Prof. Dr. H. Brenner **Osnabrück WS 2011/2012**

Mathematics for natural sciences I

Exercise sheet 29

Warm-up-exercises

Exercise 29.1. Find all solutions to the ordinary differential equation

$$y' = -\frac{y}{t}\,.$$

Exercise 29.2. Find all solutions to the ordinary differential equation

$$y' = \frac{y}{t^2}\,.$$

Exercise 29.3. Find all solutions to the ordinary differential equation

$$y' = e^t y\,.$$

Exercise 29.4. Find the solutions of the inhomogeneous linear differential equation

$$y' = y + 7\,.$$

Exercise 29.5. Find the solutions to the inhomogeneous linear differential equation

$$y' = y + \frac{\sinh t}{\cosh^2 t}\,.$$

Exercise 29.6. Let

$$f : I \longrightarrow \mathbb{R}_+$$

be a differentiable function on the interval $I \subseteq \mathbb{R}$. Find a homogeneous linear ordinary differential equation for which f is a solution.

Exercise 29.7. Let

$$y' = g(t)y$$

be a homogeneous linear ordinary differential equation with a function g differentiable infinitely many times and let y be a differentiable solution.

a) Prove that y is also infinitely differentiable.

b) Let $y(t_0) = 0$ for a time-point t_0. Prove, using the formula

$$(f \cdot g)^{(n)} = \sum_{k=0}^{n} \binom{n}{k} f^{(k)} \cdot g^{(n-k)},$$

that $y^{(n)}(t_0) = 0$ for all $n \in \mathbb{N}$.

Exercise 29.8. a) Find all solutions for the ordinary differential equation ($t \in \mathbb{R}_+$)

$$y' = \frac{y}{t}.$$

b) Find all solutions for the ordinary differential equation ($t \in \mathbb{R}_+$)

$$y' = \frac{y}{t} + t^7.$$

c) Solve the initial value problem

$$y' = \frac{y}{t} + t^7 \text{ and } y(1) = 5.$$

The following statement is called the *superposition principle* for inhomogeneous linear differential equations. It says in particular that the difference of two solutions of an inhomogeneous linear differential equation is a solution of the corresponding homogeneous linear differential equation.

Exercise 29.9. Let $I \subseteq \mathbb{R}$ be a real interval and let

$$g, h_1, h_2 : I \longrightarrow \mathbb{R}$$

be functions. Let y_1 be a solution to the differential equation $y' = g(t)y + h_1(t)$ and let y_2 be a solution to the differential equation $y' = g(t)y + h_2(t)$. Prove that $y_1 + y_2$ is a solution to the differential equation

$$y' = g(t)y + h_1(t) + h_2(t).$$

Hand-in-exercises

Exercise 29.10. (2 points)

Confirm by computation that the function

$$y(t) = c\frac{\sqrt{t-1}}{\sqrt{t+1}}$$

found in Example 29.7 satisfies the differential equation

$$y' = y/(t^2 - 1).$$

Exercise 29.11. (3 points)

Find all solutions to the ordinary differential equation

$$y' = \frac{y}{t^2 - 3}\,.$$

Exercise 29.12. (5 points)

Solve the initial value problem

$$y' = \frac{t}{t^2 + 2} y \text{ with } y(3) = 7\,.$$

Exercise 29.13. (3 points)

Find the solutions to the inhomogeneous linear differential equation

$$y' = y + e^{2t} - 4e^{-3t} + 1\,.$$

Exercise 29.14. (5 points)

Find the solutions to the inhomogeneous linear differential equation

$$y' = \frac{y}{t} + \frac{t^3 - 2t + 5}{t^2 - 3}\,.$$