CONVERSIONS & TABLES:

Power

7.1 POWER GENERALLY

7.1.1 Definition

Power is a rate of doing work. It requires more power to move a load quickly than slowly, although the work done may be the same in each case. If a force or a load moves over a distance, energy is consumed and work is done. In other words work = force × distance, and rate of work, or power = work ÷ time.

7.1.2 Units

In the past, before the development of mechanical engines, power was measured in relation to the rate of work of the horse. Hence the unit of power came to be known as the horsepower (hp). The scientific unit of power is the watt (W), and because this is small it is very often expressed as the kilowatt (kW). The unit of work or energy corresponding to the watt is the joule (J), which is defined as the work done when a force of one newton (1N) moves through a distance of one metre (1 m). Watts = joules per second.

$$\begin{aligned} J &= N \times m \\ W &= J/s \\ kW &= 1000\ W \end{aligned}$$

The horsepower, which was originally defined in terms of pound-foot units, is equivalent to 746 W. The metric hp is 735.5 W.

7.1.3 Brake Horsepower

The brake horsepower of a power unit defines its power output. The name derives from the original method of measuring the output power of an engine by causing it to work against a brake whose loading could be measured. The efficiency of an engine is the ratio of its power output to its power input.

7.2 LIVE POWER

7.2.1 Human Power

While the human physique is capable of short bursts of intense power, the sustained energy output over a prolonged period, such as might be required for pumping or load haulage or other similar labouring activity, is about 60 W.

7.2.2 Animal Power

Draught animals can provide power for ploughing and other agricultural operations, transport and water lifting. Table 7.1 gives the power output of some animals.

Table 7.1 Animal power.

Animal	*Weight (kg)*	*Draft force (kg)*	*Ave. speed (m/sec)*	*Power (W)*
Heavy horse	680–1200	50–120	0·7–1·25	350–1500
Light horse	400–700	45–78	0·8–1·40	370–1080
Mule	350–500	50–60	0·9–1·00	450– 600
Donkey	200–300	30–40	0·7	250
Cow	400–600	50–60	0·7	350
Bullock	500–900	60–80	0·6–0·80	360– 640

Source: Reproduced from *Economics and Power Requirements of Small Irrigation Pumps in Bangladesh and Egypt*, by David Birch, Institute of Irrigation Studies, University of Southampton, 1979. *Sources of information:* (1) S. B. Watt, *Chinese Chain and Washer Pumps*, IT Publications; and (2) *Civil Engineers* in *Management in the Community*, a Conference at N.E. London Polytechnic.

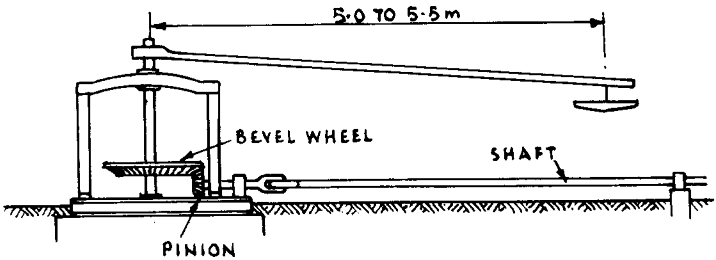


Figure 7.1 Horse gear.

It is often assumed that an animal working on a circular track can exert a force of one tenth of its weight at a speed of 0·7 m/s. To enable an animal to walk in a circular track the diameter of the walk should not be less than 7·5 m; 9–10 m would be still better. Figure 7.1 gives a sketch of animal gear suitable for water raising.

7.3 WIND POWER

7.3.1 Power in the Wind

The power which can be extracted from the wind by a windmill is, theoretically:

$$W = 0{\cdot}373\ AV^3\ \text{W}$$

where A is the swept area of the rotating blades in square metres and V is the velocity of the wind in metres per second. A well-made propeller or rotor will extract 30–40 per cent of the wind energy; if the efficiency is, say, 0·35, the power is given by:

$$W = 0{\cdot}131\ AV^3\ \text{W}$$

7.3.2 Windmills

Although the initial cost of a windmill is higher than the equivalent powered petrol or diesel engine, having no fuel costs and minimal maintenance, running costs are very low. A well-designed windmill will start in a light breeze of under 2 m/sec velocity, and must be able to withstand winds of up to 30 m/sec. A moderate breeze is about 7 m/sec, a strong breeze is about 12 m/sec and 15 m/sec is a near gale. Towers are usually from 6 to 20 m in height, and the top of the tower must always be at least 5 m higher than any surrounding obstructions to wind flow.

7.3.3 Windmills for Pumping

The most common use of windmills is for pumping water, but they are also used for generating electricity. When used for pumping, the pump must be matched to the wind machine. Storage capacity for pumped water is necessary as winds are variable and the machine may not work every day. Table 7.2 gives some wind pump performance figures for Kijito windmill pumps manufactured in Kenya.

*Table 7.2 Wind pump performance.**

Rotor dia. (m)	*Total head (m)* 15	30	45	60	90	120	150	180	210	240
3·7	45	20	13	9	6	4	—	—	—	—
4·9	75	30	20	15	10	6	4	3	—	—
6·1	125	55	35	25	17	11	7	5	3	—
7·3	180	80	52	38	24	16	11	8	5	3

* Output in m^3/day for an average wind speed of 3.5 m/sec.

7.4 WATER POWER

7.4.1 Power from Water

The theoretical power in kilowatts developed by a flow of water of Q l/sec falling through a vertical distance H m is given by:

$$W = \frac{QH}{102}\ \text{kW}$$

In practice energy is lost through mechanical and electrical friction, and sometimes, through leakage of water. The actual power output is less than the theoretical power, and the ratio of actual to theoretical power is the efficiency of the system converting the power. If k = efficiency, then actual power is given by:

$$W = k \cdot \frac{QH}{102}\ \text{kW}$$

7.5 OTHER FORMS OF POWER

7.5.1 Solar Power

The most practical form of solar power development for use on a small scale in rural areas is the solar or photovoltaic cell which transforms sunlight directly into electrical energy. In this system, silicon solar cells, usually 100 mm dia., are connected in groups known as modules, several modules being mounted in a panel known as an array.

Under a solar irradiance of 1 kW/m^2 a single cell will give about 2 ampères (A) at 0·5 volts (V), or 1 W of electrical energy. An array of 144 cells will therefore produce about 100–120 W for an irradiance of 1 kW/m^2.

7.5.2 Solar Pumping

Solar pumps have been used successfully in a number of countries. Initial costs are high and in 1982 were about US$20 per peak watt electrical output. Operation is very cheap because there are no fuel costs, and very little maintenance is required.

7.5.3 Petrol and Diesel Engines

The small petrol and diesel engine is still popular as a power unit in rural areas. Rising fuel costs make operation expensive. The brake horsepower of an engine is the power developed at its drive shaft, under normal operating conditions.

Engines are usually rated by the manufacturers at sea level and at an ambient temperature of 15°C. If engines are used at higher altitudes or higher ambient temperatures, the power output is reduced. The percentage reductions to be applied to diesel engines are given in Table 7.4.

*Table 7.4 Diesel engine reduction of power with altitude and temperature.**

Metres above sea level	Temperature (°C) 15	20	25	30	35	40	45
500	1·9	2·8	3·7	4·6	5·5	6·4	7·3
1000	6·9	7·8	8·7	9·6	10·5	11·4	12·3
1500	11·9	12·8	13·7	14·6	15·5	16·4	17·3
2000	16·9	17·8	18·7	19·6	20·5	21·4	22·3
2500	21·9	22·8	23·7	24·6	25·5	26·4	27·3

* Percentage reduction of power for heights above sea level and temperatures above 15°C.

For continuous use, reduce the normal rating by 25 per cent.

Example

A diesel engine rated at 18 bhp is to be used at an altitude of 2000 m above sea level, during daylight hours when the average ambient temperature is 22·5°C. From Table 7.4 the percentage reduction is between 17·8 and 18·7, which, by interpolation is 18·25, and the reduced output is therefore $(100-18{\cdot}25)\times 18/100 = 0{\cdot}8175 \times 18 = 14{\cdot}7$ hp. If the engine is to be run continuously, this should be reduced by a further 25%, giving 11 hp.

Specific fuel consumption for a diesel engine is 0·25 l/bhp/hour at full load. At threequarters load it is 0·26 l/bhp/hour and at half load, 0·27 l/bhp/hour.

7.5.4 Electric Motors

Wherever electric power is available, electric motors can be used. Power supply is usually alternating (a.c.), and motors are rated in kilovolt amps (kVA). Power in kilowatts = kVA × *power factor*. A very wide range of electric motors is available commercially and professional advice should be sought in choosing a motor for a particular purpose. It is very important to specify the frequency of an a.c. supply, together with the operating voltage, and whether the supply is single-phase or three-phase.

7.6 MECHANICAL AIDS

7.6.1 Buoyancy

The buoyancy of a floating object is its resistance to sinking and is measured by the minimum weight required to sink the object, so that it is totally immersed. The uplift on a body immersed in water is equal to the weight of the volume of water which it displaces.

The buoyancy of a closed cylindrical drum of circumference C and length L, both in metres, is approximately $80\,C^2L-W$ in kilogrammes, where W is the weight of the drum in kilogrammes. In practice the safe buoyancy is 90 per cent of the total buoyancy.

7.6.2 Mechanical Advantage Gained by Blocks

The theoretical gain achieved by a hoist consisting of a fixed block and a moveable block is the number of ropes leading to or from the moveable block. In practice energy is used in overcoming friction in the blocks and the actual force P required to lift a weight W is given by:

$$P = \frac{W}{G}(1 + fn)$$

where G is the theoretical gain as above, f is a friction coefficient equal to 0·17 for tackle in average condition, or 0·2 or more for tackle in poor condition, and n is the total number of sheaves (wheels in the blocks) in the tackle.

Basic SI units, prefixes, and most common derived SI units used

Basic SI units

Quantity	*Basic unit*	*Symbol*
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampère	A
Temperature	kelvin	K

SI prefixes

Prefix	*Symbol*	*Factor*	*Prefix*	*Symbol*	*Factor*
exa	E	10^{18}	deci	d	10^{-1}
peta	P	10^{15}	centi	c	10^{-2}
tera	T	10^{12}	milli	m	10^{-3}
giga	G	10^{9}	micro	μ	10^{-6}
mega	M	10^{6}	nano	n	10^{-9}
kilo	k	10^{3}	pico	p	10^{-12}
hecto	h	10^{2}	femto	f	10^{-15}
deca	da	10^{1}	atto	a	10^{-18}

Most common derived SI units

Quantity	*Unit*	*Symbol*
Area	square metre	m^2
Volume (contents)	cubic metre	m^3
Speed	metre per second	m/s
Acceleration	metre per second, squared	m/s^2
Frequency	hertz	Hz (= s^{-1})
Pressure	pascal	Pa (= N/m^2)
Volume flow	cubic metre per second	m^3/s
Mass flow	kilogram per second	kg/s
Density (specific mass)	kilogram per cubic metre	kg/m^3
Force	newton	N (= $kg.m/s^2$)
Energy/heat/work	joule	J (= N.m)*
Power/energy flow	watt	W (J/s)
Energy flux	watt per square metre	W/m^2
Calorific value (heat of combustion)	joule per kilogram	J/kg
Specific heat capacity	joule per kilogram kelvin	J/kg K
Voltage	volt	V (= W/A)

* NB The joule can also be written in the form watt second (1J = 1W.s)

Conversion of non-SI units to SI units

Although academic scientists and engineers may be strict in their use of SI units for their calculations, a number of non-SI units are still in everyday use. For example, engines are still sold by cc (cubic centimetres) and hp (horse power), and water-pumping windmill manufacturers often quote in terms of cubic feet of the same type of equipment there is not always consistency. In order to be able to compare different manufacturers' products, therefore, it is important to be able convert the different data to a common unit. The following tables give some useful conversion factors for many of the common non-SI units.

Length

Unit (symbol)	millimetre (mm)	metre (m)	kilometre (km)	inch (in.)	foot (ft)	mile (m.)
	1	0.001	10^{-6}	0.0394	0.0033	5.4×10^{-7}
	1000	1	0.001	39.4	3.28	5.4×10^{-4}
	10^{6}	1000	1	39360	3280	0.5392
	25.4	0.025	2.5×10^{-5}	1	0.083	1.4×10^{-5}
	305	0.305	3.0×10^{-4}	12	1	1.9×10^{-4}
	1.6×10^{6}	1609	1.609	63360	5280	1

Area

Unit (symbol)	square metre (m^2)	hectare (ha)	square kilometre (km^2)	square foot (ft^2)	acre –	square mile (sq. m.)
	1	10^{-4}	10^{-6}	10.76	2.5×10^{-4}	3.9×10^{-7}
	10000	1	0.01	1.1×10^{5}	2.471	3.9×10^{-3}
	10^{6}	100	1	1.1×10^{7}	247.1	0.386
	0.0929	9.3×10^{-6}	9.3×10^{-8}	1	2.3×10^{-5}	3.6×10^{-8}
	4047	0.4047	4×10^{-3}	43560	1	1.6×10^{-3}
	2.6×10^{6}	259	2.590	2.8×10^{7}	640	1

Volume

Unit (symbol)	litre (l)*	cubic metre (m^3)	cubic inch (in^3)	US gallon (gal)	Imperial gallon (gal)	cubic foot (ft^3)
	1	10^{-3}	61.02	0.264	0.220	0.0353
	1000	1	6102	264	220	35.31
	0.0164	1.6×10^{-5}	1	4.3×10^{-3}	3.6×10^{-3}	5.8×10^{-4}
	3.785	3.8×10^{-3}	231.1	1	0.833	0.134
	4.546	4.5×10^{-3}	277.4	1.201	1	0.160
	28.32	0.0283	1728	7.47	6.23	1

* L in some countries

Mass

Unit (symbol)	gram (g)	kilogram (kg)	tonne (t)	pound (lb)	ton –
	1	0.001	10^{-6}	2.2×10^{-3}	9.8×10^{-7}
	1000	1	0.001	2.205	9.8×10^{-4}
	10^{6}	1000	1	2205	0.984
	453.6	0.4536	4.5×10^{-4}	1	4.5×10^{-4}
	10^{6}	1016	1.016	2240	1

Velocity

Unit (symbol)	metres per second (m/s)	kilometres per hour (km/h)	feet per second (ft/s)	miles per hour (mph)	knots (kt)
	1	3.60	3.28	2.237	1.942
	0.278	1	0.912	0.621	0.539
	0.305	1.097	1	0.682	0.592
	0.447	1.609	1.467	1	0.868
	0.566	1.853	1.689	1.152	1

Frequency

Unit (symbol)	hertz (Hz)	revolutions per minute (rpm)	radians per second (rad/s)
	1	60	6.283
	0.0167	1	0.1047
	0.159	9.549	1

Flow rate

Unit (symbol)	litres per minute (l/min)	cubic metres per second (m^3/s)	Imperial gallons per minute (gal(Imp)/min)	cubic feet per second (ft^3/s)
	1	1.7×10^{-5}	0.220	5.9×10^{-4}
	60 000	1	13 206	35.315
	4.546	7.6×10^{-5}	1	2.7×10^{-3}
	1699	0.0283	373.7	1

Force

Unit (symbol)	newton (N)	kilonewton (kN)	kilogram force (kgf)	tonne force (t)	pound force (lbf)	ton force –
	1	0.001	0.102	1×10^{-4}	0.225	1×10^{-4}
	1000	1	102	0.102	225	0.100
	9.807	0.010	1	0.001	2.205	9.8×10^{-4}
	9807	9.807	1000	1	2205	0.984
	4.448	0.004	0.5436	4.5×10^{-4}	1	4.5×10^{-4}
	9964	9.964	1016	1.1016	2240	1

Torque

Unit (symbol)	newton-metre (Nm)	kilonewton-metre (kNm)	foot-pound (ft.lb)
	1	0.001	0.738
	1000	1	738
	1.365	1.4×10^{-3}	1

Work/heat/energy (smaller quantities)

Unit (symbol)	calorie (cal)	joule (J)	watt-hour (Wh)	British Thermal Unit (BTU)	footpound force (ft.lbf)	horsepower-hour (hp.h)
	1	4.182	1.2×10^{-3}	3.9×10^{-3}	3.088	1.6×10^{-6}
	0.239	1	2.8×10^{-4}	9.4×10^{-4}	0.7376	3.7×10^{-7}
	860.4	3600	1	3.414	2655	1.3×10^{-3}
	252	1055	2.93	1	778	3.9×10^{-4}
	0.324	1.356	3.8×10^{-4}	1.3×10^{-3}	1	5.0×10^{-7}
	6.4×10^{5}	2.6×10^{6}	745.7	2546	2.0×10^{6}	1

Work/heat/energy (larger quantities)

Unit (symbol)	kilocalorie (kcal)	megajoule (MJ)	kilowatt hour (kWh)	British Thermal Unit (BTU)	horsepower-hour (hp.h)
	1	4.2×10^{-3}	1.2×10^{-3}	3.968	1.6×10^{-3}
	239	1	0.2887	947.8	0.3725
	860.4	3.600	1	3414	1.341
	0.252	1.1×10^{-3}	2.9×10^{-4}	1	3.9×10^{-4}
	641.6	2.685	0.7457	2546	1

Power

Unit (symbol)	watt (W or J/s)	kilowatt (kW)	metric horse-power (CV)	foot-pound per second (ft.lbf/s)	horse-power (hp)	British Thermal Units per minute (BTU/min)
	1	0.001	1.4×10^{-3}	0.7376	1.3×10^{-3}	0.0569
	1000	1	1.360	737.6	1.341	56.9
	735	0.735	1	558	1.014	41.8
	1.356	1.4×10^{-3}	1.8×10^{-3}	1	1.8×10^{-3}	0.077
	746	0.746	0.9860	550	1	42.44
	17.57	0.0176	0.0239	12.96	0.0236	1

Power flux

Unit (symbol)	watts per square metre (W/m^2)	kilowatts per square metre (kW/m^2)	horsepower per square foot (hp/ft^2)
	1	0.001	1.2×10^{-4}
	1000	1	0.1246
	8023	8.023	1

Calorific value (heat of combustion)

Unit (symbol)	calories per gram (cal/g)	megajoules per kilogram (MJ/kg)	British thermal units per pound (BTU/lb)
	1	4.2×10^{-3}	1.8
	239	1	430
	0.556	2.3×10^{-3}	1

Density (specific mass) and (net) calorific value (heat of combustion) of fuels

	Density (kg/m^3)	Calorific value (MJ/kg)
LPG	560	45.3
Gasoline (petrol)	720	44.0
Kerosene	806	43.1
Diesel oil	850	42.7
Fuel oil	961	40.1
Wood, oven-dried	varies	16–20
Natural gas	—	$103 m^3$ at 1013 mbar, 0°C = 39.36×10^9 J

NB These values are approximate since the fuels vary in composition and this affects both the density and calorific value.

Replacement values

When trying to compare different fuel options, energy planners often use replacement values, which indicate in a specific situation how much fuel it would take to replace another one. For example, the tonne coal equivalent (tce) would be used to say how much coal it would take to replace a given quanity of oil or natural gas. The table below gives some of the most common equivalence values.

Fuel	Unit	Tonnes of coal equivalent (tce)	Tonnes of oil equivalent (toe)	Barrels of oil equivalent (boe)	GJ*
Coal	tonne	1.00	0.70	5.05	29.3**
Firewood (air-dried)	tonne	0.46	0.32	2.34	13.6
Kerosene	tonne	1.47	1.03	7.43	43.1
Natural gas	$1000 m^3$	1.19	0.83	6.00	34.8
Gasoline (petrol)	barrel***	0.18	0.12	0.90	5.2
Gasoil/diesel	barrel***	0.20	0.14	1.00	5.7

* GJ/tonne is numerically equivalent to MJ/kg
** The energy content of 1 tce and 1 toe varies. The values used here are the European Community norms:
1 tce = 29.31×10^9 J and 1 toe = 41.868×10^9 J
*** 1 barrel of oil = 42 US gallons = $0.158987 m^3$

Power equivalents

	Mtoe/yr	*Mbd*	*Mtce/yr*	*GW_{th}*	*PJ/yr*
Mtoe/yr	1	0.02	1.55	1.43	45
Mbd	50	1	77	71	2235
Mtce/yr	0.65	0.013	1	0.92	29
GW_{th}	0.70	0.014	1.09	1	32
PJ/yr	0.02	4.5×10^{-4}	0.034	0.031	1

Mtoe/yr = Million tonnes of oil per year
Mbd = Million barrels of oil per day
Mtce/yr = Million tonnes of coal equivalent per year
GW_{th} = Gigawatts thermal (see page 203 for further information)
PJ/yr = Petrajoules per year

USEFUL CONVERSION FORMULAE & TABLES

To convert Pounds (lb) to Kilogrammes (kg) multiply by 0.4536 and kilogrammes to pounds multiply by 2.205

Water 1 litre weighs 1.00 kg/2.2 lb.
1 gallon weighs 4.53 kg/10 lb.

Gallons to litres: Multiply by 4.546
Litres to gallons: Multiply by 0.22
1 cu ft holds 6¼ gallons & weighs 62.3lb
To convert cubic feet to cubic metres multiply by 0.028

The central figure in the table represents either of the two outside columns, as the case may be i.e. 1 gallon = 4.546 litres or 1 litre = 0.22 gallons.

Litres		Gallons
4.546	1	0.220
9.092	2	0.440
13.638	3	0.660
18.184	4	0.880
22.730	5	1.100
27.276	6	1.320
31.822	7	1.540
36.368	8	1.760
40.914	9	1.980

Petrol 1 litre weighs 0.73 kg/1.61 lb.
1 gallon weighs 3.36 kg/7.4 lb.

Diesel 1 litre weighs 0.84 kg/1.85 lb
1 gallon weighs 3.86 kg/8.5 lb.

Measurements

To convert inches to centimetres (cm) multiply by 2.54 and centimetres to inches multiply by 0.393

1 inch = 25.4 millimetres (mm)
1 foot = 30.48 centimetres (cm)
1 yard = 0.9144 metre (m)
1 mile = 1.6093 kilometres (km)
1 millimetre = 0.03937 inch
1 centimetre = 0.0328 foot (ft)
1 metre = 1.094 yards (yd)
1 kilometre = 0.62137 mile

The central figure in the table represents either of the two outside columns, as the case may be i.e. 1 inch = 2.54 centimetres or 1 centimetre = 0.394 inches.

Centimetres		Inches
2.540	1	0.394
5.080	2	0.787
7.620	3	1.181
10.160	4	1.575
12.700	5	1.969
15.240	6	2.362
17.780	7	2.756
20.320	8	3.150
22.860	9	3.543

Metres		Yards
0.914	1	1.094
1.829	2	2.187
2.743	3	3.281
3.658	4	4.374
4.572	5	5.468
5.486	6	6.562
6.401	7	7.655
7.315	8	8.749
8.230	9	9.843

Speed
To convert knots to miles per hour, multiply by 1.15

Distance
To convert miles to kilometres multiply by 1.609 and kilometres to miles multiply by 0.621

Temperature
To convert Centigrade to Fahrenheit divide by 5, multiply by 9 and add 32. To convert Fahrenheit to Centigrade deduct 32, divide by 9 and multiply by 5.

PLEASE NOTE: The above are only RECOMMENDATIONS and NO LIABILITY can be accepted for ANY DECISION based on these figures. The conversion between Imperial and Metric sizes is approximate.

Easy Conversions
Metres into yards . . . add one-tenth
Yards into metres . . . deduct one-tenth
Kilometres into miles . . . multiply by 5 and divide by 8
Miles into kilometres . . . multiply by 8 and divide by 5
Litres into pints . . . multiply by 7 and divide by 4
Pints into litres . . . multiply by 4 and divide by 7
Litres into gallons . . . multiply by 2 and divide by 9
Gallons into litres . . . multiply by 9 and divide by 2
Kilogrammes into pounds . . . divide by 9 and multiply by 20
Pounds into kilogrammes . . . divide by 20 and multiply by 9

10 millimetres		**(equal to 1 centimetre)**
10 centimetres	**about four inches**	
1 metre	**a long pace (a little more than a yard)**	
1 kilometre	**1000 metres (a little more than half a mile)**	
10 square metres	**about 12 square yards**	
1 hectare	**= 10 000 square metres (about two-and-a-half acres)**	
50 grams	**a little less than two ounces**	
500 grams	**a little more than one pound**	
1 tonne	**= 1000 kilograms (a little less than one ton)**	
500 millilitres	**a little less than one pint**	
1 litre	**about one-and-three-quarter pints**	
200 litres	**the volume of a 44-gallon drum.**	

MEASUREMENTS (USING THE BODY) AND OTHER COMMON REFERENCES.

Often when you want to measure something you find that you have left your ruler behind. One ruler that you can't lose is yourself. Different parts of your body can be conveniently used as a ruler. These measurements may be a little different for different people. Here are some typical measurements for one person. You can check your own. If you don't need to be exact these will give you a close measurement.

3-4 cm	The distance from the tip of your thumb to first knuckle.
8 cm	The distance across your hand at the widest part of four fingers.
10 cm	The distance across a flat hand including the thumb.
20-22cm	The distance from your thumb tip to the tip of your little finger when stretched as far apart as possible.
25 cm	The length of a bare foot, heel to big toe.
45 cm	The distance from elbow to tip of longest finger. Often you can estimate this as half a metre (50cm)
1 metre	The distance of a long pace. This distance is slightly longer than a step when walking comfortably. Measure out metre lengths for about 20 metres with chalk on concrete. Practice walking so that your toe always hits the metre mark. Soon your body will learn the feel of a metre step. This is particularly useful if you need to measure the side of a garden, or the length of a fence.

Properly used these measurements will give you within 10% of the actual distance. This is close enough for most agricultural requirements.

1 fish tin holds about ½kg (500 g) of fertilizer, salt, sugar.
1 bottle cap holds 3 cc when level.
The wood of a matchbox cover is approx. .50mm (Suitable to check the gap for spark plugs or contact points in an emergency. (A hacksaw blade is approx. 0.030", or 0.76 mm.)

Appendix

Metric conversion charts

Metric prefixes and abbreviations

The metre is used as an example below. The same prefixes apply to litres (l or lit) and grams (g). The abbreviation lit is used for litre when unqualified to avoid confusion with the numeral 1.

millimetre (mm)	0.001	one thousandth metre
centimetre (cm)	0.01	one hundredth metre
decimetre (dm)	0.1	one tenth metre
metre (m)	1	one metre
decametre (dam)	10	ten metres
hectometre (hm)	100	one hundred metres
kilometre (km)	1000	one thousand metres

Length (linear measure)

Fractions of 1 inch in millimetres

Thirty-seconds, sixteenths, eighths, quarters and one half

in	*mm*
1/32	0.8
1/16	1.6
3/32	2.4
1/8	3.2
5/32	4.0
3/16	4.8
7/32	5.6
1/4	6.3
9/32	7.1
5/16	7.9
11/32	8.7
3/8	9.5
13/32	10.3
7/16	11.1
15/32	11.9
1/2	12.7
17/32	13.5
9/16	14.3
19/32	15.1
5/8	15.9
21/32	16.7
11/16	17.5
23/32	18.3
3/4	19.0
25/32	19.8
13/16	20.6
27/32	21.4
7/8	22.2
29/32	23.0
15/16	23.8
31/32	24.6
1 inch	25.4

Twelfths, sixths and thirds

in	*mm*
1/12	2.1
1/6	4.2
1/4	6.3
1/3	8.5
5/12	10.6
1/2	12.7
7/12	14.8
2/3	16.9
3/4	19.0
5/6	21.2
11/12	23.3
1 inch	25.4

Note

Find the Imperial figure you wish to convert in the **heavy** type central column and read off the metric equivalent in the right-hand column and vice versa.
For example:
10 inches = 254 millimetres and 10mm = 0.39in.

Conversion from inches is only taken up to 40 in the chart below, see next chart for continuation.

Inches/millimetres

in		*mm*
0.04	**1**	25.4
0.08	**2**	50.8
0.12	**3**	76.2
0.16	**4**	101.6
0.20	**5**	127.0
0.24	**6**	152.4
0.28	**7**	177.8
0.31	**8**	203.2
0.35	**9**	228.6
0.39	**10**	254.0
0.43	**11**	279.4
0.47	**12**	304.8
0.51	**13**	330.2
0.55	**14**	355.6
0.59	**15**	381.0
0.63	**16**	406.4
0.67	**17**	431.8
0.71	**18**	457.2
0.75	**19**	482.6
0.79	**20**	508.0
0.83	**21**	533.4
0.87	**22**	558.8
0.91	**23**	584.2
0.94	**24**	609.6
0.98	**25**	635.0
1.02	**26**	660.4
1.06	**27**	685.8
1.10	**28**	711.2
1.14	**29**	736.6
1.18	**30**	762.0
1.22	**31**	787.4
1.26	**32**	812.8
1.30	**33**	838.2
1.34	**34**	863.6
1.38	**35**	889.0
1.42	**36**	914.4
1.46	**37**	939.8
1.50	**38**	965.2
1.54	**39**	990.6
1.57	**40**	1016.0
1.97	**50**	
2.36	**60**	
2.76	**70**	
3.15	**80**	
3.54	**90**	
3.94	**100**	
7.87	**200**	
11.81	**300**	
15.75	**400**	
19.68	**500**	
23.62	**600**	
27.56	**700**	
31.50	**800**	
35.43	**900**	
39.37	**1000**	

Imperial measurements are expressed below in yards, feet and inches rather than in decimals for convenience if converting with rulers or measuring tapes which do not include decimal readings.

Feet/metres

ft	*in*		*m*
3	3	**1**	0.30
6	7	**2**	0.61
9	10	**3**	0.91
13	1	**4**	1.22
16	5	**5**	1.52
19	8	**6**	1.83
23	0	**7**	2.13
26	3	**8**	2.44
29	6	**9**	2.74
32	10	**10**	3.05
65	7	**20**	6.10
98	5	**30**	9.14
131	3	**40**	12.19
164	0	**50**	15.24
196	10	**60**	18.29
229	8	**70**	21.34
262	6	**80**	24.38
295	3	**90**	27.43
328	1	**100**	30.48

Yards/metres

yd	*ft*	*in*		*m*
1	0	3	**1**	0.9
2	0	7	**2**	1.8
3	0	10	**3**	2.7
4	1	1	**4**	3.7
5	1	5	**5**	4.6
6	1	8	**6**	5.5
7	2	0	**7**	6.4
8	2	3	**8**	7.3
9	2	6	**9**	8.2
10	2	10	**10**	9.1
21	2	7	**20**	18.3
32	2	5	**30**	27.4
43	2	3	**40**	36.6
54	2	0	**50**	45.7
65	1	10	**60**	54.9
76	1	8	**70**	64.0
87	1	6	**80**	73.2
98	1	3	**90**	82.3
109	1	1	**100**	91.4

Quick conversion factors – length

Terms are set out in full in the left-hand column except where clarification is necessary.

1 inch (in)	= 25.4mm/2.54cm
1 foot (ft)/12in	= 304.8mm/30.48cm/0.3048m
1 yard (yd)/3ft	= 914.4mm/91.44cm/0.9144m
1 mile (mi)/1760yd	= 1609.344m/1.609km
1 millimetre (mm)	= 0.0394in
1 centimetre (cm)/10mm	= 0.394in
1 metre (m)/100cm	= 39.37in/3.281ft/1.094yd
1 kilometre (km)/1000m	= 1093.6yd/0.6214mi

Quick conversion factors – area

1 square inch (sq in)	= 645.16sq mm/ 6.4516sq cm
1 square foot (sq ft)/144sq in	= 929.03sq cm
1 square yard (sq yd)/9sq ft	= 8361.3sq cm/ 0.8361sq m
1 acre (ac)/4840sq yd	= 4046.9sq m/0.4047ha
1 square mile (sq mi)640ac	= 259ha
1 square centimetre (sq cm)/ 100 square millimetre (sq mm)	= 0.155sq in
1 square metre (sq m)/ 10,000sq cm	= 10.764sq ft/1.196sq yd
1 are (a)/100sq m	= 119.60sq yd/0.0247ac
1 hectare (ha)/100a	= 2.471ac/0.00386sq mi

Quick conversion factors – volume

1 cubic inch (cu in)	= 16.3871cu cm
1 cubic foot (cu ft)/ 1728cu in	= 28.3168cu dm/0.0283cu m
1 cubic yard (cu yd)/ 27cu ft	= 0.7646cu m
1 cubic centimetre (cu cm)/ 1000 cubic millimetres (cu mm)	= 0.0610cu in
1 cubic decimetre (cu dm)/ 1000cu cm	= 61.024cu in/0.0353cu ft
1 cubic metre (cu m)/ 1000cu dm	= 35.3146cu ft/1.308cu yd
1cu cm	= 1 millilitre (ml)
1cu dm	= 1 litre (lit) See **Capacity**

Area (square measure)

As millimetre numbers would be unwieldy for general use, square or cubic inches have been converted to square or cubic centimetres. Conversion from square inches is only taken up to 150 in the first chart below; see next chart for continuation.

Square inches/square centimetres

sq in		sq cm
0.2	**1**	6.5
0.3	**2**	12.9
0.5	**3**	19.4
0.6	**4**	25.8
0.8	**5**	32.3
0.9	**6**	38.7
1.1	**7**	45.2
1.2	**8**	51.6
1.4	**9**	58.1
1.6	**10**	64.5
3.1	**20**	129.0
4.7	**30**	193.5
6.2	**40**	258.1
7.8	**50**	322.6
9.3	**60**	387.1
10.9	**70**	451.6
12.4	**80**	516.1
14.0	**90**	580.6
15.5	**100**	645.2
17.1	**110**	709.7
18.6	**120**	774.2
20.2	**130**	838.7
21.7	**140**	903.2
23.3	**150**	967.7
31.0	**200**	
46.5	**300**	
62.0	**400**	
77.5	**500**	
93.0	**600**	
108.5	**700**	
124.0	**800**	
139.5	**900**	
155.0	**1000**	

Square feet/square metres

sq ft		sq m
10.8	**1**	0.09
21.5	**2**	0.19
32.3	**3**	0.28
43.1	**4**	0.37
53.8	**5**	0.46
64.6	**6**	0.56
75.3	**7**	0.65
86.1	**8**	0.74
96.9	**9**	0.84
107.6	**10**	0.93
215.3	**20**	1.86
322.9	**30**	2.79
430.6	**40**	3.72
538.2	**50**	4.65
645.8	**60**	5.57
753.5	**70**	6.50
861.1	**80**	7.43
968.8	**90**	8.36
1076.4	**100**	9.29

Square yards/square metres

sq yd		sq m
1.2	**1**	0.8
2.4	**2**	1.7
3.6	**3**	2.5
4.8	**4**	3.3
6.0	**5**	4.2
7.2	**6**	5.0
8.4	**7**	5.9
9.6	**8**	6.7
10.8	**9**	7.5
12.0	**10**	8.4
23.9	**20**	16.7
35.9	**30**	25.1
47.8	**40**	33.4
59.8	**50**	41.8
71.8	**60**	50.2
83.7	**70**	58.5
95.7	**80**	66.9
107.6	**90**	75.3
119.6	**100**	83.6

Volume (cubic measure)

Cubic inches/cubic centimetres

cu in		cu cm
0.06	**1**	16.4
0.12	**2**	32.8
0.18	**3**	49.2
0.24	**4**	65.5
0.31	**5**	81.9
0.37	**6**	98.3
0.43	**7**	114.7
0.49	**8**	131.1
0.55	**9**	147.5
0.61	**10**	163.9
1.22	**20**	327.7
1.83	**30**	491.6
2.44	**40**	655.5
3.05	**50**	819.4
3.66	**60**	983.2
4.27	**70**	1147.1/1.15cu dm
4.88	**80**	1311.0/1.31cu dm
5.49	**90**	1474.8/1.47cu dm
6.10	**100**	1638.7/1.64cu dm
12.20	**200**	3277.4/3.28cu dm
18.31	**300**	4916.1/4.92cu dm
24.41	**400**	6554.8/6.55cu dm
30.51	**500**	8193.5/8.19cu dm
36.61	**600**	9832.2/9.83cu dm
42.72	**700**	11470.9/11.47cu dm
48.82	**800**	13109.7/13.11cu dm
54.92	**900**	14748.4/14.75cu dm
61.02	**1000**	16387.1/16.39cu dm
122.05	**2000**	32774.1/32.77cu dm

Cubic feet/cubis decimetres

cu ft		cu dm
.04	**1**	28.3
0.07	**2**	56.6
0.11	**3**	85.0
0.14	**4**	113.3
0.18	**5**	141.6
0.21	**6**	169.9
0.25	**7**	198.2
0.28	**8**	226.5
0.32	**9**	254.9
0.35	**10**	283.2
0.71	**20**	566.3
1.06	**30**	849.5
1.41	**40**	1132.7/1.13cu m
1.77	**50**	1415.8/1.42cu m
2.12	**60**	1699.0/1.70cu m
2.47	**70**	1982.2/1.98cu m
2.83	**80**	2265.3/2.27cu m
3.18	**90**	2548.5/2.55cu m
3.53	**100**	2831.7/2.83cu m

Cubic yards/cubic metres

cu yd		cu m
1.3	**1**	0.8
2.6	**2**	1.5
3.9	**3**	2.3
5.2	**4**	3.1
6.5	**5**	3.8
7.8	**6**	4.6
9.2	**7**	5.4
10.5	**8**	6.1
11.8	**9**	6.9
13.1	**10**	7.6
26.2	**20**	15.3
39.2	**30**	22.9
52.3	**40**	30.6
65.4	**50**	38.2
78.5	**60**	45.9
91.6	**70**	53.5
104.6	**80**	61.2
117.7	**90**	68.8
130.8	**100**	76.5

Capacity

Fluid ounces/millilitres

fl oz		
0.04	**1**	28.4
0.07	**2**	56.8
0.11	**3**	85.2
0.14	**4**	113.6
0.18	**5**	142.1
0.21	**6**	170.5
0.25	**7**	198.9
0.28	**8**	227.3
0.32	**9**	255.7
0.35	**10**	284.1
0.70	**20**	568.2
1.06	**30**	852.4
1.41	**40**	1136.5/1.136 lit
1.76	**50**	1420.6/1.421 lit

Pints/litres

pt		lit
1.8	**1**	0.6/568ml
3.5	**2**	1.1
5.3	**3**	1.7
7.0	**4**	2.3
8.8	**5**	2.8
10.6	**6**	3.4
12.3	**7**	4.0
14.1	**8**	4.5
15.8	**9**	5.1
17.6	**10**	5.7

Gallons/litres

gal		lit
0.2	**1**	4.5
0.4	**2**	9.1
0.7	**3**	13.6
0.9	**4**	18.2
1.1	**5**	22.7
1.3	**6**	27.3
1.5	**7**	31.8
1.8	**8**	36.4
2.0	**9**	40.9
2.2	**10**	45.5
4.4	**20**	90.9
6.6	**30**	136.4
8.8	**40**	181.8
11.0	**50**	227.3
13.2	**60**	272.8
15.4	**70**	318.2
17.6	**80**	363.7
19.8	**90**	409.1
22.0	**100**	454.6

Weight

Ounces/grams

oz		g
0.04	**1**	28.3
0.07	**2**	56.7
0.11	**3**	85.0
0.14	**4**	113.4
0.18	**5**	141.7
0.21	**6**	170.1
0.25	**7**	198.4
0.28	**8**	226.8
0.32	**9**	255.1
0.35	**10**	283.5
0.39	**11**	311.8
0.42	**12**	340.2
0.46	**13**	368.5
0.49	**14**	396.9
0.53	**15**	425.2
0.56	**16**	453.6
0.71	**20**	567.0
1.06	**30**	850.5
1.41	**40**	1134.0
1.76	**50**	1417.5
2.12	**60**	1701.0
2.47	**70**	1984.5
2.82	**80**	2268.0
3.17	**90**	2551.5
3.53	**100**	2835.0

Pounds/kilograms

lb		kg
2.2	**1**	0.5
4.4	**2**	0.9
6.6	**3**	1.4
8.8	**4**	1.8
11.0	**5**	2.3
13.2	**6**	2.7
15.4	**7**	3.2
17.6	**8**	3.6
19.8	**9**	4.1
22.0	**10**	4.5
44.1	**20**	9.1
66.1	**30**	13.6
88.2	**40**	18.1
110.2	**50**	22.7
132.3	**60**	27.2
154.3	**70**	31.8
176.4	**80**	36.3
198.4	**90**	40.8
220.5	**100**	45.4

Quick conversion factors – capacity

1 fluid ounce (fl oz)	= 28.4ml
1 gill (gi)/5fl oz	= 142.1ml
1 pint (pt)/4gi	= 568.2ml/0.568 lit
1 quart (qt)/2pt	= 1.136 lit
1 gallon (gal)/4pt	= 4.546 lit
1 millilitre (ml)	= 0.035fl oz
1 litre (lit)	= 1.76pt/0.22gal
1ml	= 1 cubic centimetre (cu cm)
1 lit	= 1 cubic decimetre (cu dm) See **Volume**
1 US pint	= 5/6 Imperial pt/473.2ml/0.473 lit
1 US gallon	= 5/6 Imperial gal/3.785 lit

Quick conversion factors – weight

1 ounce (oz)	= 28.35g
1 pound (lb)/16oz	= 453.59g/0.4536kg
1 stone/14lb	= 6.35kg
1 hundredweight (cwt)/ 8 stone/112lb	= 50.80kg
1 ton/20cwt	= 1016.05kg/1.016t
1 gram (g)	= 0.035oz
1 kilogram (kg)/1000g	= 35.274oz/2.2046lb/ 2lb 3.274oz
1 tonne (t)/1000kg	= 2204.6lb/0.9842 ton

kubik-meter m^3	kubik-decimeter (= liter) dm^3	cubic-inch in^3	cubic-foot ft^3	fluid ounce (UK) oz (UK)	pint (UK) pt (UK)	gallon (UK) gal (UK)	fluid ounce (US) fl oz (US)	liquid pint (US) liq pt (US)	gallon (US) gal (US)
1	1000	61023,7	35,3146	35195	1759,75	219,97	33817,6	2113,6	264,2
10^{-3}	**1**	61,0237	0,03531	35,195	1,75975	0,21997	33,8176	2,1136	0,26417
$0,016 \cdot 10^{-3}$	0,01639	**1**	$0,579 \cdot 10^{-3}$	0,57674	0,02884	$3,605 \cdot 10^{-3}$	0,55417	0,03463	0,00433
0,02832	28,3168	1728	**1**	996,607	49,8304	6,2288	957,610	59,85	7,4805
$0,028 \cdot 10^{-3}$	0,02841	1,73387	0,00100	**1**	0,05	0,00625	0,9608	0,0601	0,00751
$0,568 \cdot 10^{-3}$	0,56826	34,6774	0,02003	20	**1**	0,125	19,2167	1,2001	0,15013
0,00455	4,5461	277,42	0,16054	160	8	**1**	153,735	9,6062	1,201
$0,03 \cdot 10^{-3}$	0,02957	1,8045	0,00104	1,0408	0,05204	0,00650	**1**	0,0625	0,00781
$0,473 \cdot 10^{-3}$	0,47313	28,875	0,01688	16,652	0,83264	0,10408	16	**1**	0,125
0,00379	3,78541	231	0,13368	133,216	6,6608	0,83267	128	8	**1**

gram	kilogram	ton	tekma	ounce	pound	short hundred-weight (US)	short ton (US)	hundred-weight (UK) (= long cwt, US)	ton (UK) (= long ton, US)
g	kg	t		oz	lb	sh. cwt	sh. ton	cwt	ton
1	0,001	10^{-6}	$0{,}102 \cdot 10^{-3}$	0,03527	0,00221	–	–	–	–
1000	**1**	0,001	0,102	35,274	2,205	0,02205	0,00110	$19{,}68 \cdot 10^{-3}$	$0{,}984 \cdot 10^{-3}$
1000000	1000	**1**	102	35274	2204,6	22,046	1,1023	19,68	0,9842
9807	9,807	0,00981	**1**	345,9	21,6205	0,2162	0,01081	0,193	0,00965
28,35	0,02835	$0{,}028 \cdot 10^{-3}$	0,00289	**1**	0,0625	$0{,}625 \cdot 10^{-3}$	$0{,}031 \cdot 10^{-3}$	$0{,}558 \cdot 10^{-3}$	$0{,}0279 \cdot 10^{-3}$
453,592	0,45359	$0{,}454 \cdot 10^{-3}$	0,04625	16	**1**	0,01	$0{,}5 \cdot 10^{-3}$	0,00893	$0{,}446 \cdot 10^{-3}$
45359	45,36	0,0454	4,625	1600	100	**1**	0,05	0,8929	0,04464
-	907,2	0,9072	92,507	32000	2000	20	**1**	17,86	0,8929
50802	50,80	0,0508	5,18	1792	112	1,12	0,056	**1**	0,05
-	1016	1,016	103,61	35840	2240	22,4	1,12	20	**1**

1 in	=	2,5400	cm
1 ft	=	0,3048	m
1 yd	=	0,9144	m
1 fathom	=	1,8288	m
1 stat mile	=	1,6093	km
1 naut mile	=	1,8532	km
1 int mile	=	1,8520	km
1 toise (Paris)	=	1,9390	m
1 ligne (Paris)	=	2,2558	mm

1 ft	=	12	in
1 yd	=	3	ft
1 fathom	=	2	yd
1 stat mile	=	1760	yd
1 naut mile	=	2026,7	yd

1 sq in	=	6,4516	cm^2
1 sq ft	=	9,2903	dm^2
1 sq yd	=	0,8361	m^2
1 acre	=	40,4684	a
1 sq mile	=	2,5899	km^2
1 acre	=	4840	sq yd

1 cu in	=	16,3871	cm^3
1 cu ft	=	28,3167	dm^3
1 cu yd	=	0,7646	m^3
1 pint	=	0,5683	dm^3
1 imp gal	=	4,5460	dm^3
1 US gal	=	3,7854	dm^3
1 US qt	=	0,9463	dm^3

1 gal	=	4	qt
1 qt	=	2	pt
1 imp gal	=	277,42	cu in
1 US gal	=	231,00	cu in
1 imp bu	=	8	imp gal
1 US bu	=	9,3092	US gal

1 oz	=	28,3495	g
1 lb	=	0,4536	kg
1 cwt	=	50,8024	kg
1 ton	=	1,0160	t
1 sh cwt	=	45,3592	kg
1 sh ton	=	0,9072	t

1 sh ton	=	2000	lb
1 ton	=	2240	lb

1 naut mile/h	=	0,5144	m/sec
1 stat mile/h	=	0,4470	m/sec
1 ft/sec	=	1,0973	km/h
1 stat mile/h	=	1,4666	ft/sec

1 sq yd/min	=	50,1675	m^2/h
1 sq ft/min	=	5,5742	m^2/h
1 sq in/sec	=	2,3226	m^2/h
1 sq yd/min	=	1,3934	dm^2/sec
1 sq ft/min	=	15,4838	cm^2/sec
1 sq in/min	=	0,1075	cm^2/sec
1 imp gal/min	=	0,2728	m^3/h
1 US gal/min	=	0,2271	m^3/h

