

# CORDIC Background (4A)

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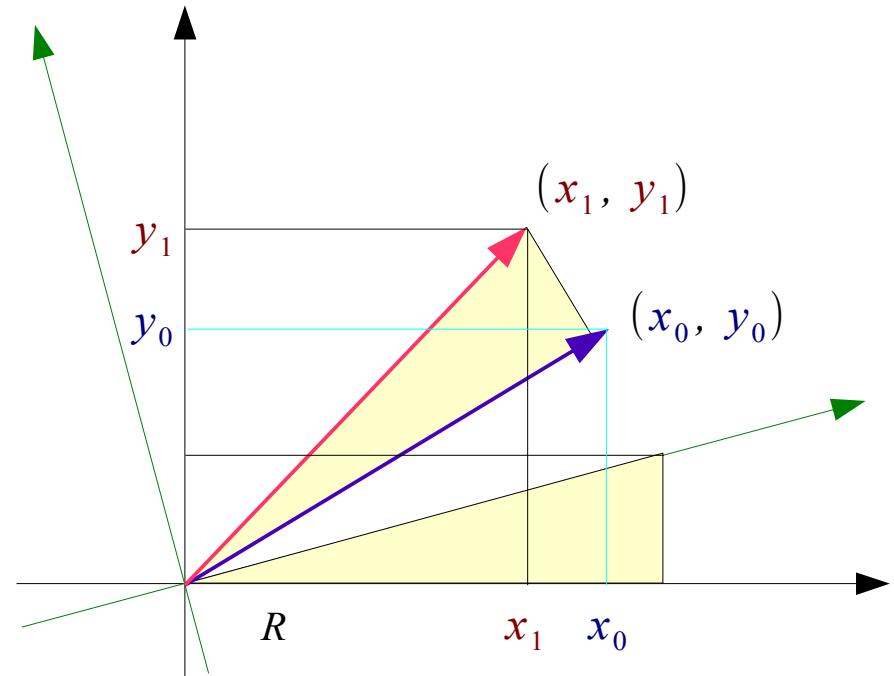
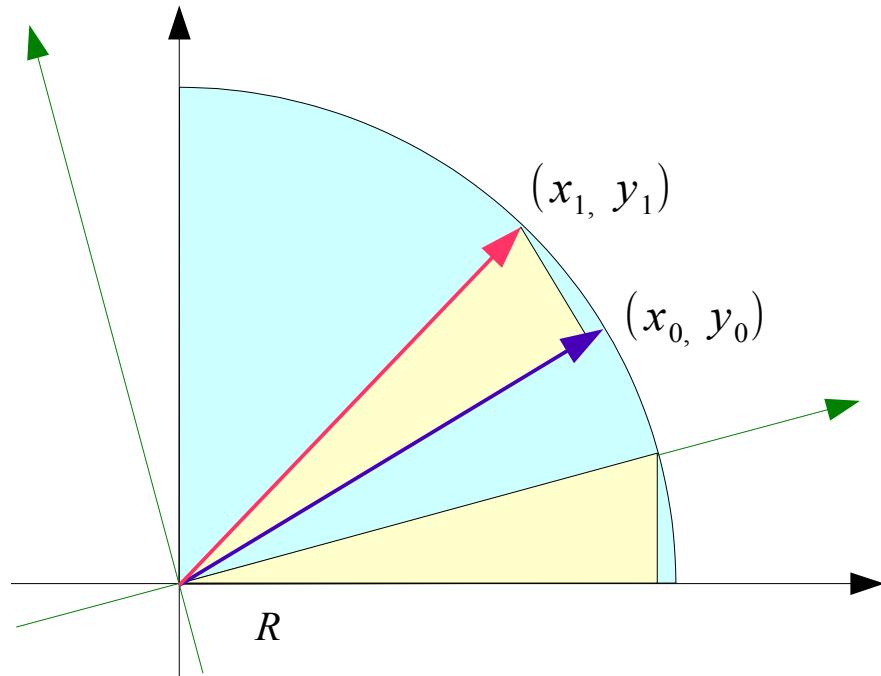
# CORDIC Background

