) \text{ impulse response delayed by } t_1-.5$$

$$+ \int \delta(v-t_1) \underline{h(t-v)} dv \rightarrow h(t-t_1) \text{ impulse response delayed by } t_1$$

$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

$$\Rightarrow y(t_1) = h(1) + h(.5) + h(0)$$

Computing $y(t_1)$: commutativity (1)

$$y(t) = \int x(v)h(t-v) dv$$

$$= \int \delta(v-t_1+1)h(t-v) dv$$

$$+ \int \delta(v-t_1+.5)h(t-v) dv$$

$$+ \int \delta(v-t_1)h(t-v) dv$$

$$\Rightarrow h(t-t_1+1)$$

$$\Rightarrow h(t-t_1+.5)$$

$$\Rightarrow h(t-t_1)$$

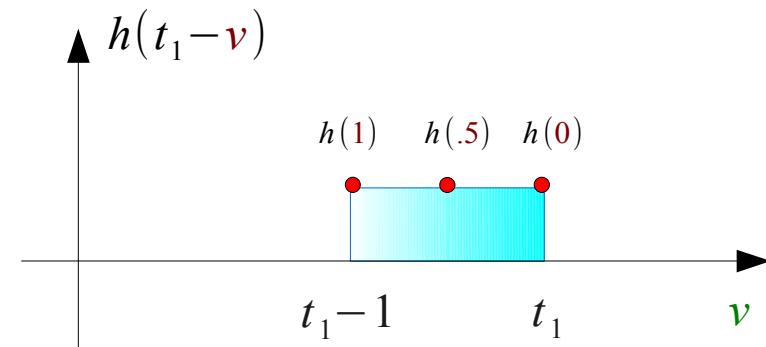
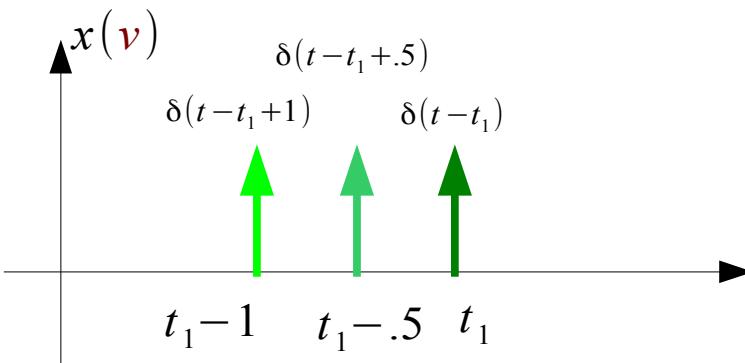
$$y(t) = \int x(t-v)h(v) dv$$

$$= \int \delta(t-v-t_1+1)h(v) dv$$

$$+ \int \delta(t-v-t_1+.5)h(v) dv$$

$$+ \int \delta(t-v-t_1)h(v) dv$$

Flip and Shift $h(t)$



Computing $y(t_1)$: commutativity (2)

$$y(t) = \int x(v)h(t-v) dv$$

$$= \int \delta(v-t_1+1)h(t-v) dv$$

$$+ \int \delta(v-t_1+.5)h(t-v) dv$$

$$+ \int \delta(v-t_1)h(t-v) dv$$

$$\Rightarrow h(t-t_1+1)$$

$$\Rightarrow h(t-t_1+.5)$$

$$\Rightarrow h(t-t_1)$$

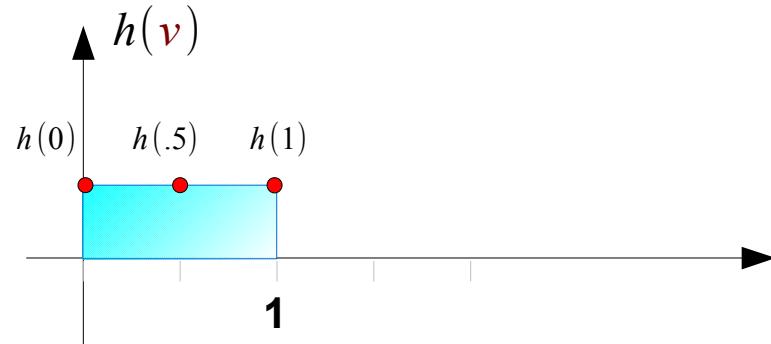
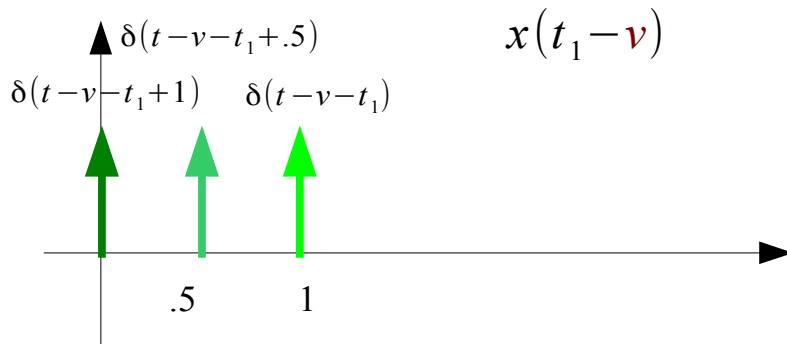
$$y(t) = \int x(t-v)h(v) dv$$