1 in Hg	=	$3,3421 \cdot 10^{-2}$	atm
1 in H_2O	=	$2,4583 \cdot 10^{-3}$	atm
1 lb/sq in	=	$6,8046 \cdot 10^{-2}$	atm
1 in Hg	=	$3,4532 \cdot 10^{-2}$	at
1 in H_2O	=	$2,5400 \cdot 10^{-3}$	at
1 lb/sq in	=	$7,0307 \cdot 10^{-2}$	at
1 in Hg	=	$3,3864 \cdot 10^{-2}$	bar
1 in H_2O	=	$2,4909 \cdot 10^{-3}$	bar
1 lb/sq in	=	$6,8948 \cdot 10^{-2}$	bar
1 Torr	=	$1,3158 \cdot 10^{-3}$	atm
1 atm	=	1,03323	kg/cm^2
1 at	=	1,0000	kg/cm^2
1 at	=	0,9678	atm
1 bar	=	1,01972	kg/cm^2
1 br atm	=	30	in Hg

1 lb/in	=	$1,7858 \cdot 10^{-1}$	kg/cm
1 lb/ft	=	1,4882	kg/m
1 lb/yd	=	$4,9606 \cdot 10^{-1}$	kg/m

1 lb/cu in	=	$2,7680 \cdot 10^{-2}$	kg/cm^3
1 lb/cu ft	=	$1,6019 \cdot 10$	kg/m^3
1 lb/cu yd	=	$5,9328 \cdot 10^{-1}$	kg/m^3
1 oz/cu in	=	1,7300	g/cm^3
1 oz/cu ft	=	1,0012	kg/m^3
1 lb/imp gal	=	$9,9779 \cdot 10^{-2}$	kg/dm^3
1 lb/US gal	=	$1,1983 \cdot 10^{-1}$	kg/dm^3
1 lb/pint	=	$7,9823 \cdot 10^{-1}$	kg/dm^3

1 lb/min	=	$2,7216 \cdot 10$	kg/h
1 lb/sec	=	1,6329	t/h

1 HK	=	0,886	IK	=	0,903	cd
1 cd	=	0,981	IK	=	1,107	HK
1 IK	=	1,019	cd	=	1,128	HK
1 bougie décimale				=	1,12	HK
1 sperm candle				=	1,14	HK
1 pentan candle				=	1,11	HK
1 carcel				=	10,75	HK
1 VK				=	1,20	HK

1 sb	=	$\pi \, 10^4$	asb
1 la	=	10^4	asb
1 m la	=	10^{-3}	la
1 ft la	=	10,764	asb
1 ft cd	=	10,764	lx
1 ph	=	10^4	lx
1 lx	=	1	lm/m^2

1 BTU	=	0,2520	IT kcal
1 BTU/lb	=	0,5555	IT kcal/kg
1 BTU/°F	=	0,4536	IT kcal/°C
1 BTU/lb °F	=	1,0000	IT kcal/kg °C
1 BTU/ft °F	=	1,4884	IT kcal/m °C
1 BTU/in °F	=	17,858	IT kcal/m °C
1 BTU/sq ft °F	=	4,8824	IT kcal/m^2 °C
1 BTU/sq in °F	=	703,07	IT kcal/m^2 °C
1 BTU/sq ft	=	2,7124	IT kcal/m^2
1 BTU/in	=	9,9211	IT kcal/m

$$T_K = t_C + 273,15 = {}^5/_9 \, T_{Rank}$$
$$T_{Rank} = t_F + 459,67 = {}^9/_5 \, T_K$$
$$t_C = {}^5/_9 \, (t_F - 32) = T_K - 273,15$$
$$t_F = {}^9/_5 \, t_C + 32 = T_{Rank} - 459,67$$

Metric conversion tables

			Millimetres to Inches		Inches to Millimetres	
Inches	Decimals	Millimetres	mm	Inches	Inches	mm
1/64	0.015625	0.3969	0.01	0.00039	0.001	0.0254
1/32	0.03125	0.7937	0.02	0.00079	0.002	0.0508
3/64	0.046875	1.1906	0.03	0.00118	0.003	0.0762
1/16	0.0625	1.5875	0.04	0.00157	0.004	0.1016
5/64	0.078125	1.9844	0.05	0.00197	0.005	0.1270
3/32	0.09375	2.3812	0.06	0.00236	0.006	0.1524
7/64	0.109375	2.7781	0.07	0.00276	0.007	0.1778
1/8	0.125	3.1750	0.08	0.00315	0.008	0.2032
9/64	0.140625	3.5719	0.09	0.00354	0.009	0.2286
5/32	0.15625	3.9687	0.1	0.00394	0.01	0.254
11/64	0.171875	4.3656	0.2	0.00787	0.02	0.508
3/16	0.1875	4.7625	0.3	0.01181	0.03	0.762
13/64	0.203125	5.1594	0.4	0.01575	0.04	1.016
7/32	0.21875	5.5562	0.5	0.01969	0.05	1.270
15/64	0.234375	5.9531	0.6	0.02362	0.06	1.524
1/4	0.25	6.3500	0.7	0.02756	0.07	1.778
17/64	0.265625	6.7469	0.8	0.03150	0.08	2.032
9/32	0.28125	7.1437	0.9	0.03543	0.09	2.286
19/64	0.296875	7.5406	1	0.03937	0.1	2.54
5/16	0.3125	7.9375	2	0.07874	0.2	5.08
21/64	0.328125	8.3344	3	0.11811	0.3	7.62
11/32	0.34375	8.7312	4	0.15748	0.4	10.16
23/64	0.359375	9.1281	5	0.19685	0.5	12.70
3/8	0.375	9.5250	6	0.23622	0.6	15.24
25/64	0.390625	9.9219	7	0.27559	0.7	17.78
13/32	0.40625	10.3187	8	0.31496	0.8	20.32
27/64	0.421875	10.7156	9	0.35433	0.9	22.86
7/16	0.4375	11.1125	10	0.39370	1	25.4
29/64	0.453125	11.5094	11	0.43307	2	50.8
15/32	0.46875	11.9062	12	0.47244	3	76.2
31/64	0.484375	12.3031	13	0.51181	4	101.6
1/2	0.5	12.7000	14	0.55118	5	127.0
33/64	0,515625	13.0969	15	0.59055	6	152.4
17/32	0.53125	13.4937	16	0.62992	7	177.8
35/64	0.546875	13.8906	17	0.66929	8	203.2
9/16	0.5625	14.2875	18	0.70866	9	228.6
37/64	0.578125	14.6844	19	0.74803	10	254.0
19/32	0.59375	15.0812	20	0.78740	11	279.4
39/64	0.609375	15.4781	21	0.82677	12	304.8
5/8	0.625	15.8750	22	0.86614	13	330.2
41/64	0.640625	16.2719	23	0.90551	14	355.6
21/32	0.65625	16.6687	24	0.94488	15	381.0
43/64	0.671875	17.0656	25	0.98425	16	406.4
11/16	0.6875	17.4625	26	1.02362	17	431.8
45/64	0.703125	17.8594	27	1.06299	18	457.2
23/32	0.71875	18.2562	28	1.10236	19	482.6
47/64	0.734375	18.6531	29	1.14173	20	508.0
3/4	0.75	19.0500	30	1.18110	21	533.4
49/64	0.765625	19.4469	31	1.22047	22	558.8
25/32	0.78125	19.8437	32	1.25984	23	584.2
51/64	0.796875	20.2406	33	1.29921	24	609.6
13/16	0.8125	20.6375	34	1.33858	25	635.0
53/64	0.828125	21.0344	35	1.37795	26	660.4
27/32	0.84375	21.4312	36	1.41732	27	685.8
55/64	0.859375	21.8281	37	1.4567	28	711.2
7/8	0.875	22.2250	38	1.4961	29	736.6
57/64	0.890625	22.6219	39	1.5354	30	762.0
29/32	0.90625	23.0187	40	1.5748	31	787.4
59/64	0.921875	23.4156	41	1.6142	32	812.8
15/16	0.9375	23.8125	42	1.6535	33	838.2
61/64	0.953125	24.2094	43	1.6929	34	863.6
31/32	0.96875	24.6062	44	1.7323	35	889.0
63/64	0.984375	25.0031	45	1.7717	36	914.4

VIII. Decimal Equivalents of Common Fractions

8ths	16ths	32ds	64ths		8ths	16ths	32ds	64ths	
			1	.015625				33	.515625
		1	2	.03125			17	34	.53125
			3	.046875				35	.546875
	1	2	4	.0625		9	18	36	.5625
			5	.078125				37	.578125
		3	6	.09375			19	38	.59375
			7	.109375				39	.609375
1	2	4	8	.125	5	10	20	40	.625
			9	.140625				41	.640625
		5	10	.15625			21	42	.65625
			11	.171875				43	.671875
	3	6	12	.1875		11	22	44	.6875
			13	.203125				45	.703125
		7	14	.21875			23	46	.71875
			15	.234375				47	.734375
2	4	8	16	.25	6	12	24	48	.75
			17	.265625				49	.765625
		9	18	.28125			25	50	.78125
			19	.296875				51	.796875
	5	10	20	.3125		13	26	52	.8125
			21	.328125				53	.828125
		11	22	.34375			27	54	.84375
			23	.359375				55	.859375
3	6	12	24	.375	7	14	28	56	.875
			25	.390625				57	.890625
		13	26	.40625			29	58	.90625
			27	.421875				59	.921875
	7	14	28	.4375		15	30	60	.9375
			29	.453125				61	.953125
		15	30	.46875			31	62	.96875
			31	.484375				63	.984375
4	8	16	32	.5	8	16	32	64	1

Table D-1. Fraction, Decimal, and Metric Equivalents

Fractions	Decimal In.	Metric mm.	Fractions	Decimal In.	Metric mm.
1/64	.015625	.397	33/64	.515625	13.097
1/32	.03125	.794	17/32	.53125	13.494
3/64	.046875	1.191	35/64	.546875	13.891
1/16	.0625	1.588	9/16	.5625	14.288
5/64	.078125	1.984	37/64	.578125	14.684
3/32	.09375	2.381	19/32	.59375	15.081
7/64	.109375	2.778	39/64	.609375	15.478
1/8	.125	3.175	5/8	.625	15.875
9/64	.140625	3.572	41/64	.640625	16.272
5/32	.15625	3.969	21/32	.65625	16.669
11/64	.171875	4.366	43/64	.671875	17.066
3/16	.1875	4.763	11/16	.6875	17.463
13/64	.203125	5.159	45/64	.703125	17.859
7/32	.21875	5.556	23/32	.71875	18.256
15/64	.234375	5.953	47/64	.734375	18.653
1/4	.250	6.35	3/4	.750	19.05
17/64	.265625	6.747	49/64	.765625	19.447
9/32	.28125	7.144	25/32	.78125	19.844
19/64	.296875	7.54	51/64	.796875	20.241
5/16	.3125	7.938	13/16	.8125	20.638
21/64	.328125	8.334	53/64	.828125	21.034
11/32	.34375	8.731	27/32	.84375	21.431
23/64	.359375	9.128	55/64	.859375	21.828
3/8	.375	9.525	7/8	.875	22.225
25/64	.390625	9.922	57/64	.890625	22.622
13/32	.40625	10.319	29/32	.90625	23.019
27/64	.421875	10.716	59/64	.921875	23.416
7/16	.4375	11.113	15/16	.9375	23.813
29/64	.453125	11.509	61/64	.953125	24.209
15/32	.46875	11.906	31/32	.96875	24.606
31/64	.484375	12.303	63/64	.984375	25.003
1/2	.500	12.7	1	1.00	25.4

Inches	Millimeters	Inches	Millimeters	Inches	Millimeters
0.001	0.0254	0.010	0.2540	0.019	0.4826
0.002	0.0508	0.011	0.2794	0.020	0.5080
0.003	0.0762	0.012	0.3048	0.021	0.5334
0.004	0.1016	0.013	0.3302	0.022	0.5588
0.005	0.1270	0.014	0.3556	0.023	0.5842
0.006	0.1524	0.015	0.3810	0.024	0.6096
0.007	0.1778	0.016	0.4064	0.025	0.6350
0.008	0.2032	0.017	0.4318		
0.009	0.2286	0.018	0.4572		

Table D-4. Feet to Meters Conversion Table

Feet – metres
1 foot = 0.3048 m

ft.	met.	ft.	met.
1	0,305	31	9,449
2	**0,610**	**32**	**9,754**
3	0,914	33	10,058
4	**1,219**	**34**	**10,363**
5	1,524	35	10,668
6	**1,829**	**36**	**10,973**
7	2,134	37	11,278
8	**2,438**	**38**	**11,582**
9	2,743	39	11,887
10	**3,048**	**40**	**12,192**
11	3,353	41	12,497
12	**3,658**	**42**	**12,802**
13	3,962	43	13,106
14	**4,267**	**44**	**13,441**
15	4,572	45	13,716
16	**4,877**	**46**	**14,021**
17	5,182	47	14,326
18	**5,486**	**48**	**14,630**
19	5,791	49	14,935
20	**6,096**	**50**	**15,240**
21	6,401	51	15,545
22	**6,706**	**52**	**15,850**
23	7,010	53	16,154
24	**7,315**	**54**	**16,459**
25	7,620	55	16,764
26	**7,925**	**56**	**17,069**
27	8,230	57	17,374
28	**8,534**	**58**	**17,678**
29	8,839	59	17,983
30	**9,144**	**60**	**18,288**

Table D-5. Meters to Feet Conversion Table

Metres – Feet
1 metre = 3.2808 feet

met.	feet
1	3,28
2	**6,56**
3	9,84
4	**13,12**
5	16,40
6	**19,69**
7	22,97
8	**26,25**
9	29,53
10	**32,81**
11	36,09
12	**39,37**
13	42,65
14	**45,93**
15	49,21
16	**52,49**
17	55,77
18	**59,06**
19	62,34
20	**65,62**

Table D-6. Inches to Centimeters Conversion Table

Inches – centimetres
1 inch = 2.54 cm

inches	cm
1	2,54
2	**5,08**
3	7,62
4	**10,16**
5	12,70
6	**15,24**
7	17,78
8	**20,32**
9	22,86
10	**25,40**
11	27,94
12	**30,48**

Conversions: length

Use of the tables: the number to be converted, which is made up by adding the unit at the side of a line to the unit at the head of a column, is converted to the number in the position where line and column meet. For example, 11 in = 10 in + 1 in = 27.940 cm

Inches to Centimetres   1 in = 2.54 cm

in → / in ↓	0	1	2	3	4	5	6	7	8	9	← in / in ↓
	cm	cm	cm	cm	cm	cm	cm	cm	cm	cm	
0	0.000	2.540	5.080	7.620	10.160	12.700	15.240	17.780	20.320	22.860	0
10	25.400	27.940	30.480	33.020	35.560	38.100	40.640	43.180	45.720	48.260	10
20	50.800	53.340	55.880	58.420	60.960	63.500	66.040	68.580	71.120	73.660	20
30	76.200	78.740	81.280	83.820	86.360	88.900	91.440	93.980	96.520	99.060	30
40	101.600	104.140	106.680	109.220	111.760	114.300	116.840	119.380	121.920	124.460	40
50	127.000	129.540	132.080	134.620	137.160	139.700	142.240	144.780	147.320	149.860	50
60	152.400	154.940	157.480	160.020	162.560	165.100	167.640	170.180	172.720	175.260	60
70	177.800	180.340	182.880	185.420	187.960	190.500	193.040	195.580	198.120	200.660	70
80	203.200	205.740	208.280	210.820	213.360	215.900	218.440	220.980	223.520	226.060	80
90	228.600	231.140	233.680	236.220	238.760	241.300	243.840	246.380	248.920	251.460	90
100	254.000										100

in → / in ↓	0	10	20	30	40	50	60	70	80	90	← in / in ↓
	cm	cm	cm	cm	cm	cm	cm	cm	cm	cm	
0	0.000	25.400	50.800	76.200	101.600	127.000	152.400	177.800	203.200	228.600	0
100	254.000	279.400	304.800	330.200	355.600	381.000	406.400	431.800	457.200	482.600	100
200	508.000	533.400	558.800	584.200	609.600	635.000	660.400	685.800	711.200	736.600	200
300	762.000	787.400	812.800	838.200	863.600	889.000	914.400	939.800	965.200	990.600	300
400	1016.000	1041.400	1066.800	1092.200	1117.600	1143.000	1168.400	1193.800	1219.200	1244.600	400
500	1270.000	1295.400	1320.800	1346.200	1371.600	1397.000	1422.400	1447.800	1473.200	1498.600	500
600	1524.000	1549.400	1574.800	1600.200	1625.600	1651.000	1676.400	1701.800	1727.200	1752.600	600
700	1778.000	1803.400	1828.800	1854.200	1879.600	1905.000	1930.400	195 .800	1981.200	2006.600	700
800	2032.000	2057.400	2082.800	2108.200	2133.600	2159.000	2184.400	2209.800	2235.200	2260.600	800
900	2286.000	2311.400	2336.800	2362.200	2387.600	2413.000	2438.400	2463.800	2489.200	2514.600	900
1000	2540.000										1000

Centimetres to Inches   1 cm = 0.393 701 in

cm → / cm ↓	0	1	2	3	4	5	6	7	8	9	← cm / cm ↓
	in	in	in	in	in	in	in	in	in	in	
0	0.000	0.394	0.787	1.181	1.575	1.969	2.362	2.756	3.150	3.543	0
10	3.937	4.331	4.724	5.118	5.512	5.906	6.299	6.693	7.087	7.480	10
20	7.874	8.268	8.661	9.055	9.449	9.843	10.236	10.630	11.024	11.417	20
30	11.811	12.205	12.598	12.992	13.386	13.780	14.173	14.567	14.961	15.354	30
40	15.748	16.142	16.535	16.929	17.323	17.717	18.110	18.504	18.898	19.291	40
50	19.685	20.079	20.472	20.866	21.260	21.654	22.047	22.441	22.835	23.228	50
60	23.622	24.016	24.409	24.803	25.197	25.591	25.984	26.378	26.772	27.165	60
70	27.559	27.953	28.346	28.740	29.134	29.528	29.921	30.315	30.709	31.102	70
80	31.496	31.890	32.283	32.677	33.071	33.465	33.858	34.252	34.646	35.039	80
90	35.433	35.827	36.220	36.614	37.008	37.402	37.795	38.189	38.583	38.976	90
100	39.370										100

cm → / cm ↓	0	10	20	30	40	50	60	70	80	90	← cm / cm ↓
	in	in	in	in	in	in	in	in	in	in	
0	0.000	3.937	7.874	11.811	15.748	19.685	23.622	27.559	31.496	35.433	0
100	39.370	43.307	47.244	51.181	55.118	59.055	62.992	66.929	70.866	74.803	100
200	78.740	82.677	86.614	90.551	94.488	98.425	102.362	106.299	110.236	114.173	200
300	118.110	122.047	125.984	129.921	133.858	137.795	141.732	145.669	149.606	153.543	300
400	157.480	161.417	165.354	169.291	173.228	177.165	181.102	185.039	188.976	192.913	400
500	196.850	200.787	204.724	208.661	212.598	216.535	220.472	224.409	228.346	232.283	500
600	236.220	240.157	244.094	248.031	251.969	255.906	259.843	263.780	267.717	271.654	600
700	275.591	279.528	283.465	287.402	291.339	295.276	299.213	303.150	307.087	311.024	700
800	314.961	318.898	322.835	326.772	330.709	334.646	338.583	342.520	346.457	350.394	800
900	354.331	358.268	362.205	366.142	370.079	374.016	377.953	381.890	385.827	389.764	900
1000	393.701										1000

Use of the tables: the number to be converted, which is made up by adding the unit at the side of a line to the unit at the head of a column, is converted to the number in the position where line and column meet.
For example, 11 in = 10 in + 1 in = 279.400 mm

Inches to Millimetres   1 in = 25.4 mm

Note. This table can also be used for converting milli-inches (mils or 'thou') to micrometres ('microns')

in →	0	1	2	3	4	5	6	7	8	9 ←	in
↓	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	↓
0	0.000	25.400	50.800	76.200	101.600	127.000	152.400	177.800	203.200	228.600	0
10	254.000	279.400	304.800	330.200	355.600	381.000	406.400	431.800	457.200	482.600	10
20	508.000	533.400	558.800	584.200	609.600	635.000	660.400	685.800	711.200	736.600	20
30	762.000	787.400	812.800	838.200	863.600	889.000	914.400	939.800	965.200	990.600	30
40	1016.000	1041.400	1066.800	1092.200	1117.600	1143.000	1168.400	1193.800	1219.200	1244.600	40
50	1270.000	1295.400	1320.800	1346.200	1371.600	1397.000	1422.400	1447.800	1473.200	1498.600	50
60	1524.000	1549.400	1574.800	1600.200	1625.600	1651.000	1676.400	1701.800	1727.200	1752.600	60
70	1778.000	1803.400	1828.800	1854.200	1879.600	1905.000	1930.400	1955.800	1981.200	2006.600	70
80	2032.000	2057.400	2082.800	2108.200	2133.600	2159.000	2184.400	2209.800	2235.200	2260.600	80
90	2286.000	2311.400	2336.800	2362.200	2387.600	2413.000	2438.400	2463.800	2489.200	2514.600	90
100	2540.000										100

in →	0	10	20	30	40	50	60	70	80	90 ←	in
↓	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	↓
0	0.000	254.000	508.000	762.000	1016.000	1270.000	1524.000	1778.000	2032.000	2286.000	0
100	2540.000	2794.000	3048.000	3302.000	3556.000	3810.000	4064.000	4318.000	4572.000	4826.000	100
200	5080.000	5334.000	5588.000	5842.000	6096.000	6350.000	6604.000	6858.000	7112.000	7366.000	200
300	7620.000	7874.000	8128.000	8382.000	8636.000	8890.000	9144.000	9398.000	9652.000	9906.000	300
400	10160.000	10414.000	10668.000	10922.000	11176.000	11430.000	11684.000	11938.000	12192.000	12446.000	400
500	12700.000	12954.000	13208.000	13462.000	13716.000	13970.000	14224.000	14478.000	14732.000	14986.000	500
600	15240.000	15494.000	15748.000	16002.000	16256.000	16510.000	16764.000	17018.000	17272.000	17526.000	600
700	17780.000	18034.000	18288.000	18542.000	18796.000	19050.000	19304.000	19558.000	19812.000	20066.000	700
800	20320.000	20574.000	20828.000	21082.000	21336.000	21590.000	21844.000	22098.000	22352.000	22606.000	800
900	22860.000	23114.000	23368.000	23622.000	23876.000	24130.000	24384.000	24638.000	24892.000	25146.000	900
1000	25400.000										1000

Millimetres to Inches   1 mm = 0.039 370 in

Note. This table can also be used for converting micrometres ('microns') to milli-inches (mils or 'thou')

mm →	0	1	2	3	4	5	6	7	8	9 ←	mm
↓	in	in	in	in	in	in	in	in	in	in	↓
0	0.000	0.039	0.079	0.118	0.157	0.197	0.236	0.276	0.315	0.354	0
10	0.394	0.433	0.472	0.512	0.551	0.591	0.630	0.669	0.709	0.748	10
20	0.787	0.827	0.866	0.906	0.945	0.984	1.024	1.063	1.102	1.142	20
30	1.181	1.220	1.260	1.299	1.339	1.378	1.417	1.457	1.496	1.535	30
40	1.575	1.614	1.654	1.693	1.732	1.772	1.811	1.850	1.890	1.929	40
50	1.969	2.008	2.047	2.087	2.126	2.165	2.205	2.244	2.283	2.323	50
60	2.362	2.402	2.441	2.480	2.520	2.559	2.598	2.638	2.677	2.717	60
70	2.756	2.795	2.835	2.874	2.913	2.953	2.992	3.031	3.071	3.110	70
80	3.150	3.189	3.228	3.268	3.307	3.346	3.386	3.425	3.465	3.504	80
90	3.543	3.583	3.622	3.661	3.701	3.740	3.780	3.819	3.858	3.898	90
100	3.937										100

mm →	0	10	20	30	40	50	60	70	80	90 ←	mm
↓	in	in	in	in	in	in	in	in	in	in	↓
0	0.000	0.394	0.787	1.181	1.575	1.969	2.362	2.756	3.150	3.543	0
100	3.937	4.331	4.724	5.118	5.512	5.906	6.299	6.693	7.087	7.480	100
200	7.874	8.268	8.661	9.055	9.449	9.843	10.236	10.630	11.024	11.417	200
300	11.811	12.205	12.598	12.992	13.386	13.780	14.173	14.567	14.961	15.354	300
400	15.748	16.142	16.535	16.929	17.323	17.717	18.110	18.504	18.898	19.291	400
500	19.685	20.079	20.472	20.866	21.260	21.654	22.047	22.441	22.835	23 228	500
600	23.622	24.016	24.409	24.803	25.197	25.591	25.984	26.378	26.772	27.165	600
700	27.559	27.953	28.346	28.740	29.134	29.528	29.921	30.315	30.709	31.102	700
800	31.496	31.890	32.283	32.677	33.071	33.465	33.858	34.252	34.646	35.039	800
900	35.433	35.827	36.220	36.614	37.008	37.402	37.795	38.189	38.583	38.976	900
1000	39.370										1000

Conversions: length

Use of the table: the number of inches to be converted, which is made up by the number of inches at the head of a column and the fraction at the side of a line, is converted to the number in the position where line and column meet.
For example, 1 1/64 in = 1 in + 1/64 in = 25.797 mm

Inches and fractions of an inch to Millimetres 1 in = 25.4 mm

in → ↓	0	1	2	3	4	5	6	7	8	9	10	11	← in ↓
	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	
0	0.000	25.400	50.800	76.200	101.600	127.000	152.400	177.800	203.200	228.600	254.000	279.400	0
1/64	0.397	25.797	51.197	76.597	101.997	127.397	152.797	178.197	203.597	228.997	254.397	279.797	1/64
1/32	0.794	26.194	51.594	76.994	102.394	127.794	153.194	178.594	203.994	229.394	254.794	280.194	1/32
3/64	1.191	26.591	51.991	77.391	102.791	128.191	153.591	178.991	204.391	229.791	255.191	280.591	3/64
1/16	1.588	26.988	52.388	77.788	103.188	128.588	153.988	179.388	204.788	230.188	255.588	280.988	1/16
5/64	1.984	27.384	52.784	78.184	103.584	128.984	154.384	179.784	205.184	230.584	255.984	281.384	5/64
3/32	2.381	27.781	53.181	78.581	103.981	129.381	154.781	180.181	205.581	230.981	256.381	281.781	3/32
7/64	2.778	28.178	53.578	78.978	104.378	129.778	155.178	180.578	205.978	231.378	256.778	282.178	7/64
1/8	3.175	28.575	53.975	79.375	104.775	130.175	155.575	180.975	206.375	231.775	257.175	282.575	1/8
9/64	3.572	28.972	54.372	79.772	105.172	130.572	155.972	181.372	206.772	232.172	257.572	282.972	9/64
5/32	3.969	29.369	54.769	80.169	105.569	130.969	156.369	181.769	207.169	232.569	257.969	283.369	5/32
11/64	4.366	29.766	55.166	80.566	105.966	131.366	156.766	182.166	207.566	232.966	258.366	283.766	11/64
3/16	4.762	30.162	55.562	80.962	106.362	131.762	157.162	182.562	207.962	233.362	258.762	284.162	3/16
13/64	5.159	30.559	55.959	81.359	106.759	132.159	157.559	182.959	208.359	233.759	259.159	284.559	13/64
7/32	5.556	30.956	56.356	81.756	107.156	132.556	157.956	183.356	208.756	234.156	259.556	284.956	7/32
15/64	5.953	31.353	56.753	82.153	107.553	132.953	158.353	183.753	209.153	234.553	259.953	285.353	15/64
1/4	6.350	31.750	57.150	82.550	107.950	133.350	158.750	184.150	209.550	234.950	260.350	285.750	1/4
17/64	6.747	32.147	57.547	82.947	108.347	133.747	159.147	184.547	209.947	235.347	260.747	286.147	17/64
9/32	7.144	32.544	57.944	83.344	108.744	134.144	159.544	184.944	210.344	235.744	261.144	286.544	9/32
19/64	7.541	32.941	58.341	83.741	109.141	134.541	159.941	185.341	210.741	236.141	261.541	286.941	19/64
5/16	7.938	33.338	58.738	84.138	109.538	134.938	160.338	185.738	211.138	236.538	261.938	287.338	5/16
21/64	8.334	33.734	59.134	84.534	109.934	135.334	160.734	186.134	211.534	236.934	262.334	287.734	21/64
11/32	8.731	34.131	59.531	84.931	110.331	135.731	161.131	186.531	211.931	237.331	262.731	288.131	11/32
23/64	9.128	34.528	59.928	85.328	110.728	136.128	161.528	186.928	212.328	237.728	263.128	288.528	23/64
3/8	9.525	34.925	60.325	85.725	111.125	136.525	161.925	187.325	212.725	238.125	263.525	288.925	3/8
25/64	9.922	35.322	60.722	86.122	111.522	136.922	162.322	187.722	213.122	238.522	263.922	289.322	25/64
13/32	10.319	35.719	61.119	86.519	111.919	137.319	162.719	188.119	213.519	238.919	264.319	289.719	13/32
27/64	10.716	36.116	61.516	86.916	112.316	137.716	163.116	188.516	213.916	239.316	264.716	290.116	27/64
7/16	11.112	36.512	61.912	87.312	112.712	138.112	163.512	188.912	214.312	239.712	265.112	290.512	7/16
29/64	11.509	36.909	62.309	87.709	113.109	138.509	163.909	189.309	214.709	240.109	265.509	290.909	29/64
15/32	11.906	37.306	62.706	88.106	113.506	138.906	164.306	189.706	215.106	240.506	265.906	291.306	15/32
31/64	12.303	37.703	63.103	88.503	113.903	139.303	164.703	190.103	215.503	240.903	266.303	291.703	31/64
1/2	12.700	38.100	63.500	88.900	114.300	139.700	165.100	190.500	215.900	241.300	266.700	292.100	1/2
33/64	13.097	38.497	63.897	89.297	114.697	140.097	165.497	190.897	216.297	241.697	267.097	292.497	33/64
17/32	13.494	38.894	64.294	89.694	115.094	140.494	165.894	191.294	216.694	242.094	267.494	292.894	17/32
35/64	13.891	39.291	64.691	90.091	115.491	140.891	166.291	191.691	217.091	242.491	267.891	293.291	35/64
9/16	14.288	39.688	65.088	90.488	115.888	141.288	166.688	192.088	217.488	242.888	268.288	293.688	9/16
37/64	14.684	40.084	65.484	90.884	116.284	141.684	167.084	192.484	217.884	243.284	268.684	294.084	37/64
19/32	15.081	40.481	65.881	91.281	116.681	142.081	167.481	192.881	218.281	243.681	269.081	294.481	19/32
39/64	15.478	40.878	66.278	91.678	117.078	142.478	167.878	193.278	218.678	244.078	269.478	294.878	39/64
5/8	15.875	41.275	66.675	92.075	117.475	142.875	168.275	193.675	219.075	244.475	269.875	295.275	5/8
41/64	16.272	41.672	67.072	92.472	117.872	143.272	168.672	194.072	219.472	244.872	270.272	295.672	41/64
21/32	16.669	42.069	67.469	92.869	118.269	143.669	169.069	194.469	219.869	245.269	270.669	296.069	21/32
43/64	17.066	42.466	67.866	93.266	118.666	144.066	169.466	194.866	220.266	245.666	271.066	296.466	43/64
11/16	17.462	42.862	68.262	93.662	119.062	144.462	169.862	195.262	220.662	246.062	271.462	296.862	11/16
45/64	17.859	43.259	68.659	94.059	119.459	144.859	170.259	195.659	221.059	246.459	271.859	297.259	45/64
23/32	18.256	43.656	69.056	94.456	119.856	145.256	170.656	196.056	221.456	246.856	272.256	297.656	23/32
47/64	18.653	44.053	69.453	94.853	120.253	145.653	171.053	196.453	221.853	247.253	272.653	298.053	47/64
3/4	19.050	44.450	69.850	95.250	120.650	146.050	171.450	196.850	222.250	247.650	273.050	298.450	3/4
49/64	19.447	44.847	70.247	95.647	121.047	146.447	171.847	197.247	222.647	248.047	273.447	298.847	49/64
25/32	19.844	45.244	70.644	96.044	121.444	146.844	172.244	197.644	223.044	248.444	273.844	299.244	25/32
51/64	20.241	45.641	71.041	96.441	121.841	147.241	172.641	198.041	223.441	248.841	274.241	299.641	51/64
13/16	20.638	46.038	71.438	96.838	122.238	147.638	173.038	198.438	223.838	249.238	274.638	300.038	13/16
53/64	21.034	46.434	71.834	97.234	122.634	148.034	173.434	198.834	224.234	249.634	275.034	300.434	53/64
27/32	21.431	46.831	72.231	97.631	123.031	148.431	173.831	199.231	224.631	250.031	275.431	300.831	27/32
55/64	21.828	47.228	72.628	98.028	123.428	148.828	174.228	199.628	225.028	250.428	275.828	301.228	55/64
7/8	22.225	47.625	73.025	98.425	123.825	149.225	174.625	200.025	225.425	250.825	276.225	301.625	7/8
57/64	22.622	48.022	73.422	98.822	124.222	149.622	175.022	200.422	225.822	251.222	276.622	302.022	57/64
29/32	23.019	48.419	73.819	99.219	124.619	150.019	175.419	200.819	226.219	251.619	277.019	302.419	29/32
59/64	23.416	48.816	74.216	99.616	125.016	150.416	175.816	201.216	226.616	252.016	277.416	302 816	59/64
15/16	23.812	49.212	74.612	100.012	125.412	150.812	176.212	201.612	227.012	252.412	277.812	303.212	15/16
61/64	24.209	49.609	75.009	100.409	125.809	151.209	176.609	202.009	227.409	252.809	278.209	303.609	61/64
31/32	24.606	50.006	75.406	100.806	126.206	151.606	177.006	202.406	227.806	253.206	278.606	304.006	31/32
63/64	25.003	50.403	75.803	101.203	126.603	152.003	177.403	202.803	228.203	253.603	279.003	304.403	63/64

Fractions to Decimals

Fraction	Decimal equivalent	Fraction	Decimal equivalent
1/2	0.5	1/32	0.031 25
1/3	0.333 333	1/33	0.030 303
1/4	0.25	1/34	0.029 412
1/5	0.2	1/35	0.028 571
1/6	0.166 667	1/36	0.027 778
1/7	0.142 857	1/37	0.027 027
1/8	0.125	1/38	0.026 316
1/9	0.111 111	1/39	0.025 641
1/10	0.1	1/40	0.025
1/11	0.090 909	1/41	0.024 390
1/12	0.083 333	1/42	0.023 810
1/13	0.076 923	1/43	0.023 256
1/14	0.071 429	1/44	0.022 727
1/15	0.066 667	1/45	0.022 222
1/16	0.062 5	1/46	0.021 739
1/17	0.058 824	1/47	0.021 277
1/18	0.055 556	1/48	0.020 833
1/19	0.052 632	1/49	0.020 408
1/20	0.05	1/50	0.02
1/21	0.047 619	1/51	0.019 608
1/22	0.045 455	1/52	0.019 231
1/23	0.043 478	1/53	0.018 868
1/24	0.041 667	1/54	0.018 519
1/25	0.04	1/55	0.018 182
1/26	0.038 462	1/56	0.017 857
1/27	0.037 037	1/57	0.017 544
1/28	0.035 714	1/58	0.017 241
1/29	0.034 483	1/59	0.016 949
1/30	0.033 333	1/60	0.016 667
1/31	0.032 258		

Note. For the decimal equivalent of other fractions with 1 as numerator, and a number from 0.01 to 100.9 as denominator, see reciprocals, pages 144–147.

Fractions 3rds	6ths	12ths	24ths	Decimal equivalent
			1	0.041 667
		1	2	0.083 333
			3	0.125
	1	2	4	0.166 667
			5	0.208 333
		3	6	0.25
			7	0.291 667
1	2	4	8	0.333 333
			9	0.375
		5	10	0.416 667
			11	0.458 333
	3	6	12	0.5
			13	0.541 667
		7	14	0.583 333
			15	0.625
2	4	8	16	0.666 667
			17	0.708 333
		9	18	0.75
			19	0.791 667
	5	10	20	0.833 333
			21	0.875
		11	22	0.916 667
			23	0.958 333
3	6	12	24	1

Fractions 1/2's	1/4's	8ths	16ths	32nds	64ths	Decimal equivalent (all figures are exact)
					1	0.015 625
				1	2	0.031 25
					3	0.046 875
			1	2	4	0.062 5
					5	0.078 125
				3	6	0.093 75
					7	0.109 375
		1	2	4	8	0.125
					9	0.140 625
				5	10	0.156 25
					11	0.171 875
			3	6	12	0.187 5
					13	0.203 125
				7	14	0.218 75
					15	0.234 375
	1	2	4	8	16	0.25
					17	0.265 625
				9	18	0.281 25
					19	0.296 875
			5	10	20	0.312 5
					21	0.328 125
				11	22	0.343 75
					23	0.359 375
		3	6	12	24	0.375
					25	0.390 625
				13	26	0.406 25
					27	0.421 875
			7	14	28	0.437 5
					29	0.453 125
				15	30	0.468 75
					31	0.484 375
1	2	4	8	16	32	0.5
					33	0.515 625
				17	34	0.531 25
					35	0.546 875
			9	18	36	0.562 5
					37	0.578 125
				19	38	0.593 75
					39	0.609 375
		5	10	20	40	0.625
					41	0.640 625
				21	42	0.656 25
					43	0.671 875
			11	22	44	0.687 5
					45	0.703 125
				23	46	0.718 75
					47	0.734 375
	3	6	12	24	48	0.75
					49	0.765 625
				25	50	0.781 25
					51	0.796 875
			13	26	52	0.812 5
					53	0.828 125
				27	54	0.843 75
					55	0.859 375
		7	14	28	56	0.875
					57	0.890 625
				29	58	0.906 25
					59	0.921 875
			15	30	60	0.937 5
					61	0.953 125
				31	62	0.968 75
					63	0.984 375
2	4	8	16	32	64	1

CONVERSION TABLE
Feet to Inches

	2 Ft.	3 Ft.	4 Ft.	5 Ft.	6 Ft.
0″	24″	36″	48″	60″	72″
1″	25	37	49	61	
2″	26	38	50	62	
3″	27	39	51	63	
4″	28	40	52	64	
5″	29	41	53	65	
6″	30	42	54	66	
7″	31	43	55	67	
8″	32	44	56	68	
9″	33	45	57	69	
10″	34	46	58	70	
11″	35	47	59	71	

TABLE — DECIMALS OF A FOOT

For Each Sixteenth of An Inch

	0	1	2	3	4	5	6	7	8	9	10	11
0		.0833	.1667	.250	.3333	.4167	.5000	.5833	.6667	.7500	.8333	.9167
1/16	.0052	.0885	.1719	.2552	.3385	.4219	.5052	.5885	.6719	.7552	.8385	.9219
1/8	.0104	.0937	.1771	.2604	.3437	.4271	.5104	.5937	.6771	.7604	.8437	.9271
3/16	.0156	.0990	.1823	.2656	.3490	.4323	.5156	.5990	.6823	.7656	.8490	.9323
1/4	.0208	.1042	.1875	.2708	.3542	.4375	.5208	.6042	.6875	.7708	.8542	.9375
5/16	.0260	.1094	.1927	.2760	.3594	.4427	.5260	.6094	.6927	.7760	.8594	.9427
3/8	.0312	.1146	.1979	.2812	.3646	.4479	.5339	.6172	.7005	.7839	.8672	.9505
7/16	.0365	.1198	.2031	.2865	.3698	.4531	.5365	.6198	.7031	.7865	.8698	.9531
1/2	.0417	.1250	.2083	.2917	.3750	.4583	.5417	.6250	.7083	.7917	.8750	.9583
9/16	.0469	.1302	.2135	.2969	.3802	.4635	.5469	.6302	.7135	.7969	.8802	.9635
5/8	.0521	.1354	.2188	.3021	.3854	.4688	.5521	.6354	.7188	.8021	.8854	.9688
11/16	.0573	.1406	.2240	.3073	.3906	.4740	.5573	.6406	.7240	.8073	.8906	.9740
3/4	.0625	.1458	.2292	.3125	.3958	.4792	.5625	.6458	.7292	.8125	.8958	.9792
13/16	.0677	.1510	.2344	.3177	.4010	.4844	.5677	.6510	.7344	.8177	.9010	.9844
7/8	.0729	.1562	.2396	.3229	.4062	.4896	.5729	.6562	.7396	.8229	.9062	.9896
15/16	.0781	.1615	.2448	.3281	.4115	.4948	.5781	.6615	.7448	.8281	.9115	.9949

MEASURING NUTS AND BOLTS

Nuts and bolts come in a great variety of sizes. The size of the head of a bolt or nut is measured between the flat sides of the hexagon. In standard American nuts and bolts these sizes usually follow one another in 1/16" steps. Thus you have 1/2" (8/16") bolts and 9/16" bolts and 5/8" (10/16") bolts, but you don't find 2/3" bolts. Foreign cars use metric-sized nuts and bolts which are measured in the same way from flat side to flat side, but in millimeters instead of sixteenths of an inch. Some English cars use yet another way of measuring head sizes known as the "Whitworth system." Instead of measuring from flat to flat of the hexagon, Whitworth sizes measure from point to point. Fortunately most British manufacturers have switched over to metric sizes or standard American sizes.

Unfortunately, head size isn't all there is to measuring nuts and bolts. Bolts are also measured by diameter—that is, through the threads. Thus you could have a bolt with a 1/2" head and a diameter of 1/4" or one with a 1/2" head and a 3/16" diameter. Nuts are measured for diameter across the widest opening the threads make.

As if this weren't enough, nuts and bolts (and studs and screws as well) are also measured according to how coarse or fine their threads are. This measurement is made in "threads per inch." A bolt with 16 threads per inch is rather coarsely threaded. Twenty threads per inch is a fine thread.

Once in a while you will run into a nut or a bolt which has **left-hand thread.** This means it screws in counterclockwise and screws out clockwise. Often left-hand thread nuts and bolts have a little "L" marked on them. Sometimes they don't—then you just have to try turning the "wrong" way and see what happens.

WRENCH SIZES

The size of a wrench refers to the size of the nut or bolt it fits (see Appendix, p. 92, on measuring sizes of nuts and bolts). Most sets of American wrenches start at 3/8" and go up in steps of 1/16". The largest wrench in a set might be anywhere from 3/4" to 1-5/16". (What we call "American" wrenches are also known as SAE [Society of Automotive Engineers] wrenches.) Wrenches smaller than 3/8" are called "miniatures" or "ignition" wrenches. There are also metric wrenches, which you will need if you want to work on foreign cars.

Americans spend a good deal of time in school learning a chaotic system of weights and measures and forget most of it rapidly. Most of us remember that there are 12 inches in a foot and 3 feet in a yard and even that there are 5,280 feet in a mile. Hardly anyone knows how big an acre is or how many pints there are in a gallon or . . .

Most of our units of weight and measure had different origins at different times. An inch was three barleycorns laid end to end. A foot was the length of Charlemagne's foot (he had big feet). A yard was the distance from the tip of Henry I's nose to the tip of his outstretched arm. Roman legions used to count off paces as they marched—a thousand paces (*mille passuum*) was roughly 5,000 feet and the English changed the name to "mile."

As you can see, our system of measure has its roots in Roman imperialism, feudal class structure and an agricultural economy. Not very relevant to a technological industrial society.

Most countries had different and equally illogical systems of measure until the French Revolution came along. The French revolutionaries decided they needed a more orderly and convenient system and chose the meter (3.28 feet) as the standard unit of measure ("meter" comes from the Greek word for measure). They then derived all other units of measure from the meter in multiples of 10 or 1/10. A centimeter is 1/100 of a meter and a millimeter is 1/1000 of a meter. A kilometer is 1000 meters. A liter is 1000 cubic centimeters and a kilo ("key" for short) is the weight of a liter of water. Neat and easy.

Over the years other countries adopted the metric system or had it forced upon them by imperialist colonizers from one European state or another, until only the English-speaking countries were holding out. Now Great Britain and the Commonwealth are in the middle of a ten-year switchover, leaving the U.S. as the lone non-metric nation in the world. In every other country school children can learn their system of measure in a month. In the U.S. we spend years getting a partial grip on our system of measure.

What all this means to you as an auto mechanic is that you may need two sets of wrenches—one for American-built cars and another for foreign makes. Most English cars are using metric nuts and bolts by now (although a few still use the Whitworth system—see Appendix, p. 92).

Metric wrench sets start with 6 or 7 mm. (millimeters) and go up to 19 mm. in 1 mm. intervals. Sixteen and 18 mm. wrenches are not usually included in metric sets. Fortunately, 16 and 18 mm. nuts and bolts are not usually included on cars.

If you are going to do a lot of work on foreign cars you have to get yourself a set of metric combination wrenches and sockets. If it's just a question of tightening one nut on a Volkswagen, you may find that you can fake it with your American wrenches. Several sizes in the American and metric systems are close enough to be almost interchangeable. Unfortunately most of these switches work in one direction only: you can use a wrench which is slightly too big on a bolt, but you can't use one which is a little too small. The table shows you which fakes you can get away with. Try not to use it just because you're too lazy to fetch the right wrench out of your tool box!

MICKEY MOUSE'S
METRIC CONVERSION CHART

	WRENCH SIZE	FITS BOLT SIZE
American to metric	3/8	9 mm
	7/16	11 mm
	9/16	14 mm
	11/16	17 mm
	3/4	19 mm
Metric to American	10 mm	3/8
	11 mm	7/16*
	13 mm	1/2
	17 mm	5/8
	19 mm	3/4*
	21 mm	13/16

* *sometimes*

Hose Clips

Pipe clips are extremely confusingly sized by the various manufacturers and the list below will be of some help in sorting out the ridiculous situation. Use ONLY stainless steel clips.

000	3/8" — 1/2"	9 — 13mm
M00	3/8" — 5/8"	9 — 16mm
00	1/2" — 3/4"	13 — 19mm
0	5/8" — 7/8"	16 — 22mm
0X	3/4" — 1"	19 — 25mm
1A	7/8" — 1 1/8"	22 — 28mm
1	1" — 1 3/8"	25 — 35mm
1X	1 1/8" — 1 5/8"	28 — 41mm
2A	1 1/4" — 1 7/8"	32 — 48mm
2	1 1/2" — 2 1/8"	38 — 54mm
2X	1 3/4" — 2 3/8"	44 — 60mm
3	2" — 2 3/4"	50 — 70mm
3X	2 3/8" — 3 1/8"	60 — 80mm

Machine Screws

Due to standardisation over recent years, the metric equivalent of the machine screws listed below should be used as imperial sizes may be more than double the cost.

British Standard	Approximate Metric Thread equivalent	Clearance Drill Size	75% Tap Thread Drill Size
3/16" (.1875)	M5	7/32"	3/32"
1/4" (.25)	M6	17/64"	7/32"
5/16" (.3125)	M8	3/8"	17/64"
3/8" (.375)	M10	7/16"	21/64"
1/2" (0.5)	M12	9/16"	7/16"

Metal & Wire Gauge Equivalents

It is useful to have the tables to hand as there is nothing more infuriating than being unable to communicate with a supplier or to have gauge numbers with no way of converting them to really useful equivalents, such as thickness. This usually occurs at a weekend according to Murphy's Law.

