Fuzzy finite element method for one-dimensional steady state heat conduction problem

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to my

MOTHER and FATHER

with love

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A THESIS

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requirement of the award of the degree of

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DECLARATION

I hereby certify that the work which is being presented in the thesis entitled **"Fuzzy finite element method for one-dimensional steady state heat conduction problem"** in partial fulfillment of the requirement for the award of the degree of Master of Science, submitted in the Department of Mathematics, National Institute of Technology, Rourkela is an authentic record of my own work carried out under the supervision of Dr. S. Chakraverty.

The matter embodied in this thesis has not been submitted by me for the award of any other degree .

Date: (SARANGAM MAJUMDAR)

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

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Abstract

Traditional finite element method is a well-established method to solve various problems of science and engineering. Different authors have used various methods to solve governing differential equation of heat conduction problem. In this study, heat conduction in a circular rod has been considered which is made up of two different materials viz. aluminum and copper. In earlier studies parameters in the differential equation have been taken as fixed numbers which actually may not. Those parameters are found in general by some measurements or experiments. So the material properties are actually uncertain and may be considered to vary in an interval or as fuzzy and in that case complex interval arithmetic or fuzzy arithmetic has to be considered in the analysis. As such the problem is discretized into finite number of elements which depend on interval/fuzzy parameters. Representation of interval/fuzzy numbers may give the clear picture of uncertainty. Hence interval/fuzzy arithmetic is applied in the finite element method to solve a steady state heat conduction problem. Application of fuzzy finite element method in the said problem gives fuzzy system of linear equations in general. Here we have also proposed new methods to handle such type of fuzzy system of linear equations. Corresponding results are computed and has been reported here.

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Keywords

Finite Element Method (FEM)

Fuzzy, Interval Finite Element Method (IFEM)

Fuzzy Finite Element Method (FFEM)

Triangular Fuzzy Number (TFN)

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Nomenclature

K_x	Thermal Conduttivity
T_1, T_2, T_3, T_4, T_5	All of them are temperature at different nodes
q_x	Heat Flux
Q	Internal heat generation rate
U	Internal energy
A	Area of cross section.
$[K]$	Conductance matrix
$\{f_Q\}$	Nodal vector
$\{f_g\}$	Gradient boundary conditions at the element nodes
$\underline{x}$	Left value of the interval
$\bar{x}$	Right value of the interval

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Chapter -1

Introduction

Conduction of heat means transfer of heat energy within the body due to the temperature gradient. Heat spontaneously flows from a body having higher temperature to lower temperature. But in absence of external driving fluxes it approaches to thermal equilibrium. There are two types of conduction such as steady and unsteady state. Steady state conduction is a form of conduction where the temperature differences deriving by the conduction remains constant and it is independent of time. The steady state heat conduction problem is well known and its solution by exact method has been solved earlier [1]. The analysis may be difficult when heat transfer through a complicated domain. Various numerical techniques are proposed for these types of problems viz. finite difference method, finite volume method and finite element method [2, 3]. Magnus used finite difference method in his paper to model and solve the governing ground water flow rates [2], flow direction and hydraulic heads through an aquifer. Muhieddine described one dimensional phase change problem [3]. They have used vertex centered finite volume method to solve the problem. In view of the above literature, it reveals that the traditional finite element method may easily be used where the parameters or the values are exact i.e. in crisp form. But in actual practice the values may be in a region of possibility or we can say the values are uncertain. In general uncertainty may be found from limited knowledge where it is impossible to exactly describe the existing state, vagueness, no specificity and dissonance etc. or we can find it if the probabilistic description of sampling variables are not available. These be reduced by appropriate experiments still it also give the variability in the uncertain parameters give uncertain model predictions. Now the uncertainty can parameters. To handle such variability several probabilistic methods have been introduced. Monte Carlo method is used as an alternative method for this type of problem. Then finite element perturbation method is used by Nicolaï and De Baerdemaeker [4] and Nicolaï for heat conduction problem with uncertain physical parameters [5].

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Further Nicolaï found the temperature in heat conduction problem for randomly varying parameters with respect to time [6, 7]. They have used a variance propagation technique to calculate the mean and covariance of the temperatures. As we cannot find always a probabilistic description for the uncertain parameters some effort has been taken to tackle this type of problem. Hence we need the help of interval/fuzzy analysis for handling these types of data. We have used the interval arithmetic given in third section of this paper [8, 9, 10]. Then we present the traditional finite element procedure for solving the problem by taking these interval parameters [11, 12, 13]. But it is a tedious task to solve by this process. Again there is a chance of occurring weak solutions. Next the interval finite element technique is described for the said problem. Finite element method turns into a system of linear equations which is solved by a proposed iterative method. Further new methods have also been used to solve the interval system of linear equations. Next fuzzy parameters are handled using the alpha cut techniques and finally we apply the fuzzy finite element method [14, 15, 16] .The proposed techniques for system of interval and fuzzy linear system of equations are used to solve a steady state heat conduction problem [17]. Suparna Das, Diptiranjan Behera and S. Chakraverty recently find out a numerical solution of interval and fuzzy system of linear equation [18, 19]. Finally we have given the numerical results and compared the different methods.

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Chapter -2

Traditional Finite Element Formulation

The principle of conservation of energy viz.

$$E_{in} + E_{generated} = E_{increase} + E_{out} \tag{1}$$

satisfies the following heat transfer equation

$$q_x Adt + QAdxdt = \Delta U + \left(q_x + \frac{\partial q_x}{\partial x} dx \right) Adt \tag{2}$$

where, q_x is the heat flux across boundary, Q is the internal heat generation rate, U is the internal energy and A is the area of cross section.

The one dimension steady state heat conduction equation [17] may be written as

$$K_x \frac{d^2 T}{dx^2} + Q = 0. \tag{3}$$

Traditional finite element formulation [17] for equation (3) can easily be obtained as

$$[K]\{T\} = \{f_Q\} + \{f_g\} \tag{4}$$

where, $[K]$ is the conductance matrix, $\{f_Q\}$ is the nodal vector, $\{f_g\}$ is the gradient boundary conditions at the element nodes.

