Optimum decision making under uncertainty and risk

Presented by: **Gautam Mitra**

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Outline

- Background
- Decision making under uncertainty
  - Algebraic representation
  - Computational framework
  - Modelling framework
- Asset and Liability Management (ALM):
  - Introduction
  - An example
- Stochastic and Robust optimisation comes of age
- Why we need high performance solvers?
- Conclusions

Background: < the message >

- Linear and integer optimisation have been around since 1960.
- But only after 1980s with spreadsheet optimisation could you show proof of concepts
  (Business school … toy models)
- As
  1. Modelling systems (AMPL, GAMS, OPL…) supported the creation of complex models
  2. Solver systems reached high performance (CPLEX, …., ….)

  Construction of Optimisation based Decision Support Systems became a reality.

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Background:

- “The time has come ,” the walrus said “to talk of many things of gold and spice and mica and mice and how to give them wings” – Lewis Carol
- We are able to not only model
  1. Optimum decision problems under uncertainty and risk
  2. We are also able to process, solve and evaluate them for decision makers…
- We have given SP wings…SP can now fly….!
- Let us see how….!!

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Background : Optimisation – What…Why…How

- **Optimisation**
  - Optimum Decision Making
  - Optimisation Model + Solution
- **LP:**

$$
\begin{aligned}
& \max Cx \\
\textit{subject to} \quad & Ax = b \\
& x \geq 0
\end{aligned}
$$

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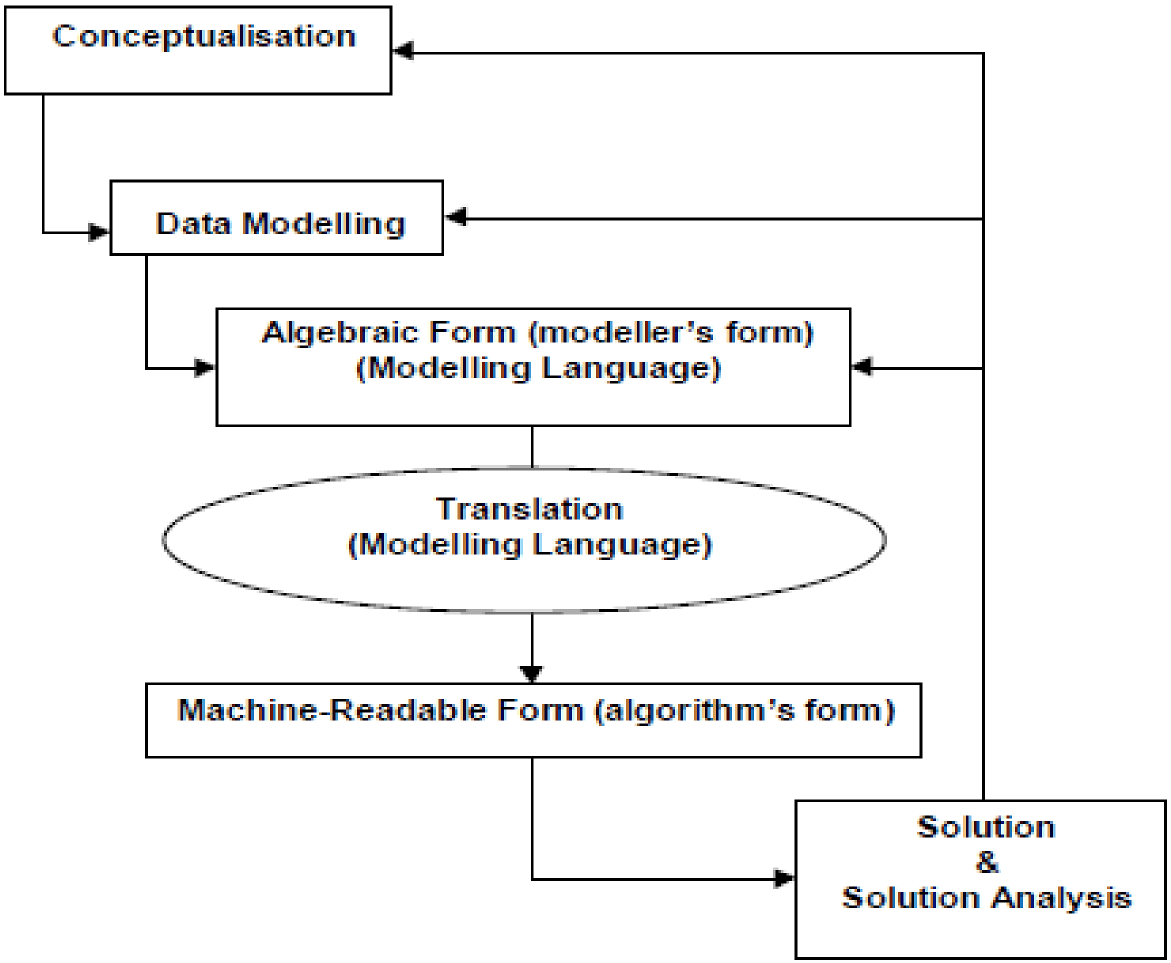
Background : Optimisation – What…Why…How



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Background: *Leading applications*

- Logistics
  - Manufacturing logistics
  - Retail logistics
- Scheduling
  - Transport scheduling (airline, bus, railways)
  - Personnel scheduling
    Energy systems
    Refinery operations scheduling
- Resource allocation
  - Capital budgeting – Project planning
    Financial planning

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Decision Making: Role of uncertainty and time

Any planning process which takes into account future outcomes has two important determinants which are:

a) **Uncertainty**

b) **Time**

Traditional approach accounts for these by net present value (NPV)

In optimisation based approach planning decisions are broken down to:

a) **Robust strategic decision followed by**

b) **Rolling contingency plans which respond to different future outcomes.**

Stochastic Programming (SP) breaks down the decision making into two parts more or less along these two lines. In SP we make:

a) **First stage decisions**

b) **Corrective (recourse) actions (decisions) as future events unfold.**

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Decision making:

optimisation models with uncertainty representation

$$
\begin{aligned}
Z \quad &= \quad \min cx \\
\textit{subject to} \quad & Ax = b \\
& x \geq 0 \\
\textit{where} \quad & A \in R^{m \times n}; c, x \in R^{n}; b \in R^{m}
\end{aligned}
$$

Notation:

Let (Ω,F,P) denote a probability space, where $\omega \in \Omega$ denotes a particular realisation of the uncertain parameters and $p(\omega)$, the corresponding probability.

Let us denote the realisations of A, b, c for a given ω as

$$(A, b, c)_{\omega} = \xi_{\omega} \quad or \quad \xi(\omega)$$

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Decision making:

Alternative models of stochastic programming

- Single stage stochastic programs …(Markowitz)
- Two stage stochastic programs with recourse
- Multi stage stochastic programs with recourse
- Chance constrained programming problem
- Integrated Chance constrained programming problem
- Robust optimisation

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Decision making:

The classical stochastic linear program with recourse

The stochastic linear program with recourse (SLPR) is stated as

$$
\begin{aligned}
Z = \ & \min \ cx + E_{\omega} Q(x, \omega) \\
\textit{subject to} \quad & Ax = b \\
& x \geq 0,
\end{aligned}
$$

where

$$
\begin{aligned}
Q(x, \omega) = \ & \min \ f(\omega) y \\
\textit{subject to} \quad & D(\omega) y = d(\omega) + B(\omega) x \\
& y \geq 0.
\end{aligned}
$$

Decision making :