Metal Gauge Number	Imperial Thickness inches	Metric Thickness mm
1	0.3	7.61
2	0.276	7.00
3	0.252	6.39
4	0.232	5.88
5	0.212	5.38
6	0.192	4.87
7	0.176	4.46
8	0.160	4.06
9	0.144	3.66
10	0.128	3.25
11	0.116	2.94
12	0.104	2.64
13	0.092	2.34
14	0.080	2.00
15	0.072	1.83
16	0.064	1.62
17	0.056	1.42
18	0.048	1.22
19	0.040	1.01
20	0.036	0.91
21	0.032	0.81
22	0.028	0.71
23	0.024	0.61
24	0.022	0.56
25	0.020	0.51
26	0.018	0.46
27	0.016	0.41
28	0.014	0.36
29	0.013	0.33
30	0.012	0.30

Standard Wire Gauge	Imperial Thickness inches	Metric Thickness mm
30	0.0124	0.314
29	0.0136	0.345
28	0.0148	0.375
27	0.0164	0.416
26	0.018	0.457
25	0.020	0.508
24	0.022	0.558
23	0.024	0.609
22	0.028	0.711
21	0.032	0.812
20	0.036	0.914
19	0.040	1.016
18	0.048	1.219
17	0.056	1.422
16	0.064	1.625
15	0.072	1.828
14	0.080	2.032
13	0.092	2.336
12	0.104	2.640
11	0.116	2.946
10	0.128	3.251
9	0.144	3.657
8	0.160	4.064
7	0.176	4.470
6	0.192	4.876
5	0.212	5.384
4	0.232	5.892
3	0.252	6.400
2	0.276	7.010
1	0.300	7.620
0	0.324	8.229
2/0	0.348	8.839
3/0	0.372	9.448
4/0	0.400	10.16
5/0	0.432	10.97
6/0	0.464	11.8
7/0	0.500	12.70

Pipe & Hoses

General Types	Bore Sizes	Outside dia.	Comments & Usage
Clear PVC, non toxic pipe	13mm-38mm (½″-1½″)	19mm-48mm (¾″-1⅞″)	Water supply and occasionally delivery. Kinks and collapses, so must not be used on suction runs. Also cannot be used where fluids are hot, nor should fuels be piped in this hose. Can be 'persuaded' to distort.
Spiral reinforced plastic pipe	19mm-30mm (¾″-1½″)		Three general types. Very and medium stiff, with moulded in spiralling, and flexible, thin wall with external spiralling. Does not kink but conversely can be difficult to get round tight bends. Used for both suction and delivery pipe runs. Heat must be used to ensure pipe clips have properly tightened down. Will not easily distort.
Low pressure rubber pipe	13mm, 19 & 25mm (½″; ¾″, & 1″)		Calor gas piping from a bibcock to, say, a gimballed cooker.
Seamless copper tube	⅛″-¾″		Calor gas and fuel pipe runs.
Reinforced, clear fuel pipe	¼″, 5/16″, ⅜″ & ½″		Suitable for petrol or diesel fuels.
Synthetic delivery pipe	13mm-25mm (½″-1″)		Hot water supply.
Reinforced rubber heater hose	13mm-25mm (½″-1″)		Engine water cooling inlet pipe.

Water pipe equivalents:-

3/8″	12mm)		3/4″	22mm)	not inter-
1/2″	15mm)	inter-	1 1/4″	35mm)	changeable &
1″	28mm)	changeable	1 1/2″	42mm)	require a
2″	54mm)				conversion coupling

Note Gas & fuel pipe sizes are still measured in Imperial sizes.

Plywood board equivalents:-

1/8″	4mm
1/4″	6mm
3/8″	9mm
1/2″	12mm
3/4″	18mm

Skin fitting equivalents

1/2″	13mm
3/4″	19mm
1″	25mm
1 1/4″	32mm
1 1/2″	38mm
2″	50mm

Timber thickness:- (approx.)

1/2″	13mm
5/8″	16mm
3/4″	19mm
7/8″	22mm
1″	25mm
1 1/4″	32mm
1 1/2″	38mm
1 3/4″	44mm
2″	50mm
2 1/2″	63mm
3″	75mm
4″	100mm
5″	125mm
6″	150mm

Note.The above sizes are for sawn timber. Planed timber looses about 1 1/2mm ($\frac{1}{16}$″) per planed surface, so a board ordered '25mm (1″) planed thickness' finishes up at about 22mm (7/8″)

Woodscrew drilling sizes:-

		Screw gauges* 6	8	10	12	14
Softwoods.	pilot drill	-	$\frac{1}{16}$″	$\frac{5}{64}$″	$\frac{5}{64}$″	$\frac{7}{64}$″
	clearance drill	-	$\frac{3}{32}$″	$\frac{5}{32}$″	$\frac{5}{32}$″	$\frac{7}{32}$″
Hardwoods.	pilot drill	$\frac{5}{64}$″	$\frac{3}{32}$″	1/8″	1/8″	$\frac{5}{32}$″
	clearance drill	$\frac{5}{32}$″	$\frac{3}{16}$″	$\frac{7}{32}$″	1/4″	1/4″

* To ascertain a screw's gauge, measure the diameter of the head of the screw, using a ruler marked in sixteenths (of an inch), multiply the measurement by two and subtract two from the result.

VII. Weights and Measures: Conversion Ratios

WEIGHTS AND MEASURES*

Tables of United States Customary Weights and Measures

Linear Measure

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards	= 1 rod (rd.), pole, or perch ($16\frac{1}{2}$ ft.)
40 rods	= 1 furlong (fur.) = 220 yards = 660 feet
8 furlongs	= 1 statute mile (mi.) = 1760 yards = 5280 feet
3 miles	= 1 league = 5280 yards = 15,840 feet
5280 feet	= 1 statute or land mile
6076.11549 feet	= 1 international nautical mile

Area Measure

Squares and cubes of units are sometimes abbreviated by using "superior" figures. For example, ft^2 means square foot, and ft^3 means cubic foot.

144 square inches	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.) = 1296 square inches
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.) = $272\frac{1}{4}$ square feet
160 square rods	= 1 acre = 4840 square yards = 43,560 square feet
640 acres	= 1 square mile (sq. mi.)
1 mile square	= 1 section (of land)
6 miles square	= 1 township = 36 sections = 36 square miles

Cubic Measure

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)

Gunter's or Surveyor's Chain Measure

7.92 inches (in.)	= 1 link (li.)
100 links	= 1 chain (ch.) = 4 rods = 66 feet
80 chains	= 1 statute mile (mi.) = 320 rods = 5280 feet

Liquid Measure

When necessary to distinguish the liquid pint or quart from the dry pint or quart, the word "liquid" or the abbreviation "liq" should be used in combination with the name or abbreviation of the liquid unit.

4 gills (gi.)	= 1 pint (pt.) (= 28.875 cubic inches)
2 pints	= 1 quart (qt.) (= 57.75 cubic inches)
4 quarts	= 1 gallon (gal.) (= 231 cubic inches) = 8 pints = 32 gills

* Source: National Bureau of Standards, U.S. Department of Commerce.

Apothecaries' Fluid Measure

60 minims (min.)	= 1 fluid dram (fl. dr.) (= 0.2256 cubic inch)
8 fluid drams	= 1 fluid ounce (fl. oz.) (= 1.8047 cubic inches)
16 fluid ounces	= 1 pint (pt.) (= 28.875 cubic inches) = 128 fluid drams
2 pints	= 1 quart (qt.) (= 57.75 cubic inches) = 32 fluid ounces = 256 fluid drams
4 quarts	= 1 gallon (gal.) (= 231 cubic inches) = 128 fluid ounces = 1024 fluid drams

Dry Measure

When necessary to distinguish the dry pint or quart from the liquid pint or quart, the word "dry" should be used in combination with the name or abbreviation of the dry unit.

2 pints (pt.)	= 1 quart (qt.) (= 67.2006 cubic inches)
8 quarts	= 1 peck (pk.) (= 537.605 cubic inches) = 16 pints
4 pecks	= 1 bushel (bu.) (= 2150.42 cubic inches) = 32 quarts

Avoirdupois Weight

When necessary to distinguish the avoirdupois dram from the apothecaries' dram, or to distinguish the avoirdupois dram or ounce from the fluid dram or ounce, or to distinguish the avoirdupois ounce or pound from the troy or apothecaries' ounce or pound, the word "avoirdupois" or the abbreviation "avdp" should be used in combination with the name or abbreviation of the avoirdupois unit.

(The "grain" is the same in avoirdupois, troy, and apothecaries' weight.)

$27^{11}/_{32}$ grains	= 1 dram (dr.)
16 drams	= 1 ounce (oz.) = $437^{1}/_{2}$ grains
16 ounces	= 1 pound (lb.) = 256 drams = 7000 grains
100 pounds	= 1 hundredweight (cwt.)†
20 hundredweights	= 1 ton (tn.) = 2000 pounds†

In "gross" or "long" measure, the following values are recognized:

112 pounds	= 1 gross or long hundredweight†
20 gross or long hundredweights	= 1 gross or long ton = 2240 pounds†

† When the terms "hundredweight" and "ton" are used unmodified, they are commonly understood to mean the 100-pound hundredweight and the 2,000-pound ton, respectively; these units may be designated "net" or "short" when necessary to distinguish them from the corresponding units in gross or long measure.

Troy Weight

24 grains	= 1 pennyweight (dwt.)
20 pennyweights	= 1 ounce troy (oz. t.) = 480 grains
12 ounces troy	= 1 pound troy (lb. t.) = 240 pennyweights = 5760 grains

Apothecaries' Weight

20 grains	= scruple (s. ap.)
3 scruples	= 1 dram apothecaries' (dr. ap.) = 60 grains
8 drams apothecaries'	= 1 ounce apothecaries' (oz. ap.) = 24 scruples = 480 grains
12 ounces apothecaries'	= 1 pound apothecaries' (lb. ap.) = 96 drams apothecaries' = 288 scruples = 5760 grains

Tables of Metric Weights and Measures

Linear Measure

10 millimeters (mm.)	= 1 centimeter (cm.)
10 centimeters	= 1 decimeter (dm.) = 100 millimeters
10 decimeters	= 1 meter (m.) = 1000 millimeters
10 meters	= 1 dekameter (dkm.)
10 dekameters	= 1 hectometer (hm.) = 100 meters
10 hectometers	= 1 kilometer (km.) = 1000 meters

Area Measure

100 square millimeters (mm^2)	= 1 square centimeter (cm^2)
10,000 square centimeters	= 1 square meter (m^2) = 1,000,000 square millimeters
100 square meters	= 1 are (a.)
100 ares	= 1 hectare (ha.) = 10,000 square meters
100 hectares	= 1 square kilometer (km^2) = 1,000,000 square meters

Volume Measure

10 milliliters (ml.)	= 1 centiliter (cl.)
10 centiliters	= 1 deciliter (dl.) = 100 milliliters
10 deciliters	= 1 liter‡ (l.) = 1000 milliliters
10 liters	= 1 dekaliter (dkl.)
10 dekaliters	= 1 hectoliter (hl.) = 100 liters
10 hectoliters	= 1 kiloliter (kl.) = 1000 liters

‡ The liter is defined as the volume occupied, under standard conditions, by a quantity of pure water having a mass of 1 kilogram. This volume is very nearly equal to 1000 cubic centimeters or 1 cubic decimeter; the actual metric equivalent is, 1 liter = 1.000028 cubic decimeters. Thus the milliliter and the liter are larger than the cubic centimeter and the cubic decimeter, respectively, by 28 parts in 1,000,000.

Cubic Measure

1000 cubic millimeters (mm^3)	= 1 cubic centimeter (cm^3)
1000 cubic centimeters	= 1 cubic decimeter (dm^3) = 1,000,000 cubic millimeters
1000 cubic decimeters	= 1 cubic meter (m^3) = 1 stere = 1,000,000 cubic centimeters = 1,000,000,000 cubic millimeters

Weight

10 milligrams (mg.)	= 1 centigram (cg.)
10 centigrams	= 1 decigram (dg.) = 100 milligrams
10 decigrams	= 1 gram (g.) = 1000 milligrams
10 grams	= 1 dekagram (dkg.)
10 dekagrams	= 1 hectogram (hg.) = 100 grams
10 hectograms	= 1 kilogram (kg.) = 1000 grams
1000 kilograms	= 1 metric ton (t.)

Tables of Equivalents

When the name of a unit is enclosed in brackets thus: [1 hand], this indicates (1) that the unit is not in general current use in the United States, or (2) that the unit is believed to be based on "custom and usage" rather than on formal definition.

Equivalents involving decimals are, in most instances, rounded off to the third decimal place except where they are exact, in which cases these exact equivalents are so designated.

Lengths

1 Angstrom (A.)	0.1 millimicron (exactly) 0.0001 micron (exactly) 0.0000001 millimeter (exactly) 0.000000004 inch
1 cable's length	120 fathoms 720 feet 219.456 meters (exactly)
1 centimeter (cm.)	0.3937 inch
1 chain (ch.) (Gunter's or surveyor's)	66 feet 20.1168 meters (exactly)
1 chain (engineer's)	100 feet 30.48 meters (exactly)
1 decimeter (dm.)	3.937 inches
1 dekameter (dkm.)	32,808 feet
1 fathom	6 feet 1.8288 meters (exactly)
1 foot (ft.)	0.3048 meters (exactly)
1 furlong (fur.)	10 chains (surveyor's) 660 feet 220 yards $^{1}/_{8}$ statute mile 201.168 meters
[1 hand]	4 inches
1 inch (in.)	2.54 centimeters (exactly)
1 kilometer (km.)	0.621 mile
1 league (land)	3 statute miles 4.828 kilometers
1 link (li.) (Gunter's or surveyor's)	7.92 inches 0.201 meter
1 link (li.) (engineer's)	1 foot 0.305 meter

1 meter (m.)	39.37 inches 1.094 yards
1 micron (μ [the Greek letter mu])	0.001 millimeter (exactly) 0.00003937 inch
1 mil	0.001 inch (exactly) 0.0254 millimeter (exactly)
1 mile (mi.) (statute or land)	5280 feet 1.609 kilometers
1 mile (mi.) (nautical, international, and new U.S. value)	1.852 kilometers (exactly) 1.150779 statute miles 6076.11549 feet
1 millimeter (mm.)	0.03937 inch
1 millimicron (mμ [the English letter *m* in combination with the Greek letter mu])	0.001 micron (exactly) 0.00000003937 inch (exactly)
1 point (typography)	0.013837 inch (exactly) 0.351 millimeter
1 rod (rd.), pole, or perch	$16\frac{1}{2}$ feet $5\frac{1}{2}$ yards 5.029 meters
1 yard (yd.)	0.9144 meter (exactly)

Areas or Surfaces

1 acre	43,560 square feet 4840 square yards 0.405 hectare
1 are (a.)	119.599 square yards 0.025 acre
1 hectare (ha.)	2.471 acres
[1 square (building)]	100 square feet
1 square centimeter (cm^2)	0.155 square inch
1 square decimeter (dm^2)	15.500 square inches
1 square foot (sq. ft.)	929.030 square centimeters
1 square inch (sq. in.)	6.452 square centimeters
1 square kilometer (km^2)	247.105 acres 0.386 square mile
1 square meter (m^2)	1.196 square yards 10.764 square feet
1 square mile (sq. mi.)	258.999 hectares
1 square millimeter (mm^2)	0.002 square inch
1 square rod (sq. rd.), sq. pole, or sq. perch	25.293 square meters
1 square yard (sq. yd.)	0.836 square meter

Capacities or Volumes

1 barrel (bbl.), liquid	31 to 42 gallons*
1 barrel (bbl.), standard, for fruits, vegetables, and other dry commodities except cranberries	7056 cubic inches 105 dry quarts 3.281 bushels, struck measure

* There are a variety of "barrels," established by law or usage. For example, federal taxes on fermented liquors are based on a barrel of 31 gallons; many state laws fix the "barrel for liquids" at $31\frac{1}{2}$ gallons; one state fixes a 36-gallon barrel for cistern measurement; federal law recognizes a 40-gallon barrel for "proof spirits"; by custom, 42 gallons comprise a barrel of crude oil or petroleum products for statistical purposes, and this equivalent is recognized "for liquids" by four states.

1 barrel (bbl.), standard, cranberry	5826 cubic inches 86 45/64 dry quarts 2.709 bushels, struck measure
1 bushel (bu.) (U.S.) (struck measure)	2150.42 cubic inches (exactly) 35.238 liters
[1 bushel, heaped (U.S.)]	2747.715 cubic inches 1.278 bushels, struck measure†

† Frequently recognized as 1 1/4 bushels, struck measure.

[1 bushel (bu.) (British Imperial) (struck measure)]	1.032 U.S. bushels, struck measure 2219.36 cubic inches
1 cord (cd.) (firewood)	128 cubic feet
1 cubic centimeter (cm^3)	0.061 cubic inch
1 cubic decimeter (dm^3)	61.023 cubic inches
1 cubic foot (cu. ft.)	7.481 gallons 28.317 cubic decimeters
1 cubic inch (cu. in.)	0.554 fluid ounce 4.433 fluid drams 16.387 cubic centimeters
1 cubic meter (m^3)	1.308 cubic yards
1 cubic yard (cu. yd.)	0.765 cubic meter
1 cup, measuring	8 fluid ounces 1/2 liquid pint
1 dekaliter (dkl.)	2.642 gallons 1.135 pecks
1 dram, fluid (or liquid) (fl. dr.) (U.S.)	1/8 fluid ounce 0.226 cubic inch 3.697 milliliters
[1 dram, fluid (fl. dr.) (British)]	0.961 U.S. fluid dram 0.217 cubic inch 3.552 milliliters
1 gallon (gal.) (U.S.)	231 cubic inches 3.785 liters 0.833 British gallon 128 U.S. fluid ounces
[1 gallon (gal.) (British Imperial)]	277.42 cubic inches 1.201 U.S. gallons 4.546 liters 160 British fluid ounces
1 gill (gi.)	7.219 cubic inches 4 fluid ounces 0.118 liter
1 hectoliter (hl.)	26.418 gallons 2.838 bushels
1 liter	1.057 liquid quarts 0.908 dry quart 61.025 cubic inches
1 milliliter (ml.)	0.271 fluid dram 16.231 minims 0.061 cubic inch
1 ounce, fluid (or liquid) (fl. oz.) (U.S.)	1.805 cubic inches 29.573 milliliters 1.041 British fluid ounces

[1 ounce, fluid (fl. oz.) (British)]	0.961 U.S. fluid ounce 1.734 cubic inches 28.412 milliliters
1 peck (pk.)	8.810 liters
1 pint (pt.), dry	33.600 cubic inches 0.551 liter
1 pint (pt.), liquid	28.875 cubic inches (exactly) 0.473 liter
1 quart (qt.), dry (U.S.)	67.201 cubic inches 1.101 liters 0.969 British quart
1 quart (qt.), liquid (U.S.)	57.75 cubic inches (exactly) 0.946 liter 0.833 British quart
[1 quart (qt.) (British)]	69.354 cubic inches 1.032 U.S. dry quarts 1.201 U.S. liquid quarts
1 tablespoon	3 teaspoons‡ 4 fluid drams 1/2 fluid ounce
1 teaspoon	1/3 tablespoon‡ 1 1/3 fluid drams‡

‡ The equivalent "1 teaspoon = 1 1/3 fluid drams" has been found by the Bureau to correspond more closely with the actual capacities of "measuring" and silver teaspoons than the equivalent "1 teaspoon = 1 fluid dram" which is given by a number of dictionaries.

Weights or Masses

1 assay ton* (AT) 29.167 grams

* Used in assaying. The assay ton bears the same relation to the milligram that a ton of 2000 pounds avoirdupois bears to the ounce troy; hence the weight in milligrams of precious metal obtained from one assay ton of ore gives directly the number of troy ounces to the net ton.

1 carat (c.) { 200 milligrams; 3.086 grains }

1 dram. apothecaries' (dr. ap.) { 60 grains; 3.888 grams }

1 dram. avoirdupois (dr. avdp.) { $27^{11}/_{32}$ (= 27.344) grains; 1.772 grams }

gamma, *see* microgram

1 grain 64.799 milligrams

1 gram (g.) { 15.432 grains; 0.035 ounce, avoirdupois }

1 hundredweight, gross or long† (gross cwt.) { 112 pounds; 50.802 kilograms }

† The gross or long ton and hundredweight are used commercially in the United States to only a limited extent, usually in restricted industrial fields. These units are the same as the British "ton" and "hundredweight."

1 hundredweight, net or short (cwt. or net cwt.) { 100 pounds; 45.359 kilograms }

1 kilogram (kg.) 2.205 pounds

1 microgram (γ [the Greek letter gamma]) 0.000001 gram (exactly)

1 milligram (mg.) 0.015 grain

1 ounce, avoirdupois (oz. avdp.) { 437.5 grains (exactly); 0.911 troy or apothecaries' ounce; 28.350 grams }

1 ounce, troy or apothecaries' (oz. t. or oz. ap.) { 480 grains; 1.097 avoirdupois ounces; 31.103 grams }

1 pennyweight (dwt.) 1.555 grams

1 pound, avoirdupois (lb. avdp.) { 7000 grains; 1.215 troy or apothecaries' pounds; 453.59237 grams (exactly) }

1 pound, troy or apothecaries' (lb. t. or lb. ap.) { 5760 grains; 0.823 avoirdupois pound; 373.242 grams }

1 scruple (s. ap.) { 20 grains; 1.296 grams }

1 ton, gross or long† (gross tn.) { 2240 pounds; 1.12 net tons (exactly); 1.016 metric tons }

1 ton, metric (t.) { 2204.623 pounds; 0.984 gross ton; 1.102 net tons }

1 ton, net or short (tn. or net tn.) { 2000 pounds; 0.893 gross ton; 0.907 metric ton }

Tables of Interrelation of Units of Measurement

Bold face type indicates exact values

Units of Length

Units	Inches	Links	Feet	Yards	Rods	Chains	Miles	Cm.	Meters
1 inch =	**1**	0.126 333	0.083 333	0.027 778	0.005 051	0.001 263	0.000 016	**2.54**	**0.025 4**
1 link =	**7.92**	**1**	**0.66**	**0.22**	**0.04**	**0.01**	**0.000 125**	20.117	**0.201 168**
1 foot =	**12**	1.515 152	**1**	0.333 333	0.060 606	0.015 152	0.000 189	**30.48**	**0.304 8**
1 yard =	**36**	4.545 45	**3**	**1**	0.181 818	0.045 455	0.000 568	**91.44**	**0.914 4**
1 rod =	**198**	**25**	**16.5**	**5.5**	**1**	**0.25**	**0.003 125**	**502.92**	**5.029 2**
1 chain =	**792**	**100**	**66**	**22**	**4**	**1**	**0.012 5**	**2011.68**	**20.116 8**
1 mile =	**63 360**	**8000**	**5280**	**1760**	**320**	**80**	**1**	**160 934.4**	**1609.344**
1 cm. =	0.3937	0.049 710	0.032 808	0.010 936	0.001 988	0.000 497	0.000 006	**1**	**0.01**
1 meter =	39.37	4.970 970	3.280 840	1.093 613	0.198 839	0.049 710	0.000 621	**100**	**1**

Units of Area

Units		Square inches	Square links	Square feet	Square yards	Square rods	Square chains
1 sq. inch	=	1	.015 942 3	0.006 944	0.000 771 605	0.000 025 5	0.000 001 594
1 sq. link	=	**62.726 4**	1	**0.435 6**	**0.0484**	**0.0016**	**0.000 1**
1 sq. foot	=	**144**	2.295 684	1	0.111 111 1	0.003 673 09	0.000 229 568
1 sq. yard	=	**1296**	20.661 16	**9**	1	0.033 057 85	0.002 066 12
1 sq. rod	=	**39 204**	**625**	**272.25**	**30.25**	1	**0.062 5**
1 sq. chain	=	**627 264**	**10 000**	**4356**	**484**	**16**	1
1 acre	=	**6 272 640**	**100 000**	**43 560**	**4840**	**160**	**10**
1 sq. mile	=	**4 014 489 600**	**64 000 000**	**27 878 400**	**3 097 600**	**102 400**	**6400**
1 sq. cm.	=	0.155 000 3	0.002 471 05	0.001 076	0.000 119 599	0.000 003 954	0.000 000 247
1 sq. meter	=	1550.003	24.710 54	10.763 91	1.195 990	0.039 536 86	0.002 471 054
1 hectare	=	15 500 031	247 105	107 639.1	11 959.90	395.368 6	24.710 54

Units		Acres	Square miles	Square centimeters	Square meters	Hectares
1 sq. inch	=	0.000 000 159 423	0.000 000 000 249 10	**6.451 6**	**0.000 645 16**	0.000 000 065
1 sq. link	=	**0.000 01**	**0.000 000 015 625**	**404.685 642 24**	0.040 468 56	0.000 004 047
1 sq. foot	=	0.000 022 956 84	0.000 000 035 870 06	**929.030 4**	**0.092 903 04**	0.000 009 290
1 sq. yard	=	0.000 206 611 6	0.000 000 322 830 6	**8 361.273 6**	**0.836 127 36**	0.000 083 613
1 sq. rod	=	**0.006 25**	**0.000 009 765 625**	**252 928.526 4**	**25.292 852 64**	0.002 529 285
1 sq. chain	=	**0.1**	**0.000 156 25**	4 046 856	**404.685 642 24**	0.040 468 564
1 acre	=	1	**0.001 562 5**	40 468 654	**4046.856 422 4**	0.404 685 642
1 sq. mile	=	**640**	1	25 899 881 103	2 589 988.11	258.998 811 034
1 sq. cm.	=	0.000 000 024 711	0.000 000 000 038 610	1	**0.0001**	**0.000 000 01**
1 sq. meter	=	0.000 247 105 4	0.000 000 386 102 2	**10 000**	1	**0.0001**
1 hectare	=	2.471 054	0.003 861 022	**100 000 000**	**10 000**	1

Units of Volume

Units		Cubic inches	Cubic feet	Cubic yards	Cubic centimeters	Cubic decimeters	Cubic meters
1 cubic inch	=	**1**	0.000 578 704	0.000 021 433	**16.387 064**	0.016 387	0.000 016 387
1 cubic foot	=	**1728**	**1**	0.037 037 04	**28 316.846 592**	28.316 847	0.028 316 847
1 cubic yard	=	**46 656**	**27**	**1**	**764 554.857 984**	764.554 858	0.764 554 858
1 cubic cm.	=	0.061 023 74	0.000 035 315	0.000 001 308	**1**	**0.001**	**0.000 001**
1 cubic dm.	=	61.023 74	0.035 314 67	0.001 307 951	**1000**	**1**	**0.001**
1 cubic meter	=	61 023.74	35.314 67	1.307 951	**1 000 000**	**1000**	**1**

Units of Capacity (Liquid Measure)

Units		Minims	Fluid drams	Fluid ounces	Gills	Liquid pints
1 minim	=	**1**	0.016 666 7	0.002 083 33	0.000 520 833	0.000 130 208
1 fluid dram	=	**60**	**1**	**0.125**	**0.031 25**	**0.007 812 5**
1 fluid ounce	=	**480**	**8**	**1**	**0.25**	**0.062 5**
1 gill	=	**1920**	**32**	**4**	**1**	**0.25**
1 liquid pint	=	**7680**	**128**	**16**	**4**	**1**
1 liquid quart	=	**15 360**	**256**	**32**	**8**	**2**
1 gallon	=	**61 440**	**1024**	**128**	**32**	**8**
1 milliliter	=	16.231	0.270 519 8	0.033 814 97	0.008 453 742	0.002 113 436
1 liter	=	16 231.19	270.519 8	33.814 97	8.453 742	2.113 436
1 cubic inch	=	265.974	4.432 900	0.554 112 6	0.138 528 1	0.034 632 03
1 cubic foot	=	459 603.1	7660.052	957.506 5	239.376 6	59.844 16

Units of Capacity (Liquid Measure) Continued. **Bold face** type indicates exact values

Units		Liquid quarts	Gallons	Milliliters	Liters	Cubic inches	Cubic feet
1 minim	=	0.000 065 104	0.000 016 276	0.061 610	0.000 061 610	0.003 760	0.000 002 176
1 fluid dram	=	**0.003 906 25**	0.000 976 562	3.696 588	0.003 696 588	0.225 586	0.000 130 547
1 fluid ounce	=	**0.031 25**	**0.007 812 5**	29.572 70	0.029 572 7	1.804 687	0.001 044 379
1 gill	=	**0.125**	**0.031 25**	118.290 8	0.118 290 8	**7.218 75**	0.004 177 517
1 liquid pint	=	**0.5**	**0.125**	473.163 2	0.473 163 2	**28.875**	0.016 710 07
1 liquid quart	=	**1**	**0.25**	946.326 4	0.946 326 4	**57.75**	0.033 420 14
1 gallon	=	**4**	**1**	3785.306	3.785 306	**231**	0.133 680 6
1 milliliter	=	0.001 056 718	0.000 264 179	**1**	**0.001**	0.061 025	0.000 035 316
1 liter	=	1.056 718	0.264 179 4	**1000**	**1**	61.025 45	0.035 315 66
1 cubic inch	=	0.017 316 02	0.004 329 004	16.386 61	0.016 386 61	**1**	0.000 578 704
1 cubic foot	=	29.922 08	7.480 519	28 316.05	28.316 05	**1728**	**1**

Units of Capacity (Dry Measure)

Units		Dry pints	Dry quarts	Pecks	Bushels	Liters	Dekaliters	Cubic inches
1 dry pint	=	**1**	**0.5**	**0.062 5**	**0.015 625**	0.550 595	0.055 060	**33.600 312 5**
1 dry quart	=	**2**	**1**	**0.125**	**0.031 25**	1.101 190	0.110 119	**67.200 625**
1 peck	=	**16**	**8**	**1**	**0.25**	8.809 521	0.880 952	**537.605**
1 bushel	=	**64**	**32**	**4**	**1**	35.238 08	3.523 808	**2150.42**
1 liter	=	1.816 217	0.908 108	0.113 514	0.028 378	**1**	**0.1**	61.025 45
1 dekaliter	=	18.162 17	9.081 084	1.135 136	0.283 784	**10**	**1**	610.254 5
1 cubic inch	=	0.029 762	0.014 881	0.001 860	0.000 465	0.016 386	0.001 639	**1**

Units of Mass not Greater than Pounds and Kilograms

Units		Grains	Apothecaries' scruples	Pennyweights	Avoirdupois drams	Apothecaries' drams	Avoirdupois ounces
1 grain	=	1	0.05	0.041 666 67	0.036 571 43	0.016 666 67	0.002 285 71
1 scruple	=	20	1	0.833 333 3	0.731 428 6	0.333 333 3	0.045 714 29
1 pennyweight	=	24	1.2	1	0.877 714 3	0.4	0.054 857 14
1 dram avdp.	=	27.343 75	1.367 187 5	1.139 323	1	0.455 729 2	0.062 5
1 dram ap.	=	60	3	2.5	2.194 286	1	0.137 142 9
1 oz. avdp.	=	437.5	21.875	18.229 17	16	7.291 667	1
1 oz. ap. or t.	=	480	24	20	17.554 29	8	1.097 143
1 lb. ap. or t.	=	5760	288	240	210.651 4	96	13.165 71
1 lb. avdp.	=	7000	350	291.666 7	256	116.666 7	16
1 milligram	=	0.015 432	0.000 771 618	0.000 643 015	0.000 564 383	0.000 257 206	0.000 035 274
1 gram	=	15.432 36	0.771 617 9	0.643 014 9	0.564 383 4	0.257 206 0	0.035 273 96
1 kilogram	=	15 432.36	771.617 9	643.014 9	564.383 4	257.206 0	35.273 96

Units		Apothecaries' or troy ounces	Apothecaries' or troy pounds	Avoirdupois pounds	Milligrams	Grams	Kilograms
1 grain	=	0.002 083 33	0.000 173 611	0.000 142 857	64.798 91	0.064 798 91	0.000 064 799
1 scruple	=	0.041 666 67	0.003 472 222	0.002 857 143	1295.978 2	1.295 978 2	0.001 295 978
1 pennyweight	=	0.05	0.004 166 667	0.003 428 571	1555.173 84	1.555 173 84	0.001 555 174
1 dram avdp.	=	0.056 966 15	0.004 747 179	0.003 906 25	1771.845 195	1.771 845 195	0.001 771 845
1 dram ap.	=	0.125	0.010 416 67	0.008 571 429	3887.934 6	3.887 934 6	0.003 887 935
1 oz. avdp.	=	0.911 458 3	0.075 954 86	0.062 5	28 349.523 125	28.349 523 125	0.028 349 52
1 oz. ap. or t.	=	1	0.083 333 333	0.068 571 43	31 103.476 8	31.103 476 8	0.031 103 47
1 lb. ap. or t.	=	12	1	0.822 857 1	373 241.721 6	373.241 721 6	0.373 241 722
1 lb. avdp.	=	14.583 33	1.215 278	1	453 592.37	453.592 37	0.453 592 37
1 milligram	=	0.000 032 151	0.000 002 679	0.000 002 205	1	0.001	0.000 001
1 gram	=	0.032 150 75	0.002 679 229	0.002 204 623	1000	1	0.001
1 kilogram	=	32.150 75	2.679 229	2.204 623	1 000 000	1000	1

Units of Mass not Less than Avoirdupois Ounces

Units		Avoirdupois ounces	Avoirdupois pounds	Short hundredweights	Short tons	Long tons	Kilograms	Metric tons
1 oz. avdp.	=	**1**	**0.0625**	**0.000 625**	**0.000 031 25**	0.000 027 902	0.028 349 523	0.000 028 350
1 lb. avdp.	=	**16**	**1**	**0.01**	**0.0005**	0.000 446 429	**0.453 592 37**	0.000 453 592
1 short cwt.	=	**1600**	**100**	**1**	**0.05**	0.044 642 86	**45.359 237**	**0.045 359 237**
1 short ton	=	**32 000**	**2000**	**20**	**1**	0.892 857 1	**907.184 74**	**0.907 184 74**
1 long ton	=	**35 840**	**2240**	**22.4**	**1.12**	**1**	**1016.046 908 8**	1.016 046 909
1 kilogram	=	35.273 96	2.204 623	0.022 046 23	0.001 102 311	0.000 094 207	**1**	**0.001**
1 metric ton	=	35 273.96	2204.623	22.046 23	1.102 311	0.984 206 5	**1000**	**1**

IX. Greek Alphabet; Roman Numerals

GREEK ALPHABET

Α	α	alpha	Ι	ι	iota	Ρ	ρ	rho
Β	β	beta	Κ	κ	kappa	Σ	σ	sigma
Γ	γ	gamma	Λ	λ	lambda	Τ	τ	tau
Δ	δ	delta	Μ	μ	mu	Υ	υ	upsilon
Ε	ε	epsilon	Ν	ν	nu	Φ	φ	phi
Ζ	ζ	zeta	Ξ	ξ	xi	Χ	χ	chi
Η	η	eta	Ο	ο	omicron	Ψ	ψ	psi
Θ	θ	theta	Π	π	pi	Ω	ω	omega

ROMAN NUMERALS

I	1	XI	11	XXX	30	CD	400
II	2	XII	12	XL	40	D	500
III	3	XIII	13	L	50	DC	600
IV	4	XIV	14	LX	60	DCC	700
V	5	XV	15	LXX	70	DCCC	800
VI	6	XVI	16	LXXX	80	CM	900
VII	7	XVII	17	XC	90	M	1,000
VIII	8	XVIII	18	C	100	MCM	1,900
IX	9	XIX	19	CC	200	MM	2,000
X	10	XX	20	CCC	300	$\overline{\text{V}}$	5,000

A dash line over a numeral multiplies the value by 1,000: thus, $\overline{\text{X}}$ = 10,000; $\overline{\text{L}}$ = 50,000; $\overline{\text{C}}$ = 100,000; $\overline{\text{D}}$ = 500,000; $\overline{\text{M}}$ = 1,000,000; $\overline{\text{CLIX}}$ = 159,000; $\overline{\text{DLIX}}$ = 559,000.

Other general rules for Roman numerals are as follows: (1) Repeating a letter repeats its value: XX = 20; CCC = 300. (2) A letter placed after one of greater value adds thereto: VI = 6; DC = 600. (3) A letter placed before one of greater value subtracts therefrom: IV = 4.

TABLE 2–5

Conversion factors between the British FPSR (foot, pound, second, rankine) system and SI (Système International) units

Quantity	FPSR Units	Multiply by	To obtain SI Units	Multiply by	To obtain FPSR Units
Mass (M)	slug	1.459×10	kg	6.852×10^{-2}	slug
Length (L)	ft	3.048×10^{-1}	m	3.281	ft
Density (ρ)	slug/ft³	5.155×10^{2}	kg/m³	1.940×10^{-3}	slug/ft³
Temperature (T)	°F + 460	5.56×10^{-1}	°C + 273	1.8	°F + 460
	°R		°K		°R
Velocity (V)	ft/sec	3.048×10^{-1}	m/sec	3.281	ft/sec
	mph	1.609	kph	6.214×10^{-1}	mph
	knot	1.853	kph	5.396×10^{-1}	knot
		0.515	m/sec	1.942	
Force (F)	lbf	4.448	N (newton)	2.248×10^{-1}	lbf
	slug ft/sec²		kg m/sec²		slug ft/sec²
Work	slug ft²/sec²	1.356	Nm	7.376×10^{-1}	slug ft²/sec²
Energy (J)	BTU		(joule)		BTU
Power (W)	slug ft²/sec³	1.356	Nm/sec	7.376×10^{-1}	slug ft²/sec³
	hp (550 ft lbf/sec)	7.456×10^{2}	(Watt)	1.341×10^{-3}	hp (550 ft lbf/sec)
Pressure (p)	slug/ft sec²	4.788×10	N/m²	2.088×10^{-2}	slug/ft sec²
	lbf/ft²		(pascal)		lbf/ft²
		4.788×10^{-4}	bar	2.088×10^{3}	
Specific Energy, etc	ft lbf/slug	9.290×10^{-2}	Nm/kg	1.076×10	ft lbf/slug
Gas Constant	ft lbf/slug° R	1.672×10^{-1}	Nm/kg° K	5.981	ft lbf/slug° R
Coef of Viscosity (μ)	slug/ft sec	4.788×10	kg/m sec	2.088×10^{-2}	slug/ft sec
Kinematic Viscosity (ν)	ft²/sec	9.290×10^{-2}	m²/sec	1.076×10	ft²/sec
Thermal Conductivity (k)	lbf/sec° R	8.007	N/sec° K	1.249×10^{-1}	lbf/sec° R
Heat Transfer Coefficient	lbf/ft sec° R	2.627×10	N/m sec° K	3.807×10^{-2}	lbf/ft sec° R
Frequency	c/sec	1.0	Hz (hertz)	1.0	c/sec

TABLE 2-6

Conversion Factors (mixed FPSR, metric and SI)

Multiply	By	To obtain
acres	0.4047	ha (= 10^4 m^2)
	43 560	ft^2
	0.0015625	mi^2
standard atmospheres (atm)	76	cm Hg
	29.92	in Hg
	1.01325	bar (= 10^5 N/m^2)
	1.033	kgf/cm^2
	14.70	lbf/in^2
	2 116	lbf/ft^2
	101 325	N/m^2
bars (bar)	0.98692	atm
	14.5038	lbf/in^2
British Thermal Unit (BTU)	0.5556	CHU
	1 055	J
	0.2520	kcal (kilocalorie)
centimetres (cm)	0.3937	in
	0.032808	ft
centimetres of mercury at 0° C (cm Hg)	0.01316	atm
	0.3937	in Hg
	0.1934	lbf/in^2
	27.85	lbf/ft^2
	135.95	kgf/m^2
centimetres per second (cm/s)	0.032808	ft/s
	1.9685	ft/min
	0.02237	mph
cubic centimetres (cm^3)	0.06102	in^3
	3.531×10^{-5}	ft^3
	0.001	litre
	2.642×10^{-4}	US gal
cubic feet (ft^3)	28317	cm^3
	0.028317	m^3
	1728	in^3
	0.037037	yd^3
	7.481	US gal
	28.32	litre
cubic feet per minute (ft^3/min)	0.472	litre/s
	0.028317	m^3/min
cubic inches (in^3)	16.39	cm^3
	1.639×10^{-5}	m^3
	5.787×10^{-4}	ft^3
	0.5541	fl oz
	0.01639	litre
	4.329×10^{-3}	US gal
	0.01732	US qt
cubic metres (m^3)	61024	in^3
	1.308	yd^3
	35.3147	ft^3
	264.2	US gal

TABLE 2-6 – *continued*

Multiply	By	To obtain
cubic metres per minute (m^3/min)	35.3147	ft^3/min
cubic yards (yd^3)	27	ft^3
	0.7646	m^3
	202	US gal
degrees (arc)	0.01745	radians
degrees per second (deg/s)	0.01745	radians/s
erg	1.0×10^{-7}	J
feet (ft)	30.48	cm
	0.3048	m
	12	in
	0.33333	yd
	0.0606061	rod
	1.894×10^{-4}	stm
	1.646×10^{-4}	nm (international)
feet per minute (ft/min)	0.01136	mph
	0.01829	km/h
	0.508	cm/s
	0.00508	m/s
feet per second (ft/s)	0.6818	mph
	1.097	km/h
	30.48	cm/s
	0.5925	knot (international)
foot-pounds (ft lbf)	0.138255	kgf m
	3.24×10^{-4}	kcal
	1.356	Nm (J)
foot-pounds per minute (ft lbf/min)	3.030×10^{-5}	hp
foot-pounds per second (ft lbf/s)	1.818×10^{-3}	hp
gallons, Imperial (Imp gal)	277.4	in^3
	1.201	US gal
	4.546	litre
	153.707	fl oz
gallons, US dry (US gal dry)	268.8	in^3
	1.556×10^{-1}	ft^3
	1.164	US gal
	4.405	litre
gallons, US liquid (US gal)	231	in^3
	0.1337	ft^3
	4.951×10^{-3}	yd^3
	3785.4	cm^3
	3.785×10^{-3}	m^3
	3.785	litre
	0.83267	Imp gal
	133.227	fl oz
US gallons per acre (gal/acre)	9.353	litre/ha
grams (g)	0.001	kg
	2.205×10^{-3}	lb
grams per centimetre (g/cm)	0.1	kg/m
	6.720×10^{-2}	lb/ft
	5.600×10^{-3}	lb/in
grams per cubic centimetre (g/cm^3)	1000	kg/m^3
	0.03613	lb/in^3
	62.43	lb/ft^3

TABLE 2-6 – *continued*

Multiply	By	To obtain
hectares (ha)	2.471	acres
	107639	ft^2
	10000	m^2
horsepower (hp)	33000	ft lbf/min
	550	ft lbf/s
	0.7457	kW
	76.04	kgf m/s
	1.014	metric hp
	745.70	Nm/s (W)
horsepower, metric	75	kgf m/s
	0.9863	hp
inches (in)	25.40	mm
	2.540	cm
	0.0254	m
	0.08333	ft
	0.027777	yd
inches of mercury at 0°C	0.033421	atm
(in Hg)	0.4912	lb/in^2
	70.73	lb/ft^2
	345.3	kg/m^2
	2.540	cm Hg
	25.40	mm Hg
	3.386×10^3	N/m^2
inch-pounds (in lbf)	0.011521	kgf m
J (joule)	0.27778×10^{-6}	kWh
	1	Nm
	1	Ws
kilograms (kg)	2.204623	lb
	35.27	oz avdp
	1000	g
kilogram-calories (kcal)	3.9683	BTU
(kilocalories)	3088	ft lbf
	426.9	kgf m
kilogram-metre² (kg m²)	3417	lb in^2
	23.729	lb ft^2
	0.7376	slug ft^2
kilograms per cubic metre	0.06243	lb/ft^3
(kg/m^3)	0.001	g/cm^3
kilograms per hectare (kg/ha)	0.892	lb/acre
kilograms per square centimetre	0.9678	atm
(kg/cm^2)	28.96	in Hg
	14.22	lbf/in^2
	2048	lbf/ft^2
kilograms per square metre	2.896×10^{-3}	in Hg
(kg/m^2)	1.422×10^{-3}	lb/in^2
	0.2048	lb/ft^2
kilometres (km)	1×10^5	cm
	3280.8	ft
	0.6214	stm
	0.53996	nm (international)
kilometers per hour (kph)	0.9113	ft/s
	58.68	ft/min

TABLE 2-6 – *continued*

Multiply	By	To obtain
	0.53996	knot (international)
	0.6214	mph
	0.27778	m/s
	16.67	m/min
kilowatt	1.34	hp
knots (knot) (international)	1	nm/h
	1.688	ft/s
	1.1508	mph
	1.852	km/h
	0.5144	m/s
litres (litre)	1000	cm^3
	61.02	in^3
	0.03531	ft^3
	33.814	fl oz
	0.2642	US gal
	0.2200	Imp gal
	1.0568	US qt
litres per hectare (litre/ha)	13.69	fl oz/acre
	0.107	US gal/acre
litres per second (litre/s)	2.12	ft^3/min
metres (m)	39.37	in
	3.280840	ft
	1.0936	yd
	0.198839	rod
	6.214×10^{-4}	stm
	5.3996×10^{-4}	nm (international)
metre-kilogram (kgf/m)	7.23301	ft lbf
	86.798	in lbf
metres per minute (m/min)	0.06	km/h
metres per second (m/sec)	3.280840	ft/s
	196.8504	ft/min
	2.237	mph
	3.6	km/h
microns	3.937×10^{-5}	in
miles (stm)	5280	ft
	1.6093	km
	1609.3	m
	0.8690	nm (international)
miles per hour (mph)	44.704	cm/s
	0.4470	m/s
	1.467	ft/s
	88	ft/min
	1.6093	km/h
	0.8690	knot (international)
millibars	2.953×10^{-2}	in Hg
	0.1	kN/m^2
millimetres (mm)	0.03937	in
millimetres of mercury at 0°C (mm Hg)	0.03937	in Hg
international nautical miles (nm)	6076	ft
	1.1508	stm
	1852	m
	1.852	km

Multiply	By	To obtain
Newton (N)	0.2248	lbf
ounces, fluid (fl oz)	8	dr fl
	29.57	cm^3
	1.805	in^3
	0.0296	litre
	0.0078	US gal
ounces, fluid per acre (fl oz/acre)	0.073	litre/ha
pounds (lb): mass	453.6	g
	0.453592	kg
	3.108×10^{-2}	slug
pounds force (lbf)	4.4482	N
	0.45359	kgf
pounds-feet (lbf ft)	1.356	Nm
pounds-feet² (lb ft²)	0.421	kg m^2
	144	lb in^2
	0.0311	slug ft^2
pounds per acre (lb/acre)	1.121	kg/ha
pounds per cubic foot (lb/ft^3)	16.02	kg/m^3
pounds per cubic inch (lb/in^3)	1728	lb/ft^3
	27.68	g/cm^3
pounds per hour per pound force (lb/h/lbf)	28.3	mg/Ns
pounds per hour per horsepower (lb/h/hp)	169	μg/J
pounds-force per square foot (lbf/ft^2)	0.1414	in Hg
	4.88243	kgf/m^2
	4.725×10^{-4}	atm
	0.048	kN/m^2
pounds per square inch (psi or lbf/in^2)	5.1715	cm Hg
	2.036	in Hg
	0.06805	atm
	0.0689476	bar
	703.1	kg/m^2
	6.89476	kN/m^2
quart, US (qt)	0.94635	litre
	57.750	in^3
	3.342×10^{-2}	ft^3
radians	57.30	deg (arc)
	0.1592	rev
radians per second (radians/s)	57.296	deg/s
	0.1592	rev/s
	9.549	rpm
revolutions (rev)	6.283	radians
revolutions per minute (rpm or rev/min)	0.1047	radians/s
revolutions per second (rev/s)	6.283	radians/s
rod	16.5	ft
	5.5	yd
	5.0292	m
slug	14.594	kg
	32.174	lb
slug feet² (slug ft^2)	1.3559	kg m^2
	4633.1	lb in^2
	32.174	lb ft^2

TABLE 2–6 – *continued*

Multiply	By	To obtain
square centimetres (cm^2)	0.1550	in^2
	0.001076	ft^2
square feet (ft^2)	929.03	cm^2
	0.092903	m^2
	144	in^2
	0.1111	yd^2
	2.296×10^{-5}	acres
square inches (in^2)	6.4516	cm^2
	6.944×10^{-3}	ft^2
square kilometres (km^2)	0.3861	stm^2
square metres (m^2)	10.76391	ft^2
	1.196	yd^2
	0.0001	ha
square miles (mi^2)	2.590	km^2
	640	acres
square rods (rod^2)	30.25	yd^2
square yards (yd^2)	0.8361	m^2
	9	ft^2
	0.0330579	rod^2
ton	2240	lb
	1016	kg
	1.016	t (tonne)
ton-force (tonf)	9.964×10^3	N
tons per square foot ($tonf/ft^2$)	107.252×10^3	kN/m^2
watt (W)	1.34×10^{-3}	hp
	10^{-3}	kW
yards (yd)	0.9144	m
	3	ft
	36	in
	0.181818	rod

TABLE 2–7

General Notation*

Axes	OX	OY	OZ
Aerodynamic forces	X	Y	Z
Angular motions	ϕ (bank)	θ (pitch)	Ψ (yaw)
Linear velocities	u	v	w
Angular velocities	p	q	r
Moments of forces	L (roll)	M (pitch)	N (yaw)
Moments of inertia	A	B	C
	(Note: $A + B \approx C$)		

Table D-3. Torque Conversion Table, Pound Feet to Newton Meters

Pound-Feet (lb.-ft.)	Newton Metres (Nm)	Newton Metres (Nm)	Pound-Feet (lb.-ft.)
1	1.356	1	0.7376
2	2.7	2	1.5
3	4.0	3	2.2
4	5.4	4	3.0
5	6.8	5	3.7
6	8.1	6	4.4
7	9.5	7	5.2
8	10.8	8	5.9
9	12.2	9	6.6
10	13.6	10	7.4
15	20.3	15	11.1
20	27.1	20	14.8
25	33.9	25	18.4
30	40.7	30	22.1
35	47.5	35	25.8
40	54.2	40	29.5
45	61.0	50	36.9
50	67.8	60	44.3
55	74.6	70	51.6
60	81.4	80	59.0
65	88.1	90	66.4
70	94.9	100	73.8
75	101.7	110	81.1
80	108.5	120	88.5
90	122.0	130	95.9
100	135.6	140	103.3
110	149.1	150	110.6
120	162.7	160	118.0
130	176.3	170	125.4
140	189.8	180	132.8
150	203.4	190	140.1
160	216.9	200	147.5
170	230.5	225	166.0
180	244.0	250	184.4

TABLE OF CONVERSION FACTORS

Because different conventions historically have been used to measure various quantities, the following tables have been compiled to sort out the different units. This first table identifies the units typically used for describing a particular quantity. For example, speed might be measured in "miles/hour".