The stiffness matrix or conductance matrix for one element for traditional finite element method is given by $\frac{k_x A}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, where k_x is the thermal conductivity, A is the area of cross-section

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and l is the length of the cylinder. As such if the domain is divided into n elements, the form of stiffness matrix is $\begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ -1 & 2 & -1 & \ddots & \vdots \\ 0 & -1 & 2 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & -1 \\ 0 & \cdots & 0 & -1 & 2 \end{bmatrix}$.

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Chapter -3

Interval Arithmetic

The interval form of the parameters may be written as

$$[\underline{x}, \bar{x}] = \{x : x \in \Re, \underline{x} \le x \le \bar{x}\}$$

where $\underline{x}$ is the left value and $\bar{x}$ is the right value of the interval respectively. We define $m = \frac{\underline{x} + \bar{x}}{2}$ is the centre and $w = \bar{x} - \underline{x}$ is the width of the interval $[\underline{x}, \bar{x}]$.

Let $[\underline{x}, \bar{x}]$ and $[\underline{y}, \bar{y}]$ be two elements then the following arithmetic are well known [8]

(i) $[\underline{x}, \bar{x}] + [\underline{y}, \bar{y}] = [\underline{x} + \underline{y}, \bar{x} + \bar{y}]$

(ii) $[\underline{x}, \bar{x}] - [\underline{y}, \bar{y}] = [\underline{x} - \bar{y}, \bar{x} - \underline{y}]$

(iii) $[\underline{x}, \bar{x}] \times [\underline{y}, \bar{y}] = [\min\{\underline{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\underline{y}, \bar{x}\bar{y}\}, \max\{\underline{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\underline{y}, \bar{x}\bar{y}\}]$

(iv) $[\underline{x}, \bar{x}] \div [\underline{y}, \bar{y}] = [\min\{\underline{x} \div \underline{y}, \underline{x} \div \bar{y}, \bar{x} \div \underline{y}, \bar{x} \div \bar{y}\}, \max\{\underline{x} \div \underline{y}, \underline{x} \div \bar{y}, \bar{x} \div \underline{y}, \bar{x} \div \bar{y}\}]$

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Chapter -4

Fuzzy number and alpha cut

Let X denote a universal set. Then, the membership function μ_A by which a fuzzy set A is usually defined as the form $\mu_A : X \to [0,1]$, where $[0,1]$ denotes the interval of real numbers from 0 to 1. Such a function is called a membership function and the set defined by it is called a fuzzy set. A fuzzy number is a convex, normalized fuzzy set $A \subseteq R$ which is piecewise continuous and has the functional value $\mu_A(x) = 1$, where $x \in X$ at precisely one element. Different types of fuzzy numbers are there. These are triangular fuzzy number, trapezoidal fuzzy number and Gaussian fuzzy number etc. Here we have discussed the said problem using triangular fuzzy number only. The membership function for triangular fuzzy number is as below.

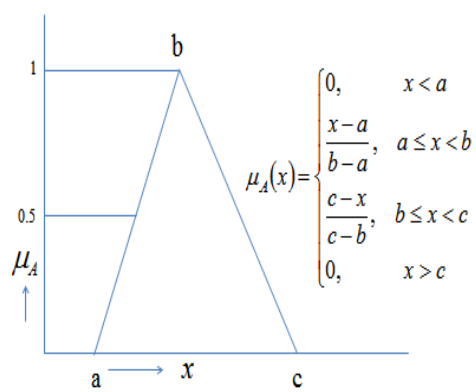

Fig.1. Triangular fuzzy number [a, b, c]

Alpha cut ($\alpha -$ cut) is an important concept of fuzzy set. Given $\alpha \in [0,1]$ then the alpha cut for above triangular fuzzy number $[a,b,c]$ may be written as $[a + (b-a)\alpha, c - (c-b)\alpha]$.

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Chapter -5

Interval and Fuzzy Finite Element Method

In this method the crisp values are replaced by interval/fuzzy and then proceeding like traditional finite element method we get a linear system of interval equations as given below.

$$\begin{bmatrix} [a_{11},b_{11}] & [a_{12},b_{12}] & \cdots & [a_{1n},b_{1n}] \\ [a_{21},b_{21}] & [a_{22},b_{22}] & \cdots & [a_{2n},b_{2n}] \\ \vdots & \vdots & \vdots & \vdots \\ [a_{n1},b_{n1}] & [a_{n2},b_{n2}] & \cdots & [a_{nn},b_{nn}] \end{bmatrix} \begin{Bmatrix} [x_1,y_1] \\ [x_2,y_2] \\ \vdots \\ [x_n,y_n] \end{Bmatrix} = \begin{Bmatrix} [c_1,d_1] \\ [c_2,d_2] \\ \vdots \\ [c_n,d_n] \end{Bmatrix} \tag{5}$$

where the member in the interval both of them either positive or negative.

The system of equations in equation (5) can be solved by direct elimination method which we generally do for the exact data. Now interval values are operated through the interval arithmetic rules. Because of the more computation involving in this procedure it is a difficult task to perform and the margin of uncertainty increases drastically. Hence we have to choose methods which give better results. To overcome the above difficulties (to a certain extent) in the following paragraph we propose two new methods.

Method-1

Equation (5) can be solved by iterative scheme like Gauss-Seidel numerical method. The new form of representation of Equation (5) is as follows

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$$[x_1, y_1]^{(k+1)} = \frac{[c_1, d_1] - [a_{12}, b_{12}][x_2, y_2]^{(k)} - [a_{13}, b_{13}][x_3, y_3]^{(k)} - \cdots - [a_{1n}, b_{1n}][x_n, y_n]^{(k)}}{[a_{11}, b_{11}]}$$

$$[x_2, y_2]^{(k+1)} = \frac{[c_2, d_2] - [a_{21}, b_{21}][x_1, y_1]^{(k)} - [a_{23}, b_{23}][x_3, y_3]^{(k)} - \cdots - [a_{2n}, b_{2n}][x_n, y_n]^{(k)}}{[a_{22}, b_{22}]}$$

$$\cdots$$

$$\cdots$$

$$\cdots$$

$$[x_n, y_n]^{(k+1)} = \frac{[c_n, d_n] - [a_{n1}, b_{n1}][x_1, y_1]^{(k)} - [a_{n2}, b_{n2}][x_2, y_2]^{(k)} - \cdots - [a_{n(n-)1}, b_{n(n-)1}][x_{n-1}, y_{n-1}]^{(k)}}{[a_{nn}, b_{nn}]}$$