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J. P. Deschamps, G. A. Bioul, G.D. Sutter, Synthesis of Arithmetic Circuits

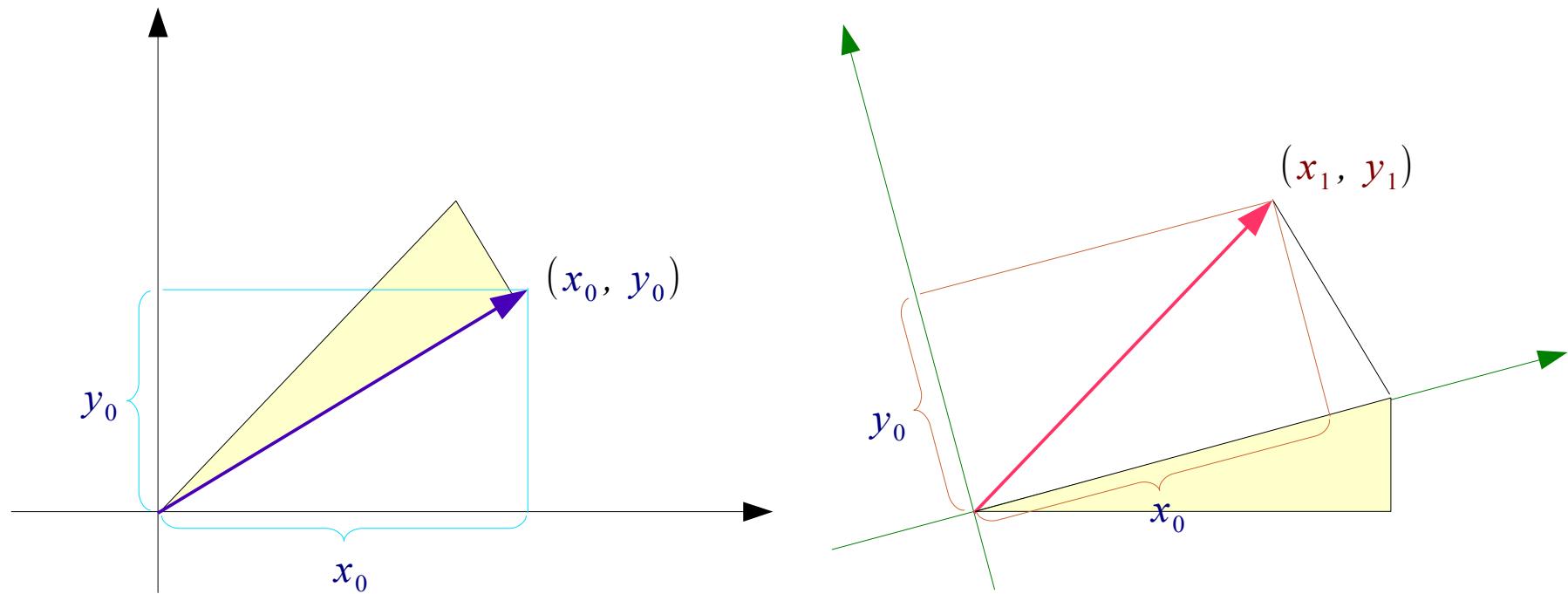
# Vector Rotation (1)