$$= \int \delta(t-v-t_1+1)h(v) dv$$

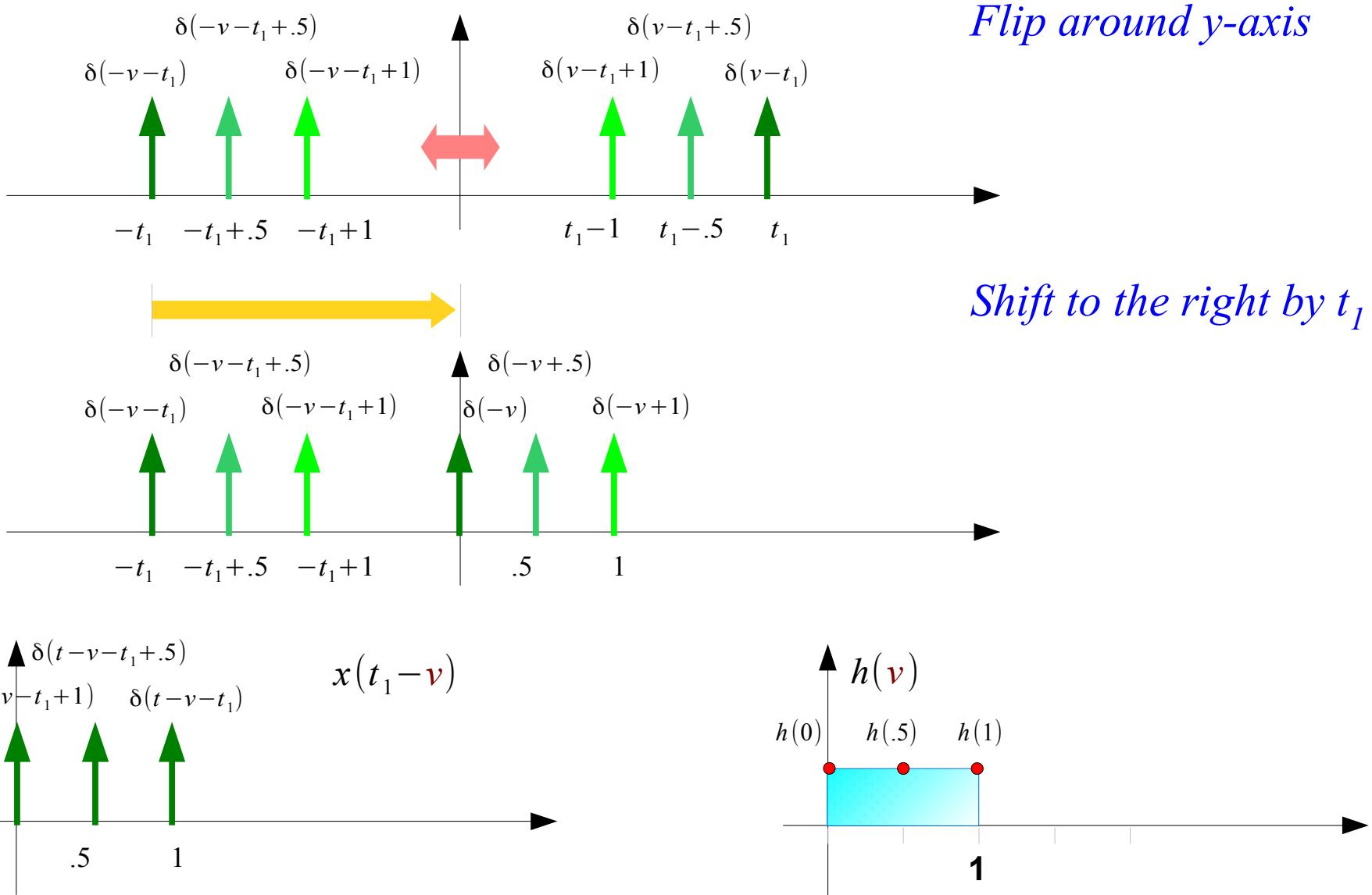
$$+ \int \delta(t-v-t_1+.5)h(v) dv$$

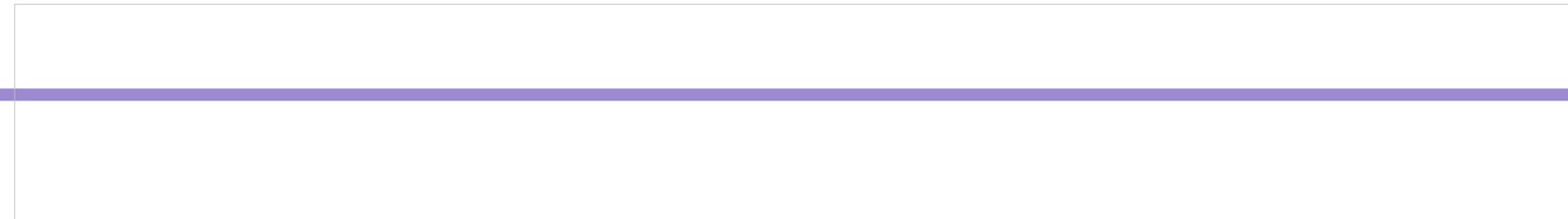
$$+ \int \delta(t-v-t_1)h(v) dv$$

Flip and shift input $x(t)$

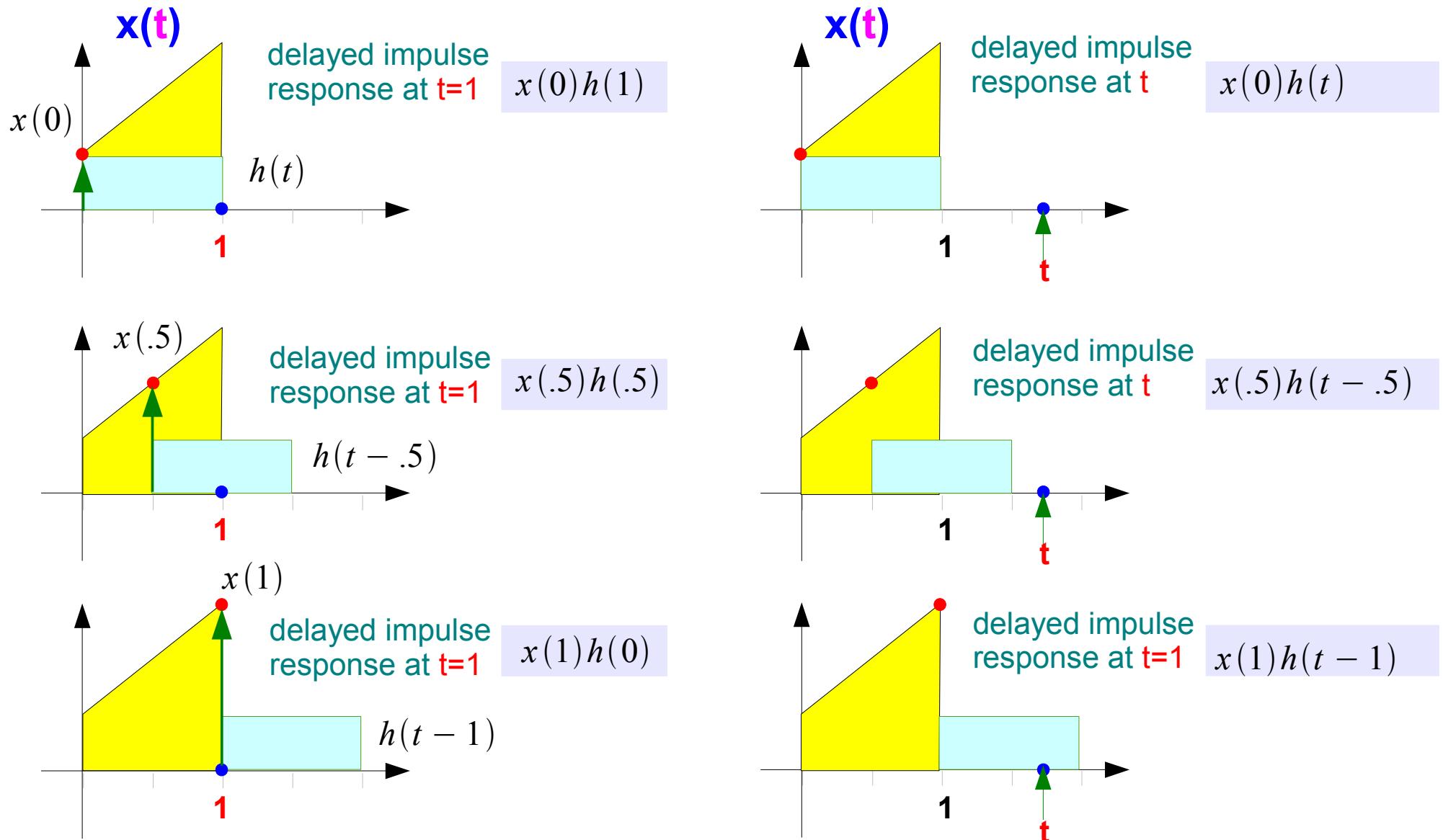


Computing $y(t_1)$: commutativity (3)