(6)

The above procedure will be more efficient if we replace the vector $[x, y]^{(k)}$ in the right side of Equation (6) element by element. So Equation (6) can be written as

$$[x_1, y_1]^{(k+1)} = \frac{[c_1, d_1] - [a_{12}, b_{12}][x_2, y_2]^{(k)} - [a_{13}, b_{13}][x_3, y_3]^{(k)} - \cdots - [a_{1n}, b_{1n}][x_n, y_n]^{(k)}}{[a_{11}, b_{11}]}$$

$$[x_2, y_2]^{(k+1)} = \frac{[c_2, d_2] - [a_{21}, b_{21}][x_1, y_1]^{(k+1)} - [a_{23}, b_{23}][x_3, y_3]^{(k)} - \cdots - [a_{2n}, b_{2n}][x_n, y_n]^{(k)}}{[a_{22}, b_{22}]}$$

$$\cdots$$

$$\cdots$$

$$\cdots$$

$$[x_n, y_n]^{(k+1)} = \frac{[c_n, d_n] - [a_{n1}, b_{n1}][x_1, y_1]^{(k+1)} - [a_{n2}, b_{n2}][x_2, y_2]^{(k+1)} - \cdots - [a_{n(n-)1}, b_{n(n-)1}][x_{n-1}, y_{n-1}]^{(k)}}{[a_{nn}, b_{nn}]}$$

(7)

Method-2

The Equation (5) can be written as

$$\sum_{j=1}^{n} (a_{ij} + \alpha_{ij}) x_j = c_j + \beta_j, \quad i = 1,2,3,...,n$$

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where $\alpha_{ij} = \dfrac{b_{ij} - a_{ij}}{N}$ and $\beta_j = \dfrac{d_j - c_j}{N}$, $N \in [1, \infty)$.

(8)

Now using any crisp method we can solve the Equation (8) and the solution for this system of equations may be written as below

$$\left[\min\left\{\lim_{n\to 1} x_1, \lim_{n\to\infty} x_1\right\}, \max\left\{\lim_{n\to 1} x_1, \lim_{n\to\infty} x_1\right\}\right], \left[\min\left\{\lim_{n\to 1} x_2, \lim_{n\to\infty} x_2\right\}, \max\left\{\lim_{n\to 1} x_2, \lim_{n\to\infty} x_2\right\}\right],$$
$$\ldots, \left[\min\left\{\lim_{n\to 1} x_n, \lim_{n\to\infty} x_n\right\}, \max\left\{\lim_{n\to 1} x_n, \lim_{n\to\infty} x_n\right\}\right].$$

As mentioned earlier in case of fuzzy finite element methods we have to convert the fuzzy parameters into alpha cut set and then solve it by the proposed method as above. In this case we get a set of solutions for different values of alpha. Thus we get the solutions in term of triangular fuzzy number where the left values of the interval and the right values of the interval all together gives a pricewise continuous function.

The example problem has been solved by crisp finite element, interval finite element and fuzzy finite element method with the help of the above two proposed methods which are discussed in the subsequent sections.

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Chapter -6

Problem and Numerical Results

We have taken a circular rod (Fig.2) having an outside diameter of 60mm, length of 1m and perfectly insulated on its circumference. The left half of the cylinder is aluminum and the right half is copper having the material thermal conductivity $200W/m-^{\circ}C$ and $389W/m-^{\circ}C$ respectively. The extreme right end of the cylinder is maintained at a temperature of $80^{\circ}C$, while the left end is subjected to a heat input rate $4000W/m^2$ as given in [17].

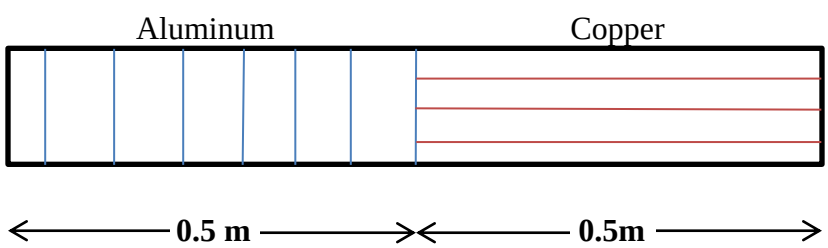

Fig.2.Model diagram for circular rod which is discretized into two equal parts having length 0.5meach.

Here the interval values for thermal conductivity viz. of aluminum has been taken as [199, 201] and for copper as [388,390]. The interval value for the heat input rate is taken as [3999, 4001]. The Problem is solved first by using the direct method and proposed iteration method (method-1) for two and four element discretization of circular rod. Corresponding results are given in different table format.

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6.1. FEM Formulation (using two elements)

(using equation (4) and the data mention in section 6) we get,

For element (1)

$$\begin{bmatrix} 1.13097 & -1.13097 \\ -1.13097 & 1.13097 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} = \begin{Bmatrix} 11.3097 \\ 0 \end{Bmatrix}$$

Similarly we obtain, For element (2)

$$\begin{bmatrix} 2.1997 & -2.1997 \\ -2.1997 & 2.1997 \end{bmatrix} \begin{Bmatrix} T_2 \\ 80 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0.0028q \end{Bmatrix}$$

Assembling the elements (1) and (2) we get ,

$$\begin{bmatrix} 1.13097 & -1.13097 & 0 \\ -1.13097 & 3.33067 & -2.1997 \\ 0 & -2.1997 & 2.1997 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ 80 \end{Bmatrix} = \begin{Bmatrix} 11.3097 \\ 0 \\ 0.0028q \end{Bmatrix}$$

Adjusting boundary condition the above turn out to

$$\begin{bmatrix} 1.13097 & -1.13097 \\ -1.13097 & 3.33067 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} = \begin{Bmatrix} 11.3097 \\ 175.976 \end{Bmatrix}$$

6.2.FEM Formulation (using four elements)

The matrix equation of for element 1 to 4 are obtained respectively as below

$$\begin{bmatrix} 2.262 & -2.262 \\ -2.262 & 2.262 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} = \begin{Bmatrix} 11.3097 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} 2.262 & -2.262 \\ -2.262 & 2.262 \end{bmatrix} \begin{Bmatrix} T_2 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

