Convolution (1B)

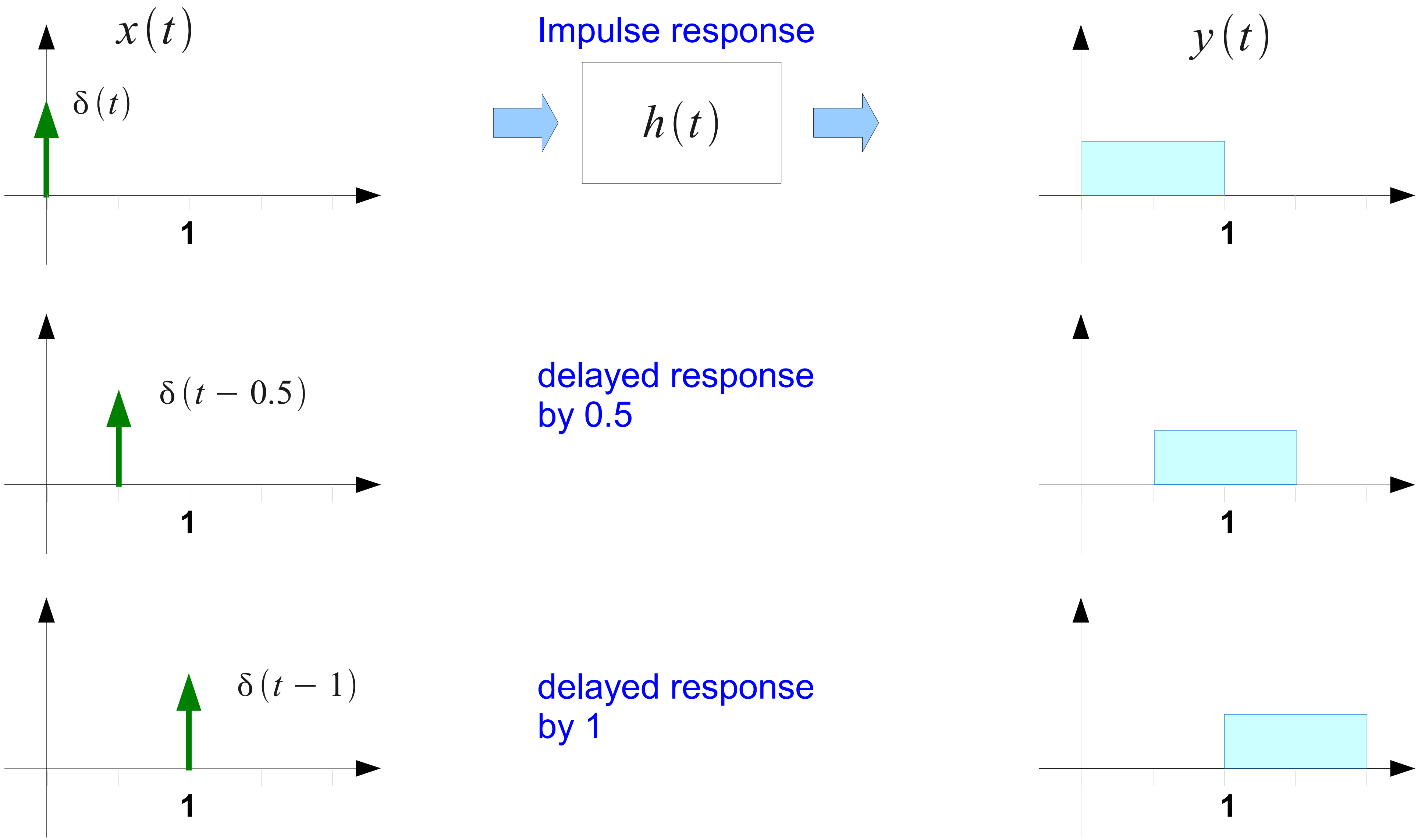
•

Young Won Lim
9/27/11

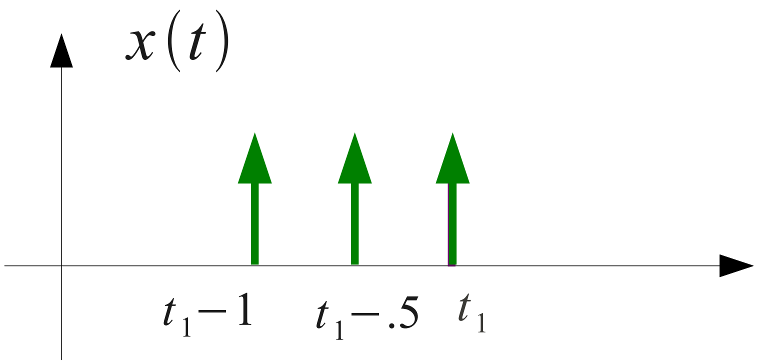
Please send corrections (or suggestions) to youngwlim@hotmail.com.

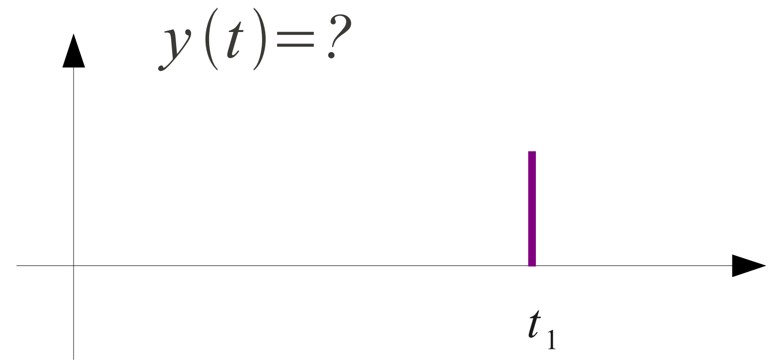
This document was produced by using OpenOffice and Octave.

Impulse Response



LTI System





$\delta(t-t_1+1) \rightarrow \boxed{h(t)} \rightarrow h(t-t_1+1)$

$\delta(t-t_1+.5) \rightarrow \boxed{h(t)} \rightarrow h(t-t_1+.5)$

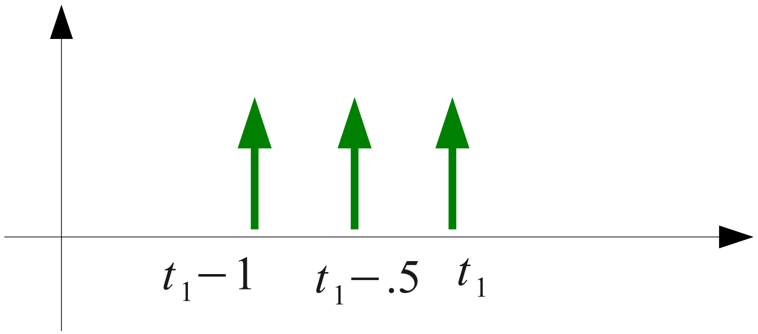
$\delta(t-t_1) \rightarrow \boxed{h(t)} \rightarrow h(t-t_1)$

$\rightarrow \boxed{h(t)} \rightarrow$

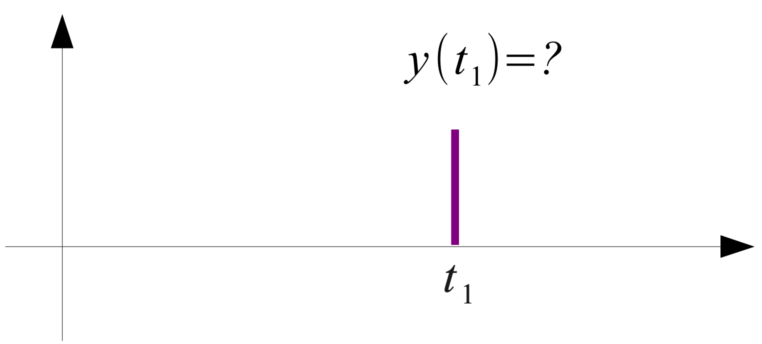
$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

