Complex Functions (1A)

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Cauchy-Riemann Condition

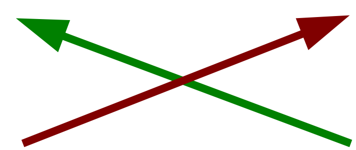
$f(z) = u(x,y) + i v(x,y)$: **analytic** in a region

⇒ in that region $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$ $\dfrac{\partial v}{\partial x} = -\dfrac{\partial u}{\partial y}$

$f(z) = u(x,y) + i v(x,y)$

$\dfrac{\partial}{\partial x}$ $\dfrac{\partial}{\partial y}$

$f(z) = u(x,y) + i v(x,y)$



$\dfrac{\partial}{\partial x}$ $\dfrac{\partial}{\partial y}$

Analytic

$$f(z) = u(x,y) + iv(x,y)$$

$u(x,y),\ v(x,y),\ \dfrac{\partial u}{\partial x},\ \dfrac{\partial u}{\partial y},\ \dfrac{\partial v}{\partial x},\ \dfrac{\partial v}{\partial y}$: continuous

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

$$\Rightarrow \quad f(z) = u(x,y) + iv(x,y)$$

: **analytic** at all points inside a region

not necessarily on the boundary

Derivatives

$f(z) = u(x,y) + iv(x,y)$: **analytic** in a region R

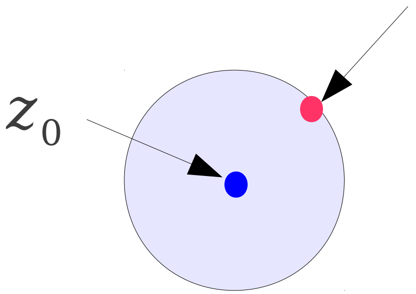
➡ derivatives of all orders at points inside region

$f'(z_0),\ f''(z_0),\ f^{(3)}(z_0),\ f^{(4)}(z_0),\ f^{(5)}(z_0),\ \cdots$

➡ Taylor series expansion about any point z_0 inside the region

The power series converges inside the circle about z_0

This circle extends to the nearest singular point



Laplace Equation

$f(z) = u(x,y) + iv(x,y)$: **analytic** in a region R

⇒ $u(x,y),\ v(x,y)$ satisfy Laplace's equation in the region

harmonic functions

$u(x,y),\ v(x,y)$ satisfy Laplace's equation in simply connected region

⇒ Real / imaginary part of an analytic function $f(z)$

Cauchy's Theorem

$f(z)$: **analytic** on and inside C ➡ $\oint_{around\,C} f(z)\,dz = 0$

simple closed curve

a continuously turning tangent

except possibly at a finite number of points

allow a finite number of corners (not smooth)

Cauchy's Integral Formula

$f(z)$: **analytic** on and inside simple close curve C

$$f(a) = \frac{1}{2\pi i} \oint \frac{f(z)}{z-a} dz$$

the value of $f(z)$

at a point $z = a$ inside C

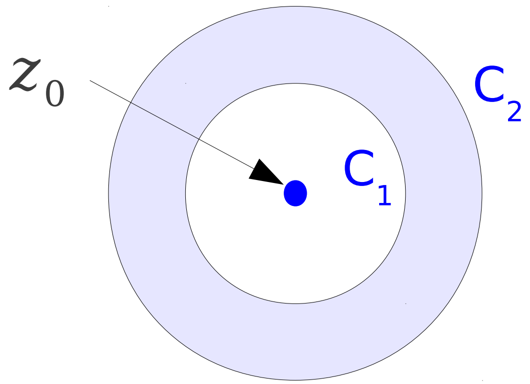
$$f(z) = \frac{1}{2\pi i} \oint \frac{f(w)}{w-z} dw$$

Laurent's Theorem

$f(z)$: **analytic** in the region R

between circles C_1, C_2

centered at z_0



$$f(z) = a_0 + a_1(z-z_0) + a_2(z-z_0)^2 + \cdots$$

$$+ \frac{b_1}{(z-z_0)} + \frac{b_2}{(z-z_0)^2} + \cdots$$

Principal part

: **convergent** in the region R

References

[1] http://en.wikipedia.org/
[2] http://planetmath.org/
[3] M.L. Boas, “Mathematical Methods in the Physical Sciences”