Most quantities can also be described in terms of the following three basic dimensions:

length	L
mass	M
time	T

For example, speed is given in terms of length divided by time, which can be written as "L/T". This description, called "dimensional analysis", is useful in determining whether an equation is correct. The product of the dimensions on each side of the equal sign must match. For example:

$$\text{Distance} = \text{Speed} \times \text{Time}$$
$$L = L/T \times T$$

The dimension on the left side of the equal sign is length, L. On the right side of the equal sign, the product of L/T times T is L, which matches the left side of the equation.

The second table is a Conversion Table, showing how to convert from one set of units to another. It might be necessary to take the reciprocal of the conversion factor or to make more than one conversion to get the desired results.

Measured Quantities and Their Common Units

Length(L)	Area(L^2)	Volume(L^3)
mile(mi.)	sq. mile(mi^2)	gallon(gal.)
yard(yd.)	sq. yard(yd^2)	quart(qt.)
foot(ft.)	sq. foot(ft^2)	pint(pt.)
inch(in.)	sq. inch(in^2)	ounce(oz.)
fathom(fath.)	acre	cu. foot(ft^3)
kilometer(km.)	sq. kilometer(km^2)	cu. yard(yd^3)
meter(m.)	sq. meter(m^2)	cu. inch(in^3)
centimeter(cm.)	sq. centimeter(cm^2)	liter(l)
micron(μ)		cu. centimeter(cm^3)
angstrom(Å)		acre-foot
		cord(cd)
		cord-foot
		barrel(bbl.)

Mass(M)	Speed(L/T)	Flow Rate(L^3/T)
pound(lb.)	feet/minute (ft./min.)	cu. feet/min.
ton(short)	feet/sec.	cu. meter/min.
ton(long)	mile/hour	liters/sec.
ton(metric)	mile/min.	gallons/min.
gram(g.)	kilometer/hr.	gallons/sec.
kilogram(kg.)	kilometer/min.	
	kilometer/sec.	

Pressure(M/L/T^2)	Energy(ML^2/T^2)	Power(ML^2/T^3)
atmosphere(atm.)	British thermal unit(Btu.)	Btu./min.
pounds/sq. inch(psi)	calories(cal.)	Btu./hour
inches of mercury	foot-pound	watt
cm. of mercury	joule	joule/sec.
feet of water	kilowatt-hour (kw-hr.)	cal./min.
	horsepower-hour (hp.-hr.)	horsepower(hp.)

Time(T)	Energy Density(M/T^2)	Power Density(M/T^3)
year(yr)	calories/sq. cm.	cal./sq. cm./min.
month	Btu./sq. foot	Btu./sq. foot/hr
day	langley	langley/min.
hour(hr.)	watthr./sq. foot	watt/sq. cm.
minute(min.)		
second(sec.)		

Table of Conversion Factors

MULTIPLY	BY	TO OBTAIN:
Acres	43560	Sq. feet
″	0.004047	Sq. kilometers
″	4047	Sq. meters
″	0.0015625	Sq. miles
″	4840	Sq. yards
Acre-feet	43560	Cu. feet
″	1233.5	Cu. meters
″	1613.3	Cu. yards
Angstroms(Å)	1×10^{-8}	Centimeters
″	3.937×10^{-9}	Inches
″	0.0001	Microns
Atmospheres(atm.)	76	Cm. of Hg(0°C)
″	1033.3	Cm. of H_2O(4°C)
″	33.8995	Ft. of H_2O(39.2°F)
″	29.92	In. of Hg(32°F)
″	14.696	Pounds/sq. inch(psi)
Barrels(petroleum, U.S.)(bbl.)	5.6146	Cu. feet
″	35	Gallons(Imperial)
″	42	Gallons(U.S.)
″	158.98	Liters
British Thermal Unit(Btu)	251.99	Calories, gm
″	777.649	Foot-pounds
″	0.00039275	Horsepower-hours
″	1054.35	Joules
″	0.000292875	Kilowatt-hours
″	1054.35	Watt-seconds
Btu/hr.	4.2	Calories/min.
″	777.65	Foot-pounds/hr.
″	0.0003927	Horsepower
″	0.000292875	Kilowatts
″	0.292875	Watts(or joule/sec.)
Btu/lb.	7.25×10^{-4}	Cal/gram
Btu/sq. ft.	0.271246	Calories/sq. cm. (or langleys)
″	0.292875	Watt-hour/sq. foot
Btu/sq. ft./hour	3.15×10^{-7}	Kilowatts/sq. meter
″	4.51×10^{-3}	Cal./sq. cm./min(or langleys/min)
″	3.15×10^{-8}	Watts/sq. cm.
Calories(cal.)	0.003968	Btu.
″	3.08596	Foot-pounds

Conversion Factors—Continued

″	1.55857×10^{-6}	Horsepower-hours
″	4.184	Joules (or watt-secs)
″	1.1622×10^{-6}	Kilowatt-hours
Calories, food unit (Cal.)	1000	Calories
Calories/min.	0.003968	Btu/min.
″	0.06973	Watts
Calories/sq. cm.	3.68669	Btu/sq. ft.
″	1.0797	Watt-hr/sq. foot
Cal./sq. cm./min.	796320.	Btu/sq. foot/hr.
″	251.04	Watts/sq. cm.
Candle power (spherical)	12.566	Lumens
Centimeters(cm.)	0.032808	Feet
″	0.3937	Inches
″	0.01	Meters
″	10.000	Microns
Cm. of Hg(0°C)	0.0131579	Atmospheres
″	0.44605	Ft. of H_2O(4°C)
″	0.19337	Pounds/sq. inch
Cm. of H_2O(4°C)	0.0009678	Atmospheres
″	0.01422	Pounds/sq. inch
Cm./sec.	0.032808	Feet/sec.
″	0.022369	Miles/hr.
Cords	8	Cord-feet
″	128(or 4×4×8)	Cu. feet
Cu. centimeters	3.5314667	Cu. feet
″	0.06102	Cu. inches
″	1×10^{-6}	Cu. meters
″	0.001	Liters
″	0.0338	Ounces(U.S. fluid)
Cu. feet(ft.^{3})	0.02831685	Cu. meters
″	7.4805	Gallons(U.S., liq.)
″	28.31685	Liters
″	29.922	Quarts(U.S., liq.)
Cu. ft. of H_2O (60°F)	62.366	Pounds of H_2O
Cu. feet/min.	471.947	Cu. cm./sec.
Cu. inches(in.^{3})	16.387	Cu. cm.
″	0.0005787	Cu. feet
″	0.004329	Gallons(U.S., liq.)
″	0.5541	Ounces(U.S., fluid)
Cu. meters	1×10^6	Cu. centimeters
″	35.314667	Cu. feet
″	264.172	Gallons(U.S., liq.)
″	1000	Liters
Cu. yard	27	Cu. feet
″	0.76455	Cu. meters
″	201.97	Gallons(U.S., liq.)
Cubits	18	Inches
Fathoms	6	Feet
″	1.8288	Meters
Feet(ft.)	30.48	Centimeters
″	12	Inches
″	0.00018939	Miles(statute)
Feet of H_2O(4°C)	0.029499	Atmospheres
″	2.2419	Cm. of Hg(0°C)
″	0.433515	Pounds/sq. inch
Feet/min.	0.508	Centimeters/second
″	0.018288	Kilometers/hr.
″	0.0113636	Miles/hr.
Foot-candles	1	Lumens/sq. foot
Foot-pounds	0.001285	Btu.
″	0.324048	Calories
″	5.0505×10^{-7}	Horsepower-hours
″	3.76616×10^{-7}	Kilowatt-hours

Furlong	220	Yards
Gallons(U.S., dry)	1.163647	Gallons(U.S., liq.)
Gallons(U.S., liq.)	3785.4	Cu. centimeters
″	0.13368	Cu. feet
″	231	Cu. inches
″	0.0037854	Cu. meters
″	3.7854	Liters
″	8	Pints(U.S., liq.)
″	4	Quarts(U.S., liq.)
Gallons/min.	2.228×10^{-3}	Cu. feet/sec.
″	0.06308	Liters/sec.
Grams	0.035274	Ounces(avdp.)
″	0.002205	Pounds(avdp.)
Grams-cm.	9.3011×10^{-8}	Btu.
Grams/meter^2	3.98	Short ton/acre
″	8.92	lbs./acre
Horsepower	42.4356	Btu./min.
″	550	Foot-pounds/sec.
″	745.7	Watts
Horsepower-hrs.	2546.14	Btu.
″	641616	Calories
″	1.98×10^6	Foot-pounds
″	0.7457	Kilowatt-hours
Inches	2.54	Centimeters
″	0.83333	Feet
In. of Hg(32°F)	0.03342	Atmospheres
″	1.133	Feet of H_2O
″	0.4912	Pounds/sq. inch
In. of Water(4°C)	0.002458	Atmospheres
″	0.07355	In. of Mercury(32°F)
″	0.03613	Pounds/sq. inch
Joules	0.0009485	Btu.
″	0.73756	Foot-pounds
″	0.0002778	Watt-hours
″	1	Watt-sec.
Kilo calories/gram	1378.54	Btu/lb
Kilograms	2.2046	Pounds(avdp.)
Kilograms/hectare	.893	lbs/acre
Kilograms/hectare	.0004465	Short ton/acre
Kilometers	1000	Meters
″	0.62137	Miles(statute)
Kilometer/hr.	54.68	Feet/min.
Kilowatts	3414.43	Btu./hr.
″	737.56	Foot-pounds/sec.
″	1.34102	Horsepower
Kilowatt-hours	3414.43	Btu.
″	1.34102	Horsepower-hours
Knots	51.44	Centimeter/sec.
″	1	Mile(nautical)/hr.
″	1.15078	Miles(Statute)/hr.
Langleys	1	Calories/sq. cm.
Liters	1000	Cu. centimeters
″	0.0353	Cu. feet
″	0.2642	Gallons(U.S., liq.)
″	1.0567	Quarts(U.S., liq.)
Lbs./acre	.0005	Short ton/acre
Liters/min.	0.0353	Cu. feet/min.
″	0.2642	Gallons(U.S., liq.)/min.
Lumens	0.079577	Candle power(spherical)
Lumens(at 5550Å)	0.0014706	Watts
Meters	3.2808	Feet
″	39.37	Inches
″	1.0936	Yards
Meters/sec.	2.24	Miles/hr.
Micron	10000	Angstroms
″	0.0001	Centimeters
Miles(statute)	5280	Feet

"	1.6093	Kilometers
"	1760	Yards
Miles/hour	44.704	Centimeter/sec.
"	88	Feet/min.
"	1.6093	Kilometer/hr.
"	0.447	Meters/second
Milliliter	1	Cu. centimeter
Millimeter	0.1	Centimeter
Ounces(avdp.)	0.0625	Pounds(avdp.)
Ounces(U.S., liq.)	29.57	Cu. centimeters
"	1.8047	Cu. inches
"	0.0625(or 1/16)	Pint(U.S., liq.)
Pints(U.S., liq.)	473.18	Cu. centimeters
"	28.875	Cu. inches
"	0.5	Quarts(U.S., liq.)
Pounds(avdp.)	0.45359	Kilograms
"	16	Ounces(avdp.)
Pounds of water	0.01602	Cu. feet of water
"	0.1198	Gallons(U.S., liq.)
Pounds/acre	0.0005	Short ton/acre
Pounds/sq. inch	0.06805	Atmospheres
"	5.1715	Cm. of mercury(0°C)
"	27.6807	In. of water(39.2°F)
Quarts(U.S., liq.)	0.25	Gallons(U.S., liq.)
"	0.9463	Liters
"	32	Ounces(U.S., liq.)
"	2	Pints(U.S., liq.)
Radians	57.30	Degrees
Sq. centimeters	0.0010764	Sq. feet
"	0.1550	Sq. inches
Sq. feet	2.2957×10^{-5}	Acres
"	0.09290	Sq. meters
Sq. inches	6.4516	Sq. centimeters
"	0.006944	Sq. feet
Sq. kilometers	247.1	Acres
"	1.0764×10^7	Sq. feet
"	0.3861	Sq. miles
Sq. meters	10.7639	Sq. feet
"	1.196	Sq. yards
Sq. miles	640	Acres
"	2.788×10^7	Sq. feet
"	2.590	Sq. kilometers
Sq. yards	9(or 3×3)	Sq. feet
"	0.83613	Sq. meters
Tons, long	1016	Kilograms
"	2240	Pounds(avdp.)
Tons(metric)	1000	Kilograms
"	2204.6	Pounds(avdp.)
Tons, metric/hectare	0.446	Short ton/acre
Tons(short)	907.2	Kilograms
"	2000	Pounds(avdp.)
Watts	3.4144	Btu./hr.
"	0.05691	Btu./min.
"	14.34	Calories/min.
"	0.001341	Horsepower
"	1	Joule/sec.
Watts/sq. cm.	3172	Btu./sq. foot/hr.
Watt-hours	3.4144	Btu.
"	860.4	Calories
"	0.001341	Horsepower-hours
Yards	3	Feet
"	0.9144	Meters

Gravitational or Engineer's System of Units

The other system of units is called the "gravitational" or "engineer's system." The unit of force is taken as the lb., and the unit of mass is called the slug. (You see, all masses have a tendency "to stay put"; they are all reluctant to move; they are very sluggish.)

A mass of 1 slug is that which when acted upon by a force of 1 lb. will move with an acceleration of 1 ft. per sec.^{2}.

But we have already seen that a force of 1 lb. will cause a mass of g lb. to move with an acceleration of 1 ft. per sec.^{2}.

Therefore, *a mass of 1 slug is equivalent to a mass of g lb.*

The mass of a body in slugs is equal to $\dfrac{\textit{its weight in lb.}}{g}$

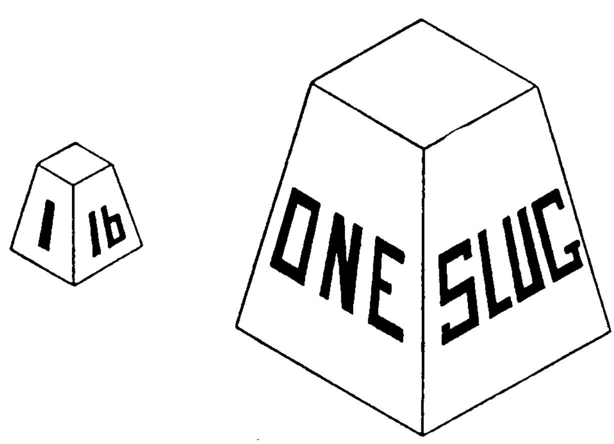


FIG. 29—THE TWO UNITS OF MASS

If a force of 1 lb. were applied to the mass of 1 lb., it would move with an acceleration of 32·2 ft. per sec.^{2}. If a force of 1 lb. were applied to the mass of 1 slug, it would move with an acceleration of 1 ft. per sec.^{2}. The slug is approximately 32·2 times as big as the pound.

Useful equations and Conversion Factors

1. Areas of a rectangle = Length x width
   Areas of a circle $= \pi \times (\text{radius})^2$

   Surface area of a cylinder
   $= 2\pi \times \text{radius} \times \text{height} + 2\pi \times (\text{radius})^2$
   Surface area of a sphere $= 4\pi\ (\text{radius})^2$

2. Volumes of a rectangular tank
   = Length x Width x Height
   Volumes of a cylinder $= \pi \times (\text{radius})^2 \times \text{Height}$
   Volumes of a sphere $= \frac{4}{3}\pi \times (\text{radius})^3$

3. Retention time $= \dfrac{\text{Volume of Tank}}{\text{Rate of flow of liquid or gas}}$

4. 1 inch = 2.54 cms. 1 m. = 3.28 feet
   1 foot = 0.305 m. 1 sq. ft. = 0.093 sq. m.
   1 sq. m. = 10.76 sq. ft. 1 cu. ft. = 28 litres = 0.028 m^3.

5. 1 gallon = 4.55 litres 1 litre = 0.22 gallons
   1 m^3 = 220 gallons

6. 1 gallon occupies 0.161 cu. ft. 1,000 litres in 1 cu. m.
   1 cu. ft. = 6.23 gallons

7. 1 gallon of water weighs 10 lbs.
   1 litre of water weighs 1 kg.

8. 1 lb. = 0.454 kg. 1 kg. = 0.221 lb.
   2240 lb. = 1 ton = 1.02 tonnes 1000 kg. = 1 tonne = 0.984 tonne

9. 1 part per million = 1 milligram/litre = 1 gm/m^3
   1% = 10,000 ppm = 10,000 mgm/litre = 10 gm/litre

10. 1 cu. ft./lb. = 0.062 m^3/kg. 1 m^3/kg. = 16.1 cu. ft./lb.

11. 1 lb./cu. ft. = 16.2 kg./m^3. 1 kg./m^3. = 0.062lb./cu. ft.

12. 1 acre = 4,840 sq. yards = 0.405 hectares
    1 hectare = 10,000 sq. metres = 2.47 acres

13. $T\,°F = \frac{5}{9}\,(T - 32)\,°C$ $T\,°C = \frac{9}{5}\,T + 32\,°F$

14. Pressure: 1 in. of water = 0.25 m. bar
    1 lb/sq. in. (psi) = 68.95 m.bar
    1 atmosphere = 1.013 m.bar

15. 1 British Thermal Unit = 0.252 Kcals.
    1 BTU = 1,055 Joules 1 Kcal = 3.97 BTU
    1 Joule = 9.5×10^{-4} BTU 1 Kcal = 4.19 kJ.

16. 100,000 BTU = 1 Therm = 29.3 kilowatt hours

17. 1 BTU/cu. ft. = 0.038 J/cm^3 (or Mega Joules/m^3)
    1 J/cm^3 = 27.0 BTU/cu. ft.

18. 1 BTU/lb. = 2320 J/kg. 1J/kg. = 4.29×10^{-3} BTU/lb.

19. 1 BTU/hr. = 0.0011 mJ/hr.

20. Heat transfer coefficient:
    1 BTU/$ft.^2$/°F/hr. = 20.44 kJ/m^2/°C/hr.

APPENDIX: Conversion Factors

To convert from:	To:	Multiply by:
Length		
centimeters (cm)	inches	0.394
feet (ft)	centimeters	30.5
inches (in)	centimeters	2.54
kilometers (km)	miles	0.621
meters (m)	feet	3.28
meters (m)	yards	1.094
miles (mi)	kilometers	1.609
millimeters (mm)	inches	0.0394
yards (yd)	meters	0.914
Area		
acres	hectares	0.405
acres	sq. meters	4047
hectares (ha)	acres	2.47
hectares (ha)	sq. meters	10,000
sq. centimeters (cm^2)	sq. inches	0.155
sq. feet (ft^2)	sq. meters	0.0929
sq. inches (in^2)	sq. centimeters	6.45
sq. kilometers (km^2)	sq. miles	0.386
sq. kilometers (km^2)	hectares	100
sq. meters (m^2)	sq. feet	10.76
sq. yards (yd^2)	sq. meters	0.836
Volume		
barrels (petroleum, bbl)	liters	159
cubic centimeters (cm^3)	cubic inches	0.0610
cubic feet (ft^3)	cubic meters	0.0283
cubic inches (in^3)	cubic centimeters	16.39
cubic meters (m^3)	cubic feet	35.3
cubic meters (m^3)	cubic yards	1.308
cubic yards (yd^3)	cubic meters	0.765
gallons (gal) US	liters	3.79
gallons (gal) Imp.	liters	4.545
gallons (gal) Imp.	gallons, US	1.20
Weight		
grams (g)	ounces, avdp.	0.0353
kilograms (kg)	pounds	2.205
ounces avdp. (oz)	grams	28.3
pounds (lb)	kilograms	0.454
tons (long)	pounds	2240
tons (long)	kilograms	1016
tons (metric)	pounds	2205
tons (metric)	kilograms	1000
tons (short)	pounds	2000
tons (short)	kilograms	907

To convert from:	To:	Multiply by:
Pressure		
atmosphere	grams/sq.cm	1033
atmosphere	pounds/sq.in	14.7
pounds/sq.in (psi)	grams/sq.cm	70.3
Energy		
British thermal units (Btu)	kilojoules	1.054
calories (cal)	joules	4.19
ergs	joules	1×10^{-7}
kilojoules (kJ)	Btu	0.948
joules (J)	calories	0.239
kilowatt-hours (kWh)	megajoules	3.6
megajoules (MJ)	kilojoules	1000
gigajoules (GJ)	megajoules	1000
terajoules (TJ)	gigajoules	1000
Energy Density		
Btu/gal	joules/cm^3	0.27
Btu/ft^3	kJ/m^3	36.5
Power		
horsepower (hp)	Btu/min	42.4
horsepower (hp)	horsepower (metric)	1.014
horsepower (hp)	kilowatts	0.746
kilowatts (kW)	horsepower	1.341
watts (W)	Btu/hour	3.41
watts (W)	joules/sec	1
Miscellaneous		
liter petrol	megajoules	35
kilogram oil	megajoules	43.2
barrel oil equivalent	gigajoules	6.1
ton coal equivalent	gigajoules	29.3
ton coal equivalent	barrels oil equivalent	4.8
pounds/acre	kilograms/hectare	1.1

A. Conversions

Speed

1 m/s	=	2.24 mph
1 mph	=	0.446 m/s
1 knot	=	1.15 mph
1 mph	=	0.870 knots

Length

1 meter	=	3.28 feet
1 foot	=	0.305 meters
1 kilometer	=	0.620 miles
1 mile	=	1.61 kilometers

Area

1 square kilometer	=	0.386 square miles
1 square kilometer	=	1,000,000 square meters
1 square kilometer	=	100 hectares
1 square mile	=	2.59 square kilometers
1 square foot	=	0.093 square meters
1 square meter	=	10.76 square feet
1 hectare	=	10,000 square meters
1 hectare	=	2.47 acres
1 acre	=	0.405 hectares
1 acre	=	4049 square meters

Volume

1 cubic meter	=	35.3 cubic feet
1 cubic feet	=	0.028 cubic meters
1 liter	=	0.264 gallons
1 gallon	=	3.78 liters
1 cubic meter	=	1000 liters
1 cubic meter	=	264 gallons
1 gallon	=	0.0038 cubic meters

Flow Rate

1 liter/second	=	0.0044 gallons/minute
1 gallon/minute	=	227 liters/second
1 cubic meter/minute	=	264 gallons/minute
1 gallon/minute	=	0.0038 cubic meters/minute

Weight

1 metric ton	=	1.10 tons
1 kilogram	=	2.20 pounds
1 pound	=	0.454 kilograms

Energy Equivalency of Common Fuels

1 kWh = 3413 BTU
= 3.41 ft^3 of natural gas
= 0.034 gallon of oil
= 0.00017 cord of wood

1 Therm = 10^5 BTU
= 100 ft^3 of natural gas
= 1 gallon of oil
= 29.3 kWh of electricity
= 0.005 cord of wood

1 gallon of oil = 1×10^5 BTU

1 cord of wood = 2×10^7 BTU

1000 ft^3 (Mcf) natural gas = 1×10^6 BTU

	J	int J	mkg	ft lb	int kWh	IT kcal	BTU	ft³ lb/in²	atm dm³	PSh	HPh	R · grd
1 J	1	$9,9981 \cdot 10^{-1}$	$1,0197 \cdot 10^{-1}$	$7,3756 \cdot 10^{-1}$	$2,7772 \cdot 10^{-7}$	$2,3884 \cdot 10^{-4}$	$9,4782 \cdot 10^{-4}$	$5,1220 \cdot 10^{-3}$	$9,8692 \cdot 10^{-3}$	$3,7767 \cdot 10^{-7}$	$3,7251 \cdot 10^{-7}$	$1,2027 \cdot 10^{-1}$
1 int J	1,0002	1	$1,0199 \cdot 10^{-1}$	$7,3770 \cdot 10^{-1}$	$2,7778 \cdot 10^{-7}$	$2,3889 \cdot 10^{-4}$	$9,4800 \cdot 10^{-4}$	$5,1229 \cdot 10^{-3}$	$9,8711 \cdot 10^{-3}$	$3,7774 \cdot 10^{-7}$	$3,7258 \cdot 10^{-7}$	$1,2029 \cdot 10^{-1}$
1 mkg	9,8066	9,8048	1	7,2330	$2,7236 \cdot 10^{-6}$	$2,3422 \cdot 10^{-3}$	$9,2949 \cdot 10^{-3}$	$5,0229 \cdot 10^{-2}$	$9,6784 \cdot 10^{-2}$	$3,7037 \cdot 10^{-6}$	$3,6530 \cdot 10^{-6}$	1,1794
1 ft lb	1,3558	1,3556	$1,3825 \cdot 10^{-1}$	1	$3,7655 \cdot 10^{-7}$	$3,2383 \cdot 10^{-4}$	$1,2851 \cdot 10^{-3}$	$6,9444 \cdot 10^{-3}$	$1,3381 \cdot 10^{-2}$	$5,1206 \cdot 10^{-7}$	$5,0505 \cdot 10^{-7}$	$1,6306 \cdot 10^{-1}$
1 int kWh	$3,6007 \cdot 10^{6}$	$3,6000 \cdot 10^{6}$	$3,6717 \cdot 10^{5}$	$2,6557 \cdot 10^{6}$	1	$8,6000 \cdot 10^{2}$	$3,4128 \cdot 10^{3}$	$1,8442 \cdot 10^{4}$	$3,5536 \cdot 10^{4}$	1,3599	1,3413	$4,3305 \cdot 10^{5}$
1 IT kcal	$4,1868 \cdot 10^{3}$	$4,1860 \cdot 10^{3}$	$4,2694 \cdot 10^{2}$	$3,0884 \cdot 10^{3}$	$1,1628 \cdot 10^{-3}$	1	3,9684	$2,1445 \cdot 10$	$4,1321 \cdot 10$	$1,5813 \cdot 10^{-3}$	$1,5596 \cdot 10^{-3}$	$5,0355 \cdot 10^{2}$
1 BTU	$1,0551 \cdot 10^{3}$	$1,0549 \cdot 10^{3}$	$1,0759 \cdot 10^{2}$	$7,7817 \cdot 10^{2}$	$2,9302 \cdot 10^{-4}$	$2,5199 \cdot 10^{-1}$	1	5,4040	$1,0413 \cdot 10$	$3,9847 \cdot 10^{-4}$	$3,9302 \cdot 10^{-4}$	$1,2689 \cdot 10^{2}$
1 ft³ lb/in²	$1,9524 \cdot 10^{2}$	$1,9520 \cdot 10^{2}$	$1,9909 \cdot 10$	$1,4400 \cdot 10^{2}$	$5,4223 \cdot 10^{-5}$	$4,6631 \cdot 10^{-2}$	$1,8505 \cdot 10^{-1}$	1	1,9268	$7,3736 \cdot 10^{-5}$	$7,2727 \cdot 10^{-5}$	$2,3481 \cdot 10$
1 atm dm³	$1,0132 \cdot 10^{2}$	$1,0131 \cdot 10^{2}$	$1,0332 \cdot 10$	$7,4734 \cdot 10$	$2,8140 \cdot 10^{-5}$	$2,4201 \cdot 10^{-2}$	$9,6038 \cdot 10^{-2}$	$5,1898 \cdot 10^{-1}$	1	$3,8268 \cdot 10^{-5}$	$3,7744 \cdot 10^{-5}$	$1,2186 \cdot 10$
1 PSh	$2,6478 \cdot 10^{6}$	$2,6473 \cdot 10^{6}$	$2,7000 \cdot 10^{5}$	$1,9529 \cdot 10^{6}$	$7,3536 \cdot 10^{-1}$	$6,3241 \cdot 10^{2}$	$2,5096 \cdot 10^{3}$	$1,3562 \cdot 10^{4}$	$2,6132 \cdot 10^{4}$	1	$9,8632 \cdot 10^{-1}$	$3,1845 \cdot 10^{5}$
1 HPh	$2,6845 \cdot 10^{6}$	$2,6840 \cdot 10^{6}$	$2,7374 \cdot 10^{5}$	$1,9800 \cdot 10^{6}$	$7,4556 \cdot 10^{-1}$	$6,4118 \cdot 10^{2}$	$2,5444 \cdot 10^{3}$	$1,3750 \cdot 10^{4}$	$2,6494 \cdot 10^{4}$	1,0139	1	$3,2287 \cdot 10^{5}$
1 R · grd	8,3147	8,3131	$8,4786 \cdot 10^{-1}$	6,1326	$2,3092 \cdot 10^{-6}$	$1,9859 \cdot 10^{-3}$	$7,8808 \cdot 10^{-3}$	$4,2588 \cdot 10^{-2}$	$8,2060 \cdot 10^{-2}$	$3,1402 \cdot 10^{-6}$	$3,0973 \cdot 10^{-6}$	1

	int kW	PS	HP
1 erg/sec	$9,9981 \cdot 10^{-11}$	$1,3596 \cdot 10^{-10}$	$1,3410 \cdot 10^{-10}$
1 int J/sec	$1,0000 \cdot 10^{-3}$	$1,3599 \cdot 10^{-3}$	$1,3413 \cdot 10^{-3}$
1 mkg/sec	$9,8048 \cdot 10^{-3}$	$1,3333 \cdot 10^{-2}$	$1,3151 \cdot 10^{-2}$
1 ft lb/sec	$1,3556 \cdot 10^{-3}$	$1,8434 \cdot 10^{-3}$	$1,8182 \cdot 10^{-3}$
1 int kW	1	1,3599	1,3413
1 IT kcal/sec	4,1860	5,6925	5,6147
1 BTU/sec	1,0549	1,4345	1,4149
1 ft³ lb/in²h	$5,4223 \cdot 10^{-5}$	$7,3736 \cdot 10^{-5}$	$7,2727 \cdot 10^{-5}$
1 atm dm³/h	$2,8140 \cdot 10^{-5}$	$3,8268 \cdot 10^{-5}$	$3,7744 \cdot 10^{-5}$
1 PS	$7,3536 \cdot 10^{-1}$	1	$9,8632 \cdot 10^{-1}$
1 HP	$7,4556 \cdot 10^{-1}$	1,0139	1
1 R grd/h	$2,3092 \cdot 10^{-6}$	$3,1402 \cdot 10^{-6}$	$3,0973 \cdot 10^{-6}$

Table D-9. Kilowatts to Horsepower Conversion Table

Kilowatts (kW) into Horsepower (HP)
1 kW = 1.3596 HP

kW	0	1	2	3	4	5	6	7	8	9	kW
0	0	1,3596	2,72	4,08	5,44	6,80	8,16	9,52	10,88	12,24	0
10	**13,60**	**14,96**	**16,32**	**17,67**	**19,03**	**20,39**	**21,75**	**23,11**	**24,47**	**25,83**	**10**
20	27.19	28,55	29,91	31,27	32,63	33,99	35,35	36,71	38,07	39,43	20
30	**40,79**	**42,15**	**43,51**	**44,87**	**46,23**	**47,59**	**48,95**	**50,31**	**51,66**	**53,02**	**30**
40	54,38	55,74	57,10	58,46	59,82	61,18	62,54	63,90	65,26	66,62	40
50	**67,98**	**69,34**	**70,70**	**72,06**	**73,42**	**74,78**	**76,14**	**77,50**	**78,86**	**80,22**	**50**
60	81,58	82,94	84,30	85,65	87,01	88,37	89,73	91,09	92,45	93,81	60
70	**95,17**	**96,53**	**97,89**	**99,25**	**100,61**	**101,97**	**103,33**	**104,69**	**106,05**	**107,41**	**70**
80	108.77	110,13	111,49	112,85	114,21	115,57	116,93	118,29	119,64	121,00	80
90	**122,36**	**123,72**	**125,08**	**126,44**	**127,80**	**129,16**	**130,52**	**131,88**	**133,24**	**134,60**	**90**
100	135,96	137,32	138,68	140,04	141,40	142,76	144,12	145,48	146,84	148,20	100

Horsepower (HP) into Kilowatts
1 HP = 0.7355 kW

HP	0	1	2	3	4	5	6	7	8	9	HP
0	0	0,7355	1,47	2,21	2,94	3,68	4,41	5,15	5,88	6,62	0
10	**7,36**	**8,09**	**8,83**	**9,56**	**10,30**	**11,03**	**11,77**	**12,50**	**13,24**	**13,97**	**10**
20	14,71	15,45	16,18	16,92	17,65	18,39	19,12	19,86	20,59	21,33	20
30	**22,07**	**22,80**	**23,54**	**24,27**	**25,01**	**25,74**	**26,48**	**27,21**	**27,95**	**28,68**	**30**
40	29,42	30,16	30,89	31,63	32,36	33,10	33,83	34,57	35,30	36,04	40
50	**36,78**	**37,51**	**38,25**	**38,98**	**39,72**	**40,45**	**41,19**	**41,92**	**42,66**	**43,39**	**50**
60	44,13	44,87	45,60	46,34	47,07	47,81	48,54	49,28	50,01	50,75	60
70	**51,49**	**52,22**	**52,96**	**53,69**	**54,43**	**55,16**	**55,90**	**56,63**	**57,37**	**58,10**	**70**
80	58,84	59,58	60,31	61,05	61,78	62,52	63,25	63,99	64,72	65,46	80
90	**66,20**	**66,93**	**67,67**	**68,40**	**69,14**	**69,87**	**70,61**	**71,34**	**72,08**	**72,81**	**90**
100	73,55	74,29	75,02	75,76	76,49	77,23	77,96	78,70	79,43	80,17	100

This table is very simple to use. Read off the tens in the vertical scale and the units in the horizontal scale. The answer is where the two lines meet.

Example (above): convert 63 kW into HP. Go down the first column until you find 60 and the across until you come to 3. The result is where the two rows join.
63 kW = 85.65 HP.
For values of over 100, the decimal point should be adjusted.

Example (below): Convert 65 HP into kW. Go down the first column until you find 60 and then across until you come to 3. The result is where the two rows join.
63 HP = 46.34 kW.
For values of over 100, the decimal point should be adjusted.

USEFUL ENERGY CONVERSION FACTORS

For example: 1 BTU has the energy of .00029 Kilo-watt-hours

One of these ↓ / Has the energy of: →	BTU	K-watt hour	Kilo-Calorie	Horse-power hour	Gallon of gasoline
BTU (British Thermal Unit)	1	.00029	.252	.00039	7.4x10^{-6}
Kilo-watt-hour	3412	1	859	1.34	.025
Kilo-calorie	3.97	.0016	1	.00156	2.5x10^{-5}
Horse-power hour	2544	.745	640	1	.0188
Gallon of gasoline	135,000	39.5	34,000	53.0	1
One pound raised one foot (one foot-pound)	.000128	3.7x10^{-7}	.00032	5.0x10^{-7}	9.5x10^{-9}
Joule (a metric unit)	.000948	2.8x10^{-7}	.00024	9.3x10^{-8}	7.0x10^{-9}
Energy collected by 40 sq. ft. S-rotor in 15 mph wind for one hour (see prob. 1.1 on page 12)	282	.0828	71.0	.110	.0021
Energy converted to electricity in line above (see prob. 1.2 on page 12)	127	.0374	32.0	.050	.00094
One gallon of water lifted 100 feet	1.06	.00031	.267	.00041	7.8x10^{-6}
Solar energy hitting one sq. ft. for one hour*	442	.129	111	.173	.0033
One gallon of water heated from 70 F to 212 F	1184	.347	298	.465	.0088
One gallon of water at 212 F boiled away	8078	2.36	2034	3.17	.060
One cu. ft. natural gas	1000	.29	252	.39	.0074
One gallon ethyl alcohol	84,000	24.6	21,100	33.0	.622
One cord of wood (average)	12,000,000	3,660	3,150,000	4,910	92.5
One ton of coal	25,000,000	7,327	6,300,000	9,830	185
One ton vehicle moving at 55 mph	257	.075	64.7	.101	.00257

WARNING: These conversion factors refer to the total energy available and not to useful work. There will always be losses in converting one form of energy into work. That is part of the Second Law of Thermodynamics. That is also why you can't expect one gallon of boiling water to move a car at over 55 mph!

*This is the solar energy available above the earth's atmosphere. There is considerable loss in going through the air.

Note on scientific notation: The superscript above the 10 refers to where the decimal point goes. For example, 7.4x10^{-6} = 0.0000074 and 2.5x10^{-5} = 0.000025

Conversion Tables

1. Conversion Factors

To Convert From	To	Multiply By
Btu	Gram calories	251.9958
Btu	Kilogram calories	.00397
Btu	Cubic centimeters atmospheres	10405.6
Btu	Cubic foot atmospheres	.36747
Btu	Foot pounds	777.649
Btu	Horsepower hours	.0003927
Btu	Kilowatt hours	.00029287
Btu/square foot	Langleys	.271
Btu/hour/square foot/°F	Watts/CM^2/°C	5.6820×10^4
Cubic foot atmospheres	Btu	2.721
Cubic feet of water	Gallons	7.4805
Cubic feet of water	Pounds	62.366
Foot pounds	Btu	.001285
Gallons of water	Cubic feet	0.13368
Gallons of water	Pounds	8.3453
Gram calories	Btu	.00397
Horsepower	Foot pounds/hour	1,980,000.
Horsepower	Foot pounds/minute	33,000.
Horsepower	Foot pounds/second	550.
Horsepower	Kilowatts	.7457
Horsepower	Watts	745.7
Horsepower hours	Btu	2546.14

To Convert From	To	Multiply By
Horsepower years	Btu	22,304,186.4
Kilogram calories	Btu	3.97
Kilowatts	Horsepower	1.34102
Kilowatt hours	Btu	3414.43
Langleys	Btu/square foot	3.69
Lumens (at 5,550 Å)	Watts	0.0014706
Months (mean calendar)	Hours	730.1
Pints (U.S., liq)	Cubic centimeters	473.18
Pints	Cubic inches	28.875
Pounds of water	Cubic feet of water	0.01602
Pounds of water	Gallons (U.S., liq)	0.1198
Watts	Btu/hour	3.4144
Watts	Btu/minute	0.05691
Watts	Calories/minute	14.34
Watts	Horsepower	0.001341
Watts/square centimeter	Btu/square feet/hour	3,172.
Watt-hours	Btu	3.4144
Watt-hours	Calories	860.4
Watt-hours	Horsepower hours	0.001341

2. Fahrenheit-Centigrade Conversion Table

The numbers in the center column, in boldface type, refer to the temperature in either Fahrenheit or Centigrade degrees. If it is desired to convert from Fahrenheit to Centigrade degrees, consider the center column as a table of Fahrenheit temperatures and read the corresponding Centigrade temperature in the column at the left. If it is desired to convert from Centigrade to Fahrenheit degrees, consider the center column as a table of Centigrade values, and read the corresponding Fahrenheit temperature on the right.

SOURCE: Clifford Strock and Richard L. Koral, eds., *Handbook of Air Conditioning, Heating, and Ventilating*, 2d ed. (New York: Industrial Press, 1965).

For conversions not covered in the table, the following formulas are used:

$F = 1.8\ C + 32 \qquad C = (F - 32) \div 1.8$

Deg C		Deg F	Deg C		Deg F
–46	**– 50**	– 58	8.9	**48**	118.4
–40	**– 40**	– 40	9.4	**49**	120.2
–34	**– 30**	– 22	10.0	**50**	122.0
–29	**– 20**	– 4	10.6	**51**	123.8
–23	**– 10**	14	11.1	**52**	125.6
–17.8	**0**	32–	11.7	**53**	127.4
–17.2	**1**	33.8	12.2	**54**	129.2
–16.7	**2**	35.6	12.8	**55**	131.0
–16.1	**3**	37.4	13.3	**56**	132.8
–15.6	**4**	39.2	13.9	**57**	134.6
–15.0	**5**	41.0	14.4	**58**	136.4
–14.4	**6**	42.8	15.0	**59**	138.2
–13.9	**7**	44.6	15.6	**60**	140.0
–13.3	**8**	46.4	16.1	**61**	141.8
–12.8	**9**	48.2	16.7	**62**	143.6
–12.2	**10**	50.0	17.2	**63**	145.4
–11.7	**11**	51.8	17.8	**64**	147.2
–11.1	**12**	53.6	18.3	**65**	149.0
–10.6	**13**	55.4	18.9	**66**	150.8
–10.0	**14**	57.2	19.4	**67**	152.6
– 9.4	**15**	59.0	20.0	**68**	154.4
– 8.9	**16**	60.8	20.6	**69**	156.2
– 8.3	**17**	62.6	21.1	**70**	158.0
– 7.8	**18**	64.4	21.7	**71**	159.8
– 7.2	**19**	66.2	22.2	**72**	161.6
– 6.7	**20**	68.0	22.8	**73**	163.4
– 6.1	**21**	69.8	23.3	**74**	165.2
– 5.6	**22**	71.6	23.9	**75**	167.0
– 5.0	**23**	73.4	24.4	**76**	168.8
– 4.4	**24**	75.2	25.0	**77**	170.6
– 3.9	**25**	77.0	25.6	**78**	172.4
– 3.3	**26**	78.8	26.1	**79**	174.2
– 2.8	**27**	80.6	26.7	**80**	176.0
– 2.2	**28**	82.4	27.2	**81**	177.8
– 1.7	**29**	84.2	27.8	**82**	179.6
– 1.1	**30**	86.0	28.3	**83**	181.4
– 0.6	**31**	87.8	28.9	**84**	183.2
0–	**32**	89.6	29.4	**85**	185.0
0.6	**33**	91.4	30.0	**86**	186.8
1.1	**34**	93.2	30.6	**87**	188.6
1.7	**35**	95.0	31.1	**88**	190.4
2.2	**36**	96.8	31.7	**89**	192.2
2.7	**37**	98.6	32.2	**90**	194.0
3.3	**38**	100.4	32.8	**91**	195.8
3.9	**39**	102.2	33.3	**92**	197.6
4.4	**40**	104.0	33.9	**93**	199.4
5.0	**41**	105.8	34.4	**94**	201.2
5.6	**42**	107.6	35.0	**95**	203.0
6.1	**43**	109.4	35.6	**96**	204.8
6.7	**44**	111.2	36.1	**97**	206.6
7.2	**45**	113.0	36.7	**98**	208.4
7.8	**46**	114.8	37.2	**99**	210.2
8.3	**47**	116.6	37.8	**100**	212.0

Wind Chill Temperature Factor

Knots	Meters/Sec.	Air Temperature											
0.0	0.0	25	20	15	10	5	0	-5	-10	-15	-20	-25	-30
		Felt Temperature											
5.0	2.5	25	19	14	9	3	-2	-7	-12	-17	-23	-28	-33
9.0	4.5	22	16	10	4	-2	-8	-14	-20	-26	-32	-36	-44
13.0	7.0	21	15	8	2	-4	-11	-18	-25	-32	-38	-45	-52
19.0	9.5	21	14	7	0	-7	-14	-21	-29	-36	-43	-50	-57
24.0	12.5	21	13	5	-2	-9	-17	-24	-32	-39	-47	-54	-61
30.0	15.5 #	20	13	5	-3	-11	-18	-26	-34	-42	-49	-57	-65
37.0	19.0	20	12	4	-4	-11	-19	-27	-35	-43	-51	-59	-68
44.0	22.5	20	12	4	-4	-12	-20	-28	-36	-44	-52	-60	-69

Notes:

Upper limit of safe crane operation.

- 21 degrees is the temperature for frozen meat in cold storage

98 % konfidens interval = 2 % signifikans niveau

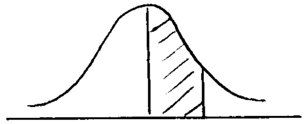


Table C Areas under the normal curve

Fractional parts of the total area (10,000) under the normal curve, corresponding to distances between the mean and ordinates which are Z standard-deviation units from the mean.