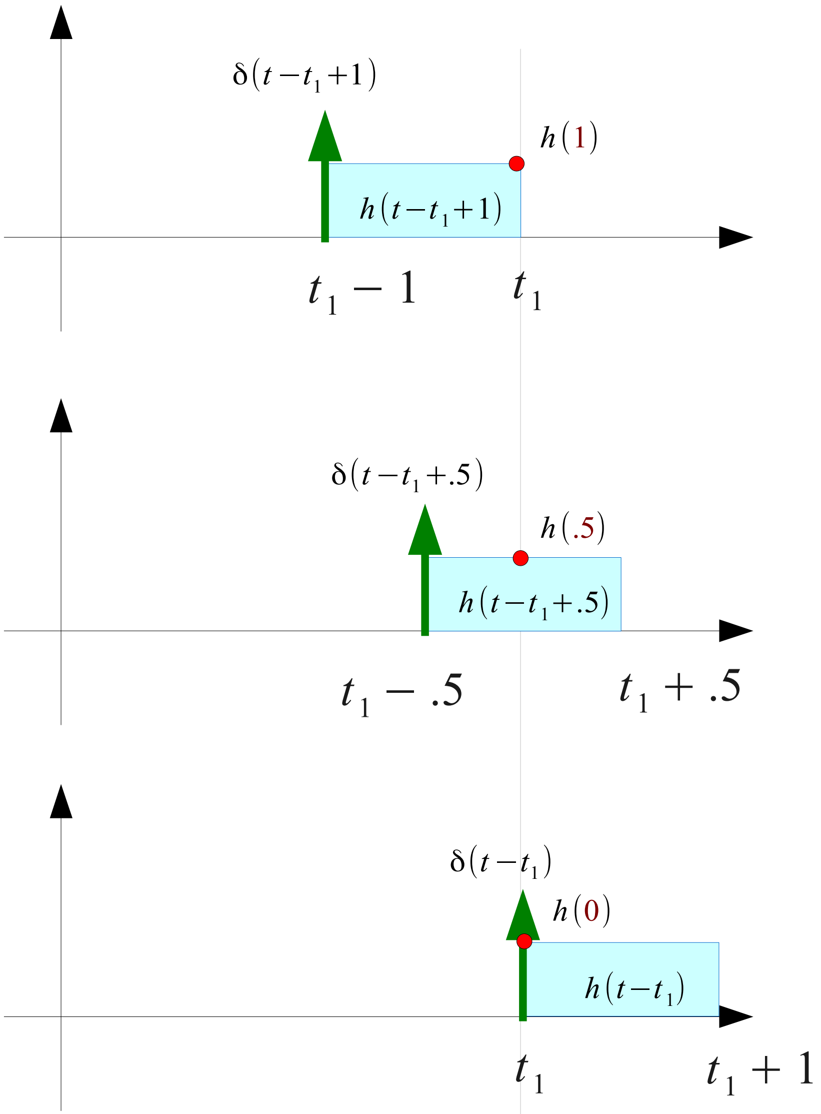
$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

Computing $y(t_1)$: Output at $t = t_1$



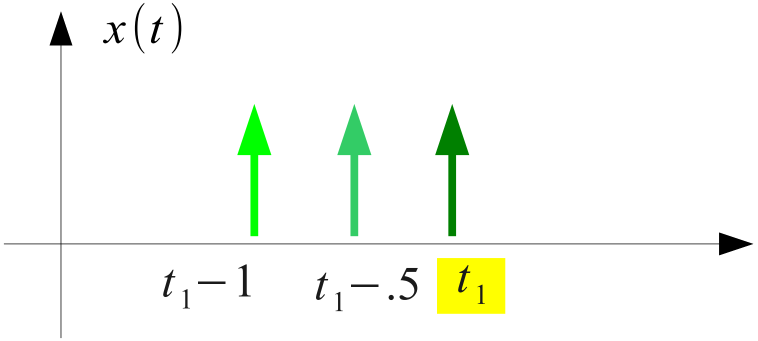
$$x(t) = \delta(t - t_1 + 1) + \delta(t - t_1 + .5) + \delta(t - t_1)$$

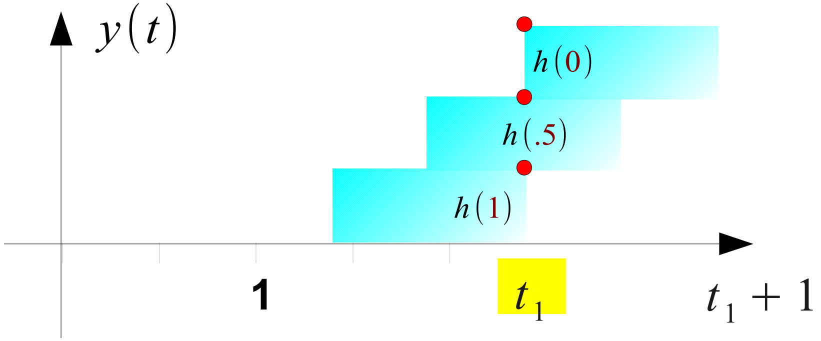




Computing $y(t_1)$: delayed impulse response h(t)

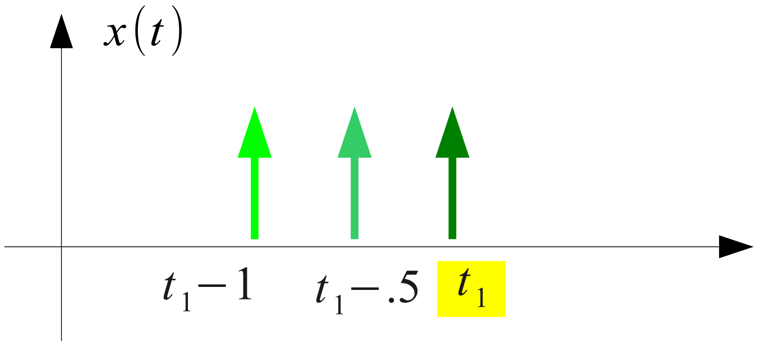
delayed impulse response – h(t)

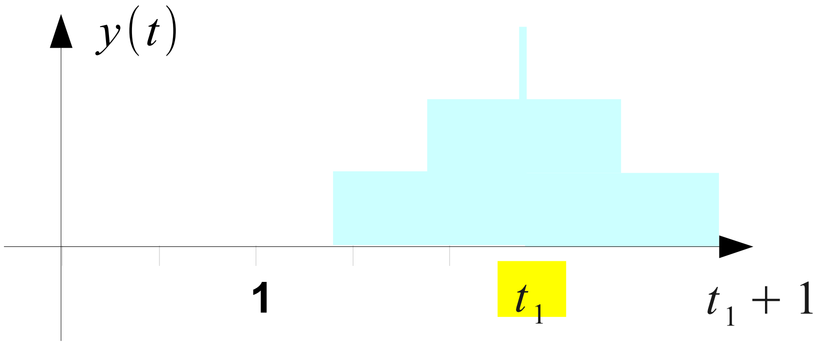




$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

$$y(t_1) = h(1) + h(.5) + h(0)$$





$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

Computing $y(t_1)$: flip and shift $x(t)$

$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

Change of variables $t \rightarrow v$

$$x(v) = \delta(v-t_1+1) + \delta(v-t_1+.5) + \delta(v-t_1)$$

Flip around y axis and then shift to the right by t $v \rightarrow t-v$

$$x(t-v) = \delta(t-v-t_1+1) + \delta(t-v-t_1+.5) + \delta(t-v-t_1)$$

$$y(t) = \int x(t-v)\, h(v)\, dv$$

$= \int \delta(t-v-t_1+1)\, h(v)\, dv$ → $h(t-t_1+1)$ impulse response delayed by t_1-1