$$y_1 = x_0 \sin \alpha + y_0 \cos \alpha$$

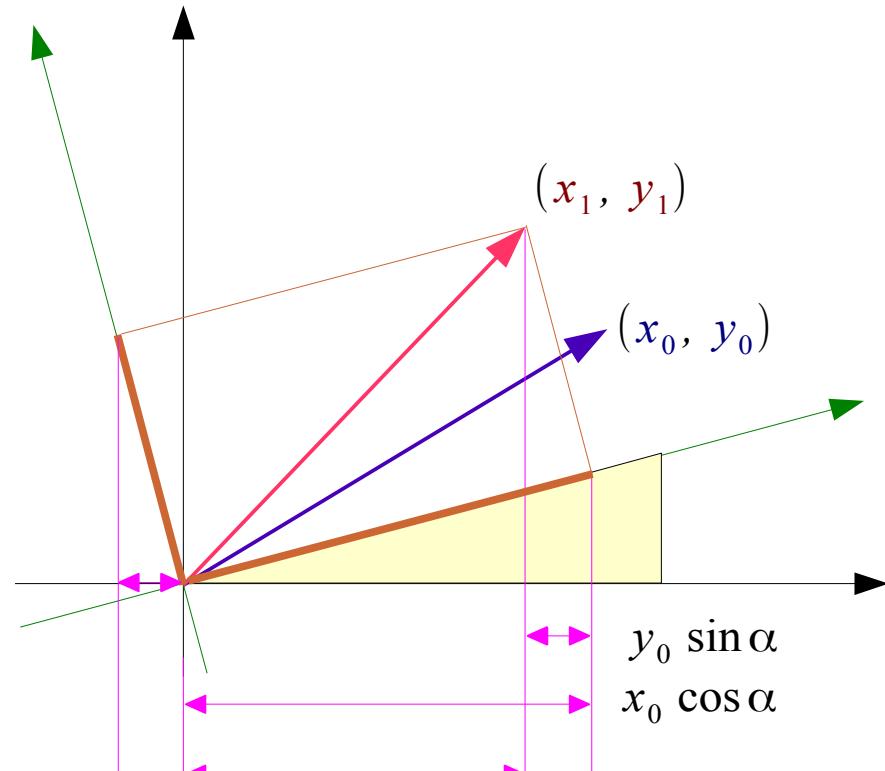


$$x_1 = x_0 \cos \alpha - y_0 \sin \alpha$$

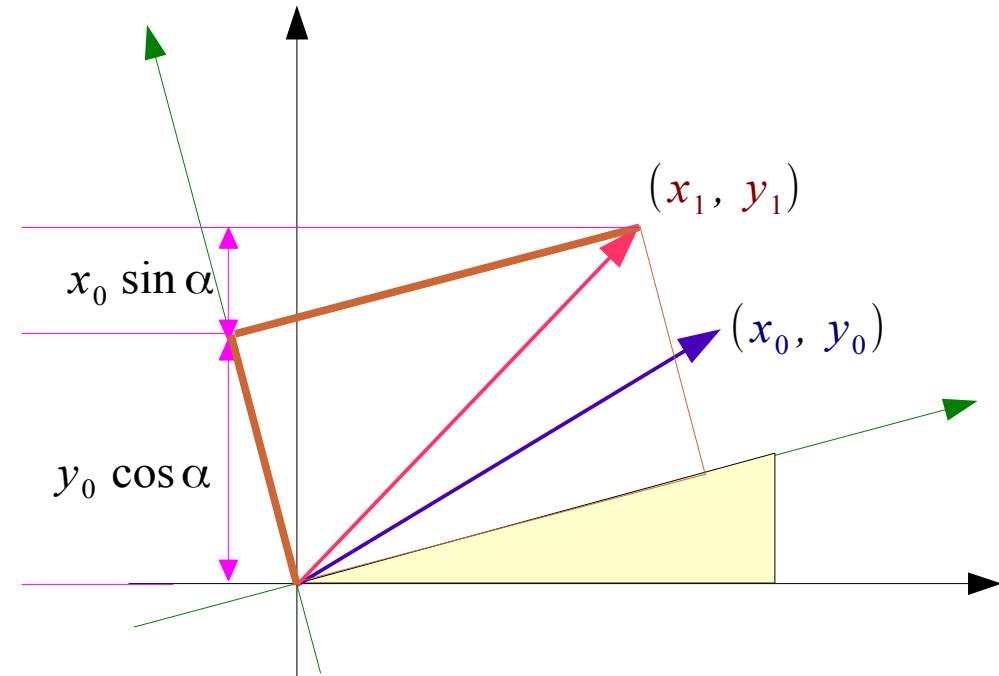
# Vector Rotation (2)