Two stage stochastic linear program

Below is a two-stage staircase problem that is transformed into a two-stage stochastic linear program:

$$
\begin{array}{lll}
\min Z & = cx + E^{\omega}\left[f y^{\omega} \right] & \\
\textit{subject to} & Ax & = b \\
 & -B^{\omega}x + D^{\omega}y^{\omega} & = d^{\omega} \\
 & x, y^{\omega} \geq 0; \quad \omega \in \Omega &
\end{array}
$$

Decision making:

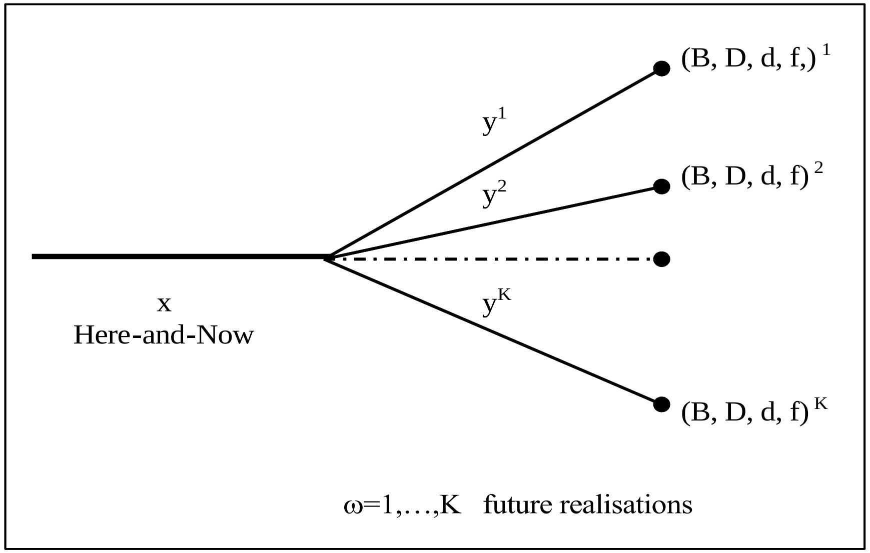
Two stage stochastic programming

Define Ω as an index set $\Omega=\{1,\ldots,K\}$, meaning that the parameter ω may take on K different values.

$$
\begin{array}{lllll}
\min Z & = & cx & + p^1 f y^1 + p^2 f y^2 + \ldots + p^K f y^K & \\
\textit{subject to} & & Ax & & = b \\
& & -B^1 x + D^1 y^1 & & = d^1 \\
& & -B^2 x + & D^2 y^2 & = d^2 \\
& & \vdots & \ddots & \vdots \\
& & -B^K x + & D^K y^K & = d^K \\
& & x, \quad y^1, \quad y^2, \quad \ldots & y^K \geq 0 & \\
& & 0 \leq p^\omega \leq 1 \quad \textit{and} & \sum p^\omega = 1.0 &
\end{array}
$$

Decision making:

Two Stage Stochastic Linear Programming



In this tree x represents the first stage (Here-and-Now) decision vector. ω =1,…,K are the possible future realisations (scenarios); the associated probabilities and demands are p, d^{ω} respectively. As future unfolds for a given scenario ω a corrective recourse action is taken; this is represented by the vector y^{ω}.

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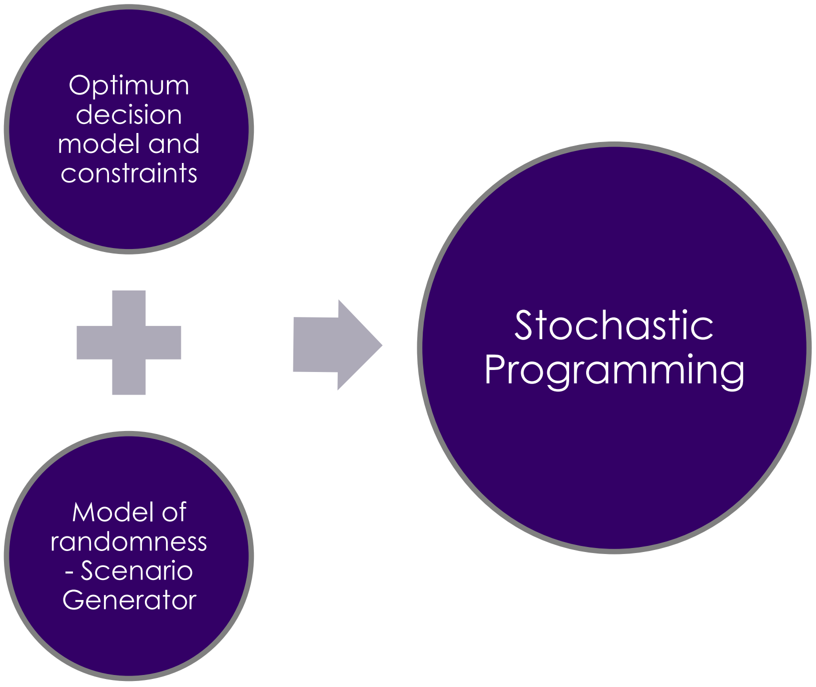
Decision making : *the SP paradigm*

- Stochastic programming seen as a combination of optimisation decision models and models of randomness



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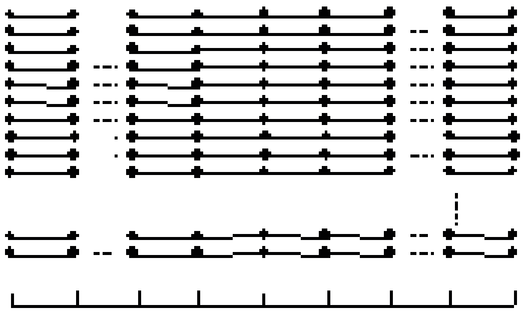
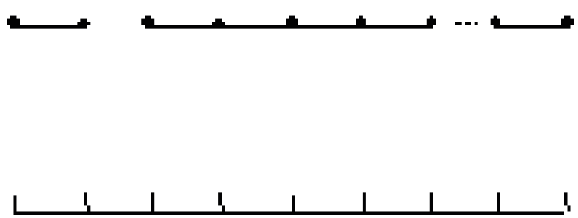
Decision making:

Stochastic processes and scenario generation

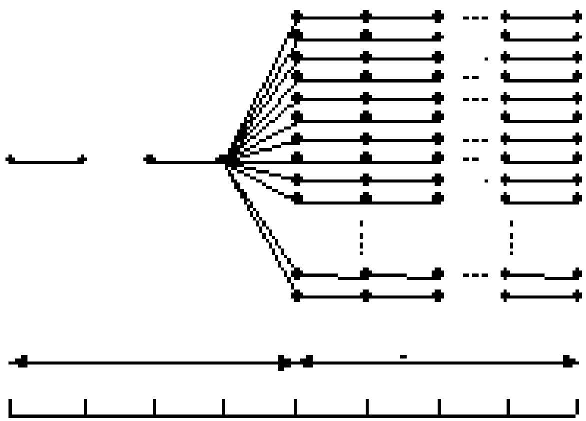
Wait and See



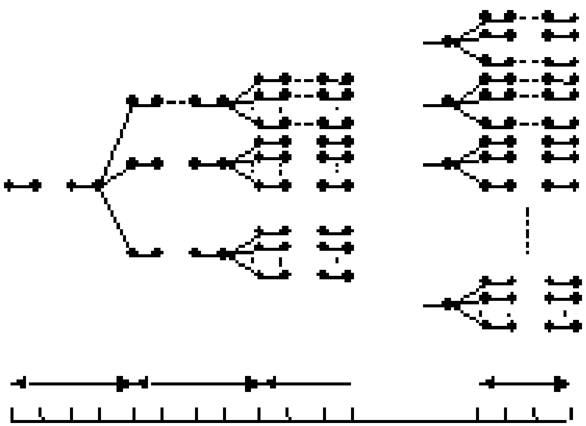
Expected Value



Two-Stage SP



Multi-Stage SP



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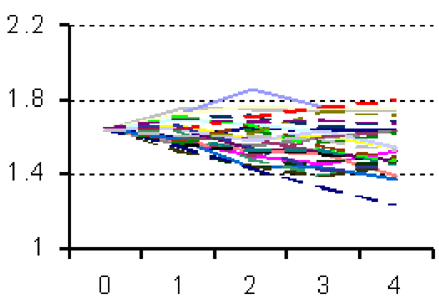
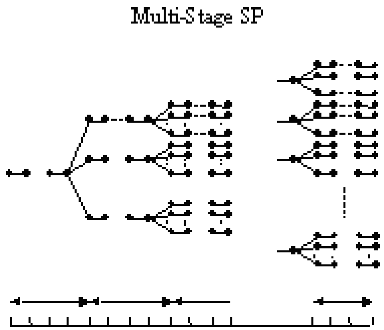
Decision making:
Stochastic processes and scenario generation