$+ \int \delta(t-v-t_1+.5)\, h(v)\, dv$ → $h(t-t_1+.5)$ impulse response delayed by $t_1-.5$

$+ \int \delta(t-v-t_1)\, h(v)\, dv$ → $h(t-t_1)$ impulse response delayed by t_1

$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

$$\Rightarrow \quad y(t_1) = h(1) + h(.5) + h(0)$$

Computing $y(t_1)$: flip and shift $h(t)$

$h(t)$

Change of variables $t \rightarrow v$

$h(v)$

Flip around y axis and then shift to the right by t $v \rightarrow t-v$

$h(t-v)$

$$y(t) = \int x(v) h(t-v)\, dv$$

$$= \int \delta(v-t_1+1) h(t-v)\, dv \rightarrow h(t-t_1+1)$$ impulse response delayed by t_1-1

$$+ \int \delta(v-t_1+.5) h(t-v)\, dv \rightarrow h(t-t_1+.5)$$ impulse response delayed by $t_1-.5$

$$+ \int \delta(v-t_1) h(t-v)\, dv \rightarrow h(t-t_1)$$ impulse response delayed by t_1

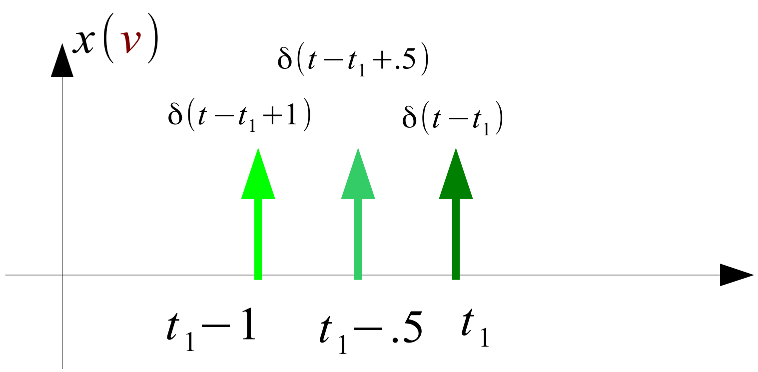
$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

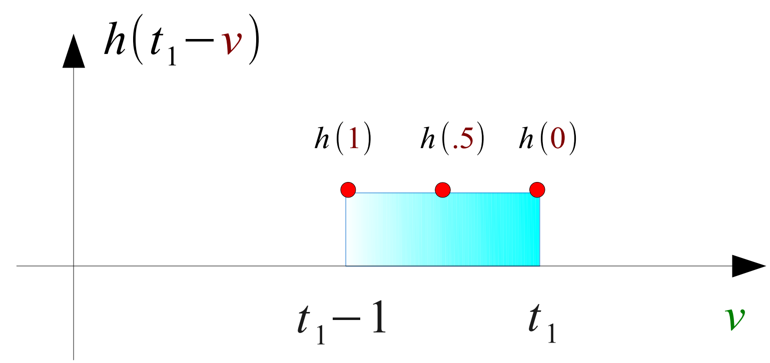
$$\Rightarrow y(t_1) = h(1) + h(.5) + h(0)$$

Computing $y(t_1)$: commutativity (1)

$y(t) = \int x(v)h(t-v)\,dv$		$y(t) = \int x(t-v)h(v)\,dv$
$= \int \delta(v-t_1+1)h(t-v)\,dv$	$\Rightarrow h(t-t_1+1) \Leftarrow$	$= \int \delta(t-v-t_1+1)h(v)\,dv$
$+ \int \delta(v-t_1+.5)h(t-v)\,dv$	$\Rightarrow h(t-t_1+.5) \Leftarrow$	$+ \int \delta(t-v-t_1+.5)h(v)\,dv$
$+ \int \delta(v-t_1)h(t-v)\,dv$	$\Rightarrow h(t-t_1) \Leftarrow$	$+ \int \delta(t-v-t_1)h(v)\,dv$

Flip and Shift h(t)

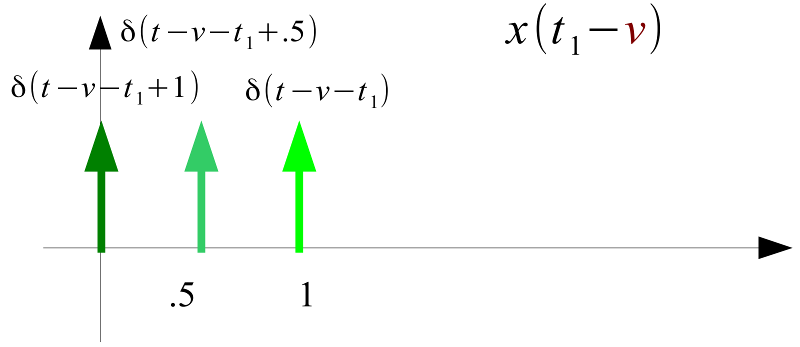


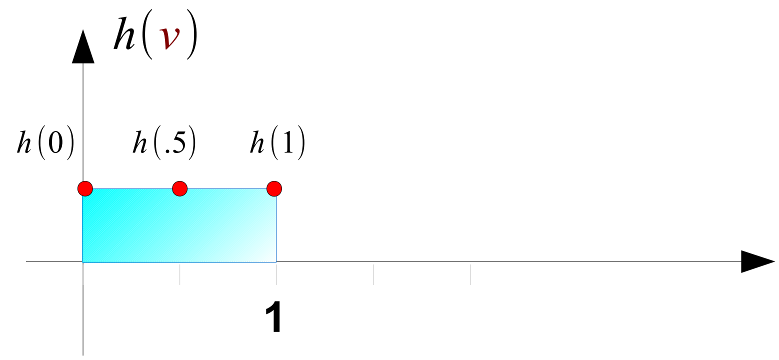


Computing $y(t_1)$: commutativity (2)