# Vector Rotation (3)

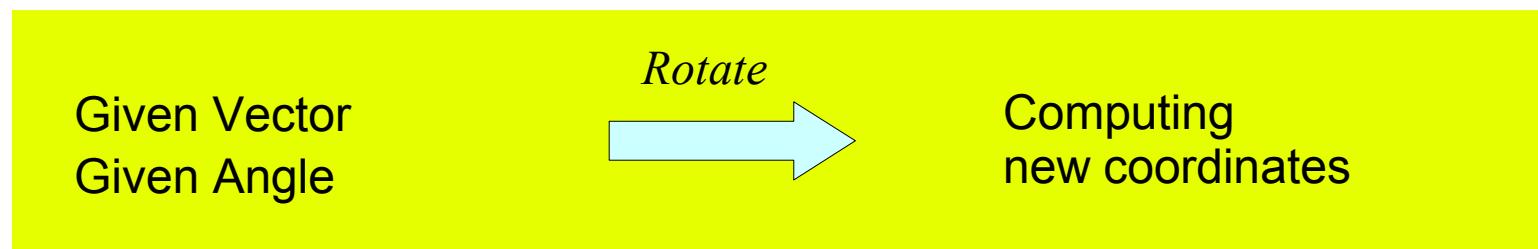


$$x_1 = x_0 \cos \alpha - y_0 \sin \alpha$$



$$y_1 = x_0 \sin \alpha + y_0 \cos \alpha$$

# CORDIC Iteration Equations



Given Unit Vector  
Given Angle  $\alpha$

Rotate

$$x = \cos \alpha$$
$$y = \sin \alpha$$

Given Vector  $(x_0, y_0)$   
Given Angle  $\alpha$

Rotate

$$x_1 = x_0 \cos \alpha - y_0 \sin \alpha$$
$$y_1 = x_0 \sin \alpha + y_0 \cos \alpha$$



$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \rightarrow \dots \rightarrow \begin{pmatrix} x_i \\ y_i \end{pmatrix} \rightarrow \begin{pmatrix} x_{i+1} \\ y_{i+1} \end{pmatrix} \rightarrow \dots \rightarrow \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

# CORDIC Iteration Equations – Pseudo-Rotation

$$\alpha_0 \rightarrow \dots \rightarrow \alpha_i \rightarrow \alpha_{i+1} \rightarrow \dots \rightarrow 0$$

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \rightarrow \dots \rightarrow \begin{pmatrix} x_i \\ y_i \end{pmatrix} \rightarrow \begin{pmatrix} x_{i+1} \\ y_{i+1} \end{pmatrix} \rightarrow \dots \rightarrow \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

Pseudo-rotation

$$x'_{i+1} = (x_i - y_i \tan \alpha_i)$$

$$y'_{i+1} = (x_i \tan \alpha_i + y_i)$$

$$x_{i+1} = x_i \cos \alpha_i - y_i \sin \alpha_i$$

$$y_{i+1} = x_i \sin \alpha_i + y_i \cos \alpha_i$$

$$x_{i+1} = \cos \alpha_i (x_i - y_i \tan \alpha_i)$$

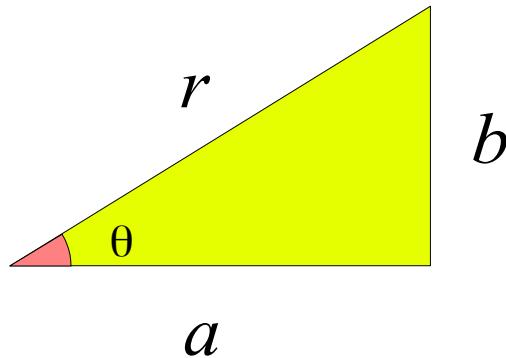
$$y_{i+1} = \cos \alpha_i (x_i \tan \alpha_i + y_i)$$