Z	.00	.01	.02	03	04	.05	06	.07	.08	.09
0.0	0000	0040	0080	0120	0159	0199	0239	0279	0319	0359
0.1	0398	0438	0478	0517	0557	0596	0636	0675	0714	0753
0.2	0793	0832	0871	0910	0948	0987	1026	1064	1103	1141
0.3	1179	1217	1255	1293	1331	1368	1406	1443	1480	1517
0.4	1554	1591	1628	1664	1700	1736	1772	1808	1844	1879
0.5	1915	1950	1985	2019	2054	2088	2123	2157	2190	2224
0.6	2257	2291	2324	2357	2389	2422	2454	2486	2518	2549
0.7	2580	2612	2642	2673	2704	2734	2764	2794	2823	2852
0.8	2881	2910	2939	2967	2995	3023	3051	3078	3106	3133
0.9	3159	3186	3212	3238	3264	3289	3315	3340	3365	3389
1.0	3413	3438	3461	3485	3508	3531	3554	3577	3599	3621
1.1	3643	3665	3686	3718	3729	3749	3770	3790	3810	3830
1.2	3849	3869	3888	3907	3925	3944	3962	3980	3997	4015
1.3	4032	4049	4066	4083	4099	4115	4131	4147	4162	4177
1.4	4192	4207	4222	4236	4251	4265	4279	4292	4306	4319
1.5	4332	4345	4357	4370	4382	4394	4406	4418	4430	4441
1.6	4452	4463	4474	4485	4495	4505	4515	4525	4535	4545
1.7	4554	4564	4573	4582	4591	4599	4608	4616	4625	4633
1.8	4641	4649	4656	4664	4671	4678	4686	4693	4699	4706
1.9	4713	4719	4726	4732	4738	4744	4750	4758	4762	4767
2.0	4773	4778	4783	4788	4793	4798	4803	4808	4812	4817
2.1	4821	4826	4830	4834	4838	4842	4846	4850	4854	4857
2.2	4861	4865	4868	4871	4875	4878	4881	4884	4887	4890
2.3	4893	4896	4898	4901	4904	4906	4909	4911	4913	4916
2.4	4918	4920	4922	4925	4927	4929	4931	4932	4934	4936
2.5	4938	4940	4941	4943	4945	4946	4948	4949	4951	4952
2.6	4953	4955	4956	4957	4959	4960	4961	4962	4963	4964
2.7	4965	4966	4967	4968	4969	4970	4971	4972	4973	4974
2.8	4974	4975	4976	4977	4977	4978	4979	4980	4980	4981
2.9	4981	4982	4983	4984	4984	4984	4985	4985	4986	4986
3.0	4986.5	4987	4987	4988	4988	4988	4989	4989	4989	4990
3.1	4990.0	4991	4991	4991	4992	4992	4992	4992	4993	4993
3.2	4993.129									
3.3	4995.166									
3.4	4996.631									
3.5	4997.674									
3.6	4998.409									
3.7	4998.922									
3.8	4999.277									
3.9	4999.519									
4.0	4999.683									
4.5	4999.966									
5.0	4999.997133									

% 9,87% interval

SOURCE: Harold O. Rugg, *Statistical Methods Applied to Education*, Houghton Mifflin Company, Boston, 1917, appendix table III, pp. 389–390, with the kind permission of the publisher.

$$Z = \frac{\bar{x} - x}{\sigma x}$$

Table D Distribution of ***t***

df	Level of significance for one-tailed test					
	.10	.05	.025	.01	.005	.0005
	Level of significance for two-tailed test					
	.20	.10	.05	.02	.01	.001
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.598
3	1.638	2.353	3.182	4.541	5.841	12.941
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.859
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.405
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.767
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646
40	1.303	1.684	2.021	2.423	2.704	3.551
60	1.296	1.671	2.000	2.390	2.660	3.460
120	1.289	1.658	1.980	2.358	2.617	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.291

SOURCE: Table D is abridged from Table III of R. A. Fisher and F. Yates, *Statistical Tables for Biological, Agricultural and Medical Research* (1948 ed.), published by Oliver & Boyd, Ltd., Edinburgh and London, by permission of the authors and publishers.

"To good to
← be true"

Afviger fra
→ Teori

Table I Distribution of χ^2

(Sandsynlighed)

Probability 10% 5% 1%

df	.99	.98	.95	.90	.80	.70	.50	.30	.20	.10	.05	.02	.01	.001
1	$.0^3157$	$.0^3628$	.00393	.0158	.0642	.148	.455	1.074	1.642	2.706	3.841	5.412	6.635	10.827
2	.0201	.0404	.103	.211	.446	.713	1.386	2.408	3.219	4.605	5.991	7.824	9.210	13.815
3	.115	.185	.352	.584	1.005	1.424	2.366	3.665	4.642	6.251	7.815	9.837	11.341	16.268
4	.297	.429	.711	1.064	1.649	2.195	3.357	4.878	5.989	7.779	9.488	11.668	13.277	18.465
5	.554	.752	1.145	1.610	2.343	3.000	4.351	6.064	7.289	9.236	11.070	13.388	15.086	20.517
6	.872	1.134	1.635	2.204	3.070	3.828	5.348	7.231	8.558	10.645	12.592	15.033	16.812	22.457
7	1.239	1.564	2.167	2.833	3.822	4.671	6.346	8.383	9.803	12.017	14.067	16.622	18.475	24.322
8	1.646	2.032	2.733	3.490	4.594	5.527	7.344	9.524	11.030	13.362	15.507	18.168	20.090	26.125
9	2.088	2.532	3.325	4.168	5.380	6.393	8.343	10.656	12.242	14.684	16.919	19.679	21.666	27.877
10	2.558	3.059	3.940	4.865	6.179	7.267	9.342	11.781	13.442	15.987	18.307	21.161	23.209	29.588
11	3.053	3.609	4.575	5.578	6.989	8.148	10.341	12.899	14.631	17.275	19.675	22.618	24.725	31.264
12	3.571	4.178	5.226	6.304	7.807	9.034	11.340	14.011	15.812	18.549	21.026	24.054	26.217	32.909
13	4.107	4.765	5.892	7.042	8.634	9.926	12.340	15.119	16.985	19.812	22.362	25.472	27.688	34.528
14	4.660	5.368	6.571	7.790	9.467	10.821	13.339	16.222	18.151	21.064	23.685	26.873	29.141	36.123
15	5.229	5.985	7.261	8.547	10.307	11.721	14.339	17.322	19.311	22.307	24.996	28.259	30.578	37.697
16	5.812	6.614	7.962	9.312	11.152	12.624	15.338	18.418	20.465	23.542	26.296	29.633	32.000	39.252
17	6.408	7.255	8.672	10.085	12.002	13.531	16.338	19.511	21.615	24.769	27.587	30.995	33.409	40.790
18	7.015	7.906	9.390	10.865	12.857	14.440	17.338	20.601	22.760	25.989	28.869	32.346	34.805	42.312
19	7.633	8.567	10.117	11.651	13.716	15.352	18.338	21.689	23.900	27.204	30.144	33.687	36.191	43.820
20	8.260	9.237	10.851	12.443	14.578	16.266	19.337	22.775	25.038	28.412	31.410	35.020	37.566	45.315
21	8.897	9.915	11.591	13.240	15.445	17.182	20.337	23.858	26.171	29.615	32.671	36.343	38.932	46.797
22	9.542	10.600	12.338	14.041	16.314	18.101	21.337	24.939	27.301	30.813	33.924	37.659	40.289	48.268
23	10.196	11.293	13.091	14.848	17.187	19.021	22.337	26.018	28.429	32.007	35.172	38.968	41.638	49.728
24	10.856	11.992	13.848	15.659	18.062	19.943	23.337	27.096	29.553	33.196	36.415	40.270	42.980	51.179
25	11.524	12.697	14.611	16.473	18.940	20.867	24.337	28.172	30.675	34.382	37.652	41.566	44.314	52.620
26	12.198	13.409	15.379	17.292	19.820	21.792	25.336	29.246	31.795	35.563	38.885	42.856	45.642	54.052
27	12.879	14.125	16.151	18.114	20.703	22.719	26.336	30.319	32.912	36.741	40.113	44.140	46.963	55.476
28	13.565	14.847	16.928	18.939	21.588	23.647	27.336	31.391	34.027	37.916	41.337	45.419	48.278	56.893
29	14.256	15.574	17.708	19.768	22.475	24.577	28.336	32.461	35.139	39.087	42.557	46.693	49.588	58.302
30	14.953	16.306	18.493	20.599	23.364	25.508	29.336	33.530	36.250	40.256	43.773	47.962	50.892	59.703

For larger values of df, the expression $\sqrt{2\chi^2} - \sqrt{2\text{df} - 1}$ may be used as a normal deviate with unit variance, remembering that the probability for χ^2 corresponds with that of a single tail of the normal curve.

SOURCE: Table I is reprinted from Table IV of R. A. Fisher and F. Yates, *Statistical Tables for Biological, Agricultural and Medical Research* (1948 ed.), published by Oliver & Boyd Ltd.,. Edinburgh and London, by permission of the authors and publishers.

	10	5	1
40	51,81	55,76	63,69
60	74,40	79,08	88,38
90	107,6	113,1	124,1
120	140,2	146,6	154,0

Some Handy Rules

Diameter of a circle × 3.1416 = Circumference.

Radius of a circle × 6.283185 = Circumference.

Square of the radius of a circle × 3.1416 = Area.

Square of the diameter of a circle × 0.7854 = Area.

Square of the circumference of a circle × 0.07958 = Area.

Half the circumference of a circle × by half its diameter = Area.

Circumference of a circle × 0.159155 = Radius.

Sqare root of the area of a circle × 0.56419 = Radius.

Circumference of a circle × 0.31831 = Diameter.

Square root a the area of a circle × 1.12838 = Diameter.

Diameter of a circle × 0.86 = Side of inscribed equilateral triangle.

Diameter of a circle × 0.7071 = Side of an inscribed square.

Circumference of a circle × 0.225 = Side of an inscribed square.

Circumference of a circle × 0.282 = Side of an equal square.

Diameter of a circle × 0.8862 = Side of an equal square.

Base of a triangle × by ½ the altitude = Area.

Multiplying both diameters and .7854 together = Area of an ellipse.

Surface of a sphere × by ⅙ of its diameter = Solidity.

Circumference of a sphere × by its diameter = Surface.

Square of the diameter of a sphere × 3.1416 = Surface.

Square of the circumference of a sphere × 0.3183 = Surface.

Cube of the diameter of a sphere × 0.5236 = Solidity.

Cube of the radius of a sphere × 4.1888 = Solidity.

Cube of the circumference of a sphere × 0.016887 = Solidity.

Square root of the surface of a sphere × 0.56419 = Diameter.

Square root of the surface of a sphere × 1.772454 = Circumference.

Cube root of the solidity of a sphere × 1.2407 = Diameter.

Cube root of the solidity of a sphere × 3.8978 = Circumference.

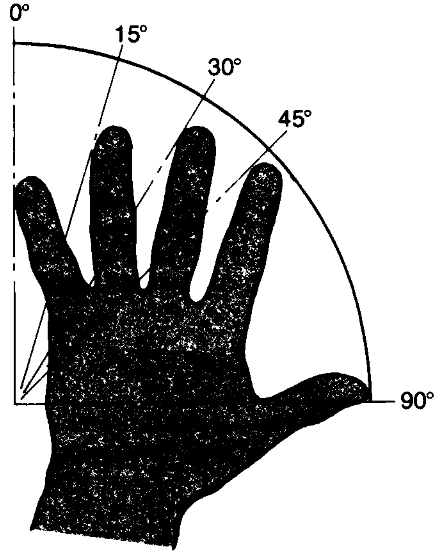
Radius of a sphere × 1.1547 = Side of inscribed cube.

Square root of (⅓ of the square of) the diameter of a sphere = Side of inscribed cube.

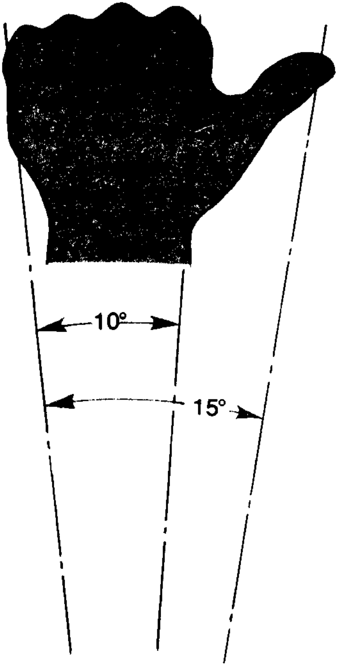
Area of its base × by ⅓ of its altitude = Solidity of a cone or pyramid, whether round, square or triangular.

Area of one of its sides × 6 = the surface of a cube.

Altitude of trapezoid × ½ the sum of it parallel sides = Area.



■ **100** *Natural hand angles*

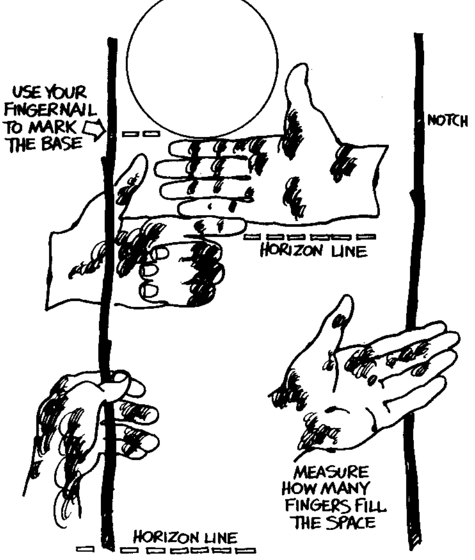


■ **101** *Natural hand angles*

Face the setting sun with arms extended and measure the distance between Ole Sol's base and the horizon in fingers (no thumbs). Each finger counts as fifteen minutes. Four of them, then, equal one hour . . . four on one hand and two on the other add up to an hour and a half . . . and so on.

If the space to be measured is greater than all eight fingers, hold a stick at arm's length with its base on the horizon and mark the position of the sun's bottom edge with your fingernail. The number of fingers it takes to fill the distance is your time.

In deep woods or hilly country where it's difficult to judge the horizon, work from an imaginary line at eye level.



Angles

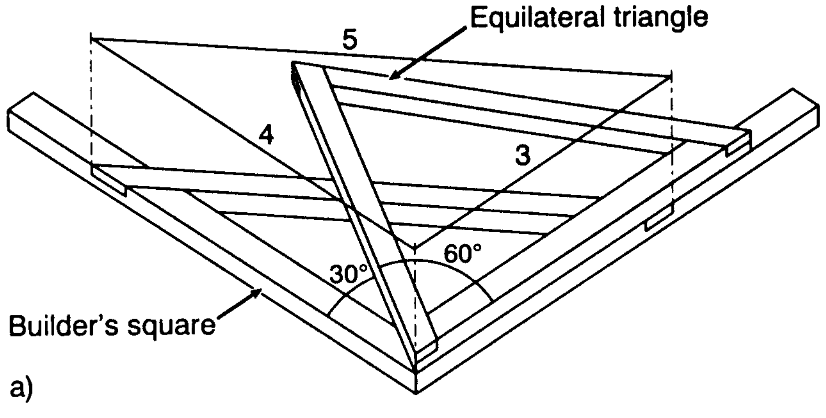
Set out a right-angle using a 3-4-5 triangle. This can be done in several ways:

- In the form of a builder's square – make a 'builder's square' from accurately cut and fixed pieces of planed wood (Figure A23.6a) or simply cut from a sheet of plywood.
- Peg out both ends of one side (3m) of the triangle. Attach a string (of 4m length) to one peg and another string (of 5m length) to the other peg. Stretch out the strings to meet each other and a right angle will be formed at the corner (Figure A23.6b).
- An alternative to string is to wrap the measuring tape itself around the pegs and position to the correct lengths.

An equilateral triangle will give an angle of 60° and, when used together with a builder's square, an angle of 30°, and multiples thereof (Figure A23.6a).

Squareness

Check the squareness of a rectangle by measuring the length of the diagonals. They should be equal.



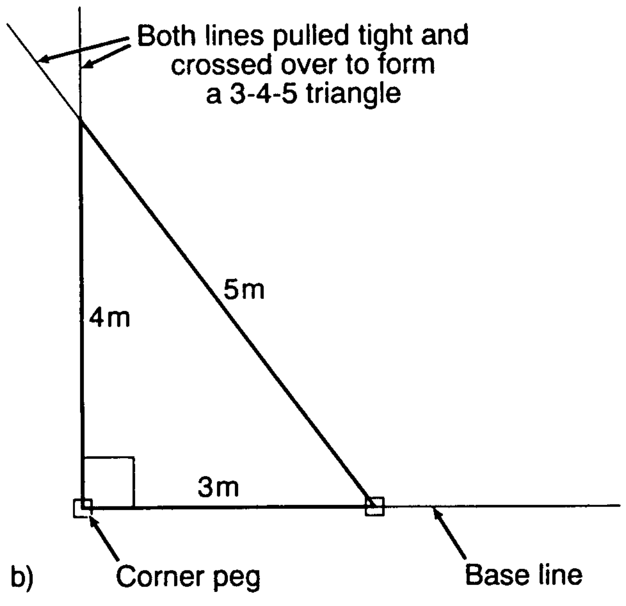
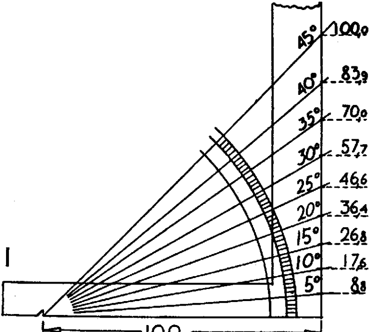


Figure A23.6 Setting out angles

1



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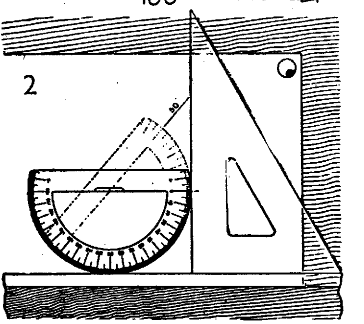


It is not necessary to use a manufactured level when one can be easily made from scrap materials. Variations of the following device have been used for the past 4000 years in the Near Eastern area. It can be made with three sticks, five nails, a stone, a wire, and a piece of string. Even this list can be reduced by substituting lashings and pegs for the nails. The gadget is basically an isosceles triangle, with the base upward. If a plumb bob is hung from the midpoint of the upper side, then this side will be level when the plumb line crosses the bottom point of the triangle. The triangle does not have to be precisely constructed because an approximation will suffice for the frame. Two nails are driven part way into the face of the frame, one at each upper corner. Then, using the wire (a string would be too stretchy) and a nail as a compass, scribe two arcs of equal radius using these corner nails as the centers. These arcs should intersect on the triangular frame near the bottom corner. This point is marked. By a series of trial-and-error arcs of smaller radii, find the midpoint of the upper side. Drive a nail part way into the frame here. From this midpoint you suspend the stone (or other weight) on the string. When using this leveling frame, the surveyor sights across the upper corner nails and his assistant watches the device from the side to see that the plumb line hangs across the lower mark, that the device is being held level. Possibly some gadgeteer can devise a system of mirrors so he can watch the plumb line himself while taking the horizontal sight. (See Fig. 19.)

The same device can be used as a simple level, not as a sighting device, when it is turned with the point up instead of down. Some adjustment will have to be made to the plumb bob so that it doesn't strike the ground. Otherwise the principle and operation remain the same. (See Fig. 20.)

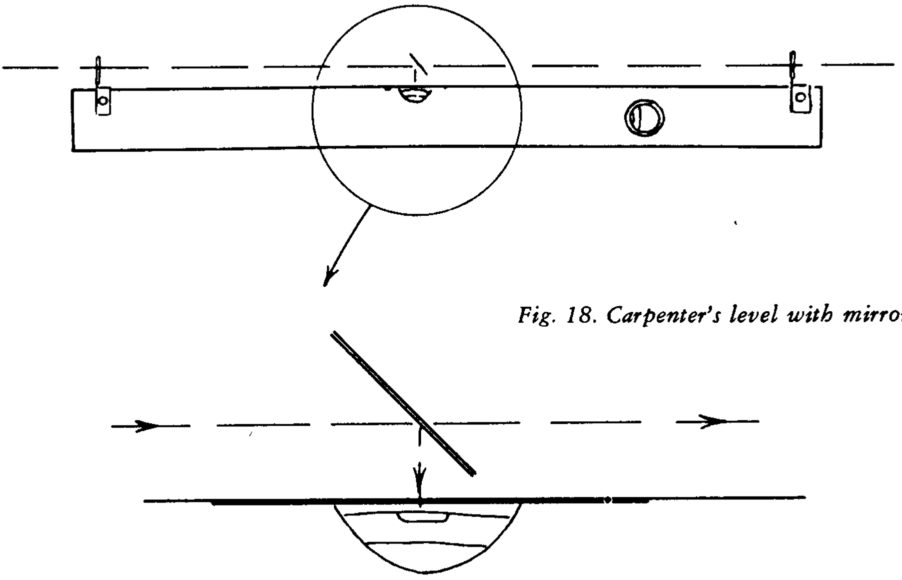


Fig. 18. Carpenter's level with mirror

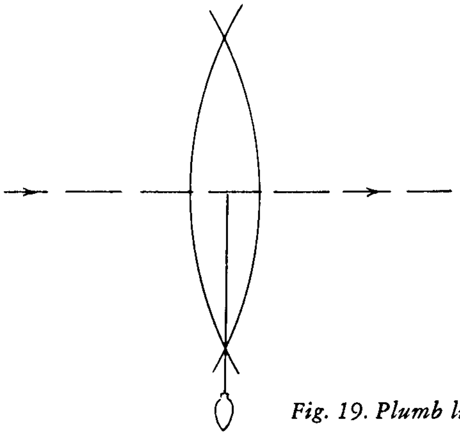


Fig. 19. Plumb line sighting level

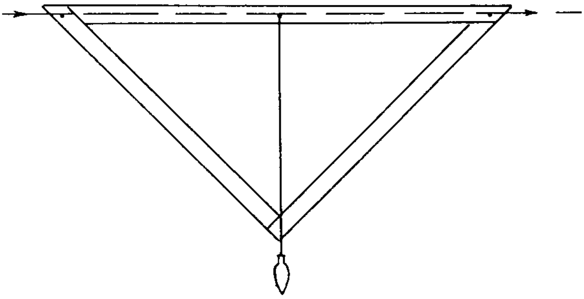
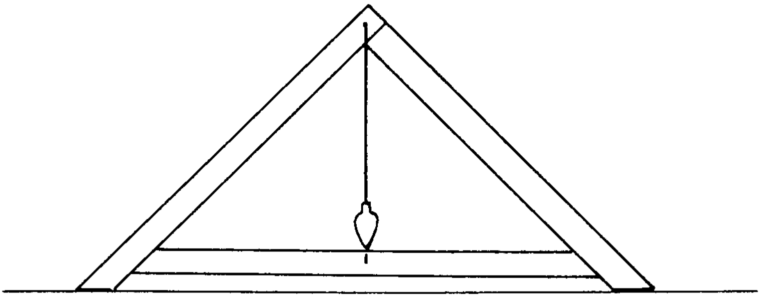
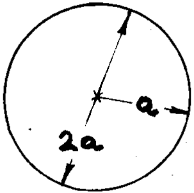


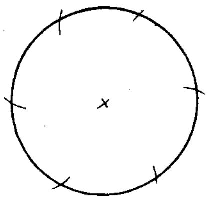
Fig. 20. Plumb line surface level



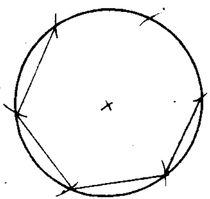
How to Draw a Hexagon:



1 DECIDE ON THE SIZE YOU WANT & SET COMPASS TO THE LENGTH of THE SIDE of HEXAGON (a). THIS WILL BE THE RADIUS of THE CIRCLE & ALSO ½ THE CIRCLE'S DIAMETER. DRAW CIRCLE.

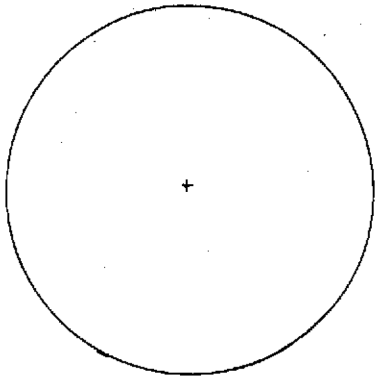


2 WITH THE COMPASS STILL SET TO THE RADIUS (a), MARK OFF THIS RADIUS 6 TIMES AROUND THE CIRCLE'S EDGE.



3 CONNECT THE 6 POINTS YOU HAVE MARKED OFF WITH STRAIGHT LINES. YOU NOW HAVE A HEXAGON, ALL of ITS SIDES ARE (a) IN LENGTH.

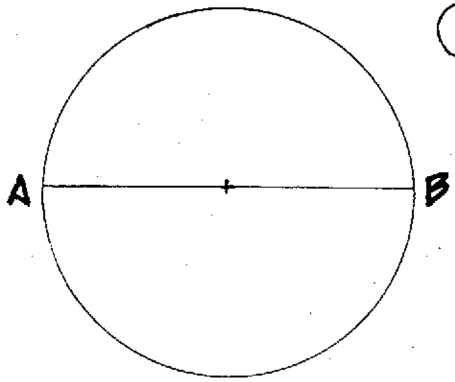
How to Draw a Pentagon:



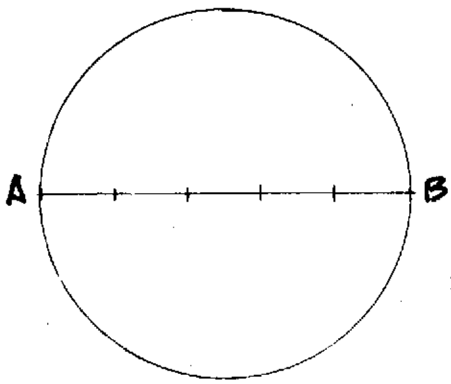
(1)

USE COMPASS TO DRAW A CIRCLE. THE PENTAGON WILL FALL WITHIN THE DIAMETER.

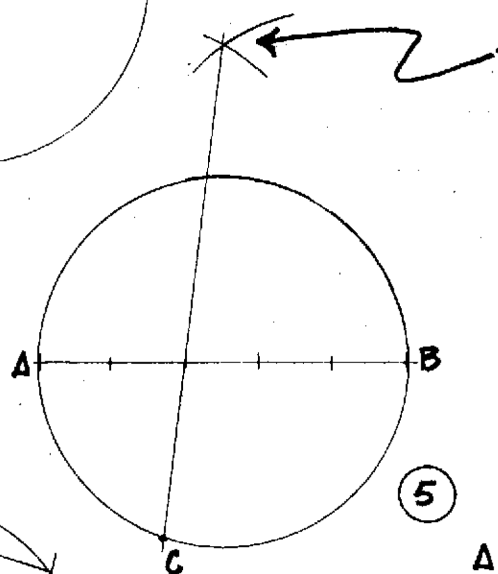
(2) DRAW THE DIAMETER HORIZONTALLY THROUGH THE CENTER: A-B



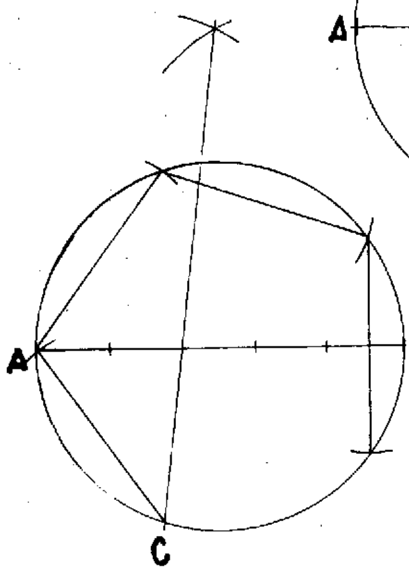
(3) DIVIDE A-B INTO FIVE EQUAL PARTS.



(4) SET THE COMPASS TO THE LENGTH of A-B. SCRIBE TWO ARCS, ONE FROM "A" & ONE FROM "B" TILL THEY INTERSECT ABOVE THE CIRCLE.
NOW DRAW A LINE FROM THIS INTERSECTION THROUGH THE <u>SECOND</u> DIVISION AND TO THE OPPOSITE RIM of CIRCLE: POINT "C".



(5)



A LINE FROM "C" TO "A" FORMS ONE SIDE of THE PENTAGON. MEASURE THIS LINE WITH THE COMPASS AND MARK THE RIM of THE CIRCLE ALL THE WAY AROUND. CONNECT ALL THE POINTS AND THEY FORM A PENTAGON.

MENSURATION. That part of mathematics which treats of the measurements of geometrical figures is called *mensuration;* it deals with the lengths of lines, the areas of surfaces, and the volumes of solids.

Lines. — A line is produced by the movement of a point. A *straight* line is one which has the same direction

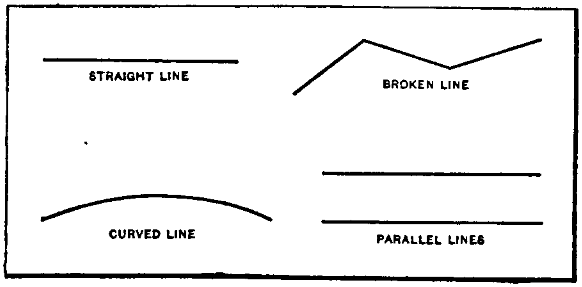
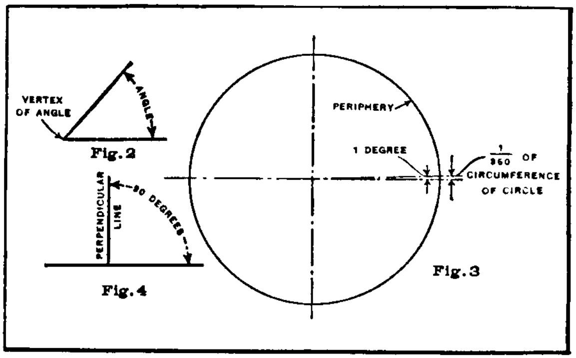


Fig. 1. Lines

throughout its length; it is also defined as the shortest distance between two points. A *broken* line is one which is made up of several straight lines, joined together, but having different directions. A *curved* line is one that constantly changes its direction; it is generally known as a *curve.* If two lines are drawn so that they are at all points at an equal distance from each other, they are said to be *parallel.* Parallel lines do not meet or intersect, no matter how far they may be extended or "produced" in either direction. (To extend a line in geometry is often called to *produce* the line.) The length of lines is measured in units of length, as inches, feet, miles, etc.

Surfaces. — A surface may be conceived of as produced by the movement of a line. A surface has two dimensions, length and breadth. The area of a surface is measured in square measure, as square inches, square



Figs. 2 to 4. Angular Measurements

feet, etc. A *plane* surface is one that is perfectly flat. A *curved* surface is one that has no plane or flat portion.

Angles. — When two lines meet as shown in Fig. 2, they form an angle with each other. The point where the two lines meet or intersect is called the *vertex* of the angle. The two lines forming the angle are called the sides of the angle. Angles are measured in degrees and subdivisions of a degree. If the circumference (periphery) of a circle is divided into 360 parts, each part is called one degree, and the angle between two lines from the center to the ends of

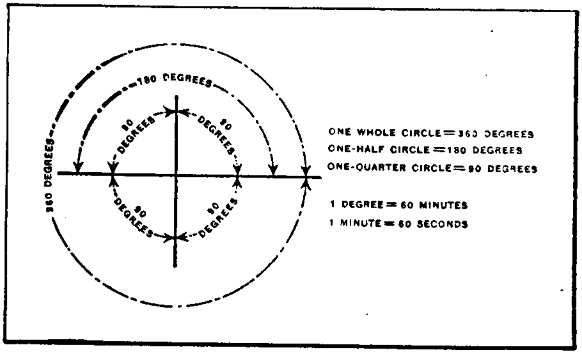


Fig. 5. Angular Measurements relative to the Circle

the small part of the circle is a one-degree angle, as shown in Fig. 3. As the whole circle contains 360 degrees, one-half of a circle contains 180 degrees, and one-quarter of a circle, 90 degrees.

A 90-degree angle is called a *right* angle. An angle larger than 90 degrees is called an *obtuse* angle, and an angle less than 90 degrees is called an *acute* angle. (See Figs. 6, 7, and 8.) Any angle which is not a right angle is called an *oblique* angle. When two lines form a right or 90-degree angle with each other, as shown in Fig. 4, one line is said to be *perpendicular* to the other.

Angles are said to be equal when they contain the same number of degrees. The angle shown in Fig. 9 and the angle in Fig. 10 are equal, because they are both 60 degrees; that the sides of the angle in Fig. 10 are longer than the sides of the angle in Fig. 9 has no influence on the angle, because of the fact that an angle is only the *difference in direction* of two lines. The angle shown in Fig. 12, which contains only 30 degrees, is only one-half of the angle in Fig. 9. One-half of a right angle is 45 degrees, as shown in Fig. 11. In Fig. 13 is shown an angle which is 120 degrees, and which can be divided into a right or 90-degree angle, and a 30-degree angle.

In order to obtain finer subdivisions for the measurement of angles than the degree, one degree is divided into 60 minutes, and one minute into 60 seconds. Any part of a degree can be expressed in minutes and seconds, for instance, $\frac{1}{2}$ degree = 30 minutes, $\frac{1}{3}$ of a degree = 20 minutes; and since $\frac{1}{4}$ of a degree = 15 minutes, $\frac{3}{4}$ of a degree = 45 minutes. In the same way, $\frac{1}{2}$ minute = 30 seconds, $\frac{1}{3}$ minute = 20 seconds, $\frac{1}{4}$ minute = 15 seconds, and $\frac{3}{4}$ minute = 45 seconds. The word "degree" is often abbreviated "deg.," or the sign (°) is used to indicate degrees; thus $60° = 60$ degrees. In the same way, $60' = 60$ minutes (min.), $60'' = 60$ seconds (sec.).

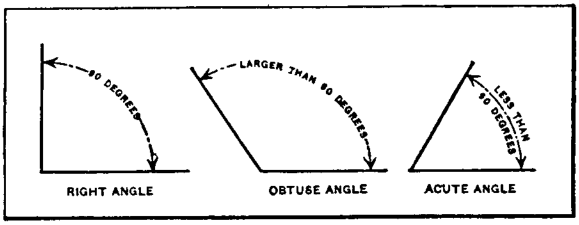
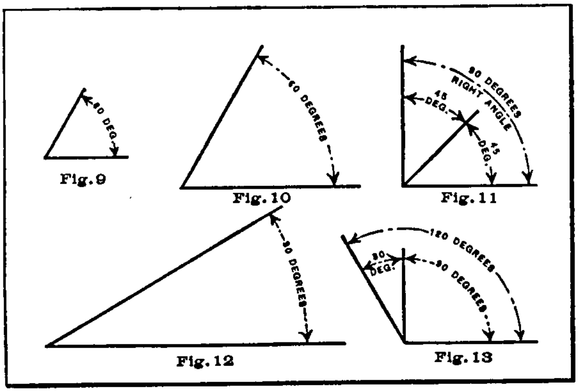


Fig. 6 **Fig. 7** **Fig. 8**

Measuring Angles in Radians. — While in practical work angles are always measured in degrees and minutes, the system for measuring angles in what is termed *circular measure* is often employed in theoretical investigations and in formulas relating to revolving bodies. In this system, the unit of measurement is the *radian*, that is, the angle at the center of a circle which embraces an arc equal in length to the length of the radius. The value of the radian in degrees equals $180 \div \pi = 57.2958$ degrees. In this measurement, then, π denotes an angle of 180 degrees, and $\pi \div 2$, an angle of 90 degrees.



Figs. 9 to 13. Angles

It is especially convenient to measure angles in radians when dealing with angular velocity. If ω = angular velocity per second of the revolving body, in radians; v = velocity of a point on the periphery of the body, in feet per second; and r = the radius, in feet, then:

$$\omega = \frac{v}{r}.$$

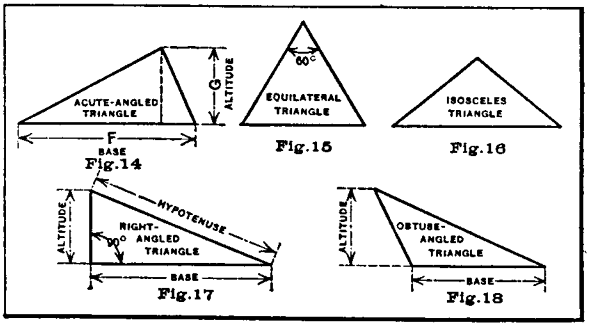
For example, assume that the velocity of a point on the periphery is 20 feet per second and the radius, 2 feet. Then the angular velocity is found as below:

$$\omega = \frac{20}{2} = 10 \text{ radians.}$$

The simple manner in which the relation between the angular velocity, the linear velocity, and the radius or diameter of the revolving body can be expressed, is the reason for using the radian as a convenient unit of angular measurement.

Triangles. — Any figure bounded by three straight lines is called a *triangle*. Any one of the three lines may be called the base, and the line drawn from the angle opposite the base at right angles to it is called the *height* or *altitude* of the triangle. In Fig. 14, if the side F is taken as the base of the triangle, then G is the altitude. If all three sides of a triangle are of equal length, as in the one shown in Fig. 15, the triangle is called *equilateral*. Each of the three angles in an equilateral triangle equals 60 degrees. If two sides are of equal length, as shown in Fig. 16, the triangle is an *isosceles* triangle.

If one angle is a right or 90-degree angle, the triangle is called a *right* or *right-angled* triangle. Such a triangle is shown in Fig. 17; the side opposite the right angle is called the *hypotenuse*. If all the angles are less than 90 degrees, the triangle is called an *acute* or *acute-angled* triangle, as shown in Fig. 14. If one of the angles is larger than 90 degrees, as shown in Fig. 18, the triangle is called an *obtuse* or *obtuse-angled* triangle. The sum of the three angles in every triangle equals 180 degrees, or two right angles.



Figs. 14 to 18. Triangles

Right-angled Triangle. — Let A = area, and a, b, and c be the sides of the triangle, as shown in Fig. 19. Then:

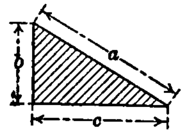


Fig. 19.

$$A = \frac{bc}{2}.$$

$$a = \sqrt{b^2 + c^2}.$$

$$b = \sqrt{a^2 - c^2}.$$

$$c = \sqrt{a^2 - b^2}.$$

As an example, assume that the sides b and c in a right-angled triangle are 6 and 8 inches. Find side a and the area.

$$a = \sqrt{b^2 + c^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10 \text{ in.}$$

$$A = \frac{b \times c}{2} = \frac{6 \times 8}{2} = \frac{48}{2} = 24 \text{ square inches.}$$

If $a = 10$ and $b = 6$, had been known, but not c, the latter would have been found as follows:

$$c = \sqrt{a^2 - b^2} = \sqrt{10^2 - 6^2} = \sqrt{100 - 36} = \sqrt{64} = 8 \text{ in.}$$

Acute-angled Triangle. — Let A = area, and a, b, c, and h be the dimensions shown in Fig. 20. Then:

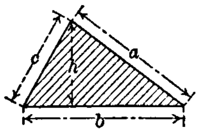


Fig. 20.

$$A = \frac{bh}{2} = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b}\right)^2}$$

If $S = \frac{1}{2}(a + b + c)$, then

$$A = \sqrt{S(S - a)(S - b)(S - c)}.$$

If $a = 10$, $b = 9$, and $c = 8$ inches, what is the area of the triangle?

$$A = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b}\right)^2} = \frac{9}{2}\sqrt{10^2 - \left(\frac{10^2 + 9^2 - 8^2}{2 \times 9}\right)^2}$$

$$= 4.5\sqrt{100 - \left(\frac{117}{18}\right)^2} = 4.5\sqrt{100 - 42.25} = 4.5\sqrt{57.75}$$

$$= 4.5 \times 7.60 = 34.20 \text{ square inches.}$$

Obtuse-angled Triangle. — Let A = area, and a, b, c, and h be the dimensions shown in Fig. 21. Then:

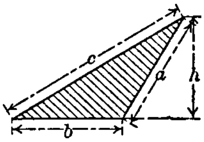


Fig. 21.

$$A = \frac{bh}{2} = \frac{b}{2}\sqrt{a^2 - \left(\frac{c^2 - a^2 - b^2}{2b}\right)^2}.$$

If $S = \frac{1}{2}(a + b + c)$, then

$$A = \sqrt{S(S - a)(S - b)(S - c)}.$$

The side $a = 5$, side $b = 4$, and side $c = 2$ inches. Find the area.

$$S = \tfrac{1}{2}(a + b + c) = \tfrac{1}{2}(5 + 4 + 2) = \tfrac{1}{2} \times 11 = 5.5.$$

$$A = \sqrt{S(S - a)(S - b)(S - c)}$$

$$= \sqrt{5.5(5.5 - 5)(5.5 - 4)(5.5 - 2)}$$

$$= \sqrt{5.5 \times 0.5 \times 1.5 \times 3.5} = \sqrt{14.437} = 3.8 \text{ sq. in.}$$

Square. — The square has four sides of equal length, and each of the four angles between the sides is a right or 90-degree angle. Let A = area, s = side, and d = diagonal of the square, as shown in Fig. 22. Then:

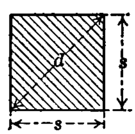


Fig. 22.

$$A = s^2.$$

$$A = \tfrac{1}{2}d^2.$$

$$s = 0.7071\,d = \sqrt{A}.$$

$$d = 1.414\,s = 1.414\sqrt{A}.$$

Assume that the side s of a square is 15 inches. Find the area and the length of the diagonal.

Area $= A = s^2 = 15^2 = 225$ square inches.

Diagonal $= d = 1.414\,s = 1.414 \times 15 = 21.21$ inches.

The area of a square is 625 square inches. Find the length of the side s and the diagonal d.

$$s = \sqrt{A} = \sqrt{625} = 25 \text{ inches.}$$

$$d = 1.414\sqrt{A} = 1.414 \times 25 = 35.35 \text{ inches.}$$

Rectangle. — The rectangle, as shown in Fig. 23, has four sides, of which those opposite each other are of equal length, and the four angles between the sides are right or 90-degree angles. Let A = area, and a, b, and d be the dimensions shown in Fig. 23. Then:

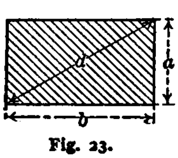


Fig. 23.

$$A = ab.$$

$$A = a\sqrt{d^2 - a^2} = b\sqrt{d^2 - b^2}.$$

$$d = \sqrt{a^2 + b^2}.$$

$$a = \sqrt{d^2 - b^2} = A \div b.$$

$$b = \sqrt{d^2 - a^2} = A \div a.$$

Assume that the side a of a rectangle is 12 inches, and the area, 70.5 square inches. Find the length of the side b, and the diagonal d.

$$b = A \div a = 70.5 \div 12 = 5.875 \text{ inches.}$$

$$d = \sqrt{a^2 + b^2} = \sqrt{12^2 + 5.875^2} = \sqrt{178.516} = 13.361 \text{ in.}$$

The sides of a rectangle are 30.5 and 11 inches long. Find the area.

Area $= a \times b = 30.5 \times 11 = 335.5$ square inches.

Parallelogram. — Any figure made up of four sides, of which those opposite are parallel, is called a *parallelogram.* The square and rectangle are parallelograms in which all the angles are right angles. In Fig. 24 is shown a parallelogram where two of the angles are less and two more than 90 degrees. A line drawn from one side of a parallelogram at *right angles* to the opposite side is called the height or altitude of the parallelogram. In Fig. 24, a is the altitude, and b is the length or base. Let A = area. Then:

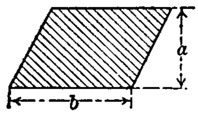


Fig. 24.

$$A = ab.$$

$$a = A \div b.$$

$$b = A \div a.$$

Note that dimension a is measured at right angles to line b.

As an example, assume that the base b of a parallelogram is 16 feet. The height a is 5.5 feet. Find the area.

Area $= A = a \times b = 5.5 \times 16 = 88$ square feet.

The area of a parallelogram is 12 square inches. The height is 1.5 inch. Find the length of the base b.

$$b = A \div a = 12 \div 1.5 = 8 \text{ inches.}$$

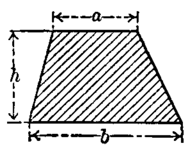


Fig. 25.

Trapezoid. — A figure bounded by four lines, of which only two are parallel, is called a *trapezoid.* The height of a trapezoid is the distance h, Fig. 25, between the two parallel lines. Let A = area and a, b, and h be the dimensions shown in Fig. 25. Then:

$$A = \frac{(a + b)h}{2}.$$

As an example, assume that side $a = 23$ feet, side $b = 32$ feet, and height $h = 12$ feet. Find the area.

$$A = \frac{(a + b)h}{2} = \frac{(23 + 32)12}{2} = \frac{55 \times 12}{2} = \frac{660}{2} = 330 \text{ sq. ft.}$$

Trapezium. — A figure bounded by four lines, no two of which are parallel, as shown in Fig. 26, is called a *trapezium.* Let A = area, and a, b, c, H, and h be the dimensions indicated in Fig. 26. Then:

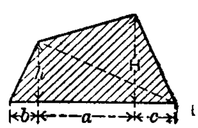


Fig. 26.

$$A = \frac{(H + h)a + bh + cH}{2}.$$

A trapezium can also be divided into two triangles as indicated by the dotted line. The area of each of these triangles is computed, and the results added to find the area of the trapezium.

As an example, let $a = 10$, $b = 2$, $c = 3$, $h = 8$, and $H = 12$ inches. Find the area.

$$A = \frac{(H + h)a + bh + cH}{2} = \frac{(12 + 8)10 + 2 \times 8 + 3 \times 12}{2}$$

$$= \frac{20 \times 10 + 16 + 36}{2} = \frac{252}{2} = 126 \text{ square inches.}$$

Regular Polygon. — Any plane figure or surface bounded by straight lines is called a *polygon*. (See POLYGONS.) If all the sides are of equal length and the angles between the sides equal, the figure is called a *regular* polygon, as shown in Fig. 27. Let A = area, n = number of sides, s = length of side, R = radius of circumscribed circle, r = radius of inscribed circle, and α and β, angles as indicated in Fig. 27. Then:

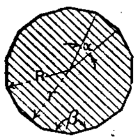


Fig. 27.

$$\alpha = 360° \div n; \; \beta = 180° - \alpha.$$

$$A = \frac{nsr}{2} = \frac{ns}{2}\sqrt{R^2 - \frac{s^2}{4}}.$$

$$R = \sqrt{r^2 + \frac{s^2}{4}}; \; r = \sqrt{R^2 - \frac{s^2}{4}}; \; s = 2\sqrt{R^2 - r^2}.$$

As an example, find the area of a polygon having 12 sides, inscribed in a circle of 8 inches radius. The length of the side s is 4.141 inches.

$$A = \frac{ns}{2}\sqrt{R^2 - \frac{s^2}{4}} = \frac{12 \times 4.141}{2}\sqrt{8^2 - \frac{4.141^2}{4}}$$
$$= 24.846\sqrt{59.713} = 24.846 \times 7.727 = 191.98 \text{ sq. in.}$$

Regular Hexagon. — Of the regular polygons with more than four sides, the *hexagon*, having six sides, is the most commonly met with in mechanical work. Let A = area; R = radius of circumscribed circle; r = radius of inscribed circle; and s = length of side. Then:

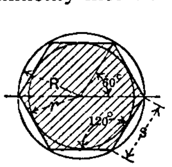


Fig. 28.

$$A = 2.598\, s^2 = 2.598\, R^2 = 3.464\, r^2.$$
$$R = s = 1.155\, r.$$
$$r = 0.866\, s = 0.866\, R.$$
$$s = R = 1.155\, r.$$

The side s of a regular hexagon is 4 inches. Find the area and the radius r of the inscribed circle.

$$A = 2.598\, s^2 = 2.598 \times 4^2 = 2.598 \times 16 = 41.568 \text{ sq. in.}$$
$$r = 0.866\, s = 0.866 \times 4 = 3.464 \text{ inches.}$$

What is the length of the side of a hexagon that is described about a circle of 5 inches radius? — Here $r = 5$. Hence,

$$s = 1.155\, r = 1.155 \times 5 = 5.775 \text{ inches.}$$

Regular Octagon. — A polygon having eight sides is called an *octagon*. Let A = area; R = radius of circumscribed circle; r = radius of inscribed circle; and s = length of side. Then:

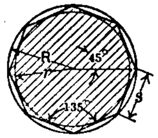


Fig. 29.

$$A = 4.828\, s^2 = 2.828\, R^2 = 3.314\, r^2.$$
$$R = 1.307\, s = 1.082\, r.$$
$$r = 1.207\, s = 0.924\, R.$$
$$s = 0.765\, R = 0.828\, r.$$

Find the area and the length of the side of an octagon that is inscribed in a circle of 12 inches diameter.