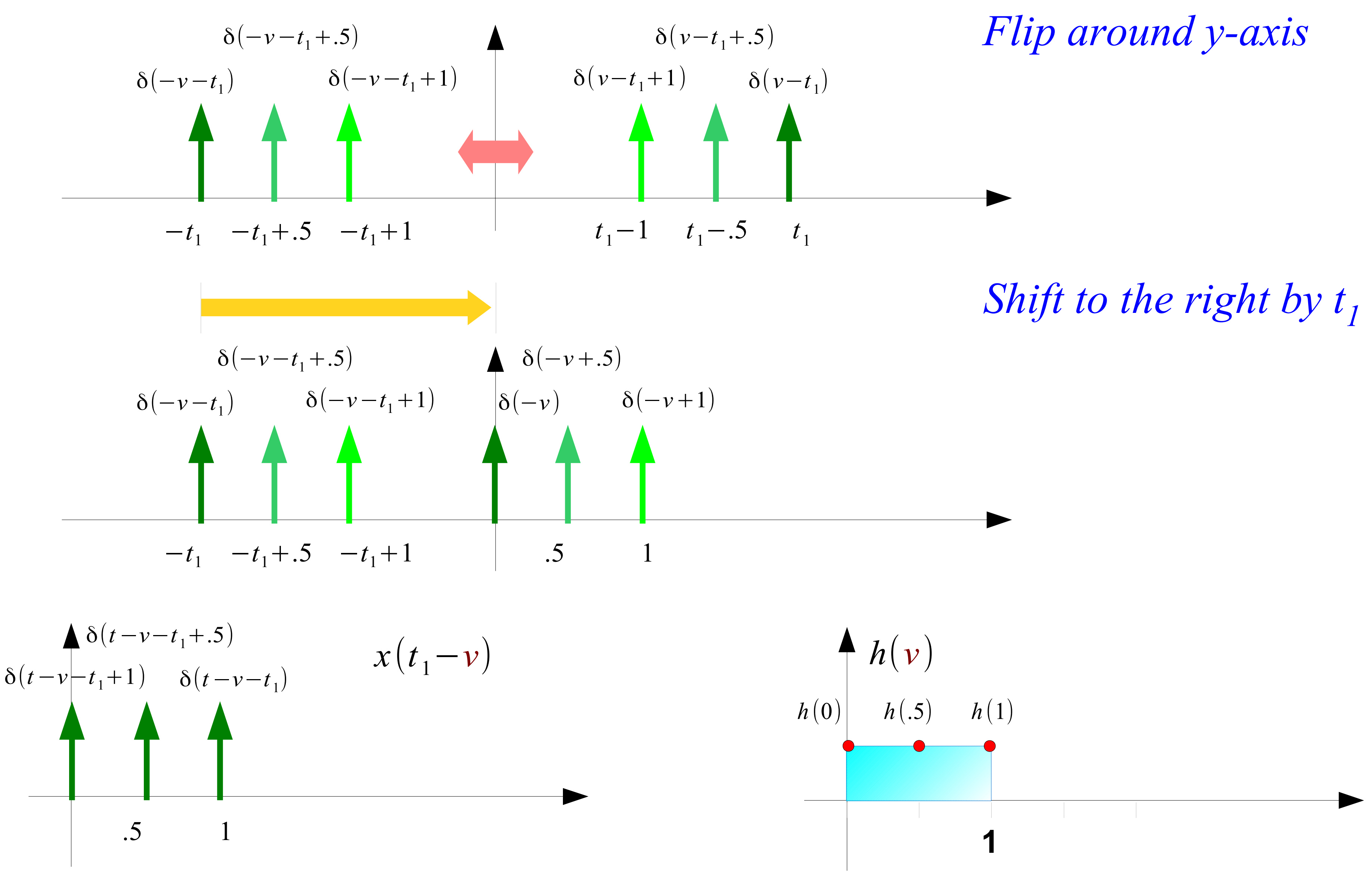
$y(t) = \int x(v) h(t-v)\, dv$		$y(t) = \int x(t-v) h(v)\, dv$
$= \int \delta(v-t_1+1) h(t-v)\, dv$	$h(t-t_1+1)$	$= \int \delta(t-v-t_1+1) h(v)\, dv$
$+ \int \delta(v-t_1+.5) h(t-v)\, dv$	$h(t-t_1+.5)$	$+ \int \delta(t-v-t_1+.5) h(v)\, dv$
$+ \int \delta(v-t_1) h(t-v)\, dv$	$h(t-t_1)$	$+ \int \delta(t-v-t_1) h(v)\, dv$

Flip and shift input x(t)