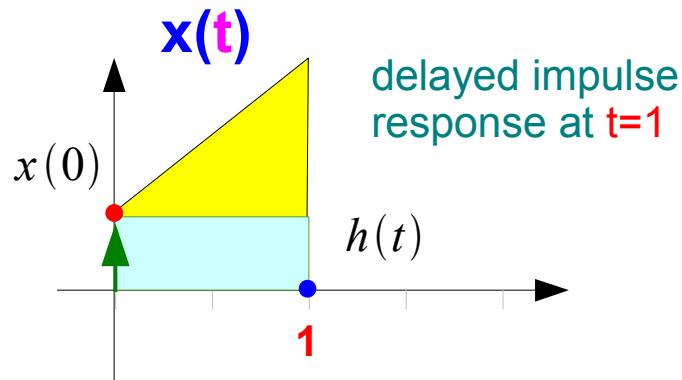




Computing $y(1)$, $y(t)$

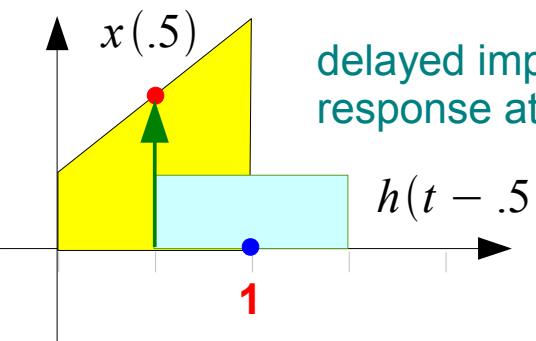


Computing $y(1)$: shift & flip $x(t)$



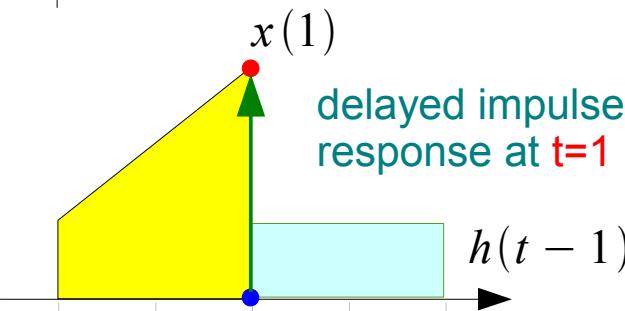
delayed impulse
response at $t=1$

$$x(0)h(1)$$



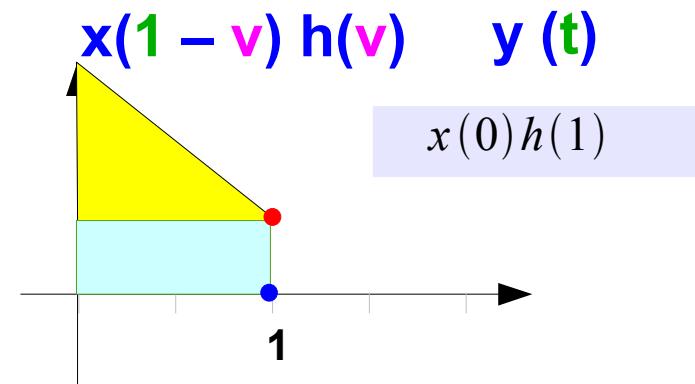
delayed impulse
response at $t=1$

$$x(.5)h(.5)$$

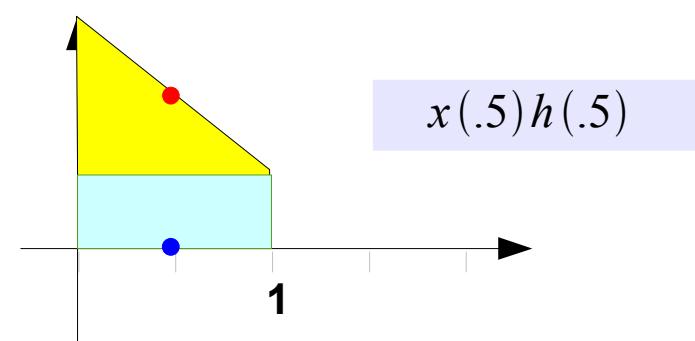