$$x_{i+1} = \frac{1}{\sqrt{1 + \tan^2 \alpha_i}} (x_i - y_i \tan \alpha_i)$$

$$y_{i+1} = \frac{1}{\sqrt{1 + \tan^2 \alpha_i}} (x_i \tan \alpha_i + y_i)$$

# $\cos \theta$

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$$r = \sqrt{a^2 + b^2}$$

$$\cos \theta = \frac{a}{r} = \frac{a}{\sqrt{a^2 + b^2}}$$

$$\cos \theta = \frac{a}{r}$$

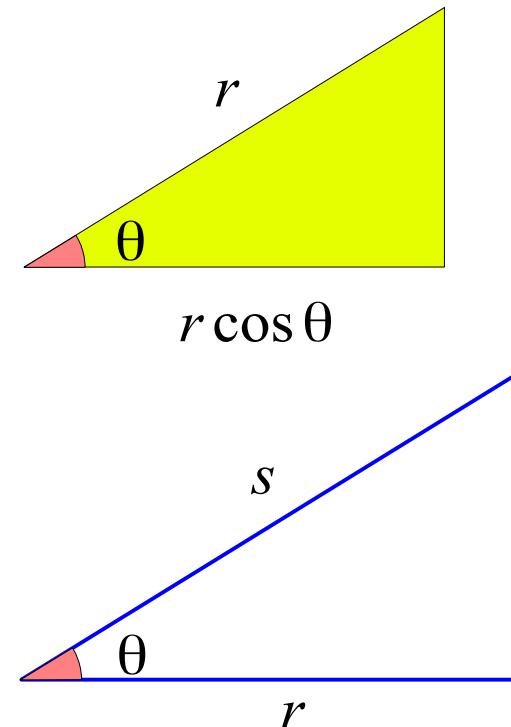
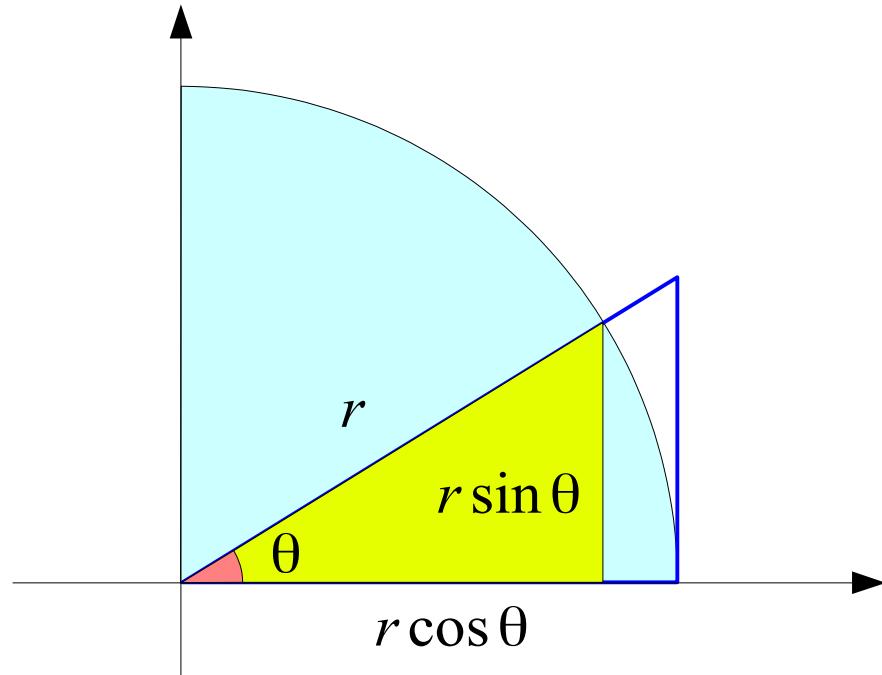
$$= \frac{1}{\sqrt{1 + (b/a)^2}}$$