Diameter of circumscribed circle = 12 inches; hence, R = 6 inches.

$$A = 2.828\, R^2 = 2.828 \times 6^2 = 2.828 \times 36 = 101.81 \text{ sq. in.}$$
$$s = 0.765\, R = 0.765 \times 6 = 4.590 \text{ inches.}$$

Circle. — The circle is a plane surface bounded by a curved line called the *periphery* or *circumference*, which is at all points at an equal distance from a point within the circle called the *center*. The distance from the center of the circle to the periphery is the *radius*, and the distance across the circle through the center is the *diameter*. It is evident that the radius is one-half of the diameter. If a line is drawn from one point on the periphery to another point, so that it does not pass through the center, it is called a *chord*.

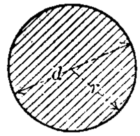


Fig. 30.

Let A = area; C = circumference; d = diameter; and r = radius. Then:

$$A = \pi r^2 = 3.1416\, r^2 = 0.7854\, d^2.$$
$$C = 2\pi r = 6.2832\, r = 3.1416\, d.$$
$$r = C \div 6.2832 = \sqrt{A \div 3.1416} = 0.564\sqrt{A}.$$
$$d = C \div 3.1416 = \sqrt{A \div 0.7854} = 1.128\sqrt{A}.$$

Length of arc for center-angle of 1° = $0.008727\, d$.
Length of arc for center-angle of $n°$ = $0.008727\, nd$.

Find the area A and circumference C of a circle with a diameter of $2\frac{3}{4}$ inches.

$$A = 0.7854\, d^2 = 0.7854 \times 2.75^2 = 0.7854 \times 2.75 \times 2.75$$
$$= 5.9396 \text{ square inches.}$$
$$C = 3.1416\, d = 3.1416 \times 2.75 = 8.6394 \text{ inches.}$$

The area of a circle is 16.8 square inches. Find its diameter.

$$d = 1.128\sqrt{A} = 1.128\sqrt{16.8} = 1.128 \times 4.099 = 4.624 \text{ in.}$$

Circular Sector. — A figure bounded by a part of the circumference of a circle and two radii, as shown in Fig. 31, is called a *circular sector*. The angle α between the radii is called the *angle* of the sector, and the length l of the circumference of the circle is called the *arc* of the sector. Let A = area of sector; r = radius; α = angle of sector in degrees; and l = length of arc. Then:

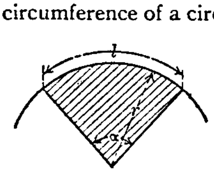


Fig. 31.

$$l = \frac{r \times \alpha \times 3.1416}{180} = 0.01745\, r\alpha = \frac{2A}{r}.$$
$$A = \tfrac{1}{2} rl = 0.008727\, \alpha r^2.$$
$$\alpha = \frac{57.296\, l}{r}; \qquad r = \frac{2A}{l} = \frac{57.296\, l}{\alpha}.$$

The radius of a circle is $1\frac{1}{2}$ inch, and angle α of a sector of the circle is 60 degrees. Find the area of the sector and the length of arc l.

$$A = 0.008727\, \alpha r^2 = 0.008727 \times 60 \times 1.5^2$$
$$= 0.5236 \times 1.5 \times 1.5 = 1.178 \text{ square inch.}$$
$$l = 0.01745\, r\alpha = 0.01745 \times 1.5 \times 60 = 1.5705 \text{ inch.}$$

Circular Segment. — A figure bounded by a part of the circumference of a circle and a chord, as shown in Fig. 32, is called a *circular segment*. The distance h from the chord to the highest point of the circular arc is called the *height* of the segment. Let A = area of segment, and c, h, l, and r be the dimensions indicated in Fig. 32; α = angle, in degrees. Then the following formulas are applicable:

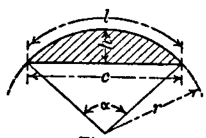


Fig. 32.

$$c = 2\sqrt{h(2r - h)}. \qquad A = \tfrac{1}{2}[rl - c(r - h)].$$

$$r = \frac{c^2 + 4h^2}{8h}. \qquad l = 0.01745\, r\alpha.$$

$$h = r - \tfrac{1}{2}\sqrt{4r^2 - c^2}. \qquad \alpha = \frac{57.296\, l}{r}.$$

The radius r of a circular segment is 60 inches and the height h is 8 inches. Find the length of the chord c.

$$c = 2\sqrt{h(2r - h)} = 2\sqrt{8 \times (2 \times 60 - 8)} = 2\sqrt{896}$$
$$= 2 \times 29.93 = 59.86 \text{ inches.}$$

If $c = 16$, and $h = 6$ inches, what is the radius of the circle of which the segment is a part?

$$r = \frac{c^2 + 4h^2}{8h} = \frac{16^2 + 4 \times 6^2}{8 \times 6} = \frac{256 + 144}{48} = \frac{400}{48} = 8\tfrac{1}{3} \text{ in.}$$

Circular Ring. — An area bounded by two concentric circles, that is, by two circles having a common center but radii of different lengths, is generally called a *circular ring* or an *annular* surface, as shown in Fig. 33. Let A = area, D = diameter, and R = radius of large circle, d = diameter, and r = radius of small circle. Then:

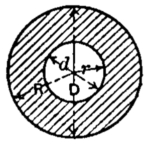


Fig. 33.

$$A = \pi(R^2 - r^2) = 3.1416(R^2 - r^2)$$
$$= 3.1416(R + r)(R - r)$$
$$= 0.7854(D^2 - d^2) = 0.7854(D + d)(D - d).$$

Let the outside diameter $D = 12$ inches and the inside diameter $d = 8$ inches. Find the area of the ring.

$$A = 0.7854(D^2 - d^2) = 0.7854(12^2 - 8^2)$$
$$= 0.7854(144 - 64) = 0.7854 \times 80$$
$$= 62.83 \text{ square inches.}$$

By the alternative formula:

$$A = 0.7854(D + d)(D - d) = 0.7854(12 + 8)(12 - 8)$$
$$= 0.7854 \times 20 \times 4 = 62.83 \text{ square inches.}$$

Circular Ring Sector. — In a circular ring sector (or *annular sector*), as shown in Fig. 34, let A = area, and α = angle of sector in degrees; let D, d, R, and r be the dimensions indicated in the illustration. Then:

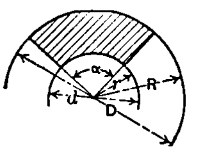


Fig. 34.

$$A = \frac{\alpha\pi}{360}(R^2 - r^2) = 0.00873\,\alpha(R^2 - r^2)$$

$$= \frac{\alpha\pi}{4 \times 360}(D^2 - d^2) = 0.00218\,\alpha(D^2 - d^2).$$

Find the area, if the outside radius $R = 5$ inches, the inside radius $r = 2$ inches, and $\alpha = 72$ degrees.

$$A = 0.00873\,\alpha(R^2 - r^2) = 0.00873 \times 72(5^2 - 2^2)$$
$$= 0.6286(25 - 4) = 0.6286 \times 21 = 13.2 \text{ sq. in.}$$

Spandrel or Fillet. — The area enclosed between a circular arc equal to one-quarter of the complete circle, and two tangents to the circle at the extreme ends of the arc, as shown in Fig. 35, is called a *spandrel or fillet*. Let A = area, and c and r be the dimensions indicated in Fig. 35. Then:

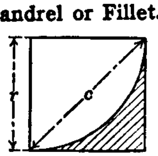


Fig. 35.

$$A = r^2 - \frac{\pi r^2}{4} = 0.215\, r^2 = 0.1075\, c^2.$$

Find the area of a spandrel, the radius of which is 0.7 inch.

$$A = 0.215\, r^2 = 0.215 \times 0.7^2$$
$$= 0.215 \times 0.7 \times 0.7 = 0.105 \text{ square inch.}$$

If chord c were given as 2.2 inches, what would be the area?

$$A = 0.1075\, c^2 = 0.1075 \times 2.2^2$$
$$= 0.1075 \times 4.84 = 0.520 \text{ square inch.}$$

Ellipse. — The ellipse is a plane surface bounded by a curved line having two diameters or axes of different length; the longer of these is called the *major* axis, the shorter, the *minor* axis. The point where these two axes intersect is called the *center of the ellipse*. An ellipse is formed by taking a section through a cylinder in a plane at an oblique angle to the axis of the cylinder. In Fig. 36, a = one-half the major axis, and b = one-half the minor axis. Let A = the area of the ellipse, and P = the perimeter or circumference. Then:

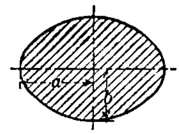


Fig. 36.

$$A = \pi ab = 3.1416\, ab.$$

The circumference of an ellipse cannot be obtained exact by any formula. An approximate formula for the perimeter is:

$$P = 3.1416\sqrt{2(a^2 + b^2)}.$$

A closer approximation is:

$$P = 3.1416\sqrt{2(a^2 + b^2) - \frac{(a - b)^2}{2.2}}.$$

The larger or major axis of an ellipse is 8 inches. The smaller or minor axis is 6 inches. Find the area and the approximate circumference. Here, then, $a = 4$, and $b = 3$.

$$A = 3.1416\, ab = 3.1416 \times 4 \times 3 = 37.699 \text{ sq. in.}$$
$$P = 3.1416\sqrt{2(a^2 + b^2)} = 3.1416 \times \sqrt{2(4^2 + 3^2)}$$
$$= 3.1416 \times \sqrt{2 \times 25} = 3.1416\sqrt{50} = 3.1416$$
$$\times 7.071 = 22.214 \text{ inches.}$$

Hyperbola. — The hyperbola is an open curve having two branches, each extending into the infinite, and is obtained by taking a section through a cone parallel to the axis, or by a section at an angle to the axis, providing the angle is *less* than the angle between the axis and the side of the cone. The area A, enclosed by the hyperbolic curve BD, its *principal* axis BC, and the line CD, perpendicular to BC as shown in Fig. 37, is found by the formula:

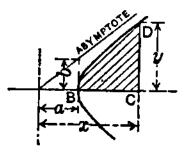


Fig. 37.

$$A = \frac{xy}{2} - \frac{ab}{2}\text{hyp. log.}\left(\frac{x}{a} + \frac{y}{b}\right),$$

in which a, b, x, and y are dimensions indicated in the illustration.

The half-axes a and b are 3 and 2 inches, respectively. Find the area shown shaded in the illustration for $x = 8$ and $y = 5$.

Inserting the known values in the formula:

$$A = \frac{8 \times 5}{2} - \frac{3 \times 2}{2} \times \text{hyp. log.}\left(\frac{8}{3} + \frac{5}{2}\right)$$
$$= 20 - 3 \times \text{hyp. log. } 5.167 = 20 - 3 \times 1.6423$$
$$= 20 - 4.927 = 15.073 \text{ sq. in.}$$

Parabola. — The parabola is an open curve having two branches, each extending into the infinite, and is obtained by taking a section through a cone parallel to the side of the cone. The length l of a parabolic arc, when p, x, and y are known, as shown in Fig. 38, is found from the formula (distance p is known as the *parameter* of the parabola):

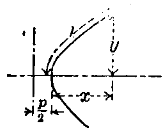


Fig. 38.

$$l = \frac{p}{2}\left[\sqrt{\frac{2x}{p}\left(1+\frac{2x}{p}\right)} + \text{hyp. log.}\left(\sqrt{\frac{2x}{p}} + \sqrt{1+\frac{2x}{p}}\right)\right].$$

When x is small in proportion to y, the following is a close approximation:

$$l = y\left[1 + \frac{2}{3}\left(\frac{x}{y}\right)^2 - \frac{2}{5}\left(\frac{x}{y}\right)^4\right], \text{ or } l = \sqrt{y^2 + \frac{4}{3}x^2}.$$

If $x = 2$ and $y = 24$ feet, what is the approximate length l of the parabolic curve?

$$l = y\left[1 + \frac{2}{3}\left(\frac{x}{y}\right)^2 - \frac{2}{5}\left(\frac{x}{y}\right)^4\right] = 24\left[1 + \frac{2}{3}\left(\frac{2}{24}\right)^2 - \frac{2}{5}\left(\frac{2}{24}\right)^4\right]$$

$$= 24\left[1 + \frac{2}{3}\times\frac{1}{144} - \frac{2}{5}\times\frac{1}{20{,}736}\right] = 24 \times 1.0046 = 24.11 \text{ ft.}$$

The area enclosed between an arc of the parabola, its axis, and a line y (Fig. 39), at right angles to the axis, is equal to two-thirds of the rectangle which has x for its base and y for its height or altitude; or, if A = area, then:

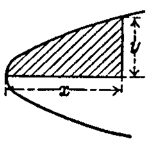


Fig. 39.

$$A = \tfrac{2}{3}xy.$$

Let the dimension x in the illustration be 15 inches, and y, 9 inches. Find the area of the shaded portion of the parabola.

$$A = \tfrac{2}{3} \times xy = \tfrac{2}{3} \times 15 \times 9 = 10 \times 9 = 90 \text{ sq. in.}$$

Segment of Parabola. — A segment of a parabola is enclosed between an arc of the parabola and a chord which intersects the arc at two points. In Fig. 40,

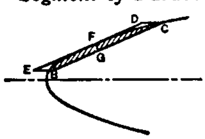


Fig. 40.

Area BFC = $\tfrac{2}{3}$ area of parallelogram $BCDE$.

If FG is the height of the segment, measured at right angles to BC, then:

Area of segment $BFC = \tfrac{2}{3}\, BC \times FG$.

The length of the chord BC = 19.5 inches. The distance between lines BC and DE, measured at right angles to BC, is 2.25 inches. This is the height of the segment. Find the area.

Area $= \tfrac{2}{3}\, BC \times FG = \tfrac{2}{3} \times 19.5 \times 2.25 = 29.25$ sq. in.

Cycloid. — The cycloid is a curve described by a given point of a circle, if the circle is rolled along a straight line. The circle which rolls upon the line is called a *generating* circle. Let A be the area enclosed between the cycloidal curve and the line upon which the generating circle rolls; l is the length of the cycloid; d is the diameter and r the radius of the generating circle. Then:

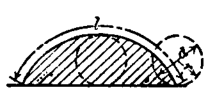


Fig. 41.

$$A = 3\pi r^2 = 9.4248\, r^2 = 2.3562\, d^2$$
$$= 3 \times \text{area of generating circle.}$$
$$l = 8\, r = 4\, d.$$

The diameter of the generating circle of a cycloid is 6 inches. Find the length l of the cycloidal curve, and the area enclosed between the curve and the base line.

$$l = 4\,d = 4 \times 6 = 24 \text{ inches.}$$
$$A = 2.3562\, d^2 = 2.3562 \times 6^2 = 2.3562 \times 36$$
$$= 84.82 \text{ square inches.}$$

Plane Surfaces of Irregular Outline. — The areas of plane surfaces of irregular outline can best be found by dividing the surfaces into a number of geometrical figures, the areas of which can be found by the general rules and formulas. The most convenient method is, as a rule, to divide the area into a number of narrow strips which may be regarded as rectangles, the base of the rectangle being the width of the strip, and a line through the center of the strip, its mean height. Surfaces of irregular outline, having straight boundary lines, are most conveniently divided into a number of triangles, the areas of which can be readily found and added together.

Cube. — The cube, Fig. 42, is a solid body having six surfaces or faces, all of which are squares; as all the faces are squares, all the sides are of equal length. Let V = volume of cube, and s = length of side. Then:

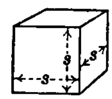


Fig. 42.

$$V = s^3.$$
$$s = \sqrt[3]{V}.$$

The side of a cube equals 9.5 inches. Find its volume.

$$\text{Volume} = V = s^3 = 9.5^3 = 9.5 \times 9.5 \times 9.5$$
$$= 857.375 \text{ cubic inches.}$$

The volume of a cube is 231 cubic inches. What is the length of the side?

$$s = \sqrt[3]{V} = \sqrt[3]{231} = 6.136 \text{ inches.}$$

Prism. — A solid body having the end faces parallel, and the lines along which the other faces intersect or meet parallel, is called a *prism*. The two parallel end faces are called *bases*. The length, height, or altitude h, Fig. 43, of a prism is the distance between the bases, measured at *right angles* to the base surfaces.

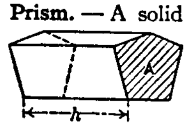


Fig. 43.

The volume of a prism equals the area of the base multiplied by the length or height h of the prism. The area of the base must, therefore, first be found before the volume can be obtained. If the base is a triangle, parallelogram, trapezoid, trapezium, or a regular polygon, its area is found by the rules given in preceding paragraphs. If it is a polygon which is not regular, it can always be divided into triangles, and the area of each of the triangles can be calculated, and these areas added together, to obtain the area of the whole polygon. Let V = volume of prism, h = height of prism measured perpendicular to the end surface, and A = area of end surface. Then:

$$V = h \times A.$$

A prism having for its base a regular hexagon, with a side s of 3 inches, is 10 inches high. Find the volume.

$$\text{Area of hexagon} = A = 2.598\, s^2 = 2.598 \times 9$$
$$= 23.382 \text{ square inches.}$$
$$\text{Volume of prism} = h \times A = 10 \times 23.382$$
$$= 233.82 \text{ cubic inches.}$$

Square Prism. — A solid body, the sides of which are all rectangles, and the ends of which are either rectangles or squares, is commonly called a *square prism*. Opposite surfaces or faces are parallel, and all the angles are right angles. A square prism is shown in Fig. 44, where a is its length, b, its height, and c, its width. Let V = volume of square prism, then:

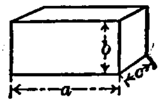


Fig. 44.

$$V = abc.$$

$$a = \frac{V}{bc}.\quad b = \frac{V}{ac}.\quad c = \frac{V}{ab}.$$

In a square prism, $a = 6$, $b = 5$, $c = 4$. Find the volume.

$$V = a \times b \times c = 6 \times 5 \times 4 = 120 \text{ cubic inches.}$$

How high should a box be made to contain 25 cubic feet, if it is 4 feet long and $2\frac{1}{2}$ feet wide? Here, $a = 4$, $c = 2.5$, and $V = 25$. Then:

$$b = \text{depth} = \frac{V}{ac} = \frac{25}{4 \times 2.5} = \frac{25}{10} = 2.5 \text{ feet.}$$

Pyramid. — A solid body having a polygon for the base and a number of triangles all having a common vertex for the sides is called a *pyramid*. In Fig. 45 a pyramid is shown where the base has six sides and the side surfaces are made up of triangles having two equal sides. If a line is drawn from the vertex of the pyramid at *right angles* to the base, the length of this line is the altitude or height h of the pyramid. The volume of a pyramid equals the base area multiplied by one-third of the height. It is, therefore, necessary to find the base area before the volume can be found. Let V = volume of pyramid, and h = height perpendicular to base. Then:

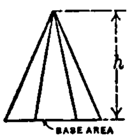


Fig. 45.

$$V = \tfrac{1}{3} h \times \text{area of base.}$$

If the base is a regular polygon with n sides, and s = length of side, r = radius of inscribed circle, and R = radius of circumscribed circle, then:

$$V = \frac{nsrh}{6} = \frac{nsh}{6}\sqrt{R^2 - \frac{s^2}{4}}.$$

A pyramid, having a height of 9 feet, has a base formed by a rectangle, the sides of which are 2 and 3 feet, respectively. Find the volume.

$$\text{Area of base} = 2 \times 3 = 6 \text{ square feet; } h = 9 \text{ feet.}$$
$$\text{Volume} = V = \tfrac{1}{3} h \times \text{area of base} = \tfrac{1}{3} \times 9 \times 6$$
$$= 18 \text{ cubic feet.}$$

Frustum of Pyramid. — A frustum of a pyramid is shown in Fig. 46. It is a pyramid from which the top has been cut off, the top surface being parallel to the base. The height h of a frustum of a pyramid is the length of a line drawn from the top surface at *right angles* to the base. The volume of a frustum of a pyramid can be found when the height, the top area, and the base area are known. Let V = volume of frustum; A_1 = area of top surface; A_2 = area of base surface; and h = height perpendicular to base. Then:

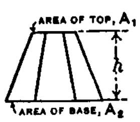


Fig. 46.

$$V = \frac{h}{3}(A_1 + A_2 + \sqrt{A_1 \times A_2}).$$

As an example, assume that a pyramid, having a height of 9 feet, has a base formed by a rectangle, the sides of which are 2 and 3 feet, respectively. This pyramid is cut off $4\frac{1}{2}$ feet from the base, the upper part being removed. The sides of the rectangle forming the top surface of the frustum are, then, 1 and $1\frac{1}{2}$ foot long, respectively. Find the volume of the frustum.

$$\text{Area of top} = A_1 = 1 \times 1\tfrac{1}{2} = 1\tfrac{1}{2} \text{ sq. ft.}$$
$$\text{Area of base} = A_2 = 2 \times 3 = 6 \text{ sq. ft.}$$

$$V = \frac{4.5}{3}(1.5 + 6 + \sqrt{1.5 \times 6}) = 1.5\,(7.5 + \sqrt{9})$$
$$= 1.5 \times 10.5 = 15.75 \text{ cubic feet.}$$

Prismoidal Formula. — The prismoidal formula is a general formula by which the volume of any prism, pyramid, or frustum of a pyramid may be found.

A_1 = area at one end of the body;
A_2 = area at the other end;
A_m = area of middle section between the two end surfaces;
h = height of body.

Then, volume V of the body is:

$$V = \frac{h}{6}(A_1 + 4\,A_m + A_2).$$

Wedge. — A solid body enclosed by a rectangular base surface, two trapezoids, and two triangles, is called a *wedge*. The two trapezoids forming the side surfaces of the wedge may also be rectangles. Let V = volume of wedge, and a, b, c, and h be the dimensions indicated in Fig. 47. The height h is measured perpendicular to the base surface. Then:

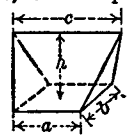


Fig. 47.

$$V = \frac{(2\,a + c)\,bh}{6}.$$

Let a = 4 inches, b = 3 inches, and c = 5 inches. The height h = 4.5 inches. Find the volume.

$$V = \frac{(2\,a + c)\,bh}{6} = \frac{(2 \times 4 + 5) \times 3 \times 4.5}{6}$$
$$= \frac{(8 + 5) \times 13.5}{6} = \frac{13 \times 13.5}{6} = \frac{175.5}{6} = 29.25 \text{ cu. in.}$$

Cylinder. — A solid body, as shown in Fig. 48, having circular and parallel end faces of equal size, is called a *cylinder*. The two parallel faces are called *bases*. The height or altitude h of a cylinder is the distance between the bases measured at *right angles* to the base surfaces. The volume of a cylinder equals the area of the base multiplied by the height. The area of the base must, therefore, first be found before the volume can be obtained. Let V = volume of cylinder; S = area of cylindrical surface; d = diameter, and r = radius of cylinder; and h = height measured perpendicular to base. Then:

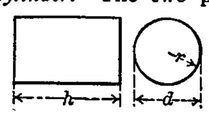


Fig. 48.

$$V = 3.1416\,r^2h = 0.7854\,d^2h.$$
$$S = 6.2832\,rh = 3.1416\,dh.$$

Total area A of cylindrical surface and end surfaces:

$$A = 6.2832\,r\,(r + h) = 3.1416\,d\,(\tfrac{1}{2}\,d + h).$$

The diameter of a cylinder is $2\frac{1}{2}$ inches. The length or height is 20 inches. Find the volume and the area of the cylindrical surface S.

$$V = 0.7854\,d^2h = 0.7854 \times 2\tfrac{1}{2}^2 \times 20$$
$$= 0.7854 \times 6.25 \times 20 = 98.17 \text{ cubic inches.}$$
$$S = 3.1416\,dh = 3.1416 \times 2\tfrac{1}{2} \times 20 = 157.08 \text{ sq. in.}$$

Portion of a Cylinder. — The volume of a portion of a cylinder, cut off by an inclined plane as shown in Fig. 49, may be found by the following formulas. Let V = volume of portion of cylinder, S = area of cylindrical surface, and d, r, h_1, and h_2 be the dimensions shown in Fig. 49. Then:

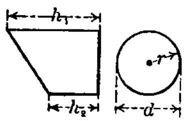


Fig. 49.

$$V = 1.5708\,r^2\,(h_1 + h_2) = 0.3927\,d^2\,(h_1 + h_2).$$
$$S = 3.1416\,r\,(h_1 + h_2) = 1.5708\,d\,(h_1 + h_2).$$

A cylinder 5 inches in diameter is cut off at an angle, as shown in the illustration. Dimension $h_1 = 6$, and $h_2 = 4$ inches. Find the volume and the area S of the cylindrical surface.

$$V = 0.3927\,d^2\,(h_1 + h_2) = 0.3927 \times 5^2 \times (6 + 4)$$
$$= 0.3927 \times 25 \times 10 = 98.175 \text{ cubic inches.}$$
$$S = 1.5708\,d\,(h_1 + h_2) = 1.5708 \times 5 \times 10$$
$$= 78.54 \text{ square inches.}$$

The volume of a portion of a cylinder so cut by an inclined plane that part of the circular base surface is removed also, as shown in Fig. 50, may be found as follows. Let V = volume of portion of cylinder, S = area of cylindrical surface, and the other dimensions be as shown in Fig. 50. Then:

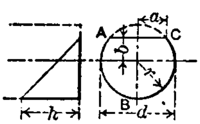


Fig. 50.

$$V = \left(\frac{2}{3}a^3 \pm b \times \text{area } ABC\right)\frac{h}{r \pm b}.$$

$$S = (ad \pm b \times \text{length of arc } ABC)\,\frac{h}{r \pm b}.$$

Use + when base area is larger, and − when base area is less than one-half the base circle.

Find the volume of a cylinder so cut off that line AC passes through the center of the base circle — that is, the base area is a half-circle. The diameter of the cylinder = 5 inches, and height h = 2 inches.

In this case, $a = 2.5$; $b = 0$; area $ABC = \frac{1}{2} \times 0.7854 \times 5^2 = 9.82$; $r = 2.5$.

$$V = \left(\frac{2}{3} \times 2.5^3 + 0 \times 9.82\right)\frac{2}{2.5 + 0} = \frac{2}{3} \times 15.625 \times 0.8$$
$$= 8.33 \text{ cubic inches.}$$

Hollow Cylinder. — In a hollow cylinder, as shown in Fig. 51, let V = volume, and the other dimensions be as shown in the illustration. Then:

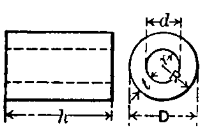


Fig. 51.

$$V = 3.1416\,h\,(R^2 - r^2) = 0.7854\,h\,(D^2 - d^2)$$
$$= 3.1416\,ht\,(2\,R - t) = 3.1416\,ht\,(D - t)$$
$$= 3.1416\,ht\,(2\,r + t) = 3.1416\,ht\,(d + t)$$
$$= 3.1416\,ht\,(R + r) = 1.5708\,ht\,(D + d).$$

A cylindrical shell, 28 inches high, is 36 inches in outside diameter, and 4 inches thick. Find its volume.

$$V = 3.1416\,ht\,(D - t) = 3.1416 \times 28 \times 4\,(36 - 4)$$
$$= 3.1416 \times 28 \times 4 \times 32 = 11{,}259.5 \text{ cubic inches.}$$

Cone. — A solid body having a circular base and the sides inclined so that they meet at a common vertex, the same as in a pyramid, is called a *cone*. (See Fig. 52.) If a line is drawn from the vertex of the cone at *right angles* to the base, the length of this line is the altitude or height h of the cone. The volume of the cone equals the base area multiplied by one-third of the height. It is, therefore, necessary to find the area of the base circle before the volume can be found.

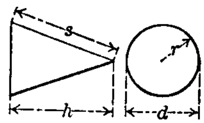


Fig. 52.

Let V = volume of cone, A = area of conical surface, and the other dimensions be as shown in Fig. 52. Then:

$$V = \frac{3.1416\,r^2h}{3} = 1.0472\,r^2h = 0.2618\,d^2h.$$
$$A = 3.1416\,r\sqrt{r^2 + h^2} = 3.1416\,rs = 1.5708\,ds.$$
$$s = \sqrt{r^2 + h^2} = \sqrt{\frac{d^2}{4} + h^2}.$$

Find the volume and area of the conical surface of a cone, the base of which is a circle of 6 inches diameter, and the height of which is 4 inches.

$$V = 0.2618\,d^2h = 0.2618 \times 6^2 \times 4 = 0.2618 \times 36 \times 4$$
$$= 37.7 \text{ cubic inches.}$$
$$A = 3.1416\,r\sqrt{r^2 + h^2} = 3.1416 \times 3 \times \sqrt{3^2 + 4^2}$$
$$= 9.4248 \times \sqrt{25} = 47.124 \text{ square inches.}$$

Frustum of Cone. — A frustum of a cone is shown in Fig. 53. It is a cone from which the top has been cut off, the top surface being a circle parallel to the base. The height h of a frustum of a cone is the length of a line drawn from the top surface at *right angles* to the base. The volume of a frustum of a cone can be found when the diameters of the top and base circles and the height are known. Let V = volume of frustum of cone, A = area of conical surface, and the other dimensions be as indicated in Fig. 53. Then:

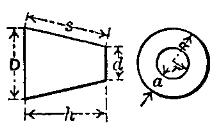


Fig. 53.

$$V = 1.0472\,h\,(R^2 + Rr + r^2) = 0.2618\,h\,(D^2 + Dd + d^2).$$
$$A = 3.1416\,s\,(R + r) = 1.5708\,s\,(D + d).$$
$$a = R - r;\ s = \sqrt{a^2 + h^2} = \sqrt{(R - r)^2 + h^2}.$$

Find the volume of a frustum of a cone of the following dimensions: D = 8 inches; d = 4 inches; h = 5 inches.

$$V = 0.2618 \times 5\,(8^2 + 8 \times 4 + 4^2)$$
$$= 0.2618 \times 5\,(64 + 32 + 16) = 0.2618 \times 5 \times 112$$
$$= 146.61 \text{ cubic inches.}$$

Sphere. — The name *sphere* is applied to a solid body shaped like a ball or globe, that is, bounded by a surface which at all points is at the same distance from a point inside of the sphere called its *center*. The diameter of a sphere is the length of a line drawn from a point on the surface through the center to the opposite side. Let V = volume of sphere; A = area of spherical surface; d = diameter, and r = radius of sphere. Then:

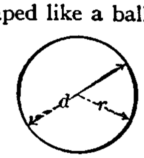


Fig. 54.

$$V = \frac{4\,\pi r^3}{3} = \frac{\pi d^3}{6} = 4.1888\,r^3 = 0.5236\,d^3.$$
$$A = 4\,\pi r^2 = \pi d^2 = 12.5664\,r^2 = 3.1416\,d^2.$$
$$r = \sqrt[3]{\frac{3\,V}{4\,\pi}} = 0.6204\sqrt[3]{V}.$$

Find the volume and surface of a sphere 6.5 inches in diameter.

$$V = 0.5236\,d^3 = 0.5236 \times 6.5^3$$
$$= 0.5236 \times 6.5 \times 6.5 \times 6.5 = 143.79 \text{ cubic inches.}$$
$$A = 3.1416\,d^2 = 3.1416 \times 6.5^2 = 3.1416 \times 6.5 \times 6.5$$
$$= 132.73 \text{ square inches.}$$

The volume of a sphere is 64 cubic inches. Find its radius.

$$r = 0.6204\sqrt[3]{64} = 0.6204 \times 4 = 2.4816 \text{ inches.}$$

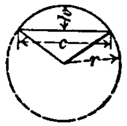


Fig. 55.

Spherical Sector. — A spherical sector is a part of a sphere bounded by a section of the spherical surface and a cone, having its vertex at the center of the sphere, as shown in Fig. 55. Let V = volume of sector, A = total area of conical and spherical surface, and c, h, and r be the dimensions indicated in the illustration. Then:

$$V = \frac{2\pi r^2 h}{3} = 2.0944\,r^2 h.$$
$$A = 3.1416\,r\,(2h + \tfrac{1}{2}c).$$
$$c = 2\sqrt{h\,(2r - h)}.$$

Find the volume of a sector of a sphere 6 inches in diameter, the height h of the sector being 1.5 inch. Also find length of chord c. — Here $r = 3$, and $h = 1.5$.

$$V = 2.0944\,r^2 h = 2.0944 \times 3^2 \times 1.5$$
$$= 2.0944 \times 9 \times 1.5 = 28.27 \text{ cubic inches.}$$
$$c = 2\sqrt{h\,(2r - h)} = 2\sqrt{1.5\,(2 \times 3 - 1.5)}$$
$$= 2\sqrt{6.75} = 2 \times 2.598 = 5.196 \text{ square inches.}$$

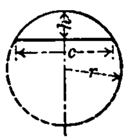


Fig. 56.

Spherical Segment. — A spherical segment is a part of a sphere bounded by a portion of the spherical surface and a plane circular base, as shown in Fig. 56. Let V = volume of segment, A = area of spherical surface of segment, and c, h, and r be the dimensions shown in Fig. 56. Then:

$$V = 3.1416\,h^2\left(r - \frac{h}{3}\right) = 3.1416\,h\left(\frac{c^2}{8} + \frac{h^2}{6}\right).$$
$$A = 2\pi r h = 6.2832\,rh = 3.1416\left(\frac{c^2}{4} + h^2\right).$$
$$c = 2\sqrt{h\,(2r - h)}; \quad r = \frac{c^2 + 4h^2}{8h}.$$

A segment of a sphere has the following dimensions: h = 2 inches; c = 5 inches. Find the volume V and the radius of the sphere of which the segment is a part.

$$V = 3.1416 \times 2 \times \left(\frac{5^2}{8} + \frac{2^2}{6}\right) = 6.2832 \times \left(\frac{25}{8} + \frac{4}{6}\right)$$
$$= 6.2832 \times 3.792 = 23.825 \text{ cubic inches.}$$
$$r = \frac{5^2 + 4 \times 2^2}{8 \times 2} = \frac{25 + 16}{16} = \frac{41}{16} = 2\tfrac{9}{16} \text{ inches.}$$

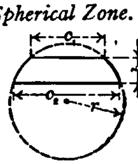


Fig. 57.

Spherical Zone. — A spherical zone is bounded by a part of a spherical surface, and by two parallel circular bases, as shown in Fig. 57, where c_1 and c_2 are the diameters of the circular bases of the zone, and h its height.

Let V = volume of zone, A = area of spherical surface, and c_1, c_2, h, and r be the dimensions shown in Fig. 57. Then:

$$V = 0.5236\,h\left(\frac{3c_1^2}{4} + \frac{3c_2^2}{4} + h^2\right).$$
$$A = 2\pi r h = 6.2832\,rh.$$
$$r = \sqrt{\frac{c_2^2}{4} + \left(\frac{c_2^2 - c_1^2 - 4h^2}{8h}\right)^2}.$$

In a spherical zone, let $c_1 = 3$; $c_2 = 4$; and $h = 1.5$ inch. Find the volume.

$$V = 0.5236 \times 1.5 \times \left(\frac{3 \times 3^2}{4} + \frac{3 \times 4^2}{4} + 1.5^2\right)$$
$$= 0.5236 \times 1.5 \times \left(\frac{27}{4} + \frac{48}{4} + 2.25\right)$$
$$= 0.5236 \times 1.5 \times 21 = 16.493 \text{ cubic inches.}$$

If a plane parallel with the end faces and passing through the center of the sphere intersects the zone, consider the zone as two zones, one zone being on each side of the center. Calculate the volume of each, and add these to find the total volume.

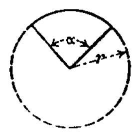


Fig. 58.

Spherical Wedge. — A spherical wedge is bounded by part of a spherical surface and two plane surfaces intersecting each other at the center of the sphere, as shown in Fig. 58. Let V = volume of wedge, A = area of spherical surface, α = center angle in degrees, and r = radius of sphere. Then:

$$V = \frac{\alpha}{360} \times \frac{4\pi r^3}{3} = 0.0116\,\alpha r^3.$$
$$A = \frac{\alpha}{360} \times 4\pi r^2 = 0.0349\,\alpha r^2.$$

Find the area of the spherical surface and the volume of a wedge of a sphere. The diameter of the sphere is 4 inches, and the center angle α is 45 degrees.

$$V = 0.0116 \times 45 \times 2^3 = 0.0116 \times 45 \times 8 = 4.176 \text{ cu. in.}$$
$$A = 0.0349 \times 45 \times 2^2 = 0.0349 \times 45 \times 4 = 6.282 \text{ sq. in.}$$

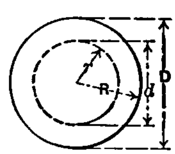


Fig. 59.

Hollow Sphere. — A hollow sphere is the shell formed between the surfaces of two spheres with the same center but different radii, as shown in Fig. 59. Let V = volume of shell of hollow sphere, and D, d, R, and r be the dimensions shown in the illustration. Then:

$$V = \frac{4\pi}{3}(R^3 - r^3) = 4.1888\,(R^3 - r^3)$$
$$= \frac{\pi}{6}(D^3 - d^3) = 0.5236\,(D^3 - d^3).$$

Find the volume of a hollow sphere, 8 inches in outside diameter, with a thickness of material of 1.5 inch.

Here $R = 4$; $r = 4 - 1.5 = 2.5$.

$$V = 4.1888\,(4^3 - 2.5^3) = 4.1888\,(64 - 15.625)$$
$$= 4.1888 \times 48.375 = 202.63 \text{ cubic inches.}$$

Ellipsoid. — A solid body of such shape that all sections through its center are ellipses is called an *ellipsoid*, as shown in Fig. 60. Let V = volume; a, b, and c are half the axes in three planes at right angles to each other. Then:

$$V = \frac{4\pi}{3} abc = 4.1888\ abc.$$

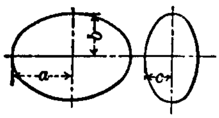


Fig. 60.

Spheroid. — If the ellipsoid is formed by an ellipse rotating about one of its axes, say, a, then b and c will be equal, and the body is called an *ellipsoid of revolution* or a *spheroid*. In this case:

$$V = 4.1888\,ab^2, \quad \text{and} \quad A = \frac{4\pi}{\sqrt{2}} b\sqrt{a^2 + b^2},$$

in which A = area of surface of spheroid.

Find the volume and area of surface of a spheroid in which a = 5, and $b = c$ = 1.5 inch.

$$V = 4.1888 \times 5 \times 1.5^2 = 4.1888 \times 5 \times 2.25$$
$$= 47.124 \text{ cubic inches.}$$

$$A = \frac{4 \times 3.1416}{\sqrt{2}} \times 1.5 \times \sqrt{5^2 + 1.5^2}$$
$$= \frac{4 \times 3.1416}{1.414} \times 1.5 \times 5.22 = 69.57 \text{ square inches.}$$

Paraboloid. — A paraboloid is formed by the rotation of a parabola about its own axis, as shown in Fig. 61. Let V = volume enclosed by paraboloidal surface and a plane surface at right angles to the axis of the paraboloid; r, h, and d are dimensions shown in the illustration. Then:

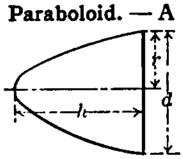


Fig. 61.

$$V = \frac{\pi}{2} r^2h = 1.5708\,r^2h = \frac{\pi}{8} d^2h = 0.3927\,d^2h.$$

Find the volume of a paraboloid in which h = 12 and d = 5 inches.

$$V = 0.3927\,d^2h = 0.3927 \times 5^2 \times 12 = 0.3927 \times 25 \times 12$$
$$= 0.3927 \times 300 = 117.81 \text{ cubic inches.}$$

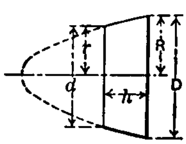


Fig. 62.

Paraboloidal Segment. — A paraboloidal segment is formed by a part of a paraboloidal surface and two plane surfaces at right angles to the axis of the paraboloid. Let V = volume of segment, the other dimensions being as shown in Fig. 62. Then:

$$V = \frac{\pi}{2} h(R^2 + r^2) = 1.5708\,h(R^2 + r^2)$$
$$= \frac{\pi}{8} h(D^2 + d^2) = 0.3927\,h(D^2 + d^2).$$

Find the volume of a segment of a paraboloid in which D = 5 inches, d = 3 inches, and h = 6 inches.

$$V = 0.3927\,h(D^2 + d^2) = 0.3927 \times 6 \times (5^2 + 3^2)$$
$$= 0.3927 \times 6 \times (25 + 9) = 0.3927 \times 6 \times 34$$
$$= 80.11 \text{ cubic inches.}$$

Relation between Volume of Certain Solids. — An interesting relation exists between the volumes of a cone, a paraboloid, a sphere, and a cylinder. If d = base diameter and height of a cone, a paraboloid, and a cylinder, and the diameter of a sphere, as shown in Fig. 63, then the volumes of these bodies are to each other as below.

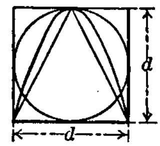


Fig. 63.

Cone : paraboloid : sphere : cylinder = $\frac{1}{3} : \frac{1}{2} : \frac{2}{3} : 1$.

Assume, as an example, that the diameter of the base of a cone, paraboloid, and cylinder is 2 inches, that the height is 2 inches, and that the diameter of a sphere is 2 inches. Then the volumes, written in formula form, are as below:

$$\overset{\text{Cone}}{\frac{3.1416 \times 2^2 \times 2}{12}} : \overset{\text{Paraboloid}}{\frac{3.1416 \times 2^2 \times 2}{8}} : \overset{\text{Sphere}}{\frac{3.1416 \times 2^3}{6}} :$$

$$\overset{\text{Cylinder}}{\frac{3.1416 \times 2^2 \times 2}{4}} = \frac{1}{3} : \frac{1}{2} : \frac{2}{3} : 1.$$

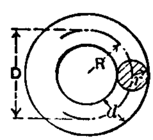


Fig. 64.

Torus. — The torus or ring is a circular or ring-shaped body having a circular cross-section, as shown in Fig. 64. Let V = volume of torus; A = area of its surface; D = its mean diameter and R = its mean radius; and d = diameter, and r = radius of cross-section. Then:

$$V = 2\pi^2Rr^2 = 19.739\,Rr^2 = \frac{\pi^2}{4} Dd^2 = 2.4674\,Dd^2.$$
$$A = 4\pi^2 Rr = 39.478\,Rr = \pi^2Dd = 9.8696\,Dd.$$

Find the volume and area of surface of a torus in which d = 1.5 and D = 5 inches.

$$V = 2.4674 \times 5 \times 1.5^2 = 2.4674 \times 5 \times 2.25 = 27.76 \text{ cu. in.}$$
$$A = 9.8696 \times 5 \times 1.5 = 74.022 \text{ square inches.}$$

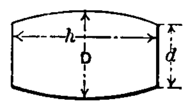


Fig. 65.

Barrel. — The exact volume of a barrel-shaped body cannot be obtained directly by a formula. The approximate volume V can be obtained, however, as below:

If the sides are bent to the arc of a circle:

$$V = \tfrac{1}{12}\pi h(2D^2 + d^2) = 0.262\,h(2D^2 + d^2).$$

If the sides are bent to the arc of a parabola:

$$V = 0.209\,h(2D^2 + Dd + \tfrac{3}{4}d^2).$$

Find the approximate contents of a barrel, the inside dimensions of which are D = 24 inches; d = 20 inches; h = 48 inches.

$$V = 0.262\,h(2D^2 + d^2) = 0.262 \times 48 \times (2 \times 24^2 + 20^2)$$
$$= 0.262 \times 48 \times (1152 + 400) = 0.262 \times 48 \times 1552$$
$$= 19{,}518 \text{ cubic inches.}$$

Pappus or Guldinus Rules. — By means of these rules, the area of any surface of revolution and the volume of any solid of revolution may be found.

Rule for Surfaces. — The area of the surface swept out by the revolution of a line ABC (see Fig. 66) about the axis DE equals the length of the line multiplied by the length of the path of its center of gravity, P.

Approximate Method. — If the line is of such a shape that it is difficult to determine its center of gravity, then the line may be divided into a number of short sections, each of which may be considered as a straight line, and the approximate areas swept out by these different sections, as computed by the rule given, may be added to find the total area. The line must lie wholly on one side of the axis of revolution and must be in the same plane.

Rule for Solids. — The volume of a solid body formed by the revolution of a surface $FGHJ$ about axis KL equals the area of the surface multiplied by the length of the path of its center of gravity. The surface must lie wholly on one side of the axis of revolution and in the same plane.

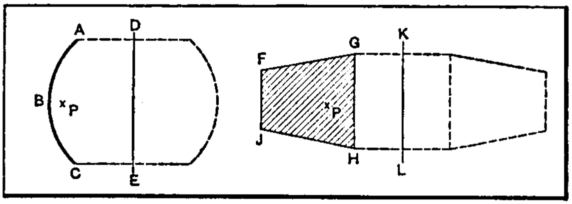


Fig. 66. Principle of Pappus Rule

By means of these rules the area and volume of a cylindrical ring or torus may be found, as an example. The torus is formed by a circle AB, Fig. 67, being rotated about axis CD. The center of gravity of the circle is at its center. Hence, with the dimensions given in the illustration, the length of the path of the center of gravity of the circle is $3.1416 \times 10 = 31.416$ inches. This multiplied by the length of the circumference of the circle, which is $3.1416 \times 3 = 9.4248$ inches, equals:

$$31.416 \times 9.4248 = 296.089 \text{ square inches,}$$

which is the area of the torus.

The volume equals the area of the circle, which is $0.7854 \times 9 = 7.0686$ square inches, multiplied by the path of the center of gravity, which is 31.416, as before; hence,

$$\text{volume} = 7.0686 \times 31.416 = 222.067 \text{ cubic inches.}$$

Application to Surface of Irregular Body.—Fig. 68 is shown in order to give an example of the approximate method based on Guldinus' rule, that can be used for finding the area of a symmetrical body. In the illustration, the dimensions in common fractions are the known dimensions; those in decimals are found by actual measurements on a figure drawn to scale. The method for finding the area is as follows: First separate such areas as are cylindrical, conical, or spherical, as these can be found by exact formulas. In the illustration, $ABCD$ is a cylinder, the area of the surface of which can be easily found. The top area EF is simply a circular area, and can thus be computed separately. The remainder of the surface generated by rotating line AF about the axis GH is found by the approximate method explained in a previous section. From point A, set off equal distances on line AF. In the present case, each division indicated is ⅛ inch long. From the central or middle point of each of these parts, draw a line at right angles to the axis of rotation GH, measure the length of these lines or diameters (the length of each is given in decimals), add all these lengths together and multiply the sum by the length of one division set off on line AF (in this case, ⅛ inch), and multiply this product by π. This gives the approximate area of the surface of revolution.