delayed impulse
response at $t=1$

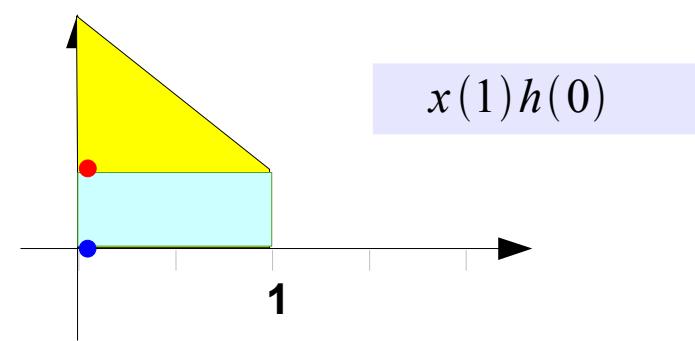
$$x(1)h(0)$$



$$x(0)h(1)$$

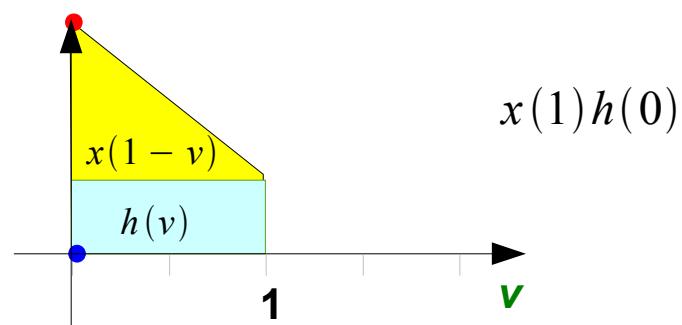
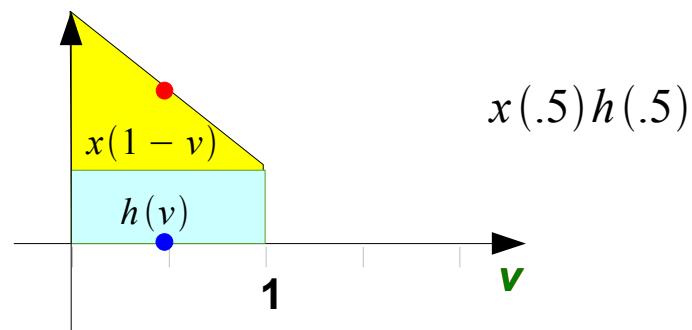
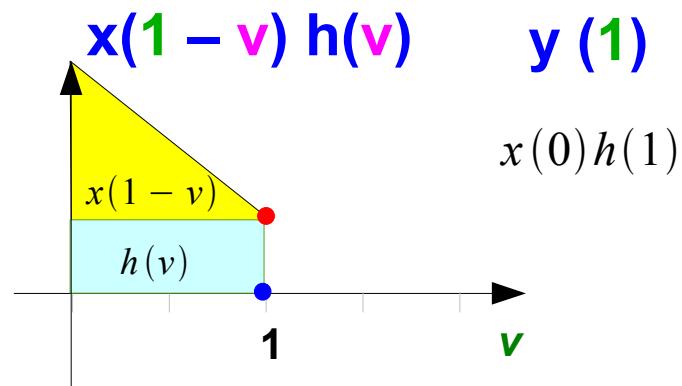


$$x(.5)h(.5)$$

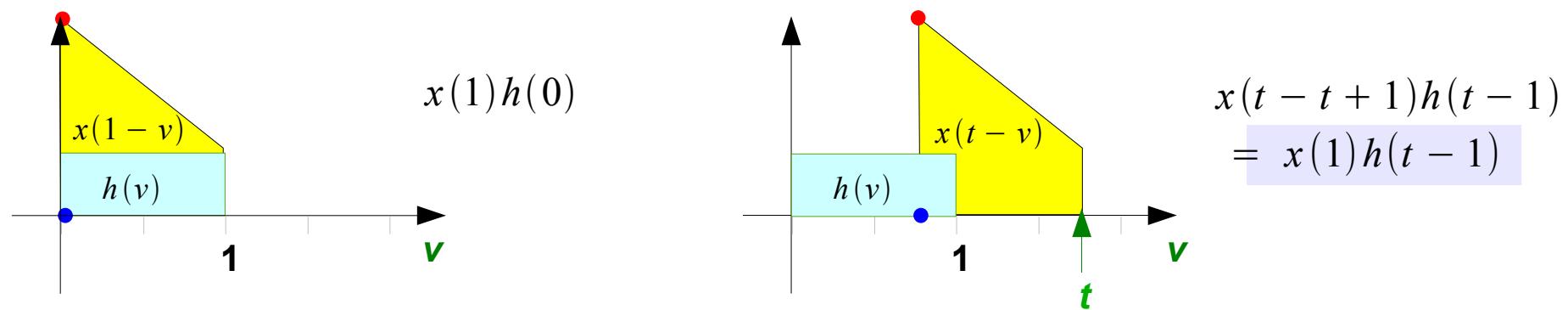
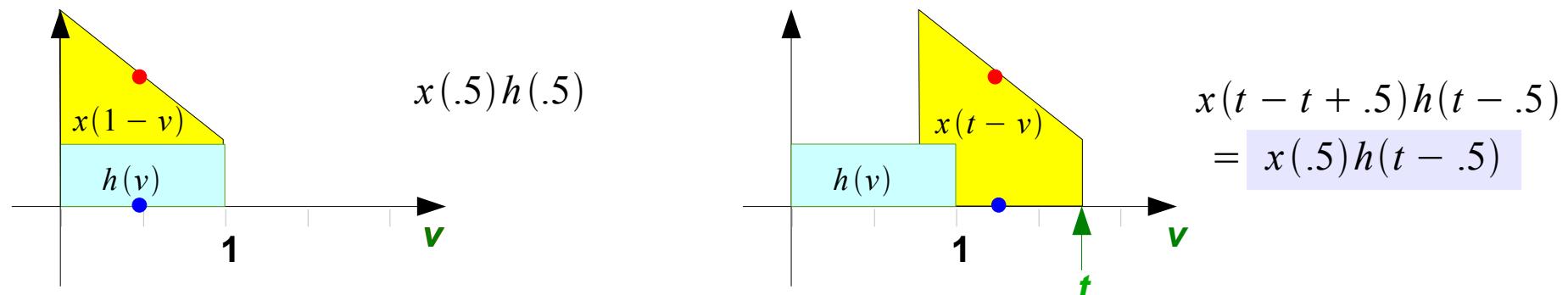
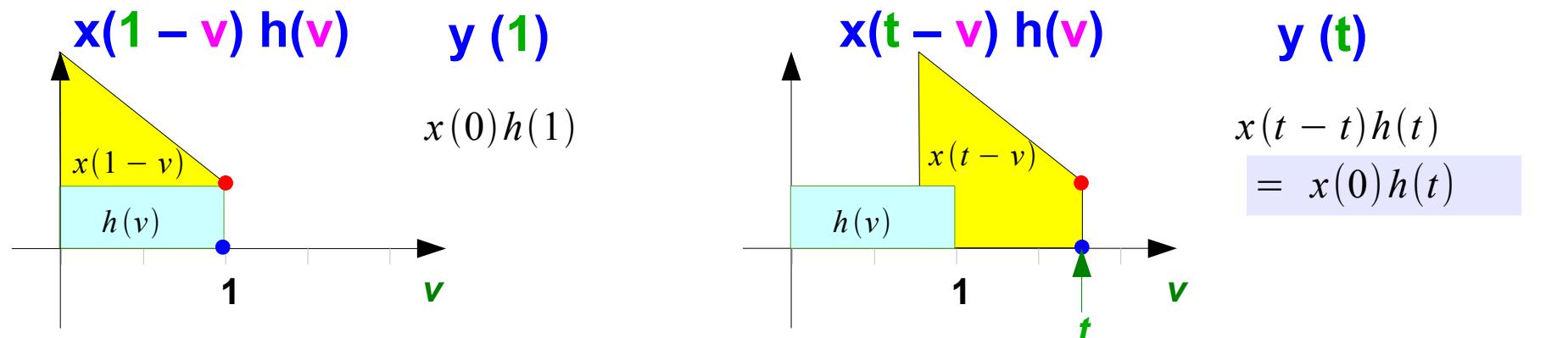