$$\sin \theta = \frac{b}{r}$$

$$= \frac{1}{\sqrt{1 + \tan^2 \theta}}$$

$$\tan \theta = \frac{b}{a}$$

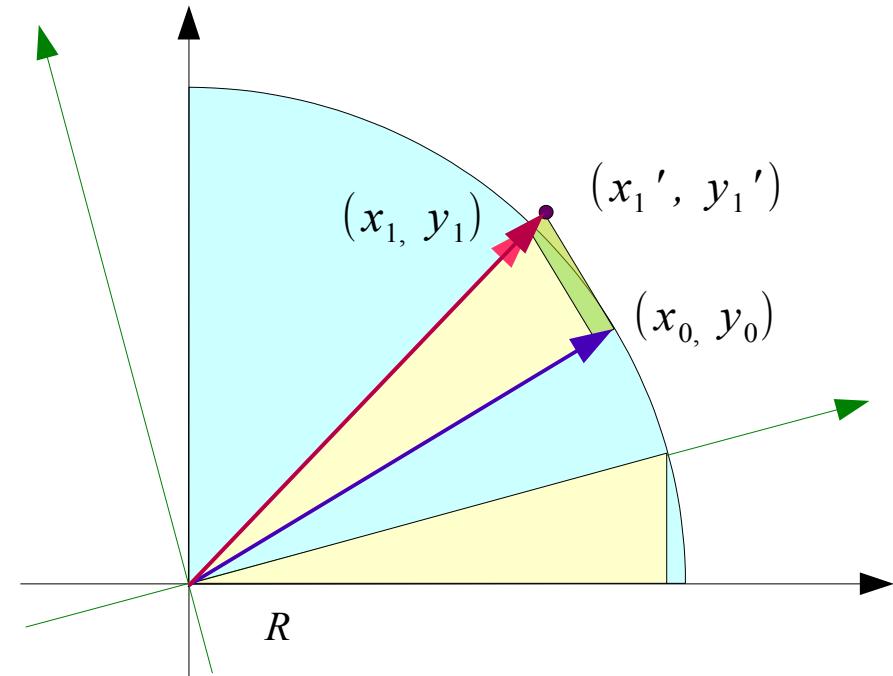
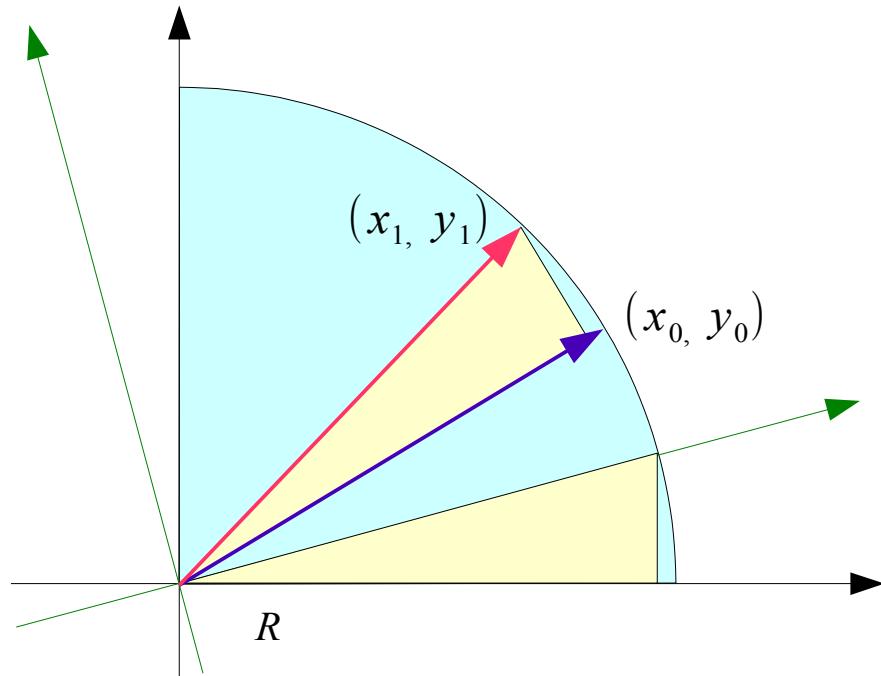
# Pseudo-rotation