- Stochastic processes generate data in form of fans/time series but SP needs trees

Fans → Mathematical structure → Trees



→ Mathematical structure →



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- **Asset and Liability Management (ALM):**
  - **Introduction**
  - **An example**
- Stochastic and Robust optimisation comes of age
- Why we need high performance solvers?
- Conclusions

Introducing ALM

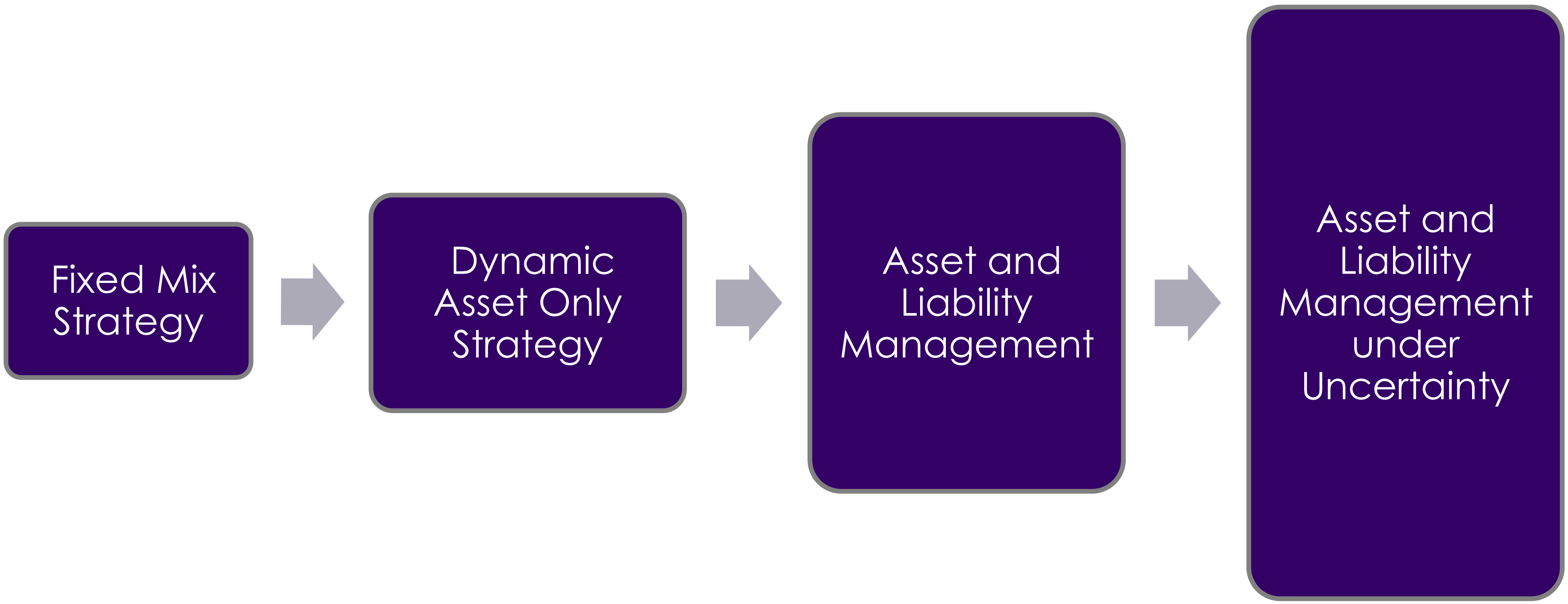
Asset and liability management is the practice of managing risks that arise due to mismatches between the assets and liabilities (assets and debts)

Asset and Liability management (ALM) is a strategic management tool to manage interest rate risk and liquidity risk faced by banks, other financial services companies and corporations

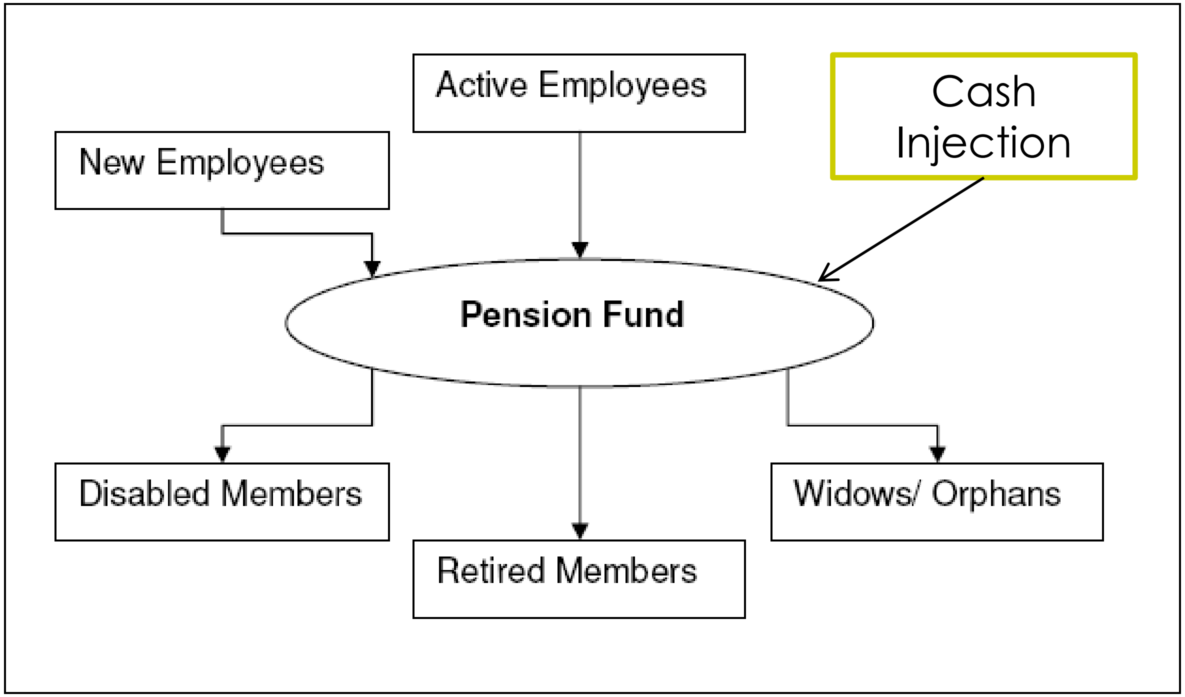
Introducing ALM

- Application areas:
  - Banks <credit portfolios>
  - Bond issuance <corporate as well as sovereign>
  - University endowments
  - Hedge funds and mutual funds
  - Wealthy individuals
  - Insurance companies and
  - Pension funds