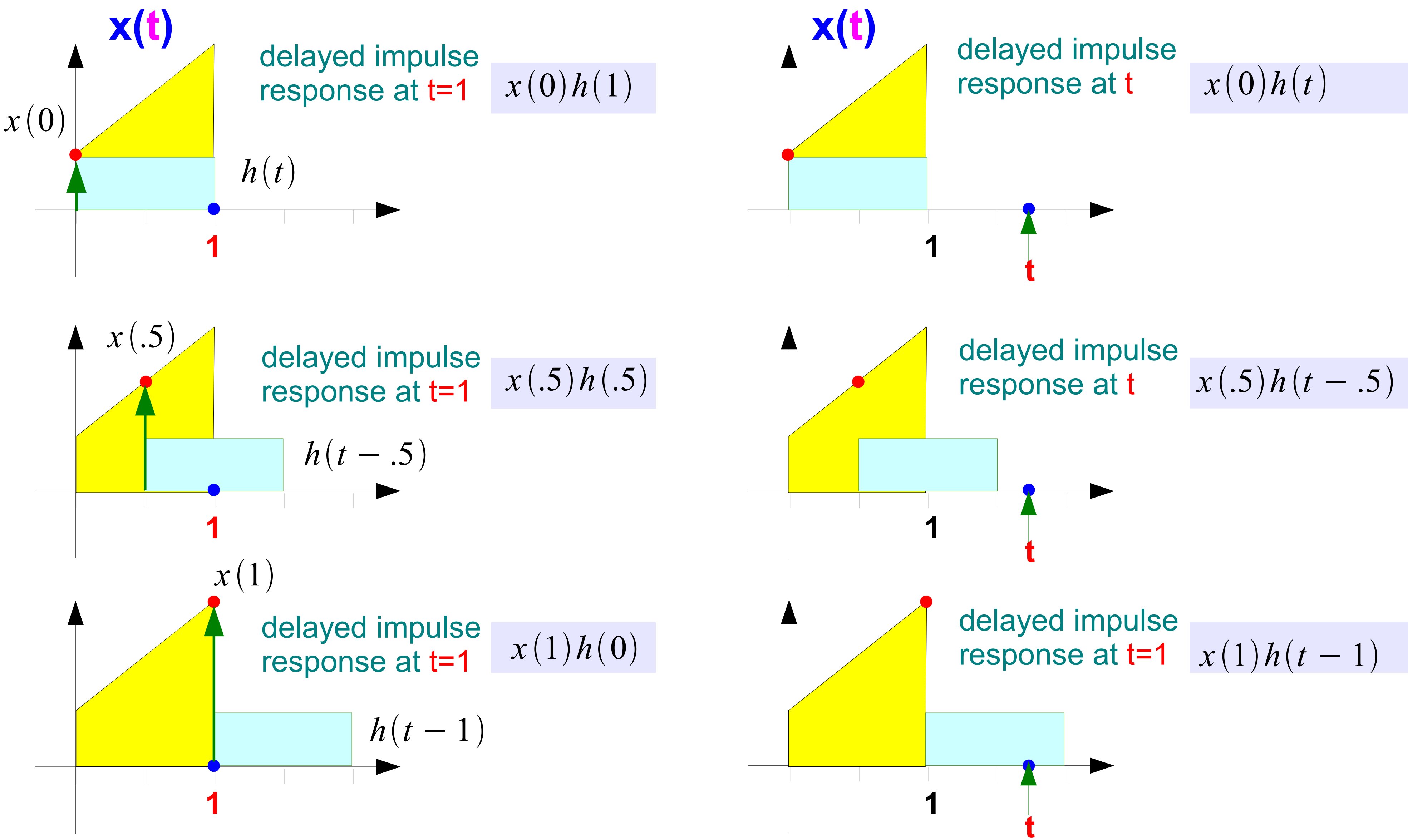




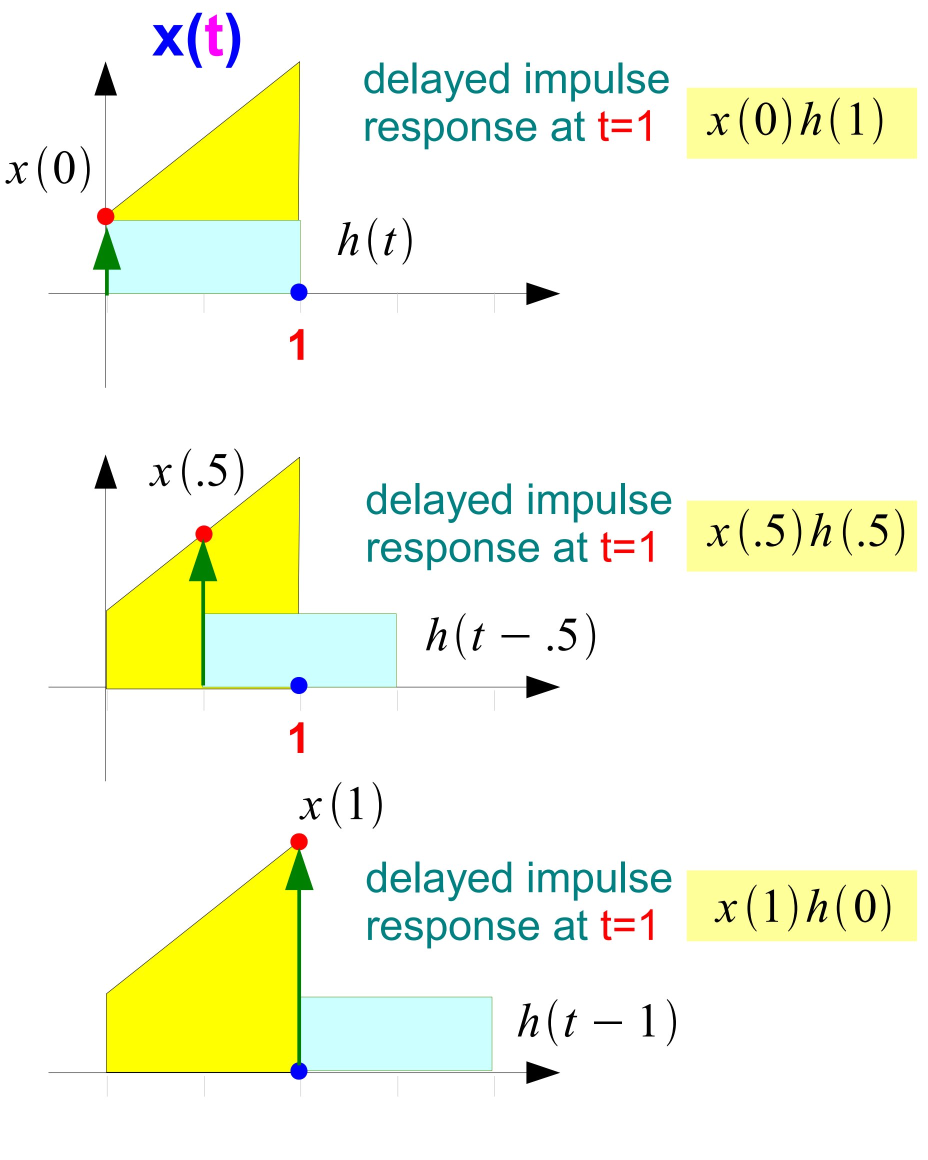
Computing $y(t_1)$: commutativity (3)

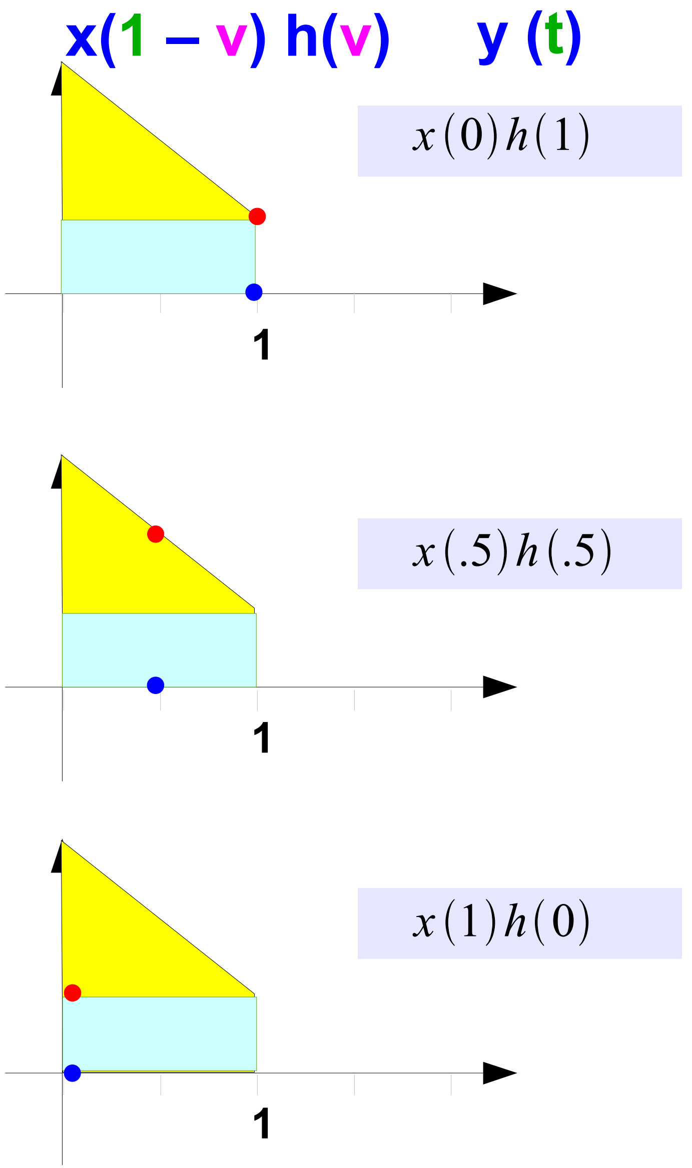


Computing y(1), y(t)



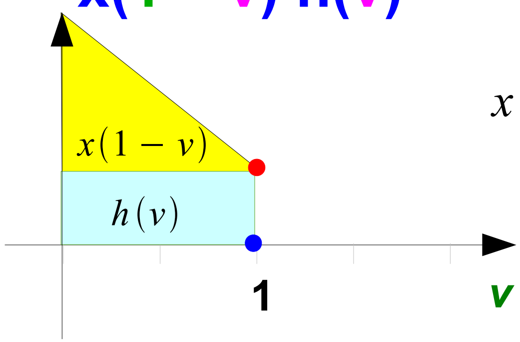
Computing y(1) : shift & flip x(t)



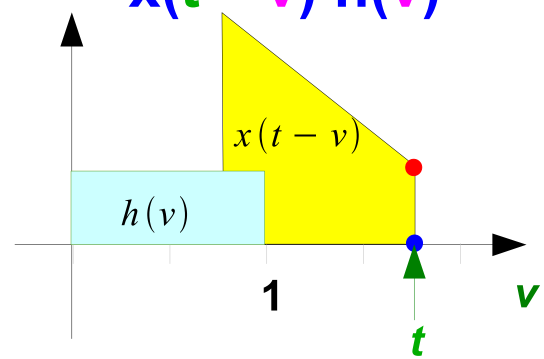


Computing y(t) : shift & flip x(t)