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$$\begin{bmatrix} 4.399 & -4.399 \\ -4.399 & 4.399 \end{bmatrix} \begin{Bmatrix} T_3 \\ T_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} 4.399 & -4.399 \\ -4.399 & 4.399 \end{bmatrix} \begin{Bmatrix} T_4 \\ 80 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0.0028q_5 \end{Bmatrix}$$

Then assembling the elements (1) , (2) ,(3) and (4) we have ,

$$\begin{bmatrix} 2.262 & -2.262 & 0 & 0 & 0 \\ -2.262 & 4.524 & -2.262 & 0 & 0 \\ 0 & -2.262 & 6.661 & -4.339 & 0 \\ 0 & 0 & -4.399 & 8.789 & -4.399 \\ 0 & 0 & 0 & -4.399 & 4.399 \end{bmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ 80 \end{pmatrix} = \begin{pmatrix} 11.3 \\ 0 \\ 0 \\ 0 \\ 0.0028q_5 \end{pmatrix}$$

Adjusting boundary condition we get,

$$\begin{pmatrix} 2.262 & -2.262 & 0 & 0 \\ -2.262 & 4.524 & -2.262 & 0 \\ 0 & -2.262 & 6.661 & -4.339 \\ 0 & 0 & -4.399 & 8.789 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{pmatrix} = \begin{pmatrix} 11.3 \\ 0 \\ 0 \\ 351.92 \end{pmatrix}$$

6.3.Interval Finite Element Formulation(using two elements)

For element (1)

$$\begin{bmatrix} [1.125,1.137] & [-1.137,-1.125] \\ [-1.137,-1.125] & [1.125,1.137] \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} = \begin{Bmatrix} [11.307,11.312] \\ [0,0] \end{Bmatrix}$$

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For element (2)

$$\begin{bmatrix} [2.194,2.205] & [-2.205,-2.194] \\ [-2.205,-2.194] & [2.194,2.205] \end{bmatrix} \begin{Bmatrix} T_2 \\ 80 \end{Bmatrix} = \begin{Bmatrix} 0 \\ [0.0028q_5, 0.0028q_5] \end{Bmatrix}$$

Assembling the elements (1) and (2) we get ,

$$\begin{bmatrix} [1.125,1.137] & [-1.137,-1.125] & [0,0] \\ [-1.137,-1.125] & [3.319,3.342] & [-2.205,-2.194] \\ [0,0] & [-2.205,-2.194] & [2.194,2.205] \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ 80 \end{Bmatrix} = \begin{Bmatrix} [11.307,11.312] \\ [0,0] \\ [0.0028q_5, 0.0028q_5] \end{Bmatrix}$$

Adjusting boundary condition we get,

$$\begin{bmatrix} [1.125,1.137] & [-1.137,-1.125] \\ [-1.137,-1.125] & [3.319,3.342] \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} = \begin{Bmatrix} [11.307,11.312] \\ [175.52,176.4] \end{Bmatrix}$$

6.4.Interval Finite Element Formulation(using fourelements)

The matrix equation of for element 1 to 4 are obtained respectively as below

$$\begin{bmatrix} [2.252,2.272] & [-2.272,-2.252] \\ [-2.272,-2.252] & [2.252,2.272] \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} = \begin{Bmatrix} [11.307,11.312] \\ [0,0] \end{Bmatrix}$$

$$\begin{bmatrix} [2.252,2.272] & [-2.272,-2.252] \\ [-2.272,-2.252] & [2.252,2.272] \end{bmatrix} \begin{Bmatrix} T_2 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} [0,0] \\ [0,0] \end{Bmatrix}$$

$$\begin{bmatrix} [4.388,4.411] & [-4.411,-4.388] \\ [-4.411,-4.388] & [4.388,4.411] \end{bmatrix} \begin{Bmatrix} T_3 \\ T_4 \end{Bmatrix} = \begin{Bmatrix} [0,0] \\ [0,0] \end{Bmatrix}$$

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$$\begin{bmatrix} [4.388,4.411] & [-4.411,-4.388] \\ [-4.411,-4.388] & [4.388,4.411] \end{bmatrix} \begin{Bmatrix} T_4 \\ 80 \end{Bmatrix} = \begin{Bmatrix} [0,0] \\ [0.0028q_5, 0.0028q_5] \end{Bmatrix}$$

Assembling the elements (1) , (2) ,(3) , (4) and adjusting boundary conditions we get,

$$\begin{pmatrix} [2.251,2.273] & [-2.273,-2.251] & [0,0] & [0,0] \\ [-2.273,-2.251] & [4.502,4.546] & [-2.273,-2.251] & [0,0] \\ [0,0] & [-2.273,-2.251] & [6.639,6.684] & [-4.411,-4.388] \\ [0,0] & [0,0] & [-4.411,-4.388] & [8.776,8.822] \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{pmatrix} = \begin{pmatrix} [11.3069,11.3126] \\ [0,0] \\ [0,0] \\ [351.04,352.88] \end{pmatrix}$$

Table 1. Comparison of results using traditional and proposed iteration method.

Temperatures (in degree Celsius)	Usual method			Method-1		
	Left	Right	Centre	left	right	Centre
T_1	87.83	103.56	95.695	89.01	101.40	95.205
T_2	83.58	97.67	90.625	84.43	96.04	90.235
T_3	80.82	90.06	85.44	81.14	89.28	85.21
T_4	80.20	85.26	82.73	80.15	85.58	82.865
T_5	80	80	80	80	80	80

Next two element and four element discretization have been considered. The proposed method (method 2) has been used to solve the final linear equations obtained from the finite element analysis. Corresponding results from Method 2 along with the results from traditional finite

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element method for crisp values are presented in Tables 2 to 5 respectively. Results of Table 2 and Table 3 are obtained for two and four element discretization respectively by taking the interval values for thermal conductivity only. Then both thermal conductivity and heat input rate are taken as interval values at a time for the analysis and the results with two and four element discretization respectively are given in Tables 4 and 5.**Table 2.** Results from two element discretization with thermal conductivity in interval form