Introducing ALM

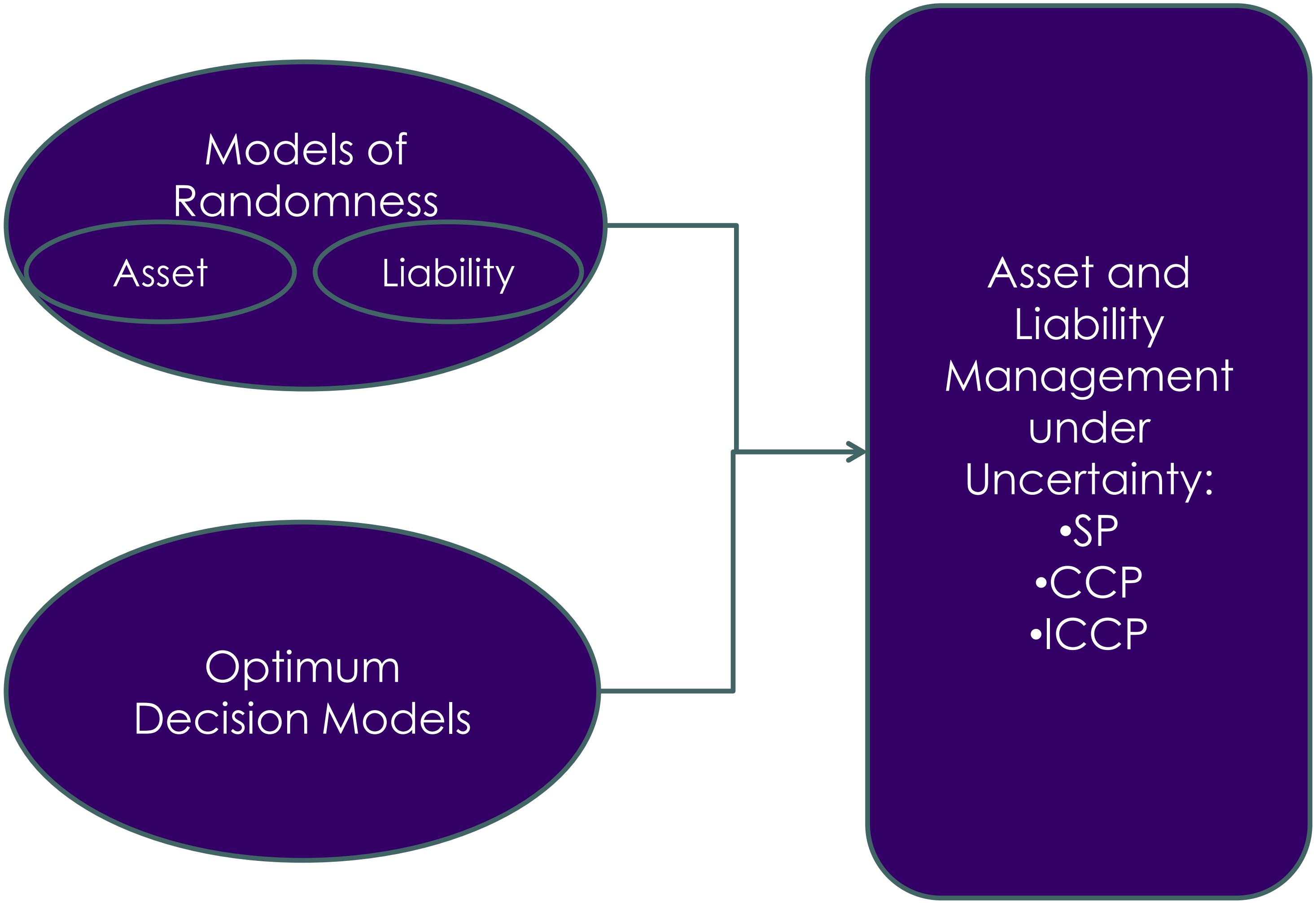


ALM example: Pension fund cash flows



- Investment: portfolio of fixed income and cash

Asset & Liability Management: ALM



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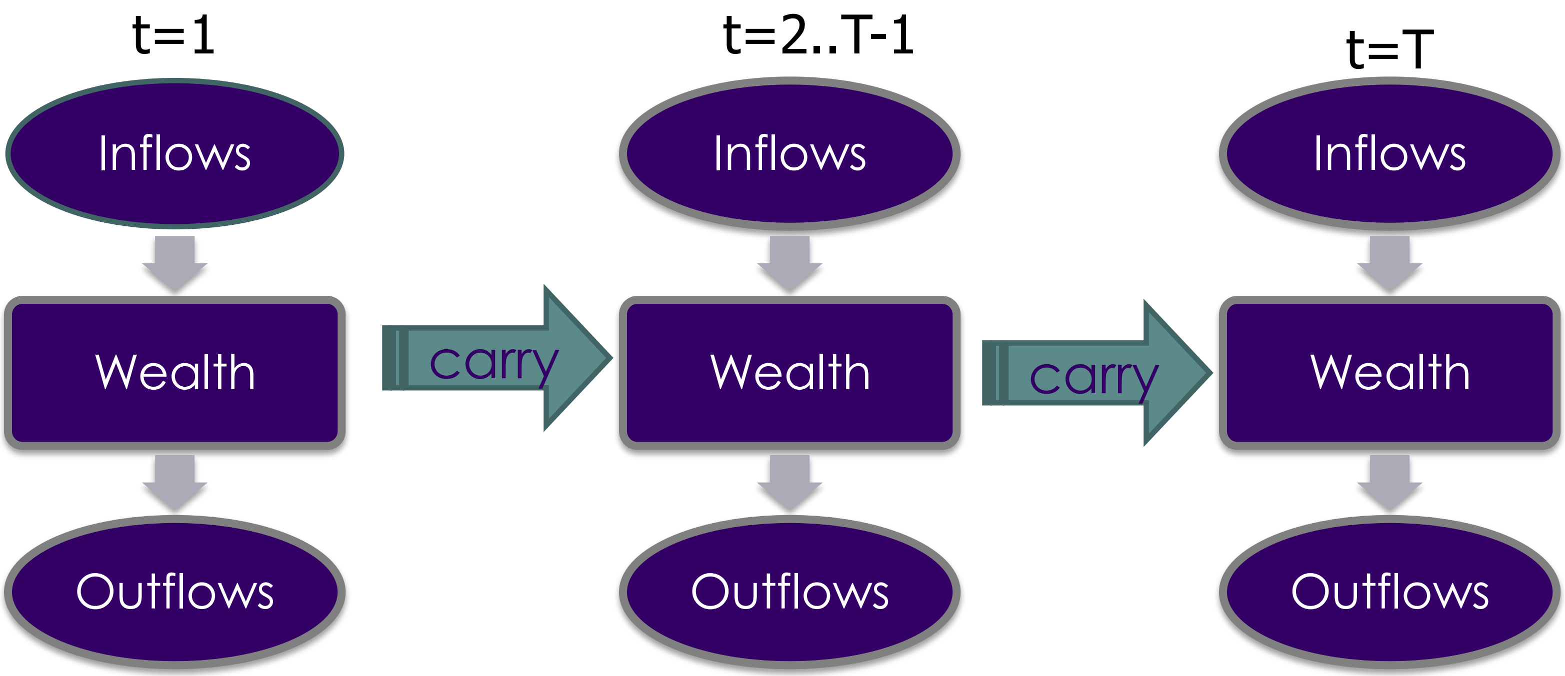
ALM : *a stochastic programming model*

Surplus Wealth = assets – PV(liabilities) – PV(goals)