$x(1 - v)\, h(v)$ **$y(1)$** **$x(t - v)\, h(v)$** **$y(t)$**

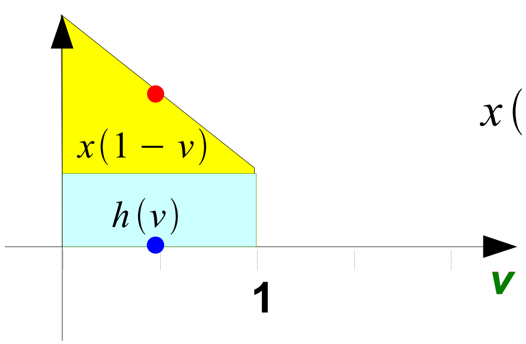


$x(0)h(1)$

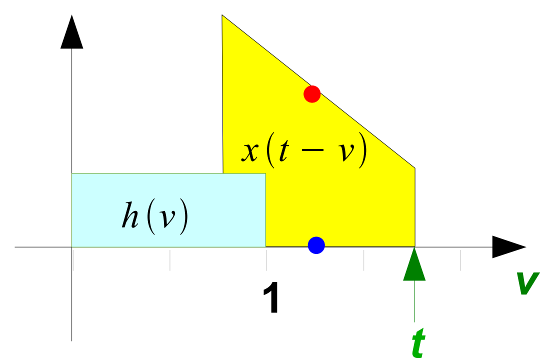


$x(t - t)h(t)$

$= x(0)h(t)$

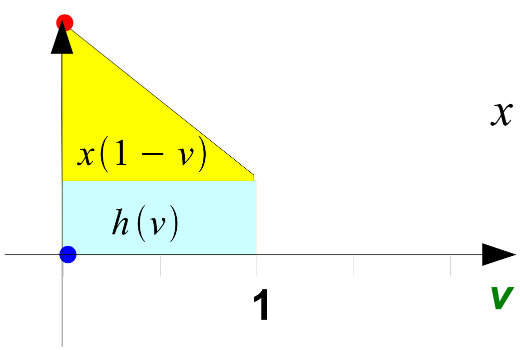


$x(.5)h(.5)$

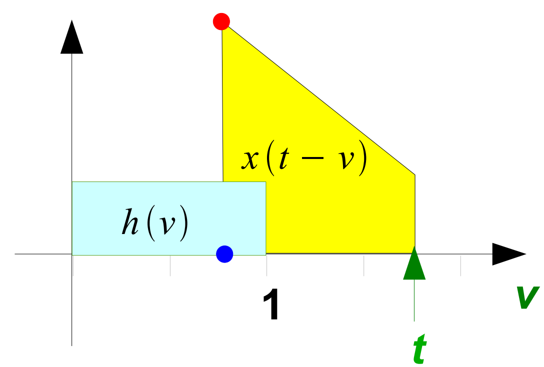


$x(t - t + .5)h(t - .5)$

$= x(.5)h(t - .5)$



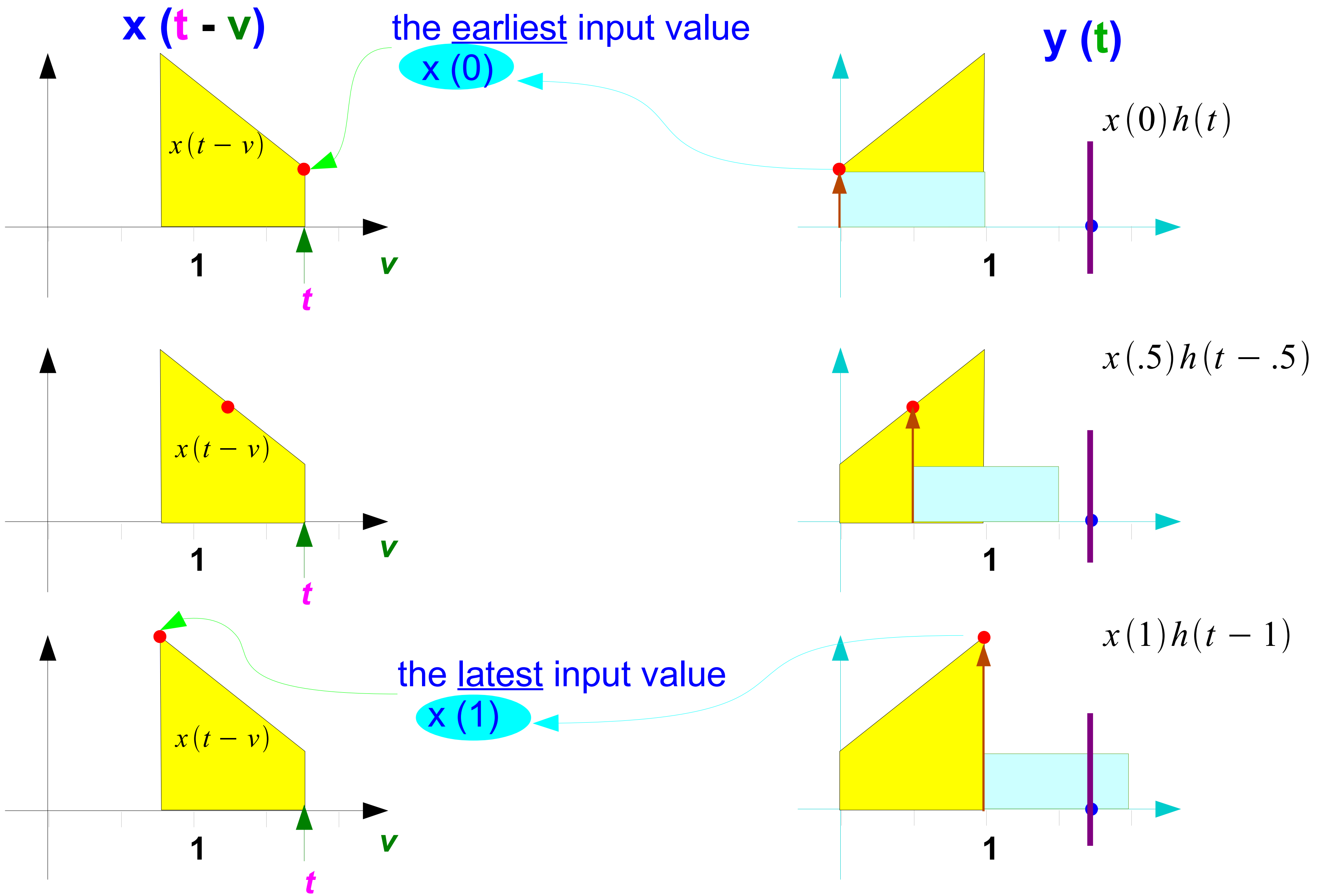
$x(1)h(0)$



$x(t - t + 1)h(t - 1)$

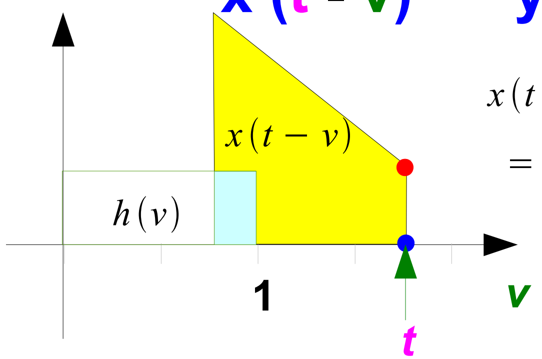
$= x(1)h(t - 1)$

Computing y(t) : the earliest and latest inputs

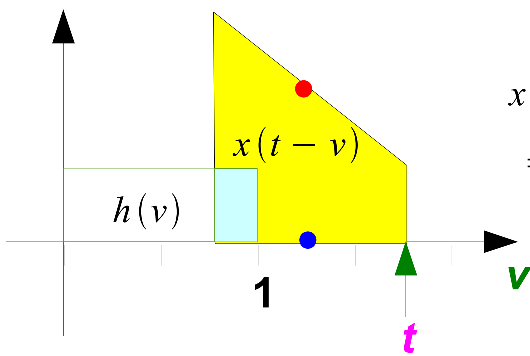


Computing y(t), y(s)