Temperatures (in degree Celsius)	Traditional Finite element method (crisp)	Method-2, Interval finite element method		
		Left	Right	Centre
T_1	95.1414	95.0785	95.2049	95.1417
T_2	85.1414	85.1282	85.1546	85.1414
T_3	80	80	80	80

Table 3. Results from four element discretization with thermal conductivity in interval form

Temperatures (in degree Celsius)	Traditional Finite element method (crisp)	Method-2, Interval finite element method		
		Left	Right	Centre
T_1	95.2980	95.0785	95.2049	95.1417
T_2	90.3024	90.1033	90.1798	90.14155
T_3	85.3068	85.1282	85.1546	85.1414
T_4	82.7380	82.5641	82.5773	82.5707
T_5	80	80	80	80

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Table 4. Results from two element discretization with thermal conductivity and heat input rate both in interval form

Temperatures (in degree Celsius)	Traditional Finite element method (crisp)	Method-2, Interval finite element method		
		Left	Right	Centre
T_1	95.1414	95.0822	95.2011	95.14165
T_2	85.1414	85.1295	85.1534	85.14145
T_3	80	80	80	80

Table 5. Results from four element discretization with thermal conductivity and heat input rate both in interval form

Temperatures (in degree Celsius)	Traditional Finite element method (crisp)	Method-2, Interval finite element method		
		Left	Right	Centre
T_1	95.2980	95.0822	95.2011	95.14165
T_2	90.3024	90.1059	90.1772	90.14155
T_3	85.3068	85.1295	85.1534	85.14145
T_4	82.7380	82.5647	82.5767	82.5707
T_5	80	80	80	80

Now the values of thermal conductivity for aluminum and copper are taken in interval form with large width viz. [197.5, 202.5] and [386.5, 391.5] respectively and the heat input rate as [3997.5, 4002.5]. Obtained results with four element discretization are incorporated in Table 6.

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Table 6. Result from four element discretization with large width for thermal conductivity of aluminum and copper heat input rate

Temperatures (in degree Celsius)	Traditional Finite element method (crisp)	Method-2, Interval finite element method		
		Left	Right	Centre
T_1	95.2980	94.9945	95.2917	95.1431
T_2	90.3024	90.0531	90.2315	90.1423
T_3	85.3068	85.1117	85.1714	85.14155
T_4	82.7380	82.5559	82.5857	82.5708
T_5	80	80	80	80

Table7. Comparison /summery of methods/results

Temperatures (in degree Celsius)	Traditional Finite element method (crisp)(for two element discretization)	Traditional Finite element method (crisp) (for four element discretization) **(series1)**	Interval analysis	Interval analysis **(series 2)**	Interval analysis **(series 3)**	Interval analysis **(series 4)**
-	-	-	Usual method(for two element discretization)	Usual method(for two element discretization	Method-1	Method-2
T_1	95.1414	95.2980	95.1756	95.695	95.205	95.1431
T_2	-	90.3024	-	90.625	90.235	90.1423
T_3	85.1414	85.3068	85.16045	85.44	85.21	85.14155
T_4	-	82.7380	-	82.73	82.865	82.5708
T_5	80	80	80	80	80	80

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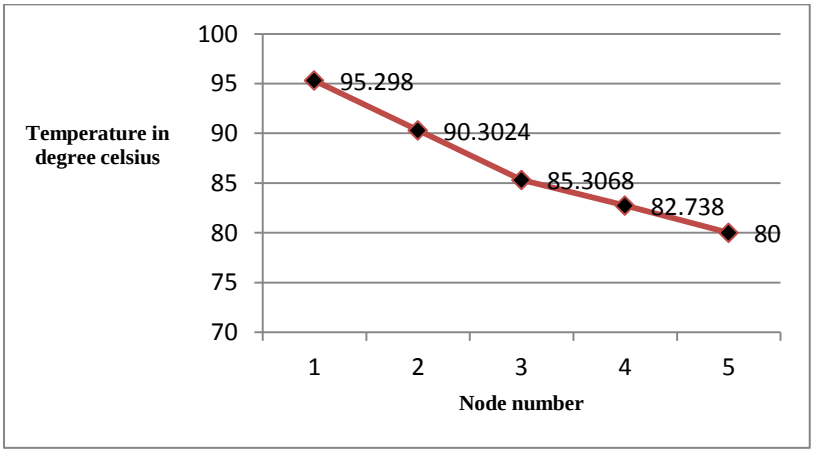

Fig.3.Graph for traditional finite element method (crisp) (for four element discretization)

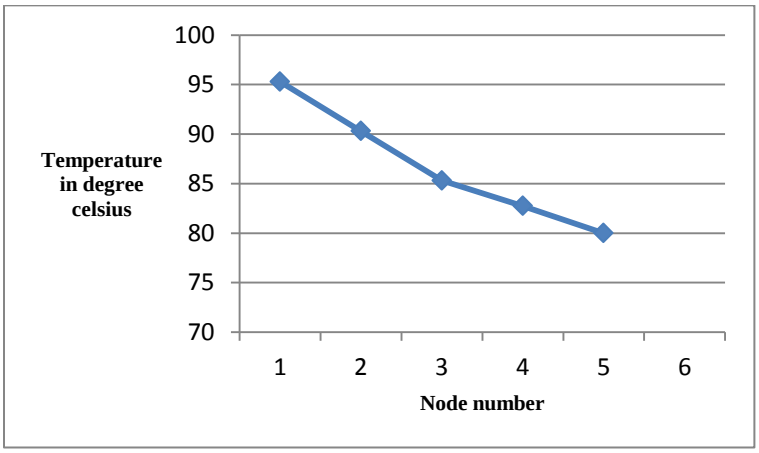

Fig.4.Graph for usual method (for four element discretization)

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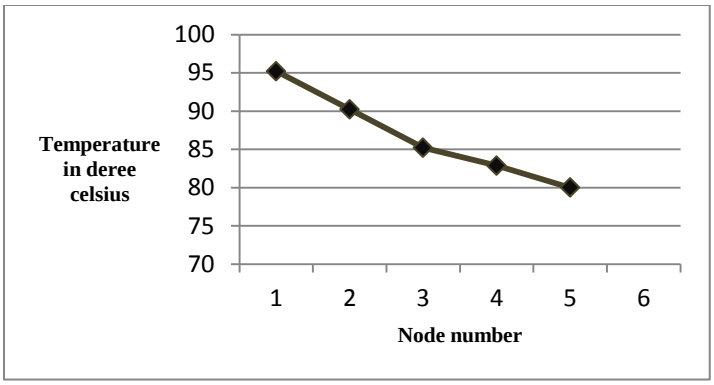