ALM: Model Components

	NOTATION	ENTITY	
INDEX (SET)	i	Assets	i=1..n
	s	Scenarios	s=1…S
	t	Timeperiod	t=1..T
	K	RiskGroup	k=1..K
PARAMETER	P_{ist}	Prices[assets, scenarios, timeperiod]	
	L_{ts}	Liability[timeperiod,secenario]	
	π_s	Probability[scenario]	
	H_{i0}	Initial Holdings[assets, timeperiod=0]	
	F_t	Funding[timeperiod]	
	1+tr	Transaction Buy	
	1- tr	Transaction Sell	
	R_k	RiskGroup Holdings	
DECISION VARIABLE	H_{ist}	Amounthold[assets, scenarios,timeperiod]	
	B_{ist}	Amountbuy[assets, scenarios,timeperiod]	
	S_{ist}	Amountsell[assets, scenarios,timeperiod]	

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ALM: *Model Components*

SAMPL syntax

		Name	SAMPL syntax
SETS	i	asset	`set asset;`
	s	scenario	`scenarioset scenario;`
PARAMETERS	P_{ist}	Prices[assets, scenarios, timeperiod]	`random param prices{assets, ... };`
	L_{ts}	Liability[timeperiod,scenario]	`param liability{timep,scenario};`
	π_s	prob[scenario]	`probability param prob{scenario};`
	tr	Sales transaction cost (%)	`param tsell;`
VARIABLES	H_{ist}	Amounthold[assets, scenarios,timeperiod]	`var hold{assets, scenarios, time};`
	B_{ist}	Amountbuy[assets, scenarios,timeperiod]	`var buy{assets, scenarios, time};`
	S_{ist}	Amountsell[assets, scenarios,timeperiod]	`var sell{assets, scenarios, time};`

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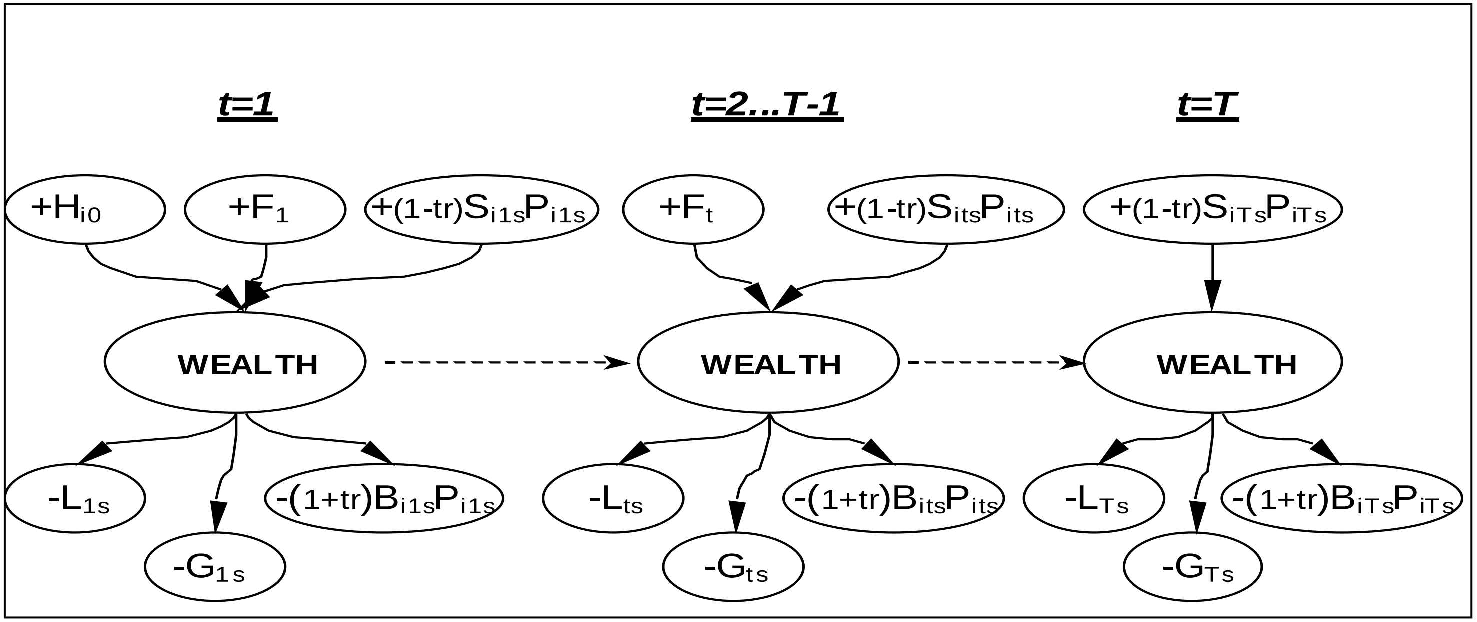
ALM: stochastic programming model…cont..

Surplus Wealth = assets – PV(liabilities) – PV(goals)



SAMPL syntax

```
subject to fundbalance1{t in 2..T-1,s in scen}:
sum{a in assets} amountbuy[t,a,s]*price[t,a,s]*tbuy
-sum{a in assets} amountsell[t,a,s]*price|
  income[t,s]-liabilities[t,s];
```

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- Why we need high performance solvers?
- Conclusions

SP comes of age: *SP software tools*

Name	Affiliation	System Name	Type
JJ Bisshop, et al.	Paragon Decision Tech.	**AIMMS**	Modelling System
A Meeraus, et al.	GAMS	**GAMS**	Modelling System
B Kristjansson	Maximal Software	**MPL**	Modelling System
R Fourer, et al.	Northwestern University	**AMPL**	Modelling System
MAH Dempster, et al.	Cambridge University	**STOCHGEN**	Modelling System
E Fragniere, et al.	University of Geneva	**SETSTOCH**	Modelling System
A King, et al.	IBM/COIN-OR	**OSL/SE, SMI**	Solver
HI Gassmann, et al.	Dalhousie University	**MSLiP**	Solver
G Infanger et. Al.	Stanford University	**DECIS**	Solver
P Kall, et al.	University of Zürich	**SLP-IOR**	Modelling System / Solver
G Mitra, et al.	Brunel University	**AMPLDev SP Edition**	Modelling System / Solver
A. Ramos	U P Comillas	**MSP & SIP**	Modelling System / Solver
A. Gavronsky	Norwegian Univ of Tech	**ALM**	Modelling System

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SP comes of age: modelling paradigms

- *Descriptive Models* as defined by a set of mathematical relations, which simply predicts how a physical, industrial or a social system may behave.

- *Normative Models* constitute the basis for (quantitative) decision making by a superhuman following an entirely rational that is, logically scrupulous set of arguments. Hence quantitative decision problems and idealised decision makers are postulated in order to define these models.

- *Prescriptive Models* involve systematic analysis of problems as carried out by normally intelligent persons who apply intuition and judgement. Two distinctive features of this approach are uncertainty analysis and preference (or value or utility) analysis.

- *Decision Models* are in some sense a derived category as they combine the concept underlying the normative models and prescriptive models.