$$x(1)h(0)$$

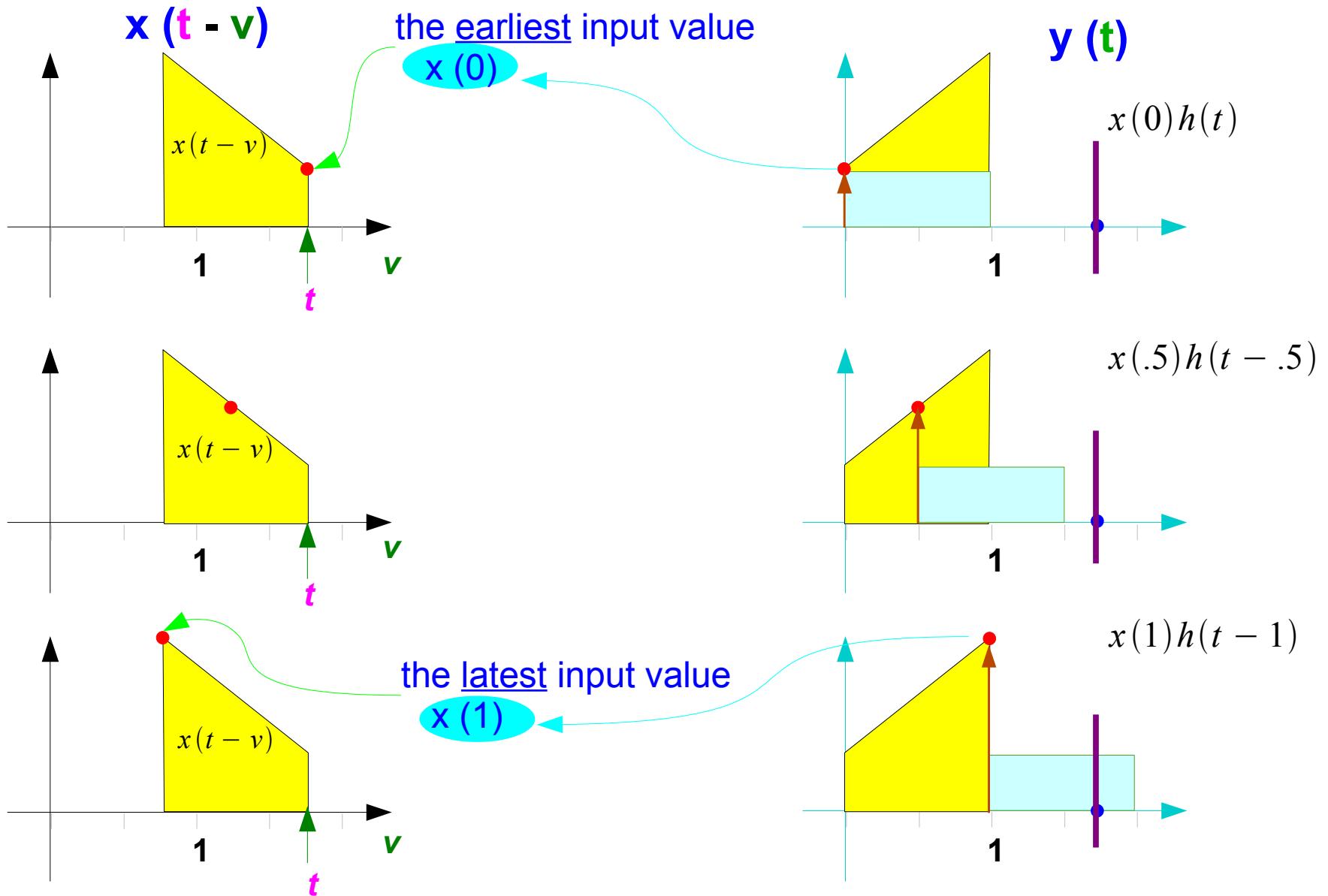
Computing $y(t)$: shift & flip $x(t)$



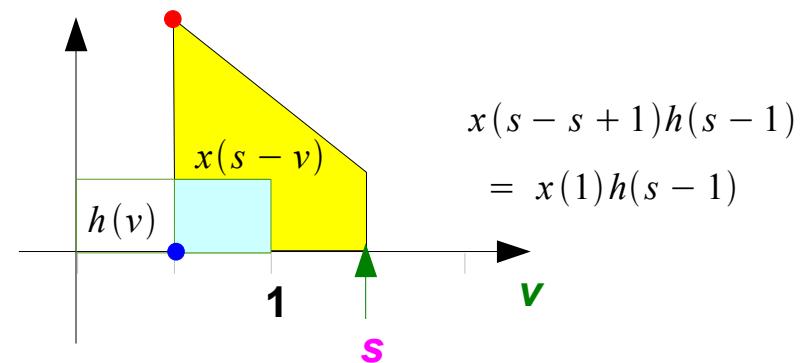
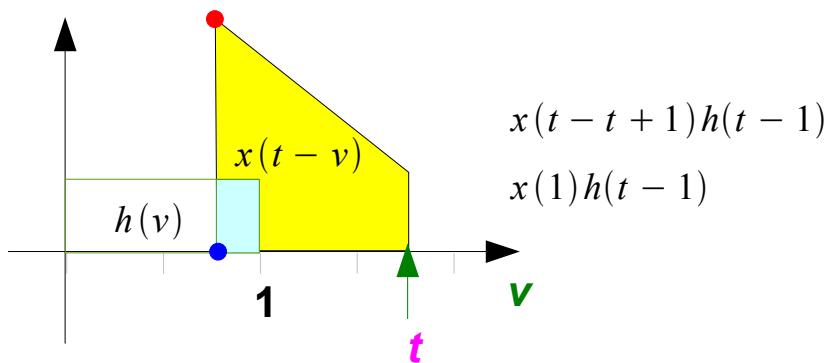
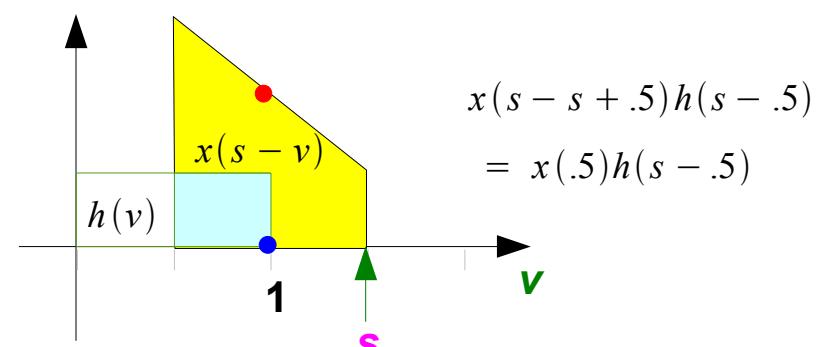