Fig.5.Graph for method-1 (for four element discretization)

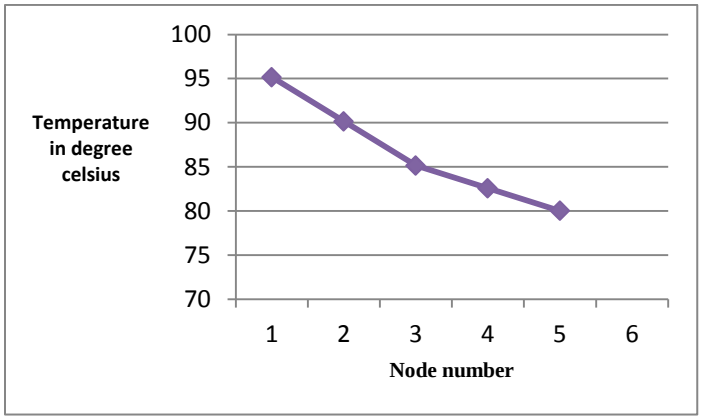

Fig.6.Graph for method-2 (for four element discretization)

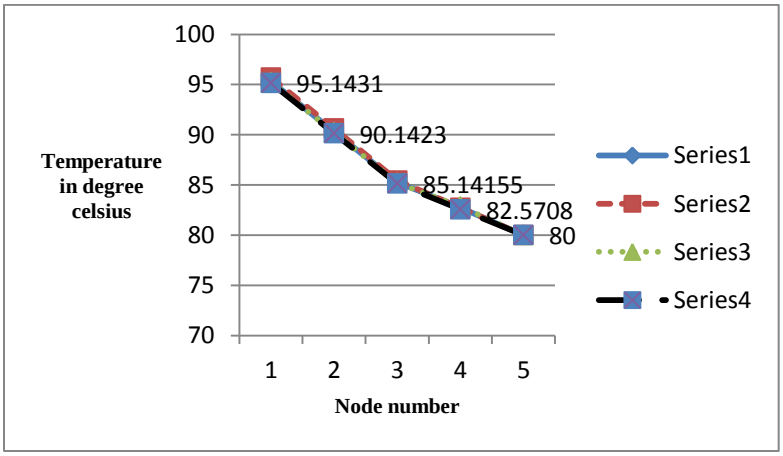

Fig.7. Compare different methods (for four element discretization)

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Next let us consider the value of thermal conductivity for aluminum as TFN [199, 200, 201], the value of thermal conductivity for copper as TFN [388, 389, 390] and the heat input rate as TFN [3999, 4000, 4001]. Obtained results in terms of fuzzy plot are depicted in Figs. 8 to 13.

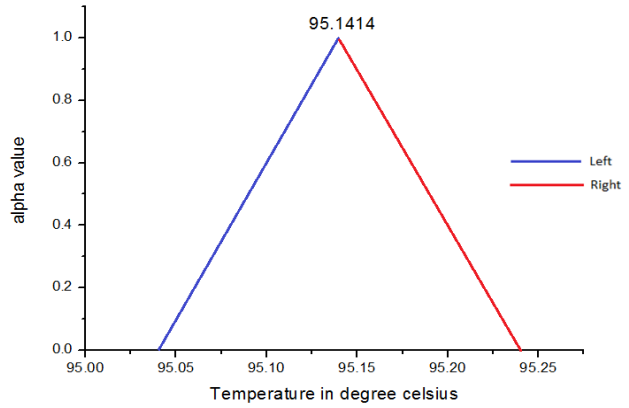

Fig.8.Fuzzy plot for temperature T_1 (For two element discritization)

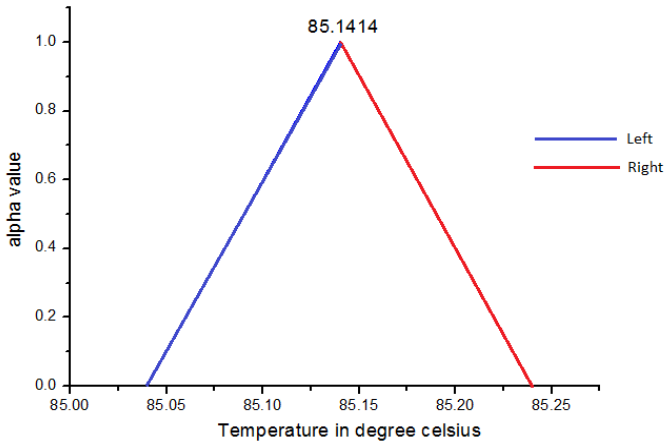

Fig.9.Fuzzy plot for temperature T_2 (For two element discritization)

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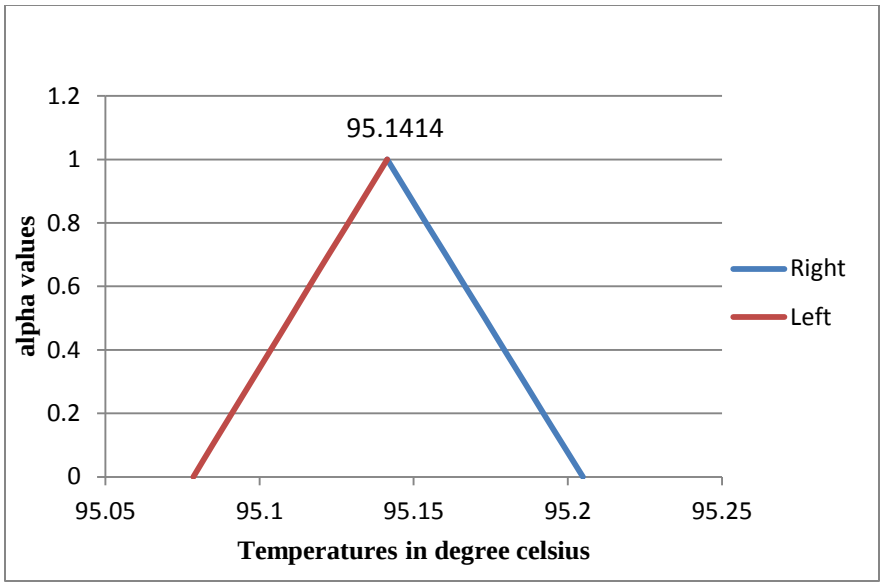