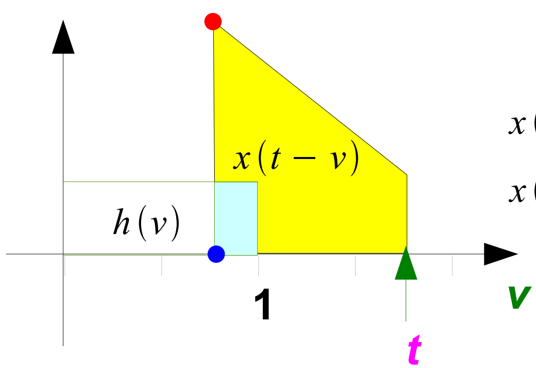
x (t - v) **y (t)**



$$x(t-t)h(t)$$
$$= x(0)h(t)$$

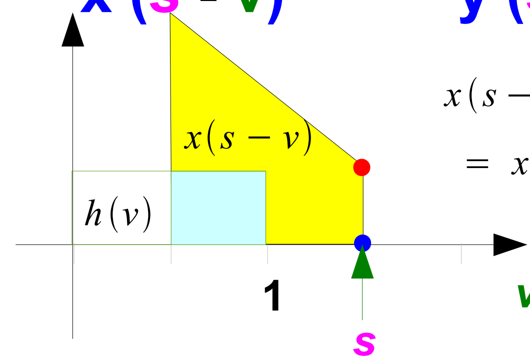


$$x(t-t+.5)h(t-.5)$$
$$= x(.5)h(t-.5)$$

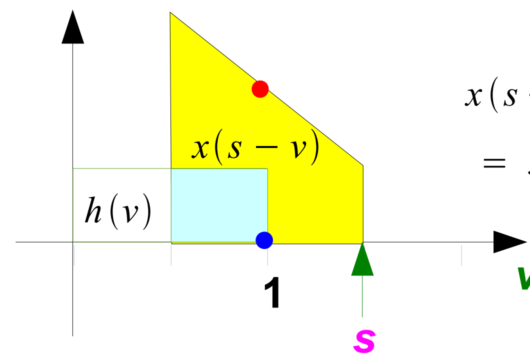


$$x(t-t+1)h(t-1)$$
$$x(1)h(t-1)$$

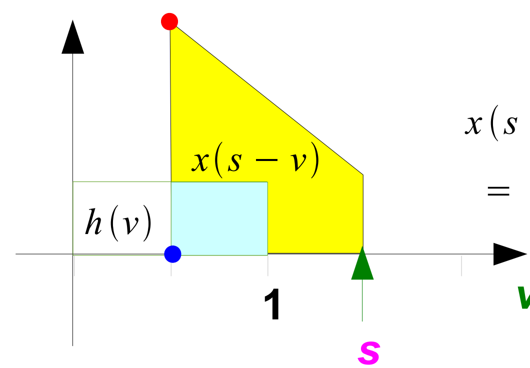
x (s - v) **y (s)**



$$x(s-s)h(s)$$
$$= x(0)h(s)$$

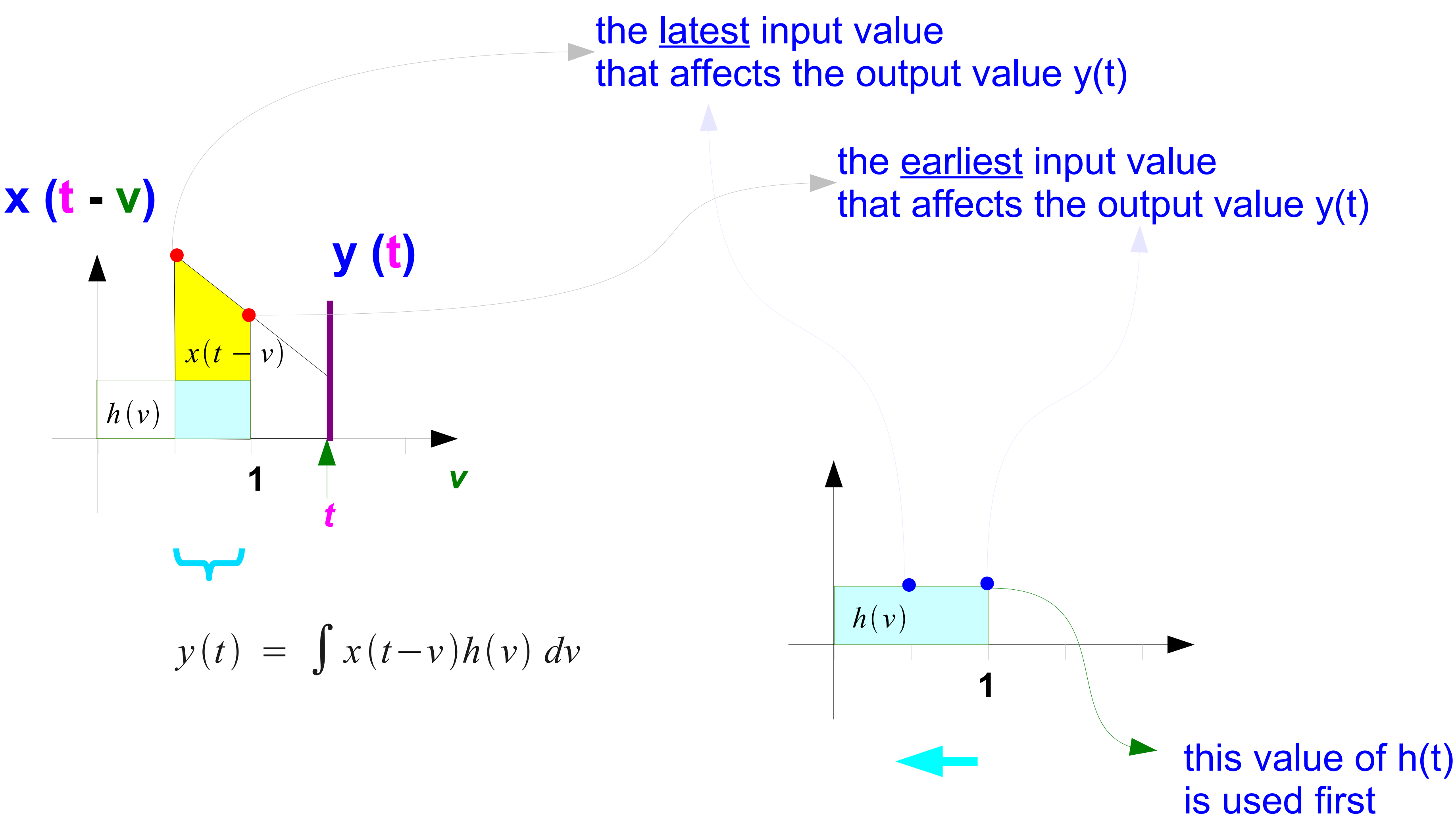