$$r : r \cos \theta = s : r$$

$$s = \frac{r}{\cos \theta}$$

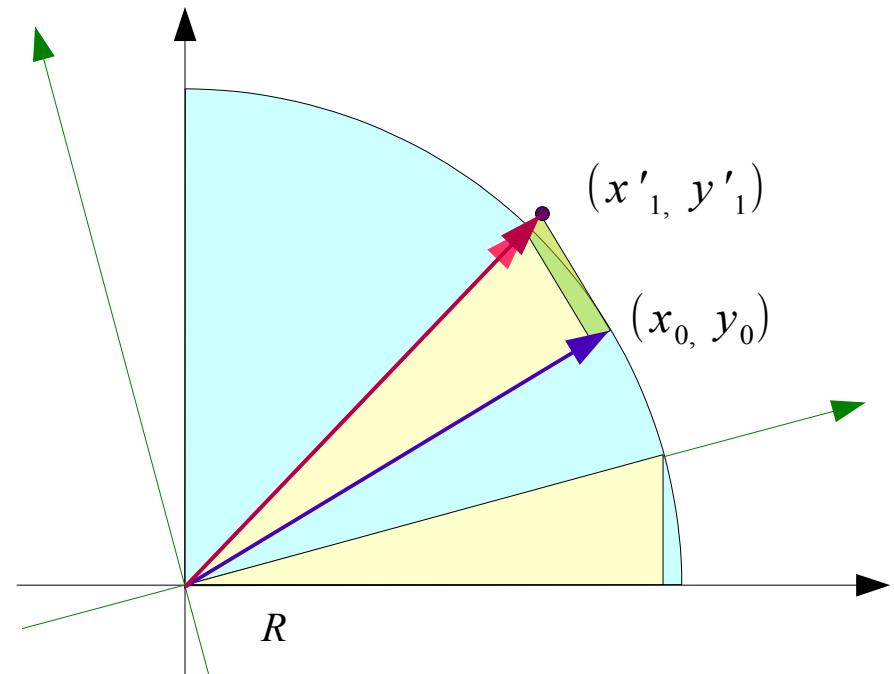
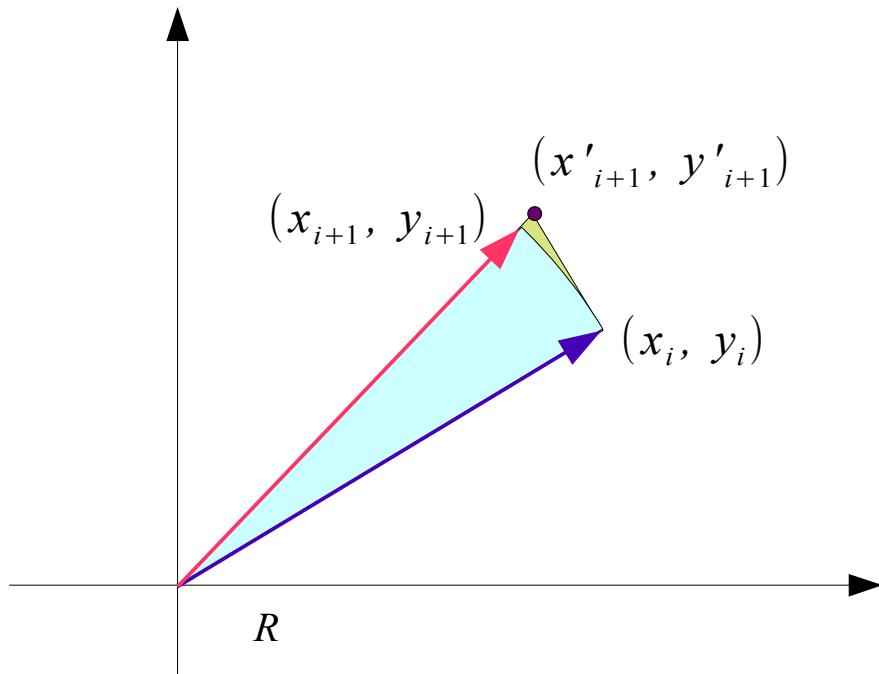
# Pseudo-rotation (1)



# Pseudo-rotation (2)

$$x'_{i+1} = x_{i+1} / \cos \alpha_i$$

$$y'_{i+1} = y_{i+1} / \cos \alpha_i$$



## References

- [1] <http://en.wikipedia.org/>
- [2] CORDIC FAQ, [www.dspguru.com](http://www.dspguru.com)
- [3] R. Andraka, A survey of CORDIC algorithms for FPGA based computers
- [4] J. S. Walther, A Unified Algorithm for Elementary Functions
- [5] J. P. Deschamps, G. A. Bioul, G.D. Sutter, Synthesis of Arithmetic Circuits