Fig. 10. Fuzzy plot for Temperature T_1(For four element discritization)

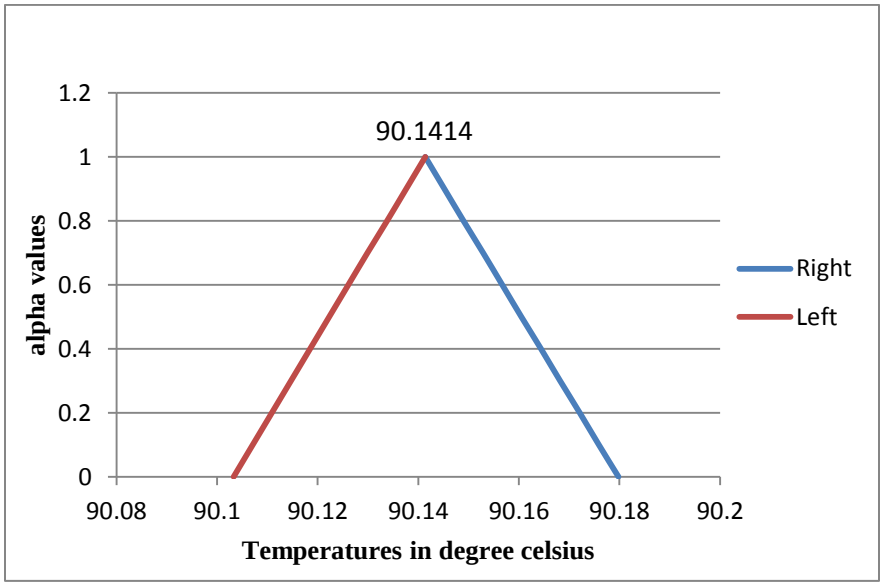

Fig.11. Fuzzy plot for Temperature T_2(For four element discritization)

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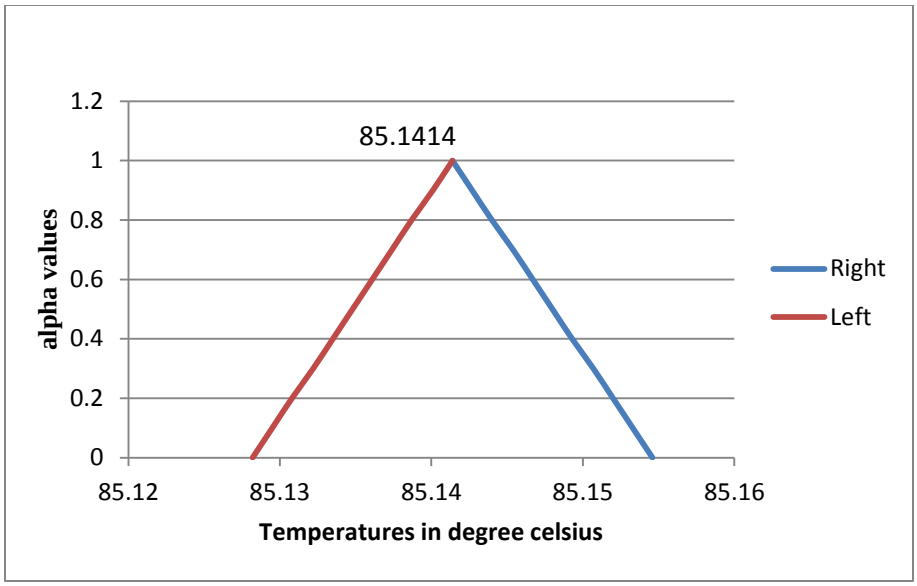

Fig. 12**.** Fuzzy plot for Temperature T_3(For four element discritization)

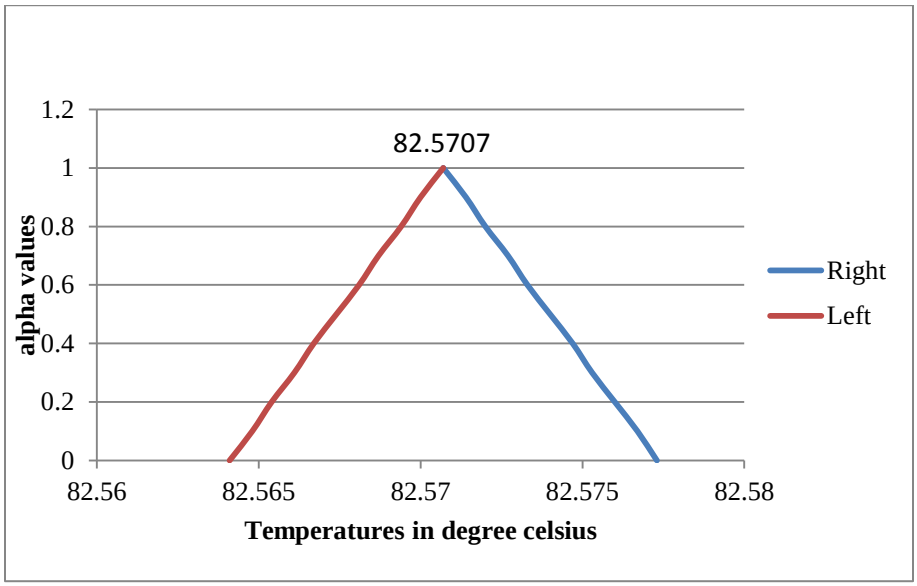

Fig. 13. Fuzzy plot for Temperature T_4(For four element discritization)

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Chapter -7

Discussion

Here we found that the marginal error caused due to the uncertain parameter used in the said problem becomes more for the usual interval finite element method which is shown in the Table-1. Whereas using method-1 we get a comparatively better result. The possibility of the solution set is decreased, which gives a better picture to predict the temperatures at the nodal point of the domain (Fig.2). Again if we consider the centre value of the solution set and comparing the two results of Table-1, method-1 gives a better approximation compare to the results of traditional finite element method (in crisp form). This may be seen by looking into Table-1 and Table-5 (column2).