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Role of uncertainty and time

Any planning process which takes into account future outcomes has two important determinants which are:

a) **Uncertainty**

b) **Time**

Traditional approach accounts for these by net present value (NPV)

Also planning decisions have been broken down to:

a) **Robust strategic decision followed by**

b) **Rolling contingency plans which respond to different future outcomes.**

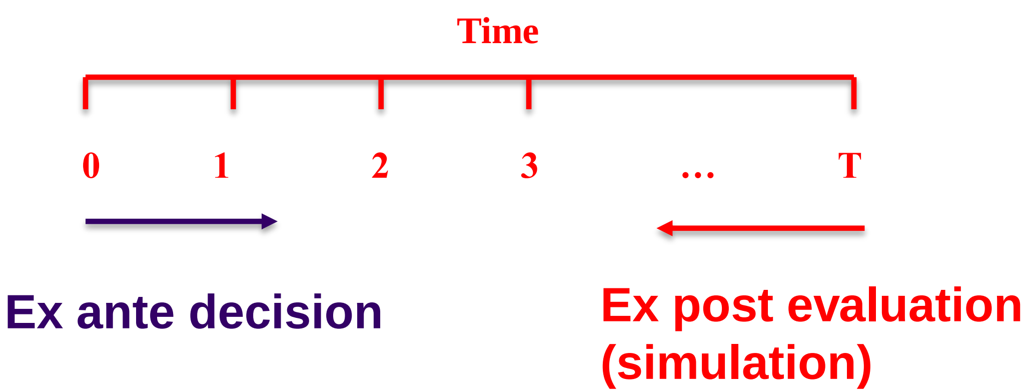
Stochastic Programming (SP) breaks down the decision making into two parts more or less along these two lines. In SP we make:

a) **First stage decisions**

b) **Corrective (recourse) actions as future events unfold.**

SP comes of age: *scope of models*

- Data Model
- Decision Model: Constrained optimisation
- Descriptive Model: Simulation and Evaluation



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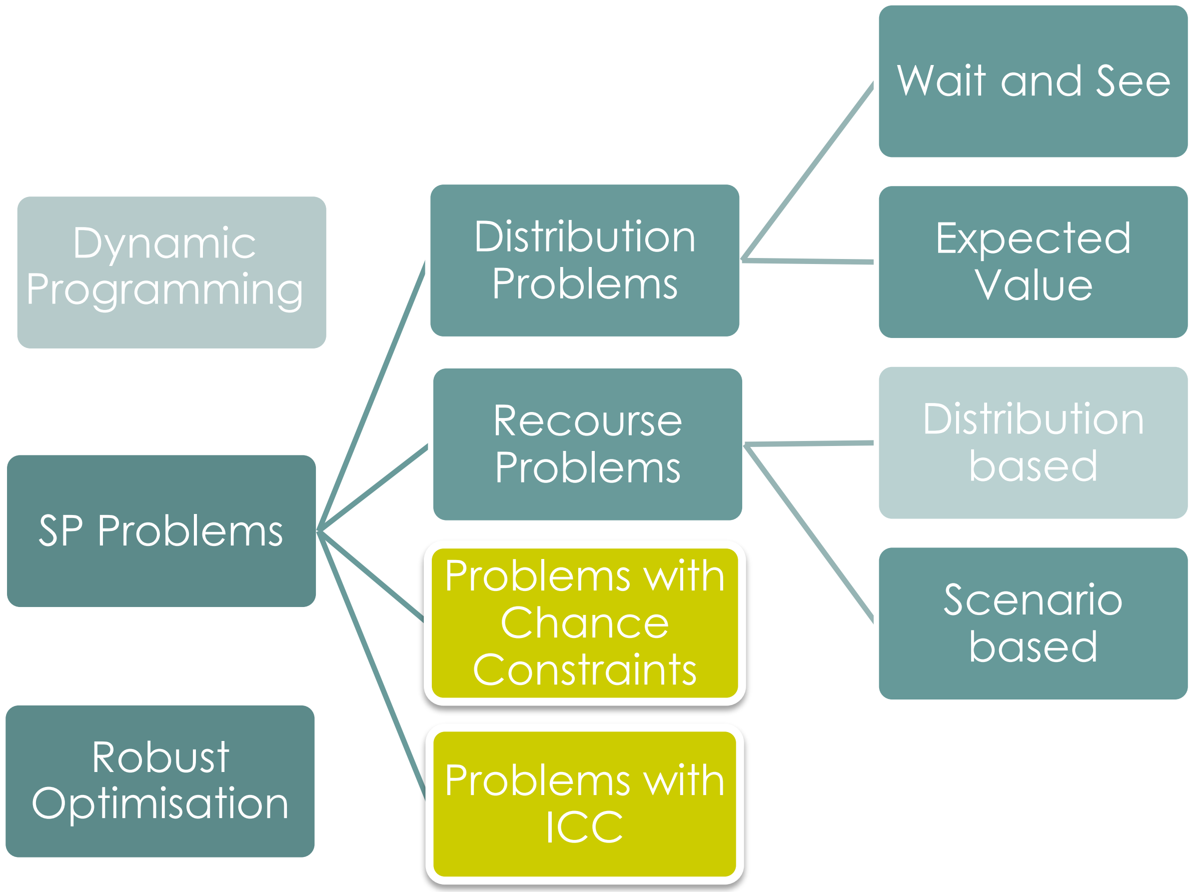
SP comes of age: *Uncertainty models*



SP comes of age:

inter-related modelling paradigms

- The modelling paradigms revisited
  - **Stage 0:** Analyse historical data [data model]
  - **Stage 1:** Create data paths for random parameter values **[descriptive models]**
  - **Stage 2:** Make decision using SP or DP **[decision models]** (data processes and decision processes are inter-twined)
  - **Stage 3:** Test the decisions using simulation Back testing, stress testing, out of sample **[descriptive models]**

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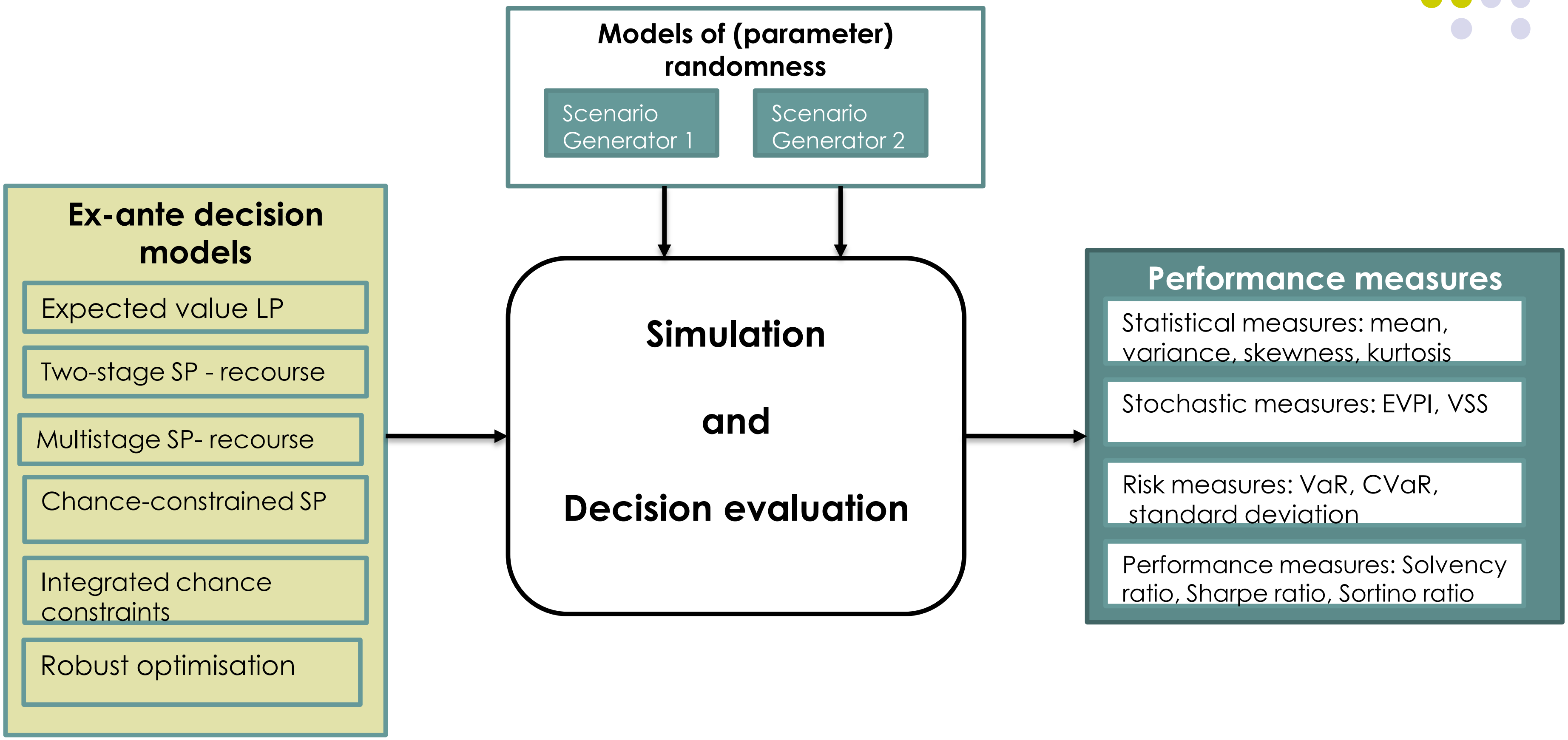
SP comes of age:
decision…simulation…evaluation engine