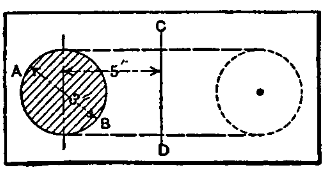


Fig. 67

In setting off divisions ⅛ inch long along line AF, the last division does not reach exactly to point F, but only to a point 0.03 inch below it. The part 0.03 inch high at the top of the cup can be considered as a cylinder of ½ inch diameter and 0.03 inch height, the area of the cylindrical surface of which is easily computed. By adding the various surfaces together, the total surface of the cup is found as below:

Cylinder, 1⅝ in. in diameter, 0.41 in. high..	2.093 sq. in.
Circle, ½ in. in diameter..................	0.196 sq. in.
Cylinder, ½ in. in diameter, 0.03 in. high..	0.047 sq. in.
Irregular surface..........................	3.868 sq. in.
Total..............................	6.204 sq. in.

Accuracy Obtainable. — In testing the accuracy of this method, the surface of a half-sphere 6 inches in diameter was found by dividing a great circle into parts ¼ inch long and finding the area by this graphical method. The exact area of this half-sphere is 56.55 square inches. The

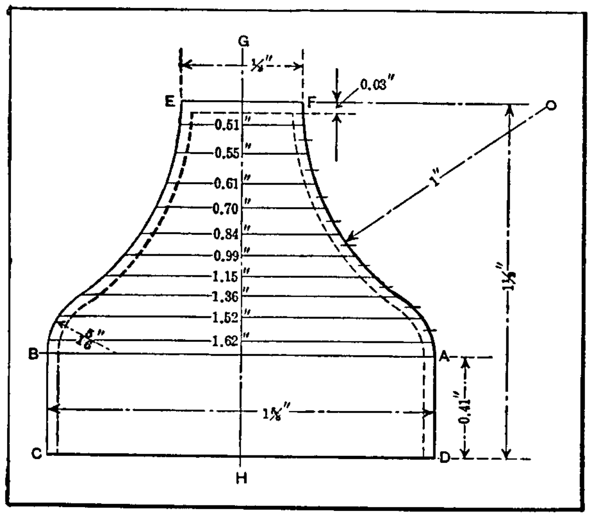
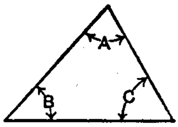


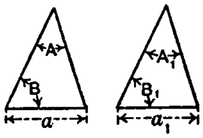
Fig. 68. Application of Pappus Rule

graphical method, using only an ordinary draftsman's scale, compass, and triangle, gave an area of 55.28 square inches, which may be considered fairly accurate for an approximate method. The half-sphere was selected for the test case because it presents one of the difficult shapes for the application of this method, as the curved outline merges from a vertical into a horizontal direction; the surface being convex throughout, the errors are also likely to be all in one direction. A more accurate result may be expected when part of the surface is convex and part concave.



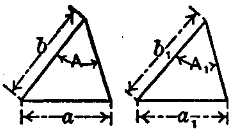
1. The sum of the three angles in a triangle always equals 180 degrees. Hence, if two angles are known, the third angle can always be found.

$$A + B + C = 180^\circ.$$
$$A = 180^\circ - (B + C).$$
$$B = 180^\circ - (A + C).$$
$$C = 180^\circ - (A + B).$$



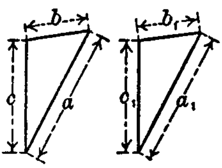
2. If one side and two angles in one triangle are equal to one side and similarly located angles in another triangle, then the remaining two sides and angle are also equal.

If $a = a_1$, $A = A_1$, and $B = B_1$, then the two other sides and the remaining angle are also equal.



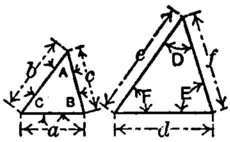
3. If two sides and one angle in one triangle are equal to two sides and a similarly located angle in another triangle, then the remaining side and angles are also equal; provided, however, that the triangles must either be both acute-angled triangles, both obtuse-angled triangles, or both right-angled triangles.

If $a = a_1$, $b = b_1$, and $A = A_1$, then the remaining side and angles are also equal, the triangles in this case being both acute-angled.



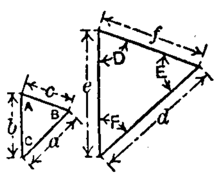
4. If the three sides in one triangle are equal to the three sides of another triangle, then the angles in the two triangles are also equal.

If $a = a_1$, $b = b_1$, and $c = c_1$, then the angles between the respective sides are also equal.



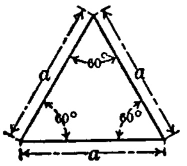
5. If the three sides of one triangle are proportional to corresponding sides in another triangle, then the triangles are called *similar*, and the angles in the one are equal to the angles in the other.

If $a : b : c = d : e : f$,
then $A = D$, $B = E$, and $C = F$.



6. If the angles in one triangle are equal to the angles of another triangle, then the triangles are similar and their corresponding sides are proportional.

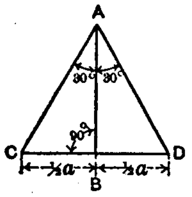
If $A = D$, $B = E$, and $C = F$, then $a : b : c = d : e : f$.



7. If the three sides in a triangle are equal — that is, if the triangle is *equilateral* — then the three angles are also equal.

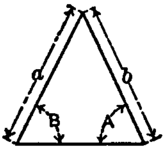
Each of the three equal angles in an equilateral triangle is 60 degrees.

If the three angles in a triangle are equal, then the three sides are also equal.



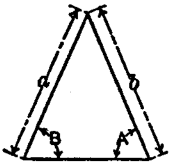
8. A line which in an equilateral triangle bisects or divides any of the angles into two equal parts, bisects also the side opposite the angle and is at right angles to it.

If line *AB* divides angle *CAD* into two equal parts, it also divides line *CD* into two equal parts and is at right angles to it.



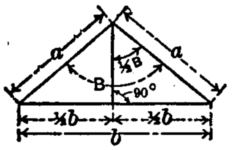
9. If two sides in a triangle are equal — that is, if the triangle is an *isosceles* triangle — then the angles opposite these sides are also equal.

If side *a* equals side *b*, then angle *A* equals angle *B*.

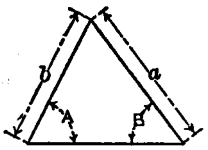


10. If two angles in a triangle are equal, then the sides opposite these angles are also equal.

If angles *A* and *B* are equal, then side *a* equals side *b*.

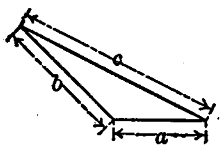


11. In an isosceles triangle, if a straight line is drawn from the point where the two equal sides meet, so that it bisects the third side or base of the triangle, then it also bisects the angle between the equal sides and is perpendicular to the base.



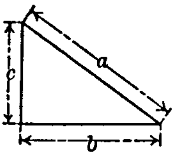
12. In every triangle, that angle is greater which is opposite a longer side. — In every triangle, that side is greater which is opposite a greater angle.

If *a* is longer than *b*, then angle *A* is greater than *B*. If angle *A* is greater than *B*, then side *a* is longer than *b*.



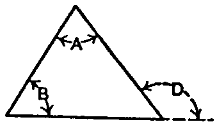
13. In every triangle, the sum of the lengths of two sides is always greater than the length of the third.

Side *a* + side *b* is always greater than side *c*.



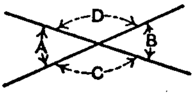
14. In a right-angled triangle, the square of the hypotenuse or the side opposite the right angle is equal to the sum of the squares on the two sides which form the right angle.

$$a^2 = b^2 + c^2.$$



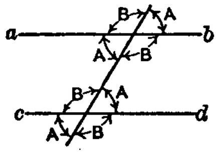
15. If one side of a triangle is produced, then the exterior angle is equal to the sum of the two interior opposite angles.

Angle $D =$ angle $A +$ angle B.



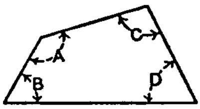
16. If two lines intersect, then the opposite angles formed by the intersecting lines are equal.

Angle A = angle B.
Angle C = angle D.



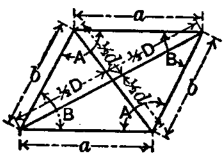
17. If a line intersects two parallel lines, then the corresponding angles formed by the intersecting line and the parallel lines are equal.

Lines ab and cd are parallel. Then all the angles designated A are equal, and all those designated B are equal.

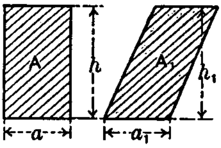


18. In any figure having four sides, the sum of the interior angles equals 360 degrees.

$$A + B + C + D = 360 \text{ degrees}.$$

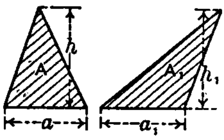


19. The sides which are opposite each other in a parallelogram are equal; the angles which are opposite each other are equal; the diagonal divides it into two equal parts. If two diagonals are drawn, they bisect each other.



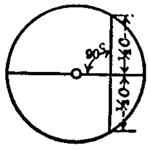
20. The areas of two parallelograms which have equal base and equal height are equal.

If $a = a_1$ and $h = h_1$, then
area A = area A_1.



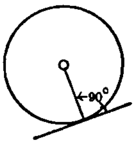
21. The areas of triangles having equal base and equal height are equal.

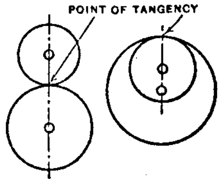
If $a = a_1$ and $h = h_1$, then
area A = area A_1.



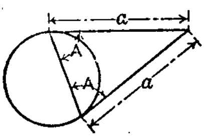
22. If a diameter of a circle is at right angles to a chord, then it bisects or divides the chord into two equal parts.

23. If a line is tangent to a circle, then it is also at right angles to a line drawn from the center of the circle to the point of tangency — that is, to a radial line through the point of tangency.

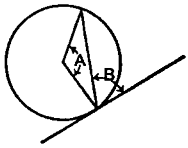




24. If two circles are tangent to each other, then the straight line which passes through the centers of the two circles must also pass through the point of tangency.

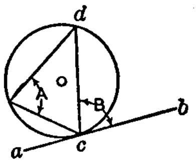


25. If from a point without a circle tangents are drawn to a circle, the two tangents are equal and make equal angles with the chord joining the points of tangency.



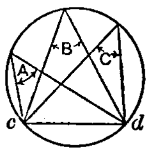
26. The angle between a tangent and a chord drawn from the point of tangency equals one-half the angle at the center subtended by the chord.

Angle $B = \frac{1}{2}$ angle A.



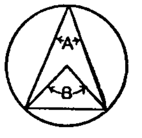
27. The angle between a tangent and a chord drawn from the point of tangency equals the angle at the periphery subtended by the chord.

Angle B, between tangent ab and chord cd, equals angle A subtended at the periphery by chord cd.



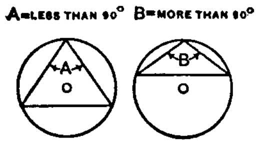
28. All angles having their vertex at the periphery of a circle and subtended by the same chord are equal.

Angles A, B, and C, all subtended by chord cd, are equal.

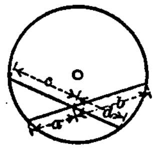


29. If an angle at the circumference of a circle, between two chords, is subtended by the same arc as the angle at the center, between two radii, then the angle at the circumference is equal to one-half of the angle at the center.

Angle $A = \frac{1}{2}$ angle B.

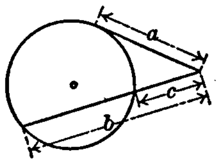


30. An angle subtended by a chord in a circular segment larger than one-half the circle is an acute angle — an angle less than 90 degrees. An angle subtended by a chord in a circular segment less than one-half the circle is an obtuse angle — an angle greater than 90 degrees.



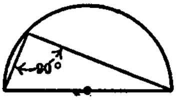
31. If two chords intersect each other in a circle, then the rectangle of the segments of the one equals the rectangle of the segments of the other.

$$a \times b = c \times d.$$



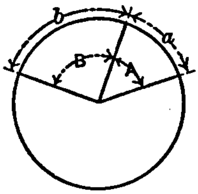
32. If from a point outside of a circle two lines are drawn, one of which intersects the circle while the other is tangent to it, then the rectangle contained by the total length of the intersecting line, and that part of it which is between the outside point and the periphery, equals the square of the tangent.

$$a^2 = b \times c.$$



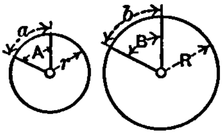
33. If a triangle is inscribed in a semi-circle, the angle opposite the diameter is a right (90-degree) angle.

All angles at the periphery of a circle, subtended by the diameter, are right (90-degree) angles.



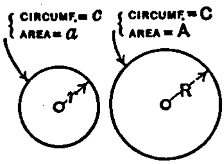
34. The length of circular arcs of the same circle are proportional to the corresponding angles at the center.

$$A : B = a : b.$$



35. The length of circular arcs having the same center angle are proportional to the length of the radii.

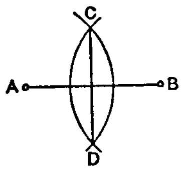
If $A = B$, then $a : b = r : R$.



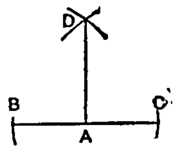
36. The circumferences of two circles are proportional to their radii.

The areas of two circles are proportional to the squares of their radii.

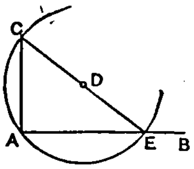
$$c : C = r : R.$$
$$a : A = r^2 : R^2.$$



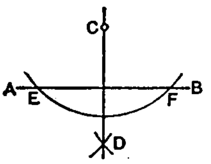
1. To divide a line *AB* into two equal parts: With the ends *A* and *B* as centers and a radius greater than one-half the line, draw circular arcs. Through the intersections *C* and *D*, draw line *CD*. This line divides *AB* into two equal parts and is also perpendicular to *AB*.



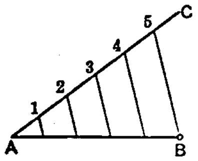
2. To draw a perpendicular to a straight line from a point *A* on that line: With *A* as a center and with any radius, draw circular arcs intersecting the given line at *B* and *C*. Then, with *B* and *C* as centers and a radius longer than *AB*, draw circular arcs intersecting at *D*. Line *DA* is perpendicular to *BC* at *A*.



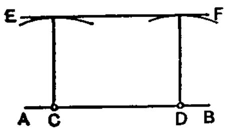
3. To draw a perpendicular line from a point *A* at the end of a line *AB*: With any point *D*, outside of the line *AB*, as a center, and with *AD* as a radius, draw a circular arc intersecting *AB* at *E*. Draw a line through *E* and *D* intersecting the arc at *C*; then join *AC*. This line is the required perpendicular.



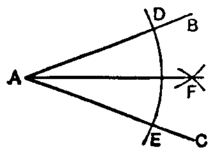
4. To draw a perpendicular to a line *AB* from a point *C* at a distance from it: With *C* as a center, draw a circular arc intersecting the given line at *E* and *F*. With *E* and *F* as centers, draw circular arcs with a radius longer than one-half the distance between *E* and *F*. These arcs intersect at *D*. Line *CD* is the required perpendicular.



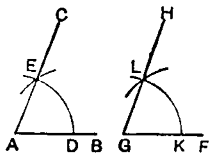
5. To divide a straight line *AB* into a number of equal parts: Let it be required to divide *AB* into five equal parts. Draw line *AC* at an angle with *AB*. Set off on *AC* five equal parts of any convenient length. Draw *B* 5 and then draw lines parallel with *B* 5 through the other division points on *AC*. The points where these lines intersect *AB* are the required division points.



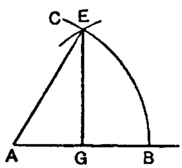
6. To draw a straight line parallel to a given line *AB*, at a given distance from it: With any points *C* and *D* on *AB* as centers, draw circular arcs with the given distance as radius. Line *EF*, drawn to touch the circular arcs, is the required parallel line.



7. To bisect or divide an angle *BAC* into two equal parts: With *A* as a center and any radius, draw arc *DE*. With *D* and *E* as centers, and a radius greater than one-half *DE*, draw circular arcs intersecting at *F*. Line *AF* divides the angle into two equal parts.

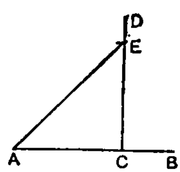


8. To draw an angle upon a line *AB*, equal to a given angle *FGH*: With point *G* as a center and with any radius, draw arc *KL*. With *A* as a center and with the same radius, draw arc *DE*. Make arc *DE* equal to *KL* and draw *AC* through *E*. Angle *BAC* then equals *FGH*.

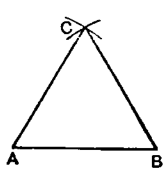


9. To lay out a 60-degree angle: With *A* as a center and any radius, draw an arc *BC*. With point *B* as a center and *AB* as a radius, draw an arc intersecting at *E* the arc just drawn. *EAB* is a 60-degree angle.

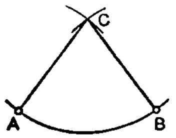
A 30-degree angle may be obtained either by dividing a 60-degree angle into two equal parts, or by drawing a line *EG* perpendicular to *AB*. Angle *AEG* is then 30 degrees.



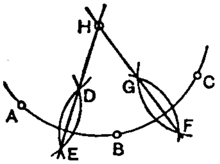
10. To draw a 45-degree angle: From point *A* on line *AB*, set off a distance *AC*. Draw the perpendicular *DC* and set off a distance *CE* equal to *AC*. Draw *AE*. Angle *EAC* is a 45-degree angle.



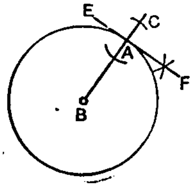
11. To draw an equilateral triangle, the length of the sides of which equals *AB*: With *A* and *B* as centers and *AB* as radius, draw circular arcs intersecting at *C*. Draw *AC* and *BC*. Then *ABC* is an equilateral triangle.



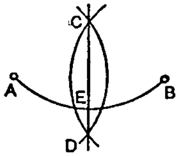
12. To draw a circular arc with a given radius through two given points *A* and *B*: With *A* and *B* as centers, and the given radius as radius, draw circular arcs intersecting at *C*. With *C* as a center, and the same radius, draw a circular arc through *A* and *B*.



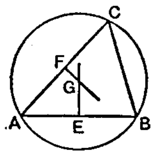
13. To find the center of a circle or of an arc of a circle: Select three points on the periphery of the circle, as *A*, *B*, and *C*. With each of these points as a center and the same radius, describe arcs intersecting each other. Through the points of intersection, draw lines *DE* and *FG*. Point *H* where lines intersect is the center of the circle.



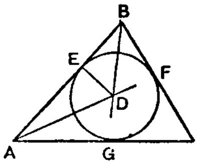
14. To draw a tangent to a circle from a given point on the circumference: Through the point of tangency *A*, draw a radial line *BC*. At point *A*, draw a line *EF* at right angles to *BC*. This line is the required tangent.



15. To divide a circular arc *AB* into two equal parts: With *A* and *B* as centers, and a radius larger than half the distance between *A* and *B*, draw circular arcs intersecting at *C* and *D*. Line *CD* divides arc *AB* into two equal parts at *E*.



16. To describe a circle about a triangle: Divide the sides *AB* and *AC* into two equal parts, and from the division points *E* and *F* draw lines at right angles to the sides. These lines intersect at *G*. With *G* as a center and *GA* as a radius, draw circle *ABC*.



17. To inscribe a circle in a triangle: Bisect two of the angles, *A* and *B*, by lines intersecting at *D*. From *D* draw a line *DE* perpendicular to one of the sides, and, with *DE* as a radius, draw circle *EFG*.

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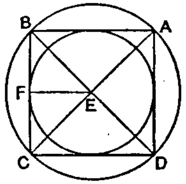
18. To describe a circle about a square and to inscribe a circle in a square: The center of both the circumscribed and inscribed circle is located at the point *E*, where the two diagonals of the square intersect. The radius of the circumscribed circle is *AE*, and of the inscribed circle, *EF*.



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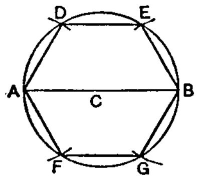
19. To inscribe a hexagon in a circle: Draw a diameter *AB*. With *A* and *B* as centers and with the radius of the circle as radius, describe circular arcs intersecting the given circle at *D*, *E*, *F*, and *G*. Draw lines *AD*, *DE*, etc., forming the required hexagon.



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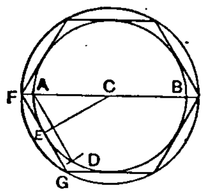
20. To describe a hexagon about a circle: Draw a diameter *AB*, and with *A* as a center and the radius of the circle as radius, cut the circumference of the given circle at *D*. Join *AD* and bisect it with radius *CE*. Through *E*, draw *FG* parallel to *AD* and intersecting line *AB* at *F*. With *C* as a center and *CF* as radius, draw a circle. Within this circle inscribe the hexagon, as in the preceding problem.



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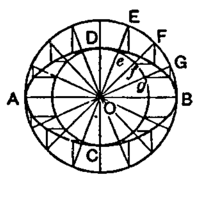
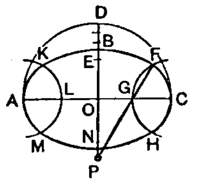
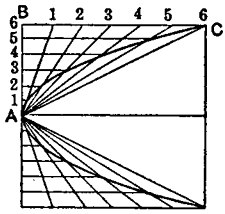
21. To describe an ellipse with the given axes *AB* and *CD*: Describe circles with *O* as a center and *AB* and *CD* as diameters. From a number of points, *E*, *F*, *G*, etc., on the outer circle, draw radii intersecting the inner circle at *e*, *f*, *g*. From *E*, *F*, and *G*, draw lines perpendicular to *AB*, and from *e*, *f*, and *g* draw lines parallel to *AB*. The intersections of these perpendicular and parallel lines are points on the curve of the ellipse.

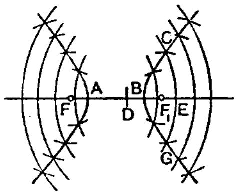




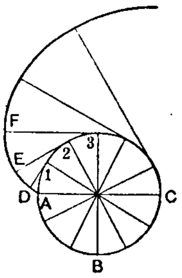
22. To construct an approximate ellipse by circular arcs: Let AC be the major axis and BN the minor. Draw half circle ADC with O as a center. Divide BD into three equal parts and set off BE equal to one of these parts. With A and C as centers and OE as radius, describe circular arcs KLM and FGH; with G and L as centers, and the same radius, describe arcs FCH and KAM. Through F and G, draw line FP, and with P as a center draw the arc FBK. Arc HNM is drawn in the same manner.



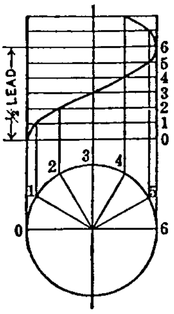
23. To construct a parabola: Divide line AB into a number of equal parts and divide BC into the same number of parts. From the division points on AB, draw horizontal lines. From the division points on BC, draw lines to point A. The points of intersection between lines drawn from points numbered alike are points on the parabola.



24. To construct a hyperbola: From focus F lay off a distance FD equal to the transverse axis, or the distance AB between the two branches of the curve. With F as a center and any distance FE greater than FB as a radius, describe a circular arc. Then with F_1 as a center and DE as a radius, describe arcs intersecting at C and G the arc just described. C and G are points on the hyperbola. Any number of points can be found in a similar manner.



25. To construct an involute: Divide the circumference of the base circle ABC into a number of equal parts. Through the division points 1, 2, 3, etc., draw tangents to the circle and make the lengths $D1$, $E2$, $F3$, etc., of these tangents equal to the actual length of the arcs $A1$, $A2$, $A3$, etc.



26. To construct a helix: Divide half the circumference of the cylinder on the surface of which the helix is to be described into a number of equal parts. Divide half the lead of the helix into the same number of equal parts. From the division points on the circle representing the cylinder, draw vertical lines, and from the division points on the lead, draw horizontal lines as shown. The intersections between lines numbered alike are points on the helix.

§ 35. The Parabola. Derivation of the Canonical Equation of the Parabola

101. *A parabola is the locus of points whose distance from a fixed point (called the focus) in the plane is equal to their distance from a fixed straight line (called the directrix* and assumed not to pass through the focus).

It is customary to denote the focus of a parabola by the letter F, and the distance from the focus to the directrix by the letter p. The quantity p is called *the parameter* of a parabola. The curve is shown in Fig. 61 (the details of the drawing are fully explained in the next few articles).

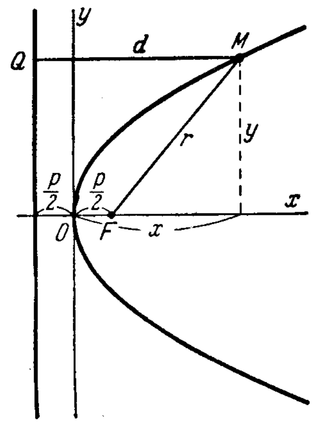


Fig. 61.

Note. In accordance with Art. 100, *a parabola is said to have eccentricity* $\varepsilon = 1$.

102. Let there be given a parabola (we assume that the parameter p is also given). Let us attach to the plane a rectangular cartesian coordinate system, whose axes are specially chosen with respect to the given parabola; namely, let the x-axis be drawn through the focus perpendicular to the directrix, the direction from the directrix to the focus adopted as positive on the x-axis, and the origin placed midway between the focus and the directrix (Fig. 61). We now proceed to derive the equation of the given parabola in this coordinate system.

Take an arbitrary point M in the plane and designate its coordinates as x and y. Let r denote the distance of the point M from the focus ($r = FM$), and d the distance of the point M from the directrix. The point M will lie on the given parabola if, and only if,

$$r = d. \tag{1}$$

In order to obtain the desired equation, it is necessary to express the variables r and d in terms of the current coordinates x, y and to substitute these expressions in (1). Note that the coordinates of the focus F are $\left(\frac{p}{2}, 0\right)$; bearing this in mind and using formula (2) of Art. 18, we find

$$r = \sqrt{\left(x - \frac{p}{2}\right)^2 + y^2}. \tag{2}$$

Denote by Q the foot of the perpendicular dropped from M upon the directrix. The coordinates of the point Q will clearly be $\left(-\frac{p}{2}, y\right)$; hence, by formula (2) of Art. 18, we obtain

$$d = MQ = \sqrt{\left(x+\frac{p}{2}\right)^2 + (y-y)^2} = x + \frac{p}{2} \tag{3}$$

(on extracting the root, we take $x+\frac{p}{2}$ with its original sign since $x+\frac{p}{2}$ is a positive number; this follows from the fact that the point $M(x, y)$ must lie on that side of the directrix where the focus is situated, that is, we must have $x > -\frac{p}{2}$, whence $x+\frac{p}{2} > 0$). Substituting expressions (2) and (3) for r and d in (1), we find

$$\sqrt{\left(x-\frac{p}{2}\right)^2 + y^2} = x + \frac{p}{2}. \tag{4}$$

The coordinates of a point $M(x, y)$ satisfy equation (4) if, and only if, the point M lies on the given parabola; accordingly, (4) is the equation of this parabola referred to the chosen coordinate system.

To reduce the equation of the parabola to a simpler form, we square both members of (4), which gives

$$x^2 - px + \frac{p^2}{4} + y^2 = x^2 + px + \frac{p^2}{4}, \tag{5}$$

or

$$y^2 = 2px. \tag{6}$$

We have derived equation (6) as a consequence of equation (4). It is easy to show that equation (4) may, in its turn, be derived as a consequence of (6). In fact, equation (5) is readily obtained from (6) by "retracing steps"; next, from (5) we get

$$\sqrt{\left(x-\frac{p}{2}\right)^2 + y^2} = \pm\left(x+\frac{p}{2}\right).$$

It remains to show that, if x, y satisfy equation (6), then the plus sign is here the only sign to choose. But this is clear since, from (6), $x = \frac{y^2}{2p}$ and, consequently, $x \geqslant 0$, so that $x+\frac{p}{2}$ is a positive number. Thus, we have come back to equation (4). Since each of equations (4) and (6) is a consequence of the other, they are equivalent. We hence conclude that *equation* (6) *is the equa-*

tion of the parabola. This equation is called the *canonical* equation of the parabola.

103. The equation $y^2 = 2px$, which represents the parabola in a certain system of rectangular cartesian coordinates is an equation of the second degree; accordingly, *the parabola is a curve of the second order.*

§ 36. Discussion of the Shape of the Parabola

104. Let us analyse the equation

$$y^2 = 2px \tag{1}$$

in order to form a clear idea of the shape of the parabola and thereby to show the correctness of its representation in Fig. 61.

Since equation (1) contains y only in an even power, the parabola represented by it is symmetrical with respect to the axis Ox. It will therefore be sufficient to investigate only the portion of the parabola which lies in the upper half-plane. This portion is represented by the equation

$$y = +\sqrt{2px}. \tag{2}$$

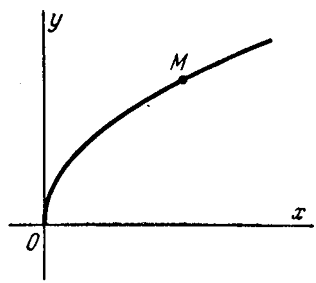


Fig. 62.

For negative values of x, equation (2) gives imaginary values of y. Consequently, no point of the parabola appears to the left of the axis Oy. For $x = 0$, we have $y = 0$. Hence the origin lies on the parabola and is its extreme "left" point. Equation (2) shows that, as x increases from zero, y continually increases. The equation also shows that, as $x \to +\infty$, $y \to +\infty$.

Thus, the variable point $M(x, y)$, which traces the portion of the parabola under consideration, moves to the "right" and "upwards", starting from the origin and receding indefinitely from both the axis Oy (to the "right") and the axis Ox ("upwards"; see Fig. 62).

Note. The following two properties of the parabola are also of importance: (1) the direction of the parabola is perpendicular to the axis Ox in the point $O(0, 0)$; (2) the portion of the parabola in the upper half-plane is convex "upwards". The graph in Fig. 62 has been drawn in accordance with these properties. Their proof will not, however, be given here, since the most natural methods for curve analysis of such kind are those furnished by the calculus.

105. Now that we have established the shape of the portion of the parabola lying in the upper half-plane, the determination of the shape of the entire parabola will present no difficulties; we have merely to reflect this portion of the curve in the axis Ox. The above-discussed Fig. 61 gives a general idea of the entire parabola represented by the equation

$$y^2 = 2px.$$

Usually the axis of symmetry of a parabola is referred to simply as its *axis* (in the case under consideration, the axis of the parabola coincides with the axis Ox). The point where a parabola cuts its axis is called *the vertex of the parabola* (in our case, the vertex is coincident with the origin). The number p, that is, the parameter of a parabola, represents the distance between the focus and the directrix. The geometric meaning of the parameter p may also be described as follows. Take some definite value of the abscissa, say $x = 1$, and find from equation (1) the corresponding values of the ordinate: $y = \pm\sqrt{2p}$. We obtain two points of the parabola, $M_1(1, +\sqrt{2p})$ and $M_2(1, -\sqrt{2p})$, symmetric with respect to the axis; the distance between these points is equal to $2\sqrt{2p}$. Thus, $2\sqrt{2p}$ is the length of the chord perpendicular to the axis and one unit of length distant from the vertex. We see that the length $(=2\sqrt{2p})$ of this chord of the parabola increases with p. Consequently, the parameter p characterises the "spread" of a parabola, provided that this "spread" is measured perpendicular to the axis at a definite distance from the vertex.

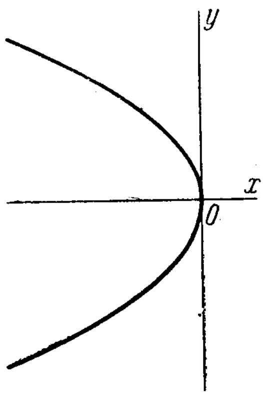


Fig. 63.

106. The equation

$$y^2 = -2px \qquad (3)$$

(where p is positive) may be reduced to the equation $y^2 = 2px$ by substituting $-x$ for x, that is, by a transformation of coordinates corresponding to a reversal of the direction of the axis Ox. Hence, the equation $y^2 = -2px$ also represents a parabola whose axis is coincident with the axis Ox and whose vertex coincides with the origin; but this parabola is situated in the left half-plane, as shown in Fig. 63.

107. By analogy with the foregoing, we may assert that each of the equations

$$x^2 = 2py, \quad x^2 = -2py$$

(where $p > 0$) represents a parabola symmetric with respect to the axis Oy, with vertex at the origin (these equations, as well as equations (1) and (3), are referred to as the canonical equations of the parabola). A parabola represented by the equation $x^2 = 2py$ is said to open upwards; a parabola represented by

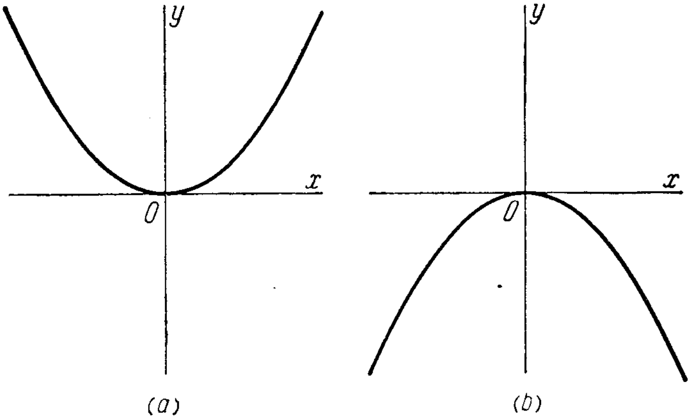
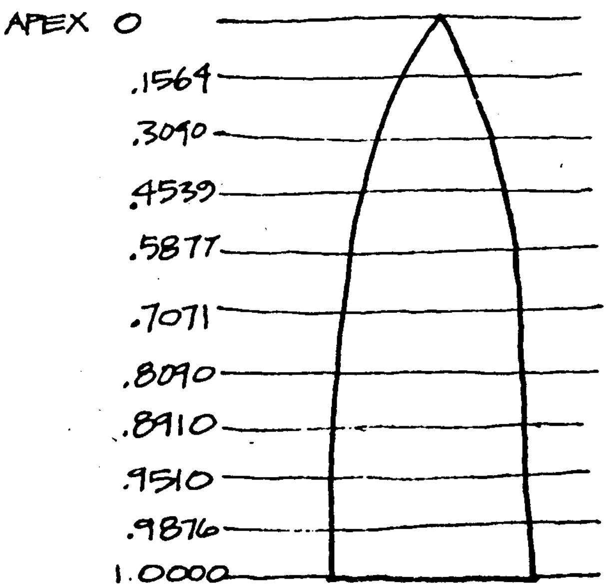


Fig. 64.

the equation $x^2 = -2py$ is said to open downwards (see Fig. 64 *a* and *b*, respectively); the use of these terms is natural and requires no further explanation.

Panel (gore) design for a hemispherical airhouse of any size:

1. *Select your diameter and find the resulting circumference (3.1416 x diameter).*
2. *Determine a maximum gore width that will go* evenly *into the circumference. This will also show the number of gores needed. Consider the available fabric widths when determining the maximum gore width.*
3. *Determine the length of each gore (circumference ÷ 4*
4. *To determine the detailed shape of the gore:*
   First, divide the gore into 10 equal sections along its length. The tenth line will be the base.
   Next, determine the width at each of these division lines by multiplying each of the following numbers times the maximum gore width found in step 2. Take the trouble to carry your figures out to 4 places and lay out your template carefully. Any error will be exagerated by the number of gores used.



Note: let a little extend below base

Triangulation is an application of the principles of trigonometry to the calculation of inaccessible lines and angles. A common occasion for its use is illustrated in Fig. 1, where the line of survey crosses a stream too wide and deep for actual measurement. Set two points A and B on line, one on each side of the stream. Estimate roughly the distance $A\,B$. Suppose the estimate is 425 ft. Set another point C, making the distance $A\,C$ equal to the estimated distance $A\,B = 425$ ft. Set the transit at A and measure the angle $B\,A\,C =$ say, 79° 00′. Next set up at the point C and measure the angle $A\,C\,B =$ say, 56° 20′. The angle $A\,B\,C$ is then determined by subtracting the sum of the angles A and C from 180°; thus, 79° 00′ + 56° 20′ = 135° 20′; 180° 00′ − 135° 20′ = 44° 40′ = the angle $A\,B\,C$. We now have a side and three angles of a triangle given, to find the other two sides $A\,B$ and $C\,B$. In trigonometry, it is demonstrated that, *in any triangle the sines of the angles are proportional to the lengths of the sides opposite to them*. In other words, $\sin A : \sin B = B\,C : A\,C$; or, $\sin A : \sin C = B\,C : A\,B$, and $\sin B : \sin C = A\,C : A\,B$.

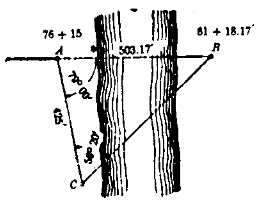


FIG. 1.

Hence, we have sin 44° 40′ : sin 56° 20′ = 425 : side $A\,B$;
sin 56° 20′ = .83228;
.83228 × 425 = 353.719;
sin 44° 40′ = .70298;
353.719 ÷ .70298 = 503.17 ft. = side $A\,B$.

Adding this distance to 76 + 15, the station of the point A, we have 81 + 18.17, the station at B.

Another case is the following: Two tangents, $A\,B$ and $C\,D$ (see Fig. 2), which are to be united by a curve, meet at some inaccessible point E. Tangents are the straight portions of a line of railroad. The angle $C\,E\,F$, which the tangents make with each other, and the distances $B\,E$ and $C\,E$ are required. Two points A and B of the tangent $A\,B$, and two points C and D of the tangent $C\,D$, being carefully located, set the transit at B, and backsighting to A, measure the angle $E\,B\,C = 21°\,45′$; set up at C, and, backsighting to D, measure the angle $E\,C\,B = 21°\,25′$. Measure the side $B\,C = 304.2$ ft.

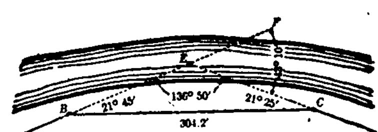


FIG. 2.

Angle $C\,E\,F$ being an exterior angle of triangle $E\,B\,C$ equals sum of $E\,B\,C$ and $E\,C\,B = 21°\,45′ + 21°\,25′ = 43°\,10′$; angle $B\,E\,C = 180° - C\,E\,F = 136°\,50′$. From trigonometry, we have:
sin 136° 50′ : sin 21° 45′ = 304.2 ft. : $C\,E$;
sin 21° 45′ = .37056;
.37056 × 304.2 = 112.724352;
sin 136° 50′ = .68412;
side $C\,E$ = 112.724352 ÷ .68412 = 164.77 ft.

Again, we find $B\,E$ by the following proportion:
sin 136° 50′ : sin 21° 25′ = 304.2 : side $B\,E$;
sin 21° 25′ = .36515;
.36515 × 304.2 = 111.07863;
sin 136° 50′ = .68412;
side $B\,E$ = 111.07863 ÷ .68412 = 162.36 ft.

A building H, Fig. 3, lies directly in the path of the line $A\,B$, which must be produced beyond H. Set a plug at B, and then turn an angle $D\,B\,C = 60°$. Set a plug at C in the line $B\,C$, at a suitable distance from B, say, 150 ft. Set up at C, and turn an angle $B\,C\,D = 60°$, and set a plug at D, 150 ft. from C. The point D will be in the prolongation of $A\,B$. Then, set up at D, and backsighting to C, turn the angle $C\,D\,D' = 120°$. $D\,D'$ will be the line required, and the distance $B\,D$ will be 150 ft., since $B\,C\,D$ is an equilateral triangle.

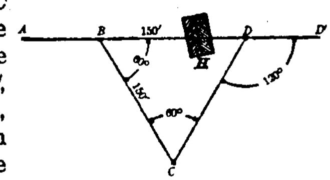


FIG. 3.

$A\,B$ and $C\,D$, Fig. 4, are tangents intersecting at some inaccessible point H. The line $A\,B$ crosses a dock $O\,P$, too wide for direct measurement, and the wharf $L\,M$. F is a point on the line $A\,B$ at the wharf crossing. It is required to find the distance $B\,H$ and the angle $F\,H\,G$. At B, an angle of 103° 30′ is turned to the left and the point E set 217′ from B = to the estimated distance $B\,F$. Setting up at E, the angle $B\,E\,F$ is found to be 39° 00′.

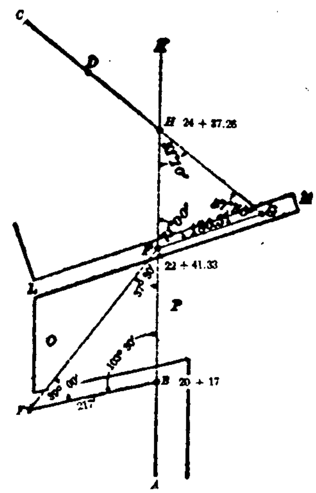


FIG. 4.

Whence, we find the angle $B\,F\,E = 180° - (103°\,30′ + 39°) = 37°\,30′$.

From trigonometry, we have
sin 37° 30′ : sin 39° 00′ = 217 ft. : side $B\,F$;
sin 39° 00′ = .62932;
.62932 × 217 = 136.56244;
sin 37° 30′ = .60876;
side $B\,F$ = 136.56244 ÷ .60876 = 224.33 ft.

Whence, we find station F to be 20 + 17 + 224.33 = 22 + 41.33. Set up at F and turn an angle $H\,F\,G = 71°\,00′$ and set up at a point G where the line $C\,D$ prolonged intersects $F\,G$. Measure the angle $F\,G\,H = 57°\,50′$, and the side $F\,G = 180.3$. The angle $F\,H\,G = 180° - (71° + 57°\,50′) = 51°\,10′$. From trigonometry we have

sin 51° 10′ : sin 57° 50′ = 180.3 : side $F\,H$.

Sin 57° 50′ = .84650; .84650 × 180.3 = 152.62395; sin 51° 10′ = .77897; side $F\,H$ = 152.62395 ÷ .77897 = 195.93 ft.; whence we find station H to be 24 + 37.26.

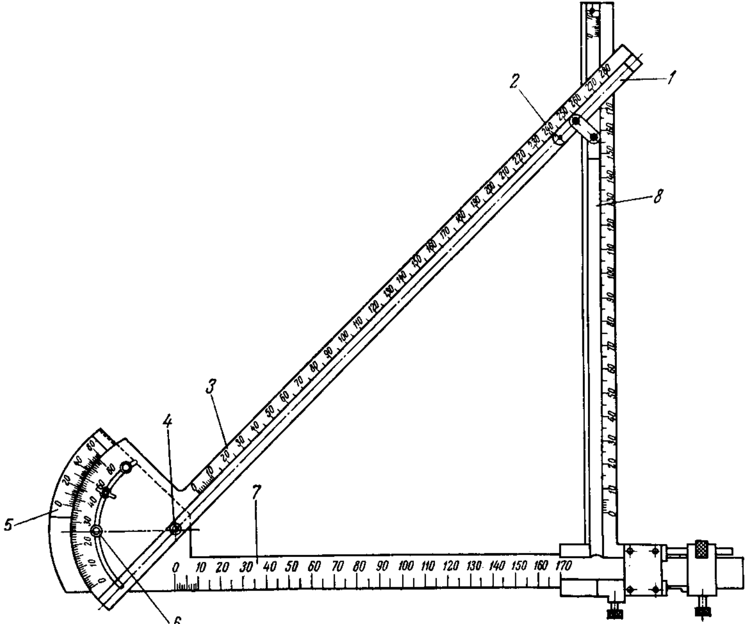


Fig. 64. Calculating triangle:

1—slide; *2*—screw; *3*—scale of hypotenuse rule; *4* and *6*—set screws *5*—vernier; *7* and *8*—cathetuses

TRIG

RIGHT-ANGLED TRIGONOMETRIC FUNCTIONS

Right-angled trigonometric functions:
The trig functions are the ratios of sides of right triangles.

Function

$$\sin \phi = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos \phi = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan \phi = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{a}{b}$$

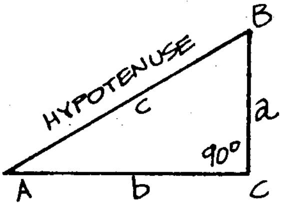
$$\cot \phi = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{b}{a}$$

$$\sec \phi = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{c}{b}$$

$$\csc \phi = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{c}{a}$$

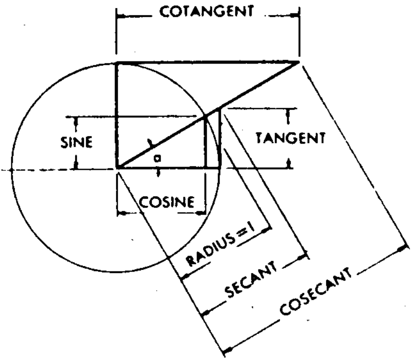
$$c^2 = a^2 + b^2$$

$$180^\circ = 90^\circ + \phi + \theta$$

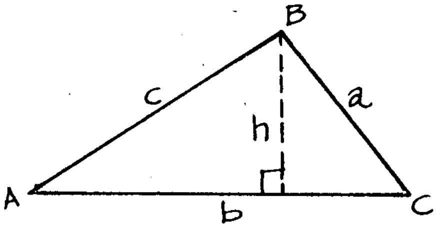


Note: See simple pocketbook Trig tables in bibliography.

The trigonometric functions may be found graphically with the aid of the following diagram:



OBLIQUE TRIANGLE TRIGONOMETRIC FUNCTIONS



Law of sines:
Lengths of the sides of a triangle are proportional to the sines of the angles opposite them:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of cosines:
The square of the length of a side of a triangle equals the sum of the squares of the lengths of the other two sides minus twice the product of these two sides times the cosines of the angle between them.

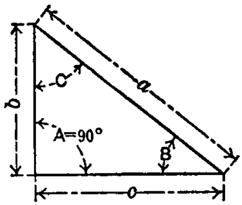
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Solving Right-angled Triangles

Formulas			

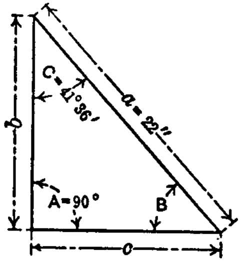


As shown in the illustration, the sides of the right-angled triangle are designated a, b, and c. The angles opposite each of these sides are designated A, B, and C, respectively.

Angle A, opposite the hypotenuse a, is the right angle, and is, therefore, always one of the known quantities.