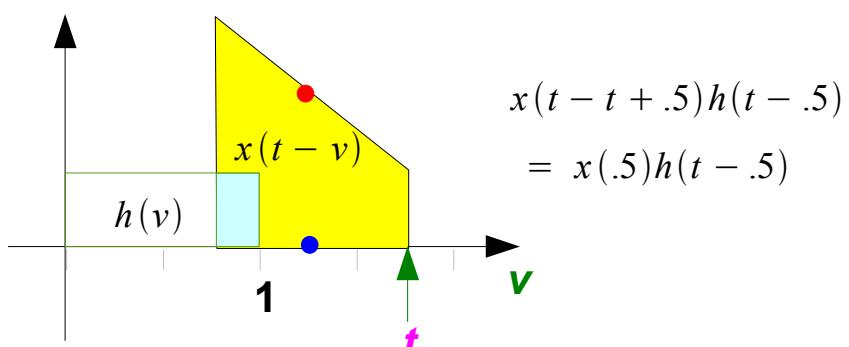
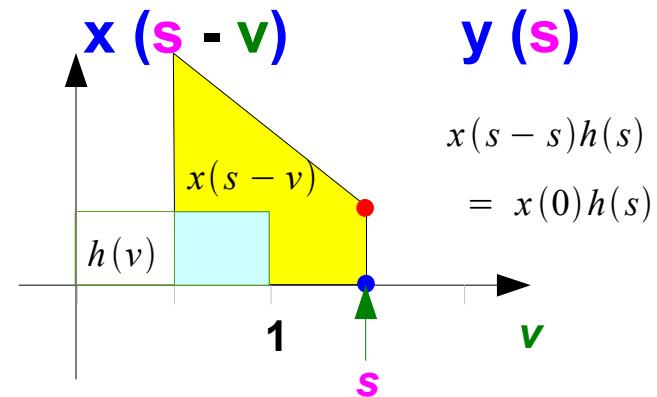
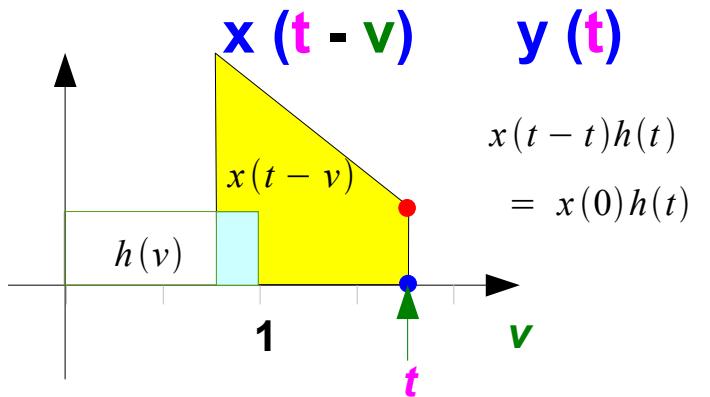
Convolution (1B)



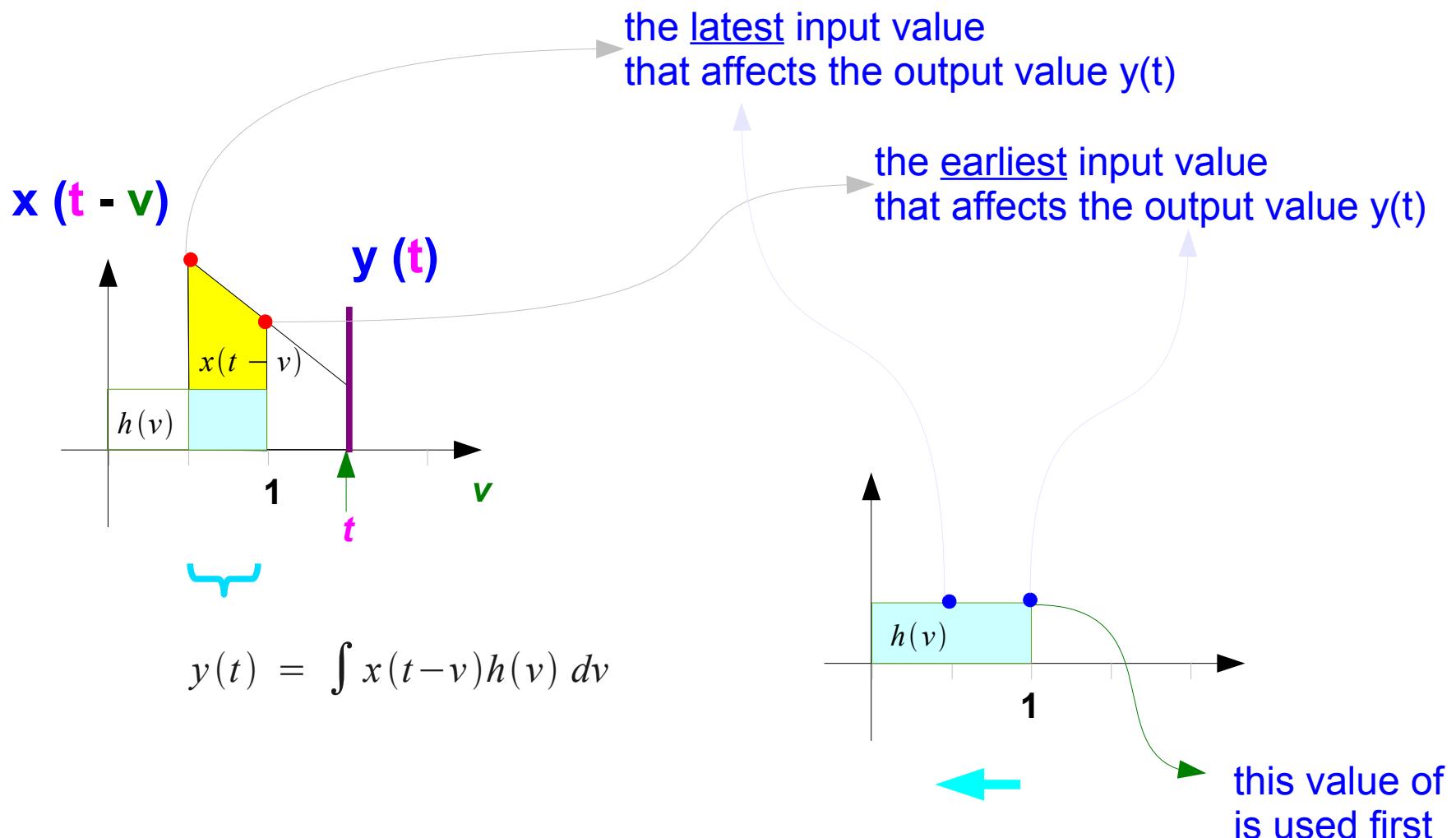
Computing $y(t)$: the earliest and latest inputs