Then the results of Table-2 and Table-3 for two element and four element discretization respectively give a comparative study about method-2 and the traditional finite element method (in crisp). Method-2 also gives a better approximation to the exact result. Again from Table-4 and Table-5 we are getting the idea of temperature distribution of domain more clearly in comparison with the usual method and method-1. Thus the possibility of temperature distribution along the rod for method-2 is far better than method-1. The centered result for method-2 (in Table-5) gives a sharp approximation to the exact result than the method-1 (in Table-1). We have seen in Table-6 that if the width of the interval is more for the uncertain parameter, method-2 giving a clear idea of temperature and it is more suitable to visualize the behavior of temperature comparing with the other methods.

Considering the uncertain parameters as fuzzy we have presented the result of the said problem in Figs.8 to 9 and also Figs.10 to 13. When the value of the alpha becomes zero the fuzzy results changes to the interval form and for the value of alpha as one, the result changes into crisp form. In Figs. 3 to 6 we get a series of narrow and peak distribution of temperatures which reflect better solution for the said problem and are very close to the solution obtained from the traditional finite element method with crisp parameters.

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Chapter -8

Conclusion

This thesis investigates in detail one-dimension steady state heat conduction problem with uncertain parameters. The parameters involved in the governing equation dictate the solution result. It is well known that the involved parameters cannot be obtained in general exactly or in crisp form. So the same are considered as uncertain in term of fuzzy/interval. As such the investigation is done by using fuzzy/interval finite element method to solve the test problem. Two proposed methods are introduced to find the numerical solution of the said problem with uncertainty. Corresponding results are given and compared with the known result in the special cases. Although the example problem seems to be simple but the main aim of this study is to develop fuzzy/interval finite element method (F/I FEM). It is worth mentioning that the F/I FEM for the said type of problems converts the problem into fuzzy/interval linear system of equations. Accordingly new methods are proposed which may very well be applicable to other complicated problems also.

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References

[1] Carlslaw, H.S., Jaeger, J.C., Conduction of Heat in Solids, second ed. OxfordUniversity Press, USA,(1986).

[2] Magnus. U. Igboekwe, N. J. Achi, Finite Difference Method of Modelling Groundwater Flow,Journal of Water Resource and Protection,(2011), 3, 192-198.

[3] MohamadMuhieddine, EdouardCanot, Ramiro March, Various Approaches For SolvingProblems in Heat Conduction With Phase Change, IJFV International Journal On FiniteVolumes, Volume 6 N°1 (2009).

[4] Nicolaï, B.M., De Baerdemaeker, J., Computation of heat conduction in materials withrandom variable thermophysical properties. Int. J. Numer. Methods Eng. 36,(1993), 523–536.

[5] Nicolaï, B.M., Scheerlinck, N., Verboven, P., De Baerdemaeker, J., Stochasticperturbation analysis of thermal food processes with random field parameters. Trans. ASAE 43,(2000).131–138.

[6] Nicolaï, B.M., Verboven, P., Scheerlinck, N., De Baerdemaeker, J., Numerical analysisof the propagation of random parameter fluctuations in time and space during thermal food processes. J. Food Eng. 38, (1999a), 259–278.

[7] Nicolaï, B.M., Verlinden, B., Beuselinck, A., Jancsok, P., Quenon, V., Scheerlinck, N.,Verboven, P., De Baerdemaeker, J., Propagation of stochastic temperature fluctuations inrefrigerated fruits. Int. J. Refrig. 22, (1999b)81–90.

[8] Cloud, Michael J.; Moore, Ramon E.; Kearfott, R. Baker, Introduction *to IntervalAnalysis*. Philadelphia: Society for Industrial and Applied Mathematics. ISBN 0-89871-669-1(2009).

[9] G. Alefeld and G. Mayer, Interval analysis: Theory and applications, J. Comput. Appl. Math., 121(2000) 421-464.

[10] R.E. Moore, Methods and Applications of Interval Analysis, SIAM studies in Applied Mathematics, Philadelphia, (1979).

[11] Kulpa Z., Pownuk A., Skalna I., Analysis of linear mechanical structures with uncertainties bymeans of interval methods. Computer Assisted Mechanics and Engineering Sciences, vol. 5,(1998), pp. 443–477.

[12] R.L. Muhanna, R.L. Mullen, Uncertainty in Mechanics Problems - Interval - Based Approach.Journal of Engineering Mechanics, Vol.127, No.6, (2001), 557-556.

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[13] A. Neumaier, Interval methods for systems of equations, Cambridge University Press, NewYork, (1990).

[14] Michael Hanss, *Applied Fuzzy Arithmetic, An Introduction with Engineering Applications*. Springer, ISBN 3-540-24201-5.(2005)

[15] Bart M. Nicolai, Jose A. Egea, NicoScheerlinck, Julio R. Banga, Ashim K. Datta, Fuzzy Finite Element Analysis of Heat Conduction Problems with Uncertain Parameters, Journal of Food Engineering 103 (2011) 38–46.

[16] Rao, S. S., Sawyer, P., "Fuzzy Finite Element Approach for Analysis of Imprecisely Defined Systems", AIAA Journal, v. 33, n. 12,(1995), pp. 2364-2370.

[17] David V. Hutton, Fundamental of Finite Element Analysis, Tata McGraw-Hill Edition (2005).

[18]Das Suparna and Chakraverty S. Numerical Solution of Interval and Fuzzy System of Linear Equations, Applications and Applied Mathematics: An International Journal (AAM), 2012 (Accepted).

[19]Behera Diptiranjan and Chakraverty S., A New Method for Solving Real and Complex Fuzzy System of Linear Equations, Computational Mathematics and Modeling, 2011 (Accepted).

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List of publication

[1]Sarangam Majumdar, Sukanta Nayak, S. Chakraverty, Fuzzy finite element method for one-dimensional steady state heat conduction problem, International Journal of Heat and Mass Transfer (under review)(2012).

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