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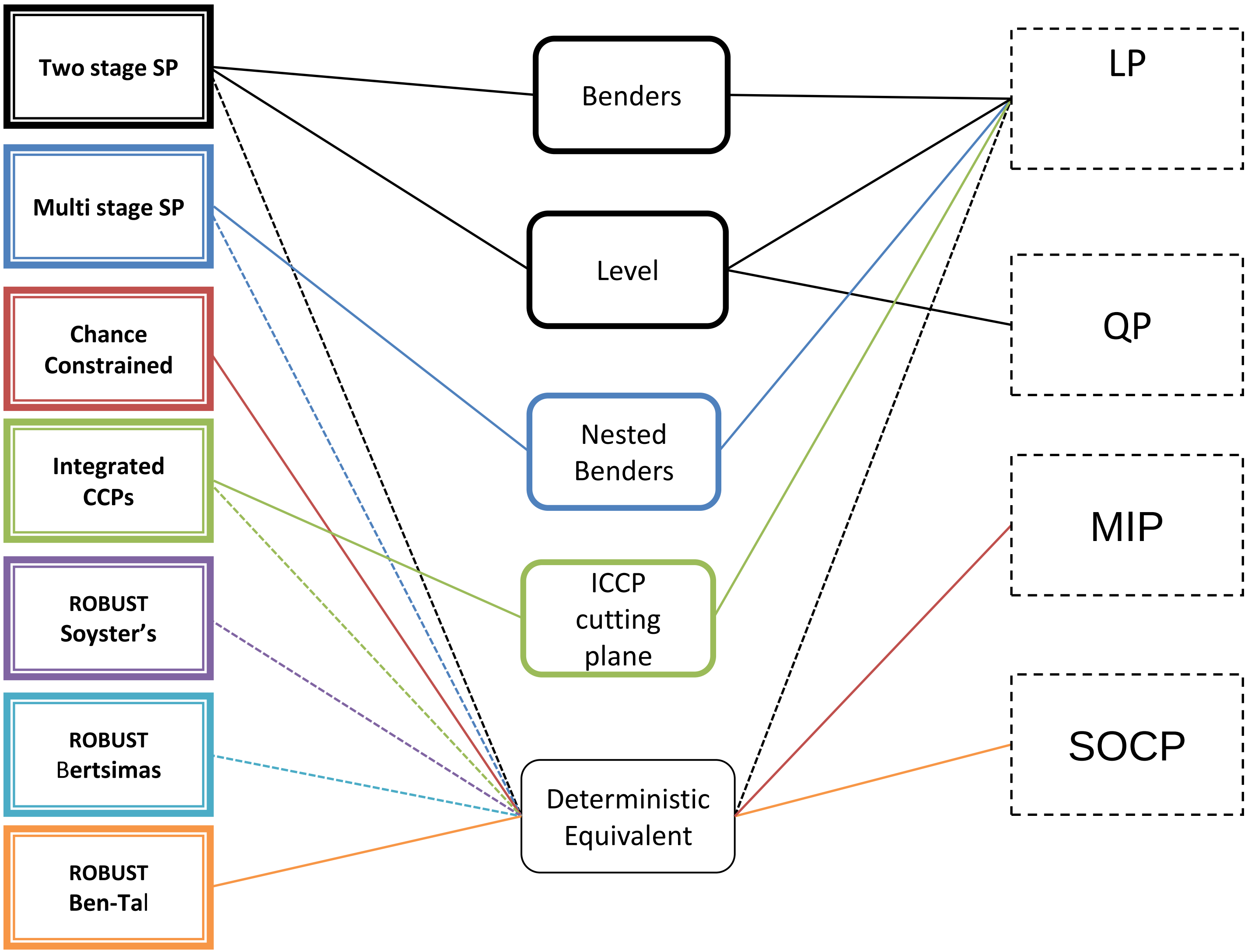
SP comes of age: Solution methods