$$x(s-s+.5)h(s-.5)$$
$$= x(.5)h(s-.5)$$



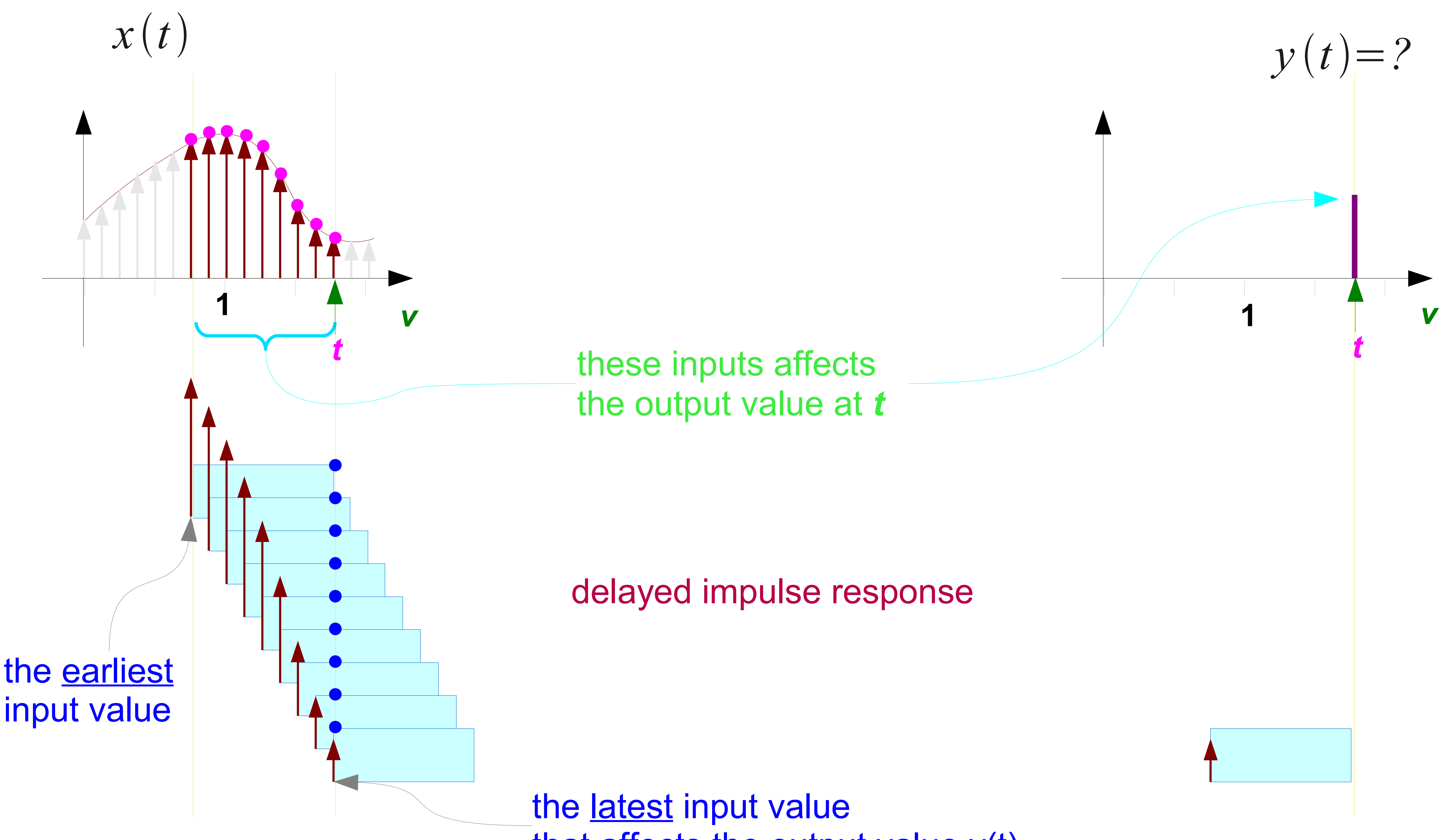
$$x(s-s+1)h(s-1)$$
$$= x(1)h(s-1)$$

Computing y(t) : earliest and latest inputs

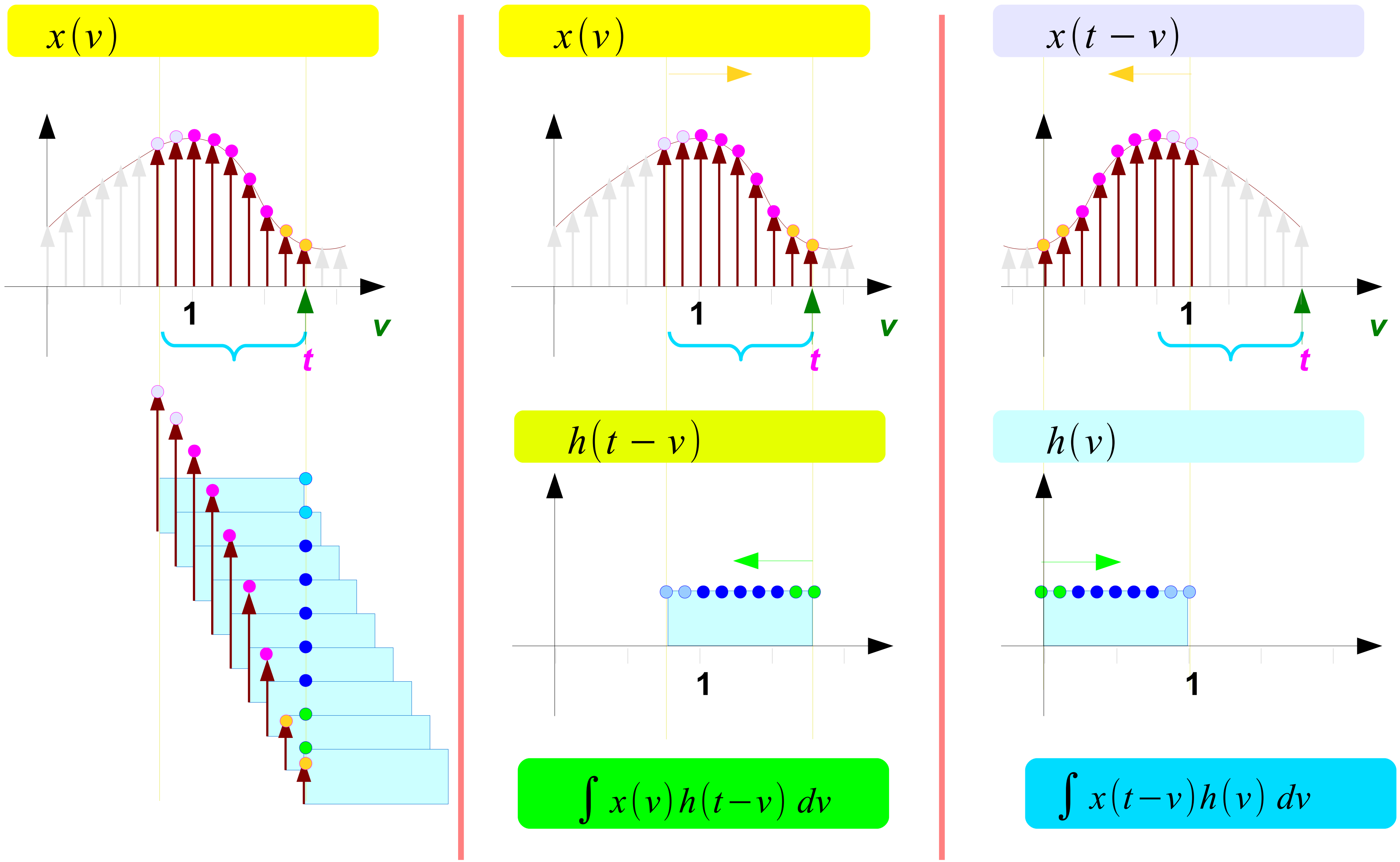


$$y(t) = \int x(t-v)h(v)\,dv$$

Computing y(t) : delayed impulse response



Computing y(t) : multiplication sequence



References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003