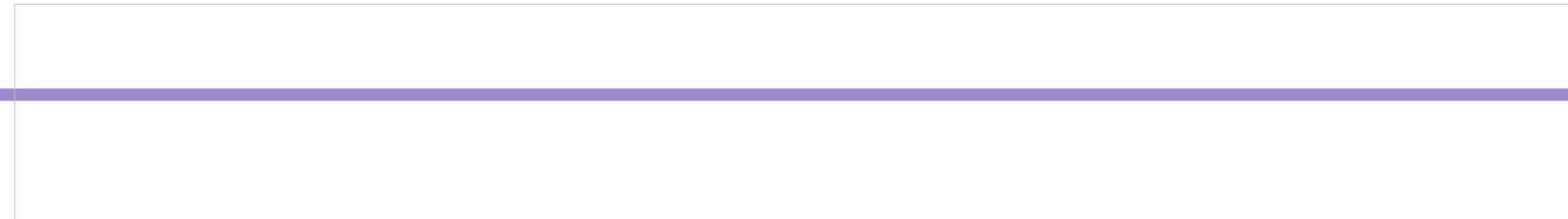


Computing $y(t)$, $y(s)$

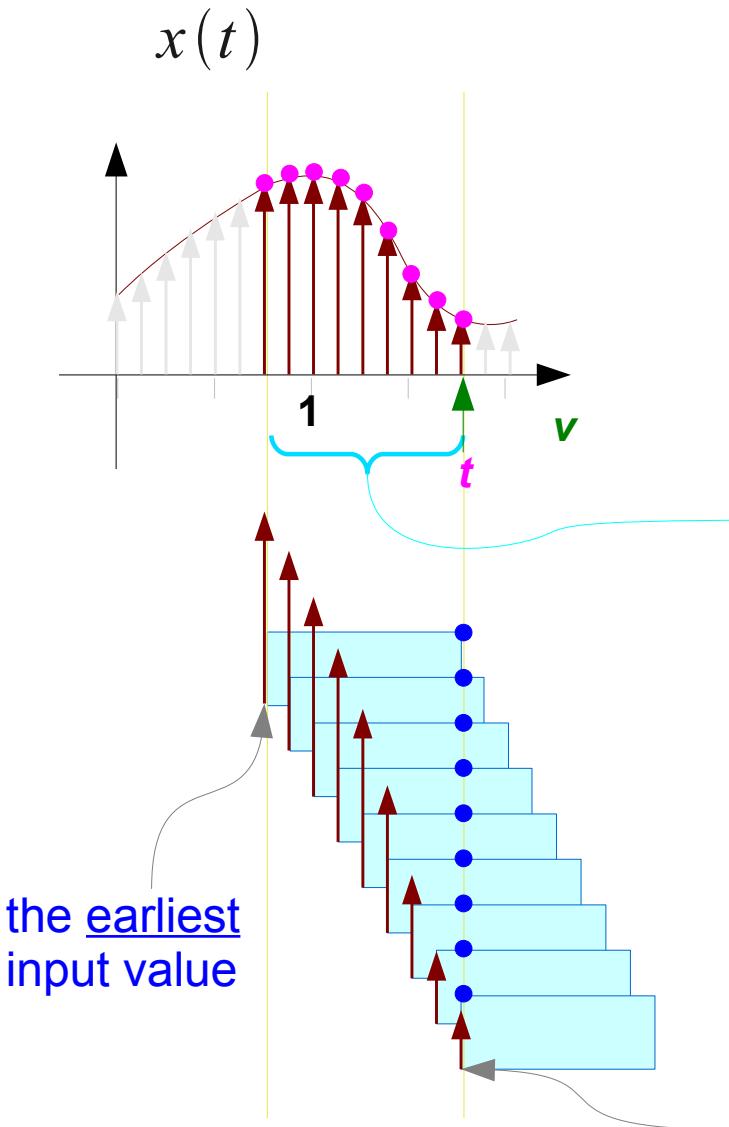


Computing $y(t)$: earliest and latest inputs





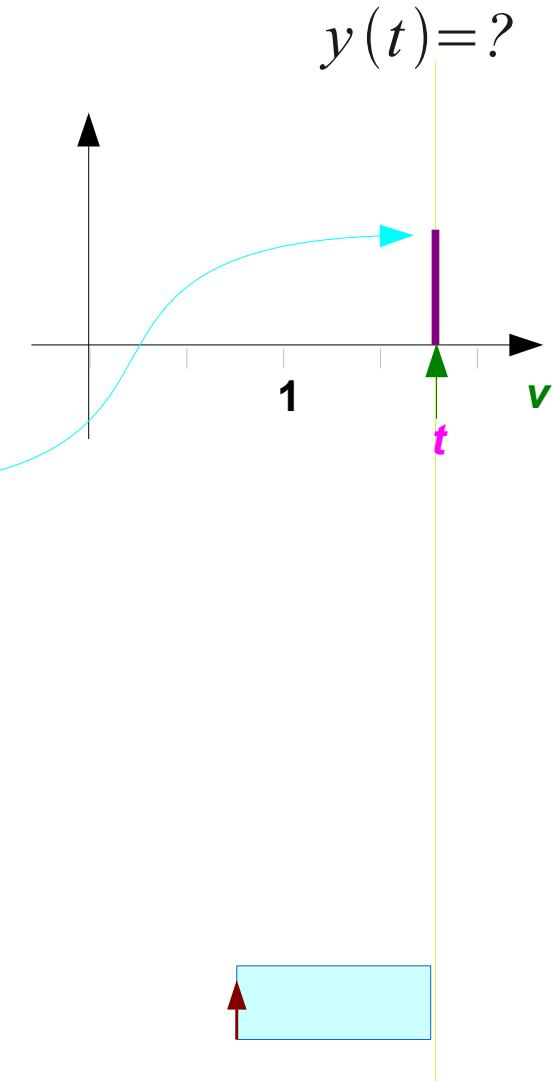
Computing $y(t)$: delayed impulse response



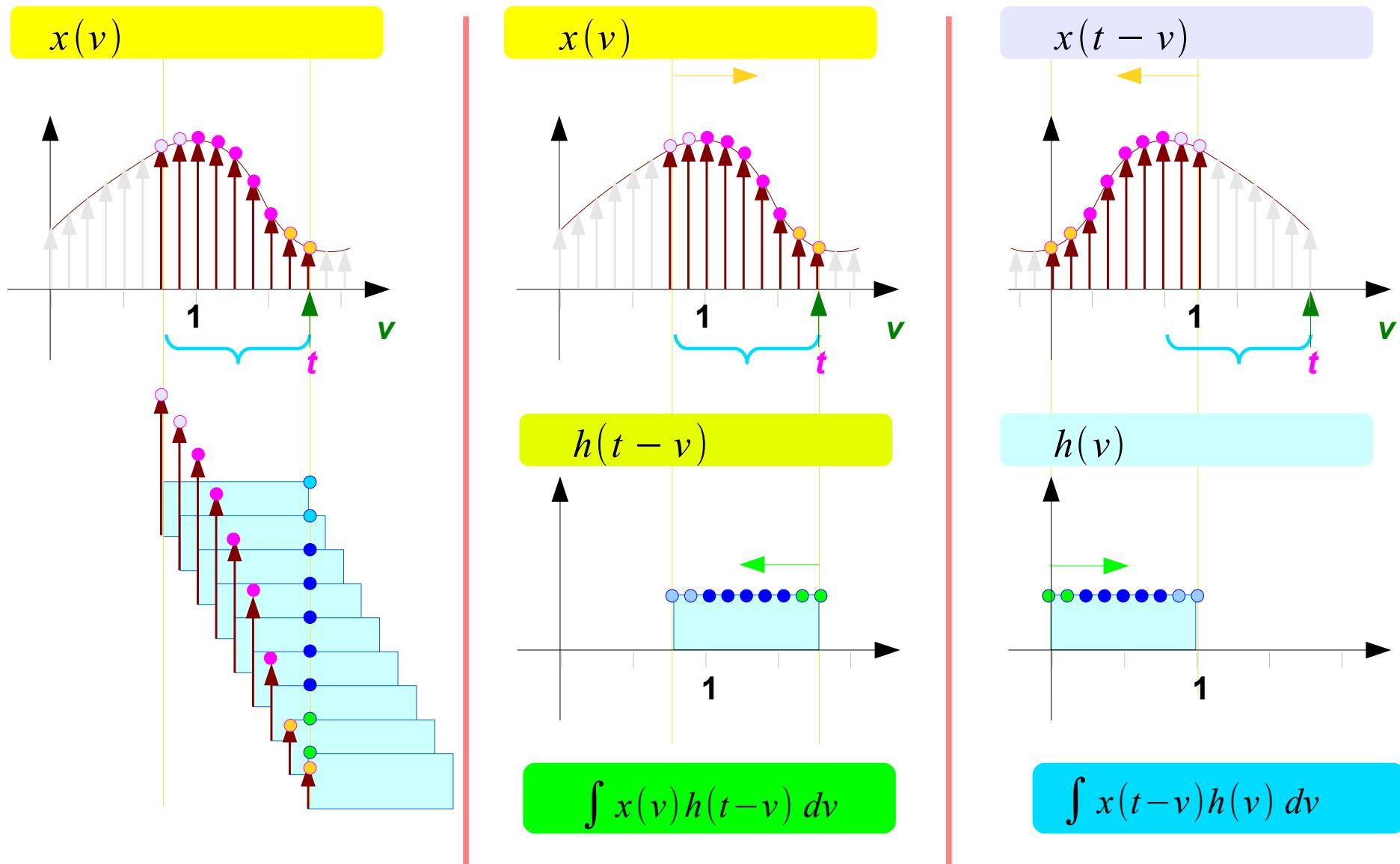
these inputs affects