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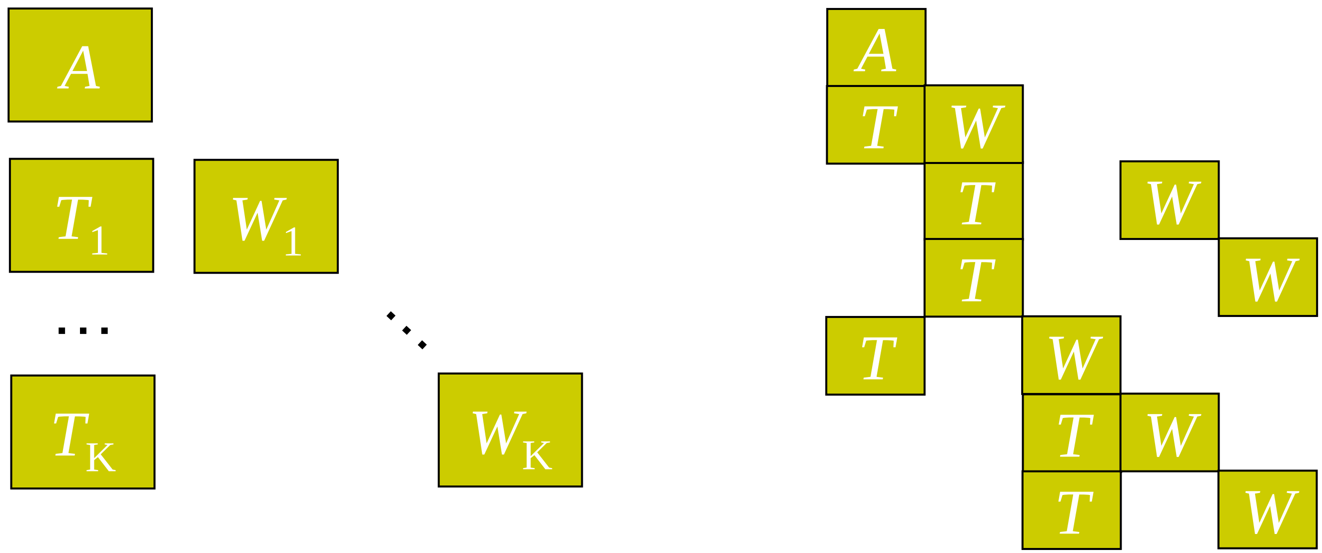
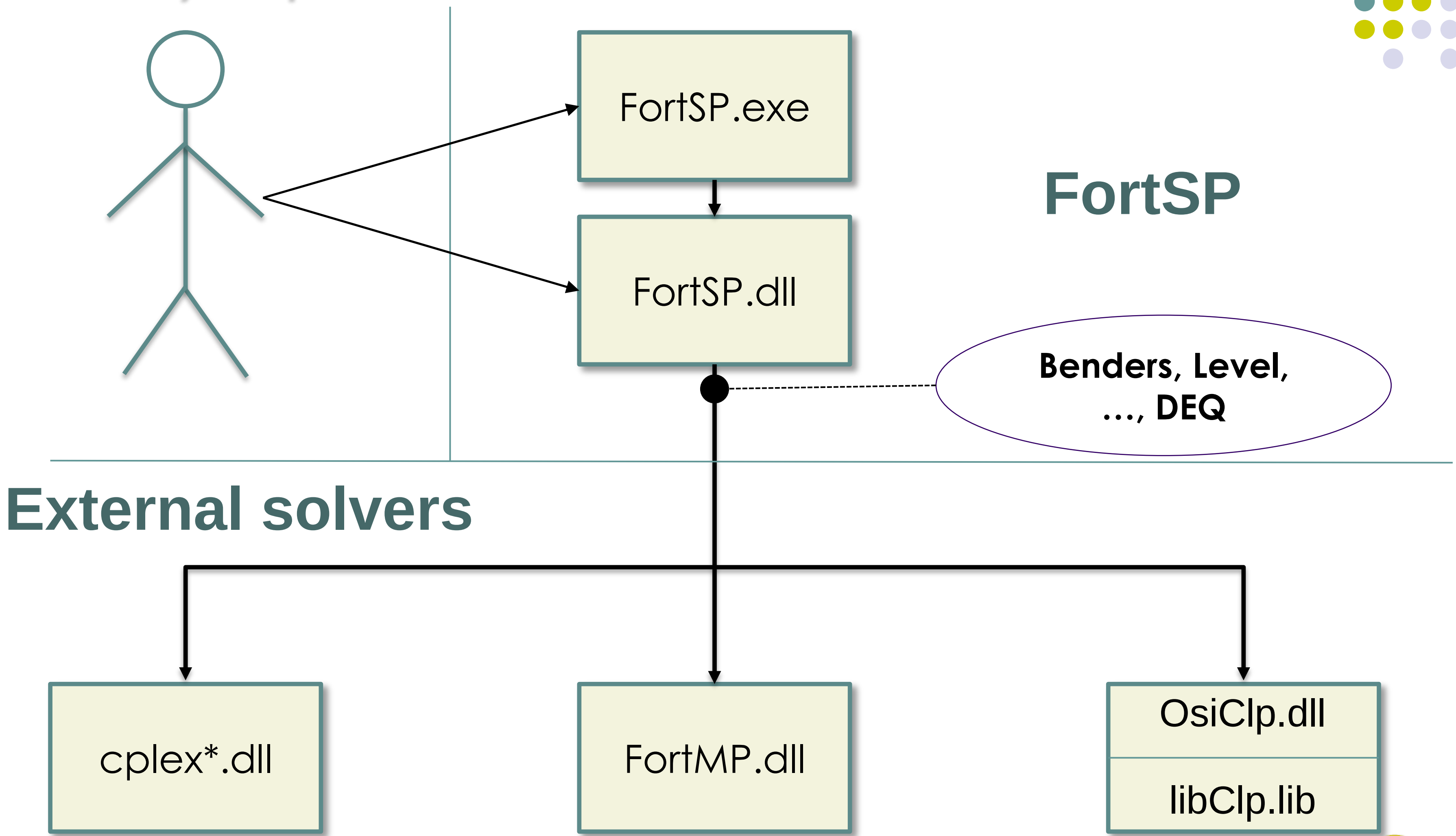
Why high performance solvers:
Problem structure



- Structure of deterministic equivalent makes it suitable for decomposition methods

- IPM can be often more appropriate than Simplex due to large size and high sparsity

Why hp solver: …Solver Architecture



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Why hp solvers: …Test problems

Name	A	W	Scen	Deterministic Equivalent Matrix	Deterministic Equivalent NNZ	Opt
pltexpA2	62×188	104×272	6	686×1820	3703	-9.47935
			16	1726×4540	9233	-9.66234
fxm2	92×114	238×343	6	1520×2172	12139	18416.8
			16	3900×5602	31239	18416.8
stormG2	185×121	528×1259	8	4409×10193	27424	15535236
			27	14441×34114	90903	15508982
			125	66185×157496	418321	15512091
			1000	528185×1259121	3341696	15802590

Name	A	W	Scen	Deterministic Equivalent Matrix	Deterministic Equivalent NNZ	Opt
saphir	32×55	8672×3916	50	433632×195855	1136153	130007211
			100	867232×391655	2272203	128798834
			200	1734432×783255	4544303	141567536
			500	4336032×1958055	11360603	137829930
			1000	8672032×3916055	22721103	133012591

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Test problems

Name	A	W	Scen	Deterministic Equivalent Matrix	NNZ	Opt
AIRL2	2×4	6×8	25	152×204	604	269665
LandS	2×4	7×12	3	23×40	92	381.853
4node	14×52	74×186	16	1198×3028	7743	423.012
			32	2382×6004	15231	423.013
			64	4750×11956	30207	423.012
			128	9486×23860	60159	423.012
			256	18958×47668	120063	425.375
			512	37902×95284	239871	429.962
			1024	75790×190516	479487	434.113
			2048	151566×380980	958719	441.738
			4096	303118×761908	1917183	446.856
			8192	606222×1523764	3834111	446.856
			16384	1212430×3047476	7667967	446.856
			32768	2424846×6094900	15335679	446.856

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Why hp solvers: Test results

Name	Scen	CPLEX Time	CPLEX Iter	Benders Time	Benders Iter	Level Time	Level Iter	Lin. Damp. Time	Lin. Damp. Iter
pltexpA2	6	0.06	14	0.04	1	0.03	1	0.03	1
	16	0.13	16	0.08	4	0.10	4	-	-
fxm2	6	0.09	17	0.29	23	0.35	15	0.36	25
	16	0.20	23	0.39	22	0.53	18	0.43	23
stormG2	8	0.38	28	0.60	23	0.83	22	0.67	25
	27	3.33	27	1.93	30	1.65	22	2.13	32
	125	12.33	57	8.38	32	4.99	19	9.75	37
	1000	189.53	109	80.20	41	34.46	18	81.07	41

Name	Scen	CPLEX Time	CPLEX Iter	Benders Time	Benders Iter	Level Time	Level Iter	Lin. Damp. Time	Lin. Damp. Iter
saphir	50	‡	‡	501.47	114	326.53	74	482.89	116
	100	‡	‡	661.64	123	462.17	71	643.67	121
	200	835.74	151	‡	‡	1186.21	96	‡	‡
	500	‡	‡	2553.64	110	1753.66	62	2548.59	113
	1000	†	†	4433.30	116	3678.84	58	4214.99	115

† Failed to solve due to insufficient memory

‡ Failed to solve due to numerical difficulties

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Conclusions: *Assertions and Commercials*

- The models and tools for decision making under uncertainty, optimum risk decisions have matured.
- For more information and to gain SP skills attend OptiRisk Workshops…..!
- The domains of supply chain logistics, energy systems planning and operation … especially financial planning are ready for SP
- Quants may use
  - SP modelling support for rapid prototyping
  - Special purpose solvers to scale up prototypes

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Conclusions: *Assertions and Commercials*

- A particularly valuable aspect of scenario based SP is the dual paradigm of
  - Ex ante optimisation decisions followed by
  - Ex poste decision evaluation
- OptiRisk is a certified partner of IBM in the UK and in India
- OptiRisk and IBM together are ready for business…!
- We will be waiting for you to call or email…!!

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Thank you....

- Thank you for your attention

- Comments and Questions please
- { not difficult or awkward ones…! }