the output value at t

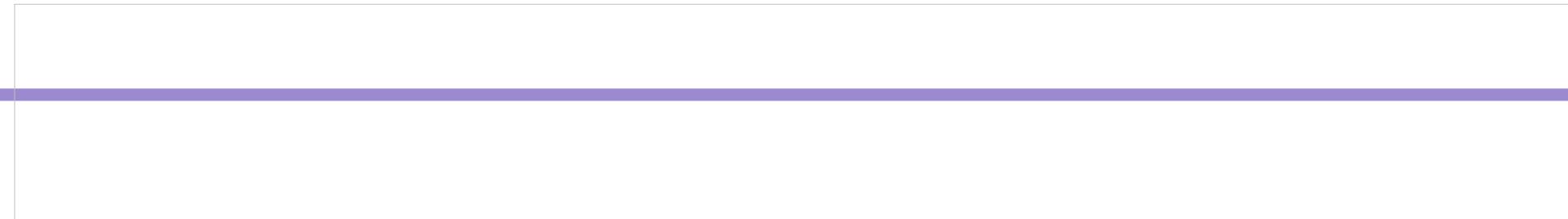
delayed impulse response

the latest
input value
that affects the output value $y(t)$



Computing $y(t)$: multiplication sequence





References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., *Signal Processing First*, Pearson Prentice Hall, 2003