Sides and Angles Known	Formulas for Sides and Angles to be Found		
Sides a and b	$c = \sqrt{a^2 - b^2}$	$\sin B = \frac{b}{a}$	$C = 90° - B$
Sides a and c	$b = \sqrt{a^2 - c^2}$	$\sin C = \frac{c}{a}$	$B = 90° - C$
Sides b and c	$a = \sqrt{b^2 + c^2}$	$\tan B = \frac{b}{c}$	$C = 90° - B$
Side a; angle B	$b = a \times \sin B$	$c = a \times \cos B$	$C = 90° - B$
Side a; angle C	$b = a \times \cos C$	$c = a \times \sin C$	$B = 90° - C$
Side b; angle B	$a = \frac{b}{\sin B}$	$c = b \times \cot B$	$C = 90° - B$
Side b; angle C	$a = \frac{b}{\cos C}$	$c = b \times \tan C$	$B = 90° - C$
Side c; angle B	$a = \frac{c}{\cos B}$	$b = c \times \tan B$	$C = 90° - B$
Side c; angle C	$a = \frac{c}{\sin C}$	$b = c \times \cot C$	$B = 90° - C$

Examples

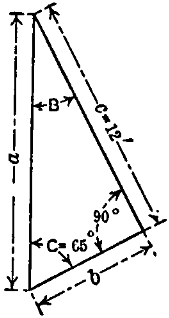


Sides and angles known:

$$a = 22 \text{ inches}; \ C = 41 \text{ degrees } 36 \text{ minutes.}$$

Then, by the formulas given herewith:

$$b = a \times \cos C = 22 \times \cos 41^\circ 36' = 22 \times 0.74780$$
$$= 16.4516 \text{ inches.}$$
$$c = a \times \sin C = 22 \times \sin 41^\circ 36' = 22 \times 0.66393$$
$$= 14.6065 \text{ inches.}$$
$$B = 90^\circ - 41^\circ 36' = 48^\circ 24'.$$

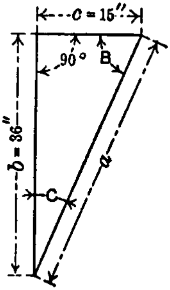


Sides and angles known:

$$c = 12 \text{ feet}; \ C = 65 \text{ degrees.}$$

Then, by the formulas:

$$a = \frac{c}{\sin C} = \frac{12}{\sin 65^\circ} = \frac{12}{0.90631} = 13.2405 \text{ feet.}$$
$$b = c \times \cot C = 12 \times \cot 65^\circ = 12 \times 0.46631$$
$$= 5.5957 \text{ feet.}$$
$$B = 90^\circ - 65^\circ = 25^\circ.$$



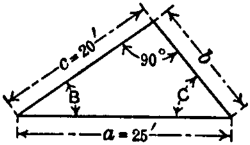
Sides known:

$$b = 36 \text{ inches}; \ c = 15 \text{ inches.}$$

Then, by the formulas:

$$a = \sqrt{b^2 + c^2} = \sqrt{36^2 + 15^2} = \sqrt{1296 + 225}$$
$$= \sqrt{1521} = 39 \text{ inches.}$$
$$\tan B = \frac{b}{c} = \frac{36}{15} = 2.4.$$

Hence,
$$B = 67^\circ 23'.$$
$$C = 90^\circ - 67^\circ 23' = 22^\circ 37'.$$



Sides known:

$$a = 25 \text{ feet}; \ c = 20 \text{ feet.}$$

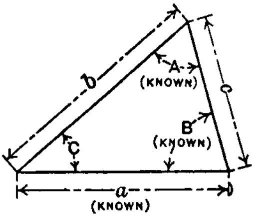
From the formulas:

$$b = \sqrt{a^2 - c^2} = \sqrt{25^2 - 20^2} = \sqrt{625 - 400}$$
$$= \sqrt{225} = 15 \text{ feet.}$$
$$\sin C = \frac{c}{a} = \frac{20}{25} = 0.8.$$

Hence,
$$C = 53^\circ 8'.$$
$$B = 90^\circ - 53^\circ 8' = 36^\circ 52'.$$

Solving Oblique-angled Triangles

Formulas



One side and two angles known.

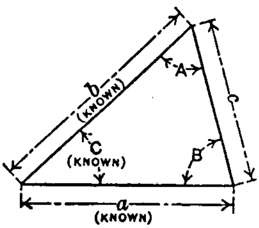
Call the known side a, the angle opposite it A, and the other known angle B. Then:

$$C = 180° - (A + B).$$

$$b = \frac{a \times \sin B}{\sin A}. \qquad c = \frac{a \times \sin C}{\sin A}.$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}.$$

If angles B and C are given, but not A, then $A = 180° - (B + C)$, the other formulas being the same.



Two sides and the angle between them known.

Call the known sides a and b and the known angle between them C. Then:

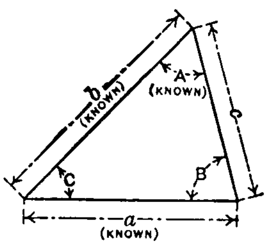
$$\tan A = \frac{a \times \sin C}{b - a \times \cos C}.$$

$$B = 180° - (A + C). \qquad c = \frac{a \times \sin C}{\sin A}.$$

Side c may also be found directly as below:

$$c = \sqrt{a^2 + b^2 - 2\,ab \times \cos C}.$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}.$$



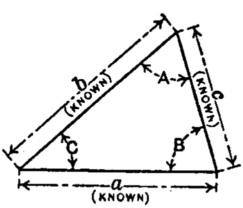
Two sides and the angle opposite one of the sides known.

Call the known angle A, the side opposite it a, and the other known side b. Then:

$$\sin B = \frac{b \times \sin A}{a}. \qquad C = 180° - (A + B).$$

$$c = \frac{a \times \sin C}{\sin A}.$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}.$$



All three sides known.

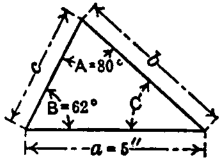
Call the sides a, b, and c, and the angles opposite them A, B, and C. Then:

$$\cos A = \frac{b^2 + c^2 - a^2}{2\,bc}.$$

$$\sin B = \frac{b \times \sin A}{a}. \qquad C = 180° - (A + B).$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}.$$

Examples



Sides and angles known:

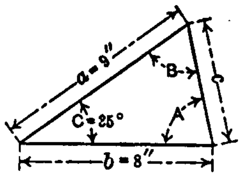
$$a = 5 \text{ inches};\ A = 80 \text{ degrees};\ B = 62 \text{ degrees}.$$

Then, by the formulas opposite:

$$C = 180° - (80° + 62°) = 180° - 142° = 38°.$$

$$b = \frac{a \times \sin B}{\sin A} = \frac{5 \times \sin 62°}{\sin 80°} = \frac{5 \times 0.88295}{0.98481} = 4.483 \text{ inches}.$$

$$c = \frac{a \times \sin C}{\sin A} = \frac{5 \times \sin 38°}{\sin 80°} = \frac{5 \times 0.61566}{0.98481} = 3.126 \text{ inches}.$$



Sides and angles known:

$$a = 9 \text{ inches};\ b = 8 \text{ inches};\ C = 35 \text{ degrees}.$$

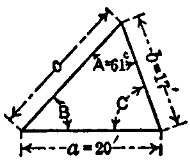
$$\tan A = \frac{a \times \sin C}{b - a \times \cos C} = \frac{9 \times \sin 35°}{8 - 9 \times \cos 35°}$$

$$= \frac{9 \times 0.57358}{8 - 9 \times 0.81915} = \frac{5.16222}{0.62765} = 8.22468.$$

Hence, $A = 83°\ 4'.$

$$B = 180° - (A + C) = 180° - 118°\ 4' = 61°\ 56'.$$

$$c = \frac{a \times \sin C}{\sin A} = \frac{9 \times 0.57358}{0.99269} = 5.2 \text{ inches}.$$



Sides and angles known:

$$a = 20 \text{ feet};\ b = 17 \text{ feet};\ A = 61 \text{ degrees}.$$

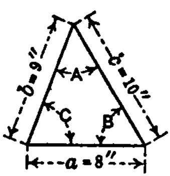
$$\sin B = \frac{b \times \sin A}{a} = \frac{17 \times \sin 61°}{20}$$

$$= \frac{17 \times 0.87462}{20} = 0.74343.$$

Hence, $B = 48°\ 1'.$

$$C = 180° - (A + B) = 180° - 109°\ 1' = 70°\ 59'.$$

$$c = \frac{a \times \sin C}{\sin A} = \frac{20 \times \sin 70°\ 59'}{\sin 61°} = \frac{20 \times 0.94542}{0.87462} = 21.62 \text{ feet}.$$



Sides known:

$$a = 8 \text{ inches};\ b = 9 \text{ inches};\ c = 10 \text{ inches}.$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2\,bc} = \frac{9^2 + 10^2 - 8^2}{2 \times 9 \times 10}$$

$$= \frac{81 + 100 - 64}{180} = \frac{117}{180} = 0.65000.$$

Hence, $A = 49°\ 27'.$

$$\sin B = \frac{b \times \sin A}{a} = \frac{9 \times 0.75984}{8} = 0.85482.$$

Hence, $B = 58°\ 44'.$

$$C = 180° - (A + B) = 180° - 108°\ 11' = 71°\ 49'.$$

x	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Δ	ADD 1′	2′	3′	4′	5′
0°	0·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	18	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332		3	6	9	12	15
2	·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506		3	6	9	12	15
3	·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680		3	6	9	12	15
4	·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854		3	6	9	12	14
5	0·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028		3	6	9	12	14
6	·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201		3	6	9	12	14
7	·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374		3	6	9	12	14
8	·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547		3	6	9	11	14
9	·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719		3	6	9	11	14
10	0·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891		3	6	9	11	14
11	·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062		3	6	9	11	14
12	·2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	17	3	6	9	11	14
13	·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402		3	6	8	11	14
14	·2419	2436	2453	2470	2487	2504	2521	2538	2554	2571		3	6	8	11	14
15	0·2588	2605	2622	2639	2656	2672	2689	2706	2723	2740		3	6	8	11	14
16	·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907		3	6	8	11	14
17	·2924	2940	2957	2974	2990	3007	3024	3040	3057	3074		3	6	8	11	14
18	·3090	3107	3123	3140	3156	3173	3190	3206	3223	3239		3	6	8	11	14
19	·3256	3272	3289	3305	3322	3338	3355	3371	3387	3404		3	5	8	11	14
20	0·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567		3	5	8	11	14
21	·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730		3	5	8	11	14
22	·3746	3762	3778	3795	3811	3827	3843	3859	3875	3891		3	5	8	11	13
23	·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	16	3	5	8	11	13
24	·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210		3	5	8	11	13
25	0·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368		3	5	8	11	13
26	·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524		3	5	8	10	13
27	·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679		3	5	8	10	13
28	·4695	4710	4726	4741	4756	4772	4787	4802	4818	4833		3	5	8	10	13
29	·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985		3	5	8	10	13
30	0·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	15	3	5	8	10	13
31	·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284		2	5	7	10	12
32	·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432		2	5	7	10	12
33	·5446	5461	5476	5490	5505	5519	5534	5548	5563	5577		2	5	7	10	12
34	·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721		2	5	7	10	12
35	0·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864		2	5	7	9	12
36	·5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	14	2	5	7	9	12
37	·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143		2	5	7	9	12
38	·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280		2	5	7	9	11
39	·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414		2	4	7	9	11
40	0·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547		2	4	7	9	11
41	·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	13	2	4	7	9	11
42	·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807		2	4	6	9	11
43	·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934		2	4	6	8	11
44	·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059		2	4	6	8	10
45	0·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181		2	4	6	8	10
46	·7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	12	2	4	6	8	10
47	·7314	7325	7337	7349	7361	7373	7385	7396	7408	7420		2	4	6	8	10
48	·7431	7443	7455	7466	7478	7490	7501	7513	7524	7536		2	4	6	8	10
49	0·7547	7559	7570	7581	7593	7604	7615	7627	7638	7649		2	4	6	8	9

O
H

x	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Δ	ADD 1′	2′	3′	4′	5′
50°	0·7660	7672	7683	7694	7705	7716	7727	7738	7749	7760		2	4	6	7	9
51	·7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	11	2	4	5	7	9
52	·7880	7891	7902	7912	7923	7934	7944	7955	7965	7976		2	4	5	7	9
53	·7986	7997	8007	8018	8028	8039	8049	8059	8070	8080		2	3	5	7	9
54	·8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	10	2	3	5	7	8
55	0·8192	8202	8211	8221	8231	8241	8251	8261	8271	8281		2	3	5	7	8
56	·8290	8300	8310	8320	8329	8339	8348	8358	8368	8377		2	3	5	6	8
57	·8387	8396	8406	8415	8425	8434	8443	8453	8462	8471		2	3	5	6	8
58	·8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	9	2	3	5	6	8
59	·8572	8581	8590	8599	8607	8616	8625	8634	8643	8652		1	3	4	6	7
60	0·8660	8669	8678	8686	8695	8704	8712	8721	8729	8738		1	3	4	6	7
61	·8746	8755	8763	8771	8780	8788	8796	8805	8813	8821		1	3	4	6	7
62	·8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	8	1	3	4	5	7
63	·8910	8918	8926	8934	8942	8949	8957	8965	8973	8980		1	3	4	5	6
64	·8988	8996	9003	9011	9018	9026	9033	9041	9048	9056		1	3	4	5	6
65	0·9063	9070	9078	9085	9092	9100	9107	9114	9121	9128		1	2	4	5	6
66	·9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	7	1	2	4	5	6
67	·9205	9212	9219	9225	9232	9239	9245	9252	9259	9265		1	2	3	4	6
68	·9272	9278	9285	9291	9298	9304	9311	9317	9323	9330		1	2	3	4	5
69	·9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	6	1	2	3	4	5
70	0·9397	9403	9409	9415	9421	9426	9432	9438	9444	9449		1	2	3	4	5
71	·9455	9461	9466	9472	9478	9483	9489	9494	9500	9505		1	2	3	4	5
72	·9511	9516	9521	9527	9532	9537	9542	9548	9553	9558		1	2	3	3	4
73	·9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	5	1	2	2	3	4
74	·9613	9617	9622	9627	9632	9636	9641	9646	9650	9655		1	2	2	3	4
75	0·9659	9664	9668	9673	9677	9681	9686	9690	9694	9699		1	1	2	3	4
76	·9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	4	1	1	2	3	3
77	·9744	9748	9751	9755	9759	9763	9767	9770	9774	9778		1	1	2	2	3
78	·9781	9785	9789	9792	9796	9799	9803	9806	9810	9813		1	1	2	2	3
79	·9816	9820	9823	9826	9829	9833	9836	9839	9842	9845		1	1	2	2	3
80	0·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	3	0	1	1	2	2
81	·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900		0	1	1	2	2
82	·9903	9905	9907	9910	9912	9914	9917	9919	9921	9923		0	1	1	1	2
83	·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	2	0	1	1	1	2
84	·9945	9947	9949	9951	9952	9954	9956	9957	9959	9960		0	1	1	1	1
85	0·9962	9963	9965	9966	9968	9969	9971	9972	9973	9974		0	0	1	1	1
86	·9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	1	0	0	1	1	1
87	·9986	9987	9988	9989	9990	9990	9991	9992	9993	9993		See Table below.				
88	·9994	9995	9995	9996	9996	9997	9997	9997	9998	9998						
89	0·9998	9999	9999	9999	9999	1·000	1·000	1·000	1·000	1·000						
90	1·0000															

Sines of Angles near 90°

° ′	sine	°	° ′	sine	°
86 48		86·80	87 46		87·7
	0·9985			0·9993	
86 54		86·91	87 56		87·9
	0·9986			0·9994	
87 01		87·02	88 05		88·0
	0·9987			0·9995	
87 08		87·13	88 16		88·2
	0·9988			0·9996	
87 15		87·25	88 29		88·4
	0·9989			0·9997	
87 22		87·37	88 43		88·7
	0·9990			0·9998	
87 30		87·50	89 00		89·0
	0·9991			0·9999	
87 38		87·63	89 25		89·4
	0·9992			1·0000	
87 46		87·78	90 00		90·0

The values in the centre columns represent the sines for all angles lying between the successive ranges shown in the outer columns. Thus sin 87° 20′ is 0·9989. For inverse use, the best angle for a given sine is the one lying midway between the adjacent ranges; if the difference is odd, choose the angle nearer 90°. Thus if sin x = 0·9988, x = 87° 12′.

For tabulated angles read the sine value in the half-line above; e.g., sin 87° 38′ = 0·9991.

x	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Δ	SUBTRACT 1′	2′	3′	4′	5′
0°	1·000	1·000	1·000	1·000	1·000	1·000	0·9999	0·9999	0·9999	0·9999	·	See table at				
1	0·9998	9998	9998	9997	9997	9997	9996	9996	9995	9995		foot of page.				
2	·9994	9993	9993	9992	9991	9990	9990	9989	9988	9987						
3	·9986	9985	9984	9983	9982	9981	9980	9979	9978	9977	1	0	0	1	1	1
4	·9976	9974	9973	9972	9971	9969	9968	9966	9965	9963		0	0	1	1	1
5	0·9962	9960	9959	9957	9956	9954	9952	9951	9949	9947		0	1	1	1	1
6	·9945	9943	9942	9940	9938	9936	9934	9932	9930	9928	2	0	1	1	1	2
7	·9925	9923	9921	9919	9917	9914	9912	9910	9907	9905		0	1	1	1	2
8	·9903	9900	9898	9895	9893	9890	9888	9885	9882	9880		0	1	1	2	2
9	·9877	9874	9871	9869	9866	9863	9860	9857	9854	9851	3	0	1	1	2	2
10	0·9848	9845	9842	9839	9836	9833	9829	9826	9823	9820		1	1	2	2	3
11	·9816	9813	9810	9806	9803	9799	9796	9792	9789	9785		1	1	2	2	3
12	·9781	9778	9774	9770	9767	9763	9759	9755	9751	9748		1	1	2	2	3
13	·9744	9740	9736	9732	9728	9724	9720	9715	9711	9707	4	1	1	2	3	3
14	·9703	9699	9694	9690	9686	9681	9677	9673	9668	9664		1	1	2	3	4
15	0·9659	9655	9650	9646	9641	9636	9632	9627	9622	9617		1	2	2	3	4
16	·9613	9608	9603	9598	9593	9588	9583	9578	9573	9568	5	1	2	2	3	4
17	·9563	9558	9553	9548	9542	9537	9532	9527	9521	9516		1	2	3	3	4
18	·9511	9505	9500	9494	9489	9483	9478	9472	9466	9461		1	2	3	4	5
19	·9455	9449	9444	9438	9432	9426	9421	9415	9409	9403		1	2	3	4	5
20	0·9397	9391	9385	9379	9373	9367	9361	9354	9348	9342	6	1	2	3	4	5
21	·9336	9330	9323	9317	9311	9304	9298	9291	9285	9278		1	2	3	4	5
22	·9272	9265	9259	9252	9245	9239	9232	9225	9219	9212		1	2	3	4	6
23	·9205	9198	9191	9184	9178	9171	9164	9157	9150	9143	7	1	2	4	5	6
24	·9135	9128	9121	9114	9107	9100	9092	9085	9078	9070		1	2	4	5	6
25	0·9063	9056	9048	9041	9033	9026	9018	9011	9003	8996		1	3	4	5	6
26	·8988	8980	8973	8965	8957	8949	8942	8934	8926	8918		1	3	4	5	6
27	·8910	8902	8894	8886	8878	8870	8862	8854	8846	8838	8	1	3	4	5	7
28	·8829	8821	8813	8805	8796	8788	8780	8771	8763	8755		1	3	4	6	7
29	·8746	8738	8729	8721	8712	8704	8695	8686	8678	8669		1	3	4	6	7
30	0·8660	8652	8643	8634	8625	8616	8607	8599	8590	8581		1	3	4	6	7
31	·8572	8563	8554	8545	8536	8526	8517	8508	8499	8490	9	2	3	5	6	8
32	·8480	8471	8462	8453	8443	8434	8425	8415	8406	8396		2	3	5	6	8
33	·8387	8377	8368	8358	8348	8339	8329	8320	8310	8300		2	3	5	6	8
34	·8290	8281	8271	8261	8251	8241	8231	8221	8211	8202		2	3	5	7	8
35	0·8192	8181	8171	8161	8151	8141	8131	8121	8111	8100	10	2	3	5	7	8
36	·8090	8080	8070	8059	8049	8039	8028	8018	8007	7997		2	3	5	7	9
37	·7986	7976	7965	7955	7944	7934	7923	7912	7902	7891		2	4	5	7	9
38	·7880	7869	7859	7848	7837	7826	7815	7804	7793	7782	11	2	4	5	7	9
39	0·7771	7760	7749	7738	7727	7716	7705	7694	7683	7672		2	4	6	7	9

Cosines of Small Angles

° ′	cosine	°	° ′	cosine	°
0 00		0·0	2 13		2·21
	1·0000			0·9992	
0 34		0·5	2 21		2·36
	0·9999			0·9991	
0 59		0·9	2 29		2·49
	0·9998			0·9990	
1 16		1·2	2 37		2·62
	0·9997			0·9989	
1 30		1·5	2 44		2·74
	0·9996			0·9988	
1 43		1·7	2 51		2·86
	0·9995			0·9987	
1 54		1·9	2 58		2·97
	0·9994			0·9986	
2 03		2·0	3 05		3·08
	0·9993			0·9985	
2 13		2·2	3 11		3·19

This table is similar to that given for sines on page 15; thus

cos 2° 40′ = 0·9989
0·9986 = cos 3° 2′

A
H.

x	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Δ	SUBTRACT 1′	2′	3′	4′	5′
40°	0·7660	7649	7638	7627	7615	7604	7593	7581	7570	7559		2	4	6	8	9
41	·7547	7536	7524	7513	7501	7490	7478	7466	7455	7443		2	4	6	8	10
42	·7431	7420	7408	7396	7385	7373	7361	7349	7337	7325		2	4	6	8	10
43	·7314	7302	7290	7278	7266	7254	7242	7230	7218	7206	12	2	4	6	8	10
44	·7193	7181	7169	7157	7145	7133	7120	7108	7096	7083		2	4	6	8	10
45	0·7071	7059	7046	7034	7022	7009	6997	6984	6972	6959		2	4	6	8	10
46	·6947	6934	6921	6909	6896	6884	6871	6858	6845	6833		2	4	6	8	11
47	·6820	6807	6794	6782	6769	6756	6743	6730	6717	6704		2	4	6	9	11
48	·6691	6678	6665	6652	6639	6626	6613	6600	6587	6574	13	2	4	7	9	11
49	·6561	6547	6534	6521	6508	6494	6481	6468	6455	6441		2	4	7	9	11
50	0·6428	6414	6401	6388	6374	6361	6347	6334	6320	6307		2	4	7	9	11
51	·6293	6280	6266	6252	6239	6225	6211	6198	6184	6170		2	5	7	9	11
52	·6157	6143	6129	6115	6101	6088	6074	6060	6046	6032		2	5	7	9	12
53	·6018	6004	5990	5976	5962	5948	5934	5920	5906	5892	14	2	5	7	9	12
54	·5878	5864	5850	5835	5821	5807	5793	5779	5764	5750		2	5	7	9	12
55	0·5736	5721	5707	5693	5678	5664	5650	5635	5621	5606		2	5	7	10	12
56	·5592	5577	5563	5548	5534	5519	5505	5490	5476	5461		2	5	7	10	12
57	·5446	5432	5417	5402	5388	5373	5358	5344	5329	5314		2	5	7	10	12
58	·5299	5284	5270	5255	5240	5225	5210	5195	5180	5165		2	5	7	10	12
59	·5150	5135	5120	5105	5090	5075	5060	5045	5030	5015	15	3	5	8	10	13
60	0·5000	4985	4970	4955	4939	4924	4909	4894	4879	4863		3	5	8	10	13
61	·4848	4833	4818	4802	4787	4772	4756	4741	4726	4710		3	5	8	10	13
62	·4695	4679	4664	4648	4633	4617	4602	4586	4571	4555		3	5	8	10	13
63	·4540	4524	4509	4493	4478	4462	4446	4431	4415	4399		3	5	8	10	13
64	·4384	4368	4352	4337	4321	4305	4289	4274	4258	4242		3	5	8	11	13
65	0·4226	4210	4195	4179	4163	4147	4131	4115	4099	4083		3	5	8	11	13
66	·4067	4051	4035	4019	4003	3987	3971	3955	3939	3923	16	3	5	8	11	13
67	·3907	3891	3875	3859	3843	3827	3811	3795	3778	3762		3	5	8	11	13
68	·3746	3730	3714	3697	3681	3665	3649	3633	3616	3600		3	5	8	11	14
69	·3584	3567	3551	3535	3518	3502	3486	3469	3453	3437		3	5	8	11	14
70	0·3420	3404	3387	3371	3355	3338	3322	3305	3289	3272		3	5	8	11	14
71	·3256	3239	3223	3206	3190	3173	3156	3140	3123	3107		3	6	8	11	14
72	·3090	3074	3057	3040	3024	3007	2990	2974	2957	2940		3	6	8	11	14
73	·2924	2907	2890	2874	2857	2840	2823	2807	2790	2773		3	6	8	11	14
74	·2756	2740	2723	2706	2689	2672	2656	2639	2622	2605		3	6	8	11	14
75	0·2588	2571	2554	2538	2521	2504	2487	2470	2453	2436		3	6	8	11	14
76	·2419	2402	2385	2368	2351	2334	2317	2300	2284	2267		3	6	8	11	14
77	·2250	2233	2215	2198	2181	2164	2147	2130	2113	2096	17	3	6	9	11	14
78	·2079	2062	2045	2028	2011	1994	1977	1959	1942	1925		3	6	9	11	14
79	·1908	1891	1874	1857	1840	1822	1805	1788	1771	1754		3	6	9	11	14
80	0·1736	1719	1702	1685	1668	1650	1633	1616	1599	1582		3	6	9	11	14
81	·1564	1547	1530	1513	1495	1478	1461	1444	1426	1409		3	6	9	11	14
82	·1392	1374	1357	1340	1323	1305	1288	1271	1253	1236		3	6	9	12	14
83	·1219	1201	1184	1167	1149	1132	1115	1097	1080	1063		3	6	9	12	14
84	·1045	1028	1011	0993	0976	0958	0941	0924	0906	0889		3	6	9	12	14
85	0·0872	0854	0837	0819	0802	0785	0767	0750	0732	0715		3	6	9	12	14
86	·0698	0680	0663	0645	0628	0610	0593	0576	0558	0541		3	6	9	12	15
87	·0523	0506	0488	0471	0454	0436	0419	0401	0384	0366		3	6	9	12	15
88	·0349	0332	0314	0297	0279	0262	0244	0227	0209	0192		3	6	9	12	15
89	0·0175	0157	0140	0122	0105	0087	0070	0052	0035	0017	18	3	6	9	12	15
90	0·0000															

x	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Δ	ADD 1′	2′	3′	4′	5′
0°	0·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	18	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332		3	6	9	12	15
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507		3	6	9	12	15
3	·0524	0542	0559	0577	0594	0612	0629	0647	0664	0682		3	6	9	12	15
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857		3	6	9	12	15
5	0·0875	0892	0910	0928	0945	0963	0981	0998	1016	1033		3	6	9	12	15
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210		3	6	9	12	15
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388		3	6	9	12	15
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566		3	6	9	12	15
9	·1584	1602	1620	1638	1655	1673	1691	1709	1727	1745		3	6	9	12	15
10	0·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926		3	6	9	12	15
11	·1944	1962	1980	1998	2016	2035	2053	2071	2089	2107		3	6	9	12	15
12	·2126	2144	2162	2180	2199	2217	2235	2254	2272	2290		3	6	9	12	15
13	·2309	2327	2345	2364	2382	2401	2419	2438	2456	2475		3	6	9	12	15
14	·2493	2512	2530	2549	2568	2586	2605	2623	2642	2661		3	6	9	12	15
15	0·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849		3	6	9	13	16
16	·2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	19	3	6	10	13	16
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230		3	6	10	13	16
18	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424		3	6	10	13	16
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620		3	7	10	13	16
20	0·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	20	3	7	10	13	17
21	·3839	3859	3879	3899	3919	3939	3959	3979	4000	4020		3	7	10	13	17
22	·4040	4061	4081	4101	4122	4142	4163	4183	4204	4224		3	7	10	14	17
23	·4245	4265	4286	4307	4327	4348	4369	4390	4411	4431		3	7	10	14	17
24	·4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	21	4	7	11	14	18
25	0·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856		4	7	11	14	18
26	·4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	22	4	7	11	15	18
27	·5095	5117	5139	5161	5184	5206	5228	5250	5272	5295		4	7	11	15	18
28	·5317	5340	5362	5384	5407	5430	5452	5475	5498	5520		4	8	11	15	19
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	23	4	8	12	15	19
30	0·5774	5797	5820	5844	5867	5890	5914	5938	5961	5985		4	8	12	16	20
31	·6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	24	4	8	12	16	20
32	·6249	6273	6297	6322	6346	6371	6395	6420	6445	6469		4	8	12	16	20
33	·6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	25	4	8	13	17	21
34	·6745	6771	6796	6822	6847	6873	6899	6924	6950	6976		4	9	13	17	21
35	0·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	26	4	9	13	17	22
36	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	27	5	9	14	18	23
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	28	5	9	14	19	23
38	·7813	7841	7869	7898	7926	7954	7983	8012	8040	8069		5	10	14	19	24
39	·8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	29	5	10	15	19	24
40	0·8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	30	5	10	15	20	25
41	·8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	31	5	10	16	21	26
42	·9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	32	5	11	16	21	27
43	·9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	33	6	11	17	22	28
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	34	6	11	17	23	28
45	1·0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	36	6	12	18	24	30
46	·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	37	6	12	18	25	31
47	·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	38	6	13	19	25	32
48	·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	40	7	13	20	27	33
49	1·1504	1544	1585	1626	1667	1708					41	7	14	20	27	34
						1708	1750	1792	1833	1875	42	7	14	21	28	35

O/A.

x	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Δ	ADD				
	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9		1′	2′	3′	4′	5′
50°	1·192	196	200	205	209	213	217	222	226	230		1	1	2	3	4
51	·235	239	244	248	253	257	262	266	271	275		1	2	2	3	4
52	·280	285	289	294	299	303	308	313	317	322		1	2	2	3	4
53	·327	332	337	342	347	351	356	361	366	371		1	2	2	3	4
54	·376	381	387	392	397	402	407	412	418	423	5	1	2	3	3	4
55	1·428	433	439	444	450	455	460	466	471	477		1	2	3	4	5
56	·483	488	494	499	505	511	517	522	528	534		1	2	3	4	5
57	·540	546	552	558	564	570	576	582	588	594	6	1	2	3	4	5
58	·600	607	613	619	625	632	638	645	651	658		1	2	3	4	5
59	·664	671	678	684	691	698	704	711	718	725		1	2	3	5	6
60	1·732	739	746	753	760	767	775	782	789	797	7	1	2	4	5	6
61	·804	811	819	827	834	842	849	857	865	873		1	3	4	5	6
62	·881	889	897	905	913	921	929	937	946	954	8	1	3	4	5	7
63	1·963	971	980	988	1·997	2·006	2·014	2·023	2·032	2·041		1	3	4	6	7
64	2·050	059	069	078	087	097	106	116	125	135	9	2	3	5	6	8
65	2·145	154	164	174	184	194	204	215	225	236	10	2	3	5	7	8
66	·246	257	267	278	289	300	311	322	333	344	11	2	4	6	7	9
67	·356	367	379	391	402	414	426	438	450	463	12	2	4	6	8	10
68	·475	488	500	513	526	539	552	565	578	592	13	2	4	6	9	11
69	·605	619	633	646	660	675	689	703	718	733	14	2	5	7	9	12
70	2·747	762	778	793	808	824	840	856	872	888	16	3	5	8	11	13
71	2·904	921	937	954	971	2·989	3·006	3·024	3·042	3·060	17	3	6	9	11	14
72	3·078	096	115	133	152	3·172					19	3	6	9	13	16
						172	191	211	230	251	20	3	7	10	13	17
73	·271	291	312	333	354	376					21	4	7	10	14	18
						376	398	420	442	465	22	4	7	11	15	18
74	·487	511	534	558	582	606					24	4	8	12	16	20
						606	630	655	681	706	25	4	8	13	17	21
75	3·732	758	785	812	839	867					27	4	9	14	18	22
						867	895	923	952	981	29	5	10	14	19	24
76	4·011	041	071	102	134	4·165					31	5	10	15	21	26
						165	198	230	264	297	33	6	11	17	22	28
77	4·331	366	402	437	474	511					36	6	12	18	24	30
						511	548	586	625	665	39	6	13	19	26	32
78	4·705	745	787	829	872	4·915					42	7	14	21	28	35
						4·915	4·959	5·005	5·050	5·097	46	8	15	23	31	38
79	5·145	193	242	292	343	5·396					50	8	17	25	33	42
						396	449	503	558	614	55	9	18	28	37	46
80	5·671	5·730	5·789	5·850	5·912	5·976						10	20	30	41	51
						5·976	6·041	6·107	6·174	6·243		11	23	34	45	56
81	6·314	6·386	6·460	6·535	6·612	6·691						13	25	38	50	63
						6·691	6·772	6·855	6·940	7·026		14	28	42	57	71
82	7·115	7·207	7·300	7·396	7·495	7·596	7·700	7·806	7·916	8·028						
83	8·144	8·264	8·386	8·513	8·643	8·777	8·915	9·058	9·205	9·357						
84	9·514	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20						
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95						
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46						
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27						
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08						
89	57·29	63·66	71·62	81·85	95·49	114·6	143·2	191·0	286·5	573·0						
90	∞															

Differences vary too rapidly for interpolation by P.P.s. See table on page 22.

P.P.s for differences exceeding 14, if not shown on this page, should be taken from the inside end cover of the book. For angles between 72° and 82° P.P.s based on actual differences should be used.

Chapter XI

Lay-Out Work

40. Plane Laying-Out

The fabrication of plugs, flanges, and other parts involves inscribing the boundaries of these parts and marking the bolt hole centres on plate steel, transferring them from the working drawings to the blank surface, i.e., laying-out.

In mass production of parts at pipe-fitting shops, lay-out work is performed with the aid of previously made templates. On small jobs and in field conditions lay-out work is done manually in conformity with geometric rules, using rulers, squares, protractors, compasses, scribers, and other tools.

The accurate dimensions of flanges or plugs, as well as the number and position of bolt holes are indicated in the working drawings. When laying-out, allowances must be provided to work the part; the margin allowed should be minimal to save metal, time, and labour required for removing excess metal.

The steel to be laid out must be cleaned from dirt and rust, and then coated with chalk paint. Such paint is prepared as follows: take 8 lit of water, add 1 kg of ground chalk, and heat the mixture obtained to the boiling point. Then add 50 g of joiner's glue previously dissolved in water.

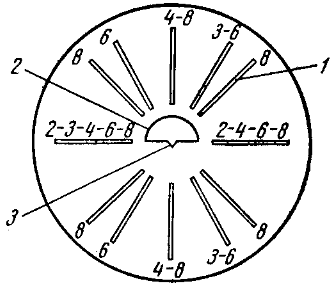


Fig. 77. Template for laying out bolt hole centres in flanges and plugs

1—slits; *2*—semicircular hole; *3*—template centre]

After adding the glue, reheat the mixture to the boiling point. In the summer, linseed oil and siccative are added to the mixture.

The chalk paint is applied to the steel surface with a painter's brush. It is good practice to use spray guns to apply the paint evenly.

When the paint dries, use a template for laying-out.

To make the template (Fig. 77) cut a round disc out of sheet steel 0.3 to 1.0 mm thick. Then cut out semicircular hole *2* and template centre *3*. Mark out lines dividing the disc into 2, 4, 6, 8, etc. equal parts. 0.3-mm wide slits *1* made along these lines are designated by figures inscribed at the tips of the slits. The figures indicate the number of circumference parts the slit corresponds to.

For laying-out the bolt hole centres on a flange or plug, draw a circle of the required radius on the blank, all the centres lying on the circumference of this circle. Then place the template on the blank surface so that its centre coincides with the centre of the inscribed circle. After that, using a sharply pointed scriber, incise lines along the appropriate slits. For example, for laying-out six holes scribe lines in the slits designated by figure *6*. The points of intersection of the scribed lines and the previously drawn circle will be the bolt hole centres. The hole centres at these points are marked with a punch.

If templates are not available, flanges and plugs, as well as bolt holes are laid out in the following way.

Draw line *AB* on the steel surface with the aid of a rule (Fig. 78). Then using the compasses, draw a circle with

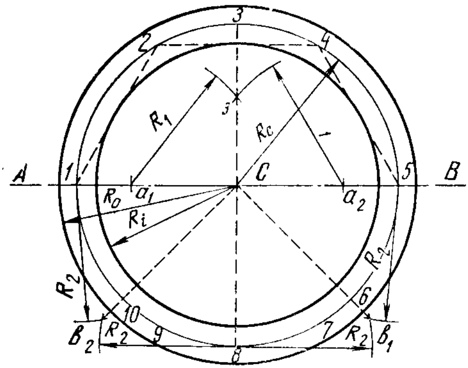


Fig. 78. Laying out holes in flanges in conformity with geometric rules

radius R_0 corresponding to the outside radius of the flange or plug, a second circle with radius R_1 corresponding to the inside radius of the flange, and finally a third circle with radius R_c corresponding to the position of the bolt hole centres. The radii are given in the working drawings.

For marking four bolt centres, lay-out work is carried out as follows. Mark two holes as the points of intersection of line *AB* and the circle with radius R_c, i. e., at points *1* and *5*, and the other two holes at the point of intersection of the same circle with the perpendicular to line *AB* erected from centre *C* of the circles. For this purpose, mark points a_1 and a_2 on line *AB* and draw arcs with radius R_1 from these points. The straight line passing through the point of intersection (a_3) of these arcs and centre *C* gives the position of the hole centres for the third and fourth bolts at the points of intersection of this line and the circle with radius R_c, i.e., at points *3* and *8*.

The bolt hole centres of six-bolt connections can be marked out very simply, since the chords, i.e., the straight lines between the centres of the bolt holes, are equal to radius R_c. For marking-out the hole centres, inscribe the circle with radius R_c, then starting at point *1* (point of intersection of this circle with line *AB*), using radius R_c, plot distances 1-2, 2-4, 4-5, 5-7, 7-9, and 9-1. Points *1*, *2*, *4*, *5*, *7*, and *9* located on the circle with radius R_c will be the hole centres.

In flanges and plugs containing eight bolt holes, first mark four hole centres and then the other four between the previously marked ones. To mark-out these centres, draw arcs with radius R_2 from adjacent centres *5* and *8* (*8* and *1*); then connect intersection points b_1 and b_2 with centre *C*. The centres of the additional bolt holes will be located at points *6* and *10* where these lines intersect the circle with radius R_c. On accomplishing lay-out work, each bolt hole centre should be fixed on the metal with a steel prick-punch and hammer.

41. Three-Dimensional Laying-Out

Three-dimensional laying-out is employed when cutting large-diameter steel pipes to obtain accurate cutting lines either at right angles to the pipe axis or at a preset angle; it is also used in producing welded steel fittings (tees, crosses, branches, etc.).

To produce cutting lines at right angles to the pipe axis, use is made of sheet steel strips from 10 to 12 cm wide, their length somewhat exceeding the circumference of the pipe being cut.

The strip must be tightly wrapped around the pipe and its ends should meet so that the cutting line formed is straight. On making sure that the strip is in firm contact with the pipe surface and its ends are properly aligned, scribe the cutting line along the strip edge. To make the eine clearer, use sharpened chalk or crayons.

In case the pipe is not cut immediately after scribing, the cutting line must be prick-punched at points spaced 15-25 cm apart.

For producing steel tees, crosses, and other fittings, use is made of sheet steel or cardboard templates.

Blanks for producing tees (Fig. 79) or crosses (in the Figure the second branch is shown as a dash line) with main pipe outer diameter D, and branch diameter d should be laid in the following manner.

The branch is connected to the main pipe along curved line ll_1 which should be scribed on the main pipe and on the branch.

For making the template, it is necessary to develop the branch (show the three-dimensional body on a plane); at first it will represent rectangle n-6^1-6^1-n. Then curved line *n-m-f-e-c-b-a...n* is laid-out on the rectangle. This line is used to contact the branch with the main pipe along line ll_1.

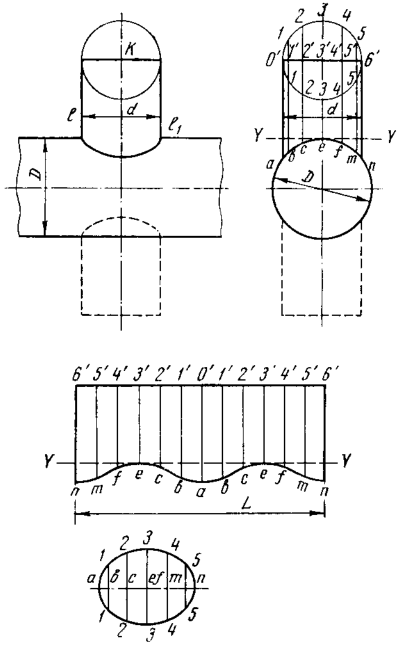


Fig. 79. Laying out blanks for fabricating tees

To make the template, proceed as follows. Draw the cross section of the tee in full size on a sheet of paper. On the butt-end of the drawn branch inscribe a circle of the same diameter as that of the actual branch. Then divide the circumference of this circle into 8 or 12 equal parts in accordance with the above-mentioned rules. From points 0′, *1*, *2*, *3*, *4*, *5*, and 6′ draw lines parallel to the branch generating line, that intersect the main pipe circumference at points *a*, *b*, *c*, *e*, *f*, *m*, *n*. Lines 0′-*a*, *1*-*b*, *2*-*c*, *3*-*e*, *4*-*f*, *5*-*m*, and 6′-*n* intersect the branch butt-end line at points 1′, 2′, 3′, etc.

After that, it is possible to make the template. For this purpose draw straight line *6′-6′* equal to the branch circumference, i.e., $L = \pi \cdot d = 3 \cdot 14d$, on a sheet of cardboard or roofing steel. Divide the line into as many equal parts as the circumference was divided into (here 12). At each division draw a line at right angles to line *6′-6′* and

Table 3

Chord Lengths in Circle (with Radius R=1) Divided into Equal Parts

Number of divisions	Chord length S	Number of divisions	Chord length S	Number of divisions	Chord length S
3	1.7321	17	0.3676	31	0.2023
4	1.4142	18	0.3473	32	0.1961
5	1.1756	19	0.3292	33	0.1901
6	1.0000	20	0.3129	34	0.1846
7	0.8678	21	0.2980	35	0.1793
8	0.7654	22	0.2845	36	0.1743
9	0.6840	23	0.2723	37	0.1697
10	0.6180	24	0.2611	38	0.1652
11	0.5635	25	0.2507	39	0.1609
12	0.5176	26	0.2411	40	0.1569
13	0.4786	27	0.2321	41	0.1531
14	0.4450	28	0.2240	42	0.1494
15	0.4158	29	0.2162	43	0.1459
16	0.3902	30	0.2091	44	0.1426

Example. Determine the chord length X between two centres of adjacent holes around the circumference of a circle with diameter $D = 960$ mm, the number of holes being 24.

The chord length X is determined from the formula

$$X = S \cdot \frac{D}{2} \text{ mm}$$

The chord length S for a circle with a radius equal to 1 is also divided into 24 parts. The number of holes being 24, table 3 gives $S = 0.2611$.

$$X = 0.2611 \cdot \frac{960}{2} = 125.3 \text{ mm}$$

on each of them plot distances corresponding to lines *6′-n*, *5′-m*, *4′-f*, etc. taken from the drawing. For obtaining the bottom contour of the branch development, connect points *m*, *n*, *f*, *e*, etc. with a smooth curve.

To obtain the required template, carefully cut the steel sheet or cardboard along the developed curve. Place the template on the steel pipe blank and scribe the curved cutting line on the branch and cut it along this line.

The holes for joining the branch to the main pipe can also be marked out with the aid of templates made in conformity with geometric rules. However when ready-made branches are available, it is good practice to place an accurately fabricated branch on the main pipe and to scribe the lines for cutting out the holes.

In lay-out work, division of the circumference into a preset number of equal parts in conformity with geometric rules (particularly if it is divided into a great number of equal parts) takes a lot of time. This work can be done quicker and more simply by using chord lengths *S* previously calculated for a circle with a radius equal to one (see Table 3).

3. DETERMINING DIMENSIONS AND LAYING OUT STOCK DEVELOPMENTS

Many parts from sheet, strip and round stock (angles, cramps, boxes, jackets, hinges, etc.) are made by bending at various angles and radii or into curvatures and other shapes.

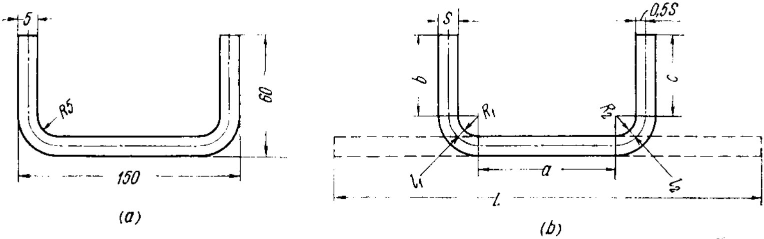


Fig. 68. Example of bending a part

In each case, in order to obtain the required size after bending, it is necessary first to determine (calculate) the size of the stock.

To determine the stock size in bending to internal curvature radii it is sufficient to sum up the straight sections and the neutral lines within the curvature.

The length of the neutral line within the curved section in bending to 90° is determined by the formula:

$$l=\left(R+\frac{S}{2}\right)\frac{\pi}{2}$$

in bending to any other angle $l = 0.0175\left(R+\frac{S}{2}\right)\alpha$,

where l = length of the neutral lines, mm
R = internal curvature radius, mm
S = stock thickness, mm
α = bending angle, deg.

The lengths of the straight sections of the stock up to the point of bending are calculated from the drawing. The dimensions are usually given on the external contour of the part, as shown in Fig. 68*a*.

The size of the stock made of sheet material 0.3 to 10 mm thick and bent to a curvature with radius 1 to 30 mm at any bending angle can be found using Table 13 by the formula

$$L=a+b+c+l \text{ (see Fig. 68}b\text{)}$$
$$l=l_1+l_2$$

where a, b, c stand for the length of the straight sections before the curvature.

l = length of the neutral lines within the curvature.

The length of the neutral line is computed by the formula $l = K\alpha$.

The value of K is given in Table 13. For computing, it is necessary to know stock thickness S, bending radius R and bending angle α, which data are commonly listed in the drawings. The value of K taken from the Table is multiplied by the bending angle α and the product is added to the lengths of the straight sections before the curvature.

How to use Table 13. To calculate the dimensions of the stock for bending to the following sizes: $a = 48$ mm; $b = 65$ mm; $\alpha = 137°$; $R = 2l$ mm and $S = 8$ mm. $L = a + b + l$. Find the given thickness figure in the horizontal row for S ($S = 8$ mm) and the bending radius in the vertical column ($R = 2l$ mm) in Table 13.

The intersection of the row and column gives $K = 0.436$.

The length of the neutral line $l = K\alpha = 0.436 \times 137 = 59.7$ mm; by summing up the numerical values of a, b and l determine the stock length

$$L = a + b + l = 48 + 65 + 59.7 = 172.7 \text{ mm}$$

Calculating the Length of Neutral Lines *l* at Any Bending Angles

Bending radius, R, mm	Factor K with Various Thickness of Material S, in mm: 0.3	0.5	0.8	1	1.2	1.5	1.6	1.8	2	2.5
1	0.020	0.022	0.024	0.026	0.028	—	—	—	—	—
2	0.038	0.039	0.042	0.044	0.045	0.048	0.049	0.051	0.052	0.057
3	0.055	0.057	0.059	0.061	0.063	0.065	0.066	0.068	0.070	0.074
4	0.072	0.074	0.077	0.079	0.080	0.083	0.084	0.086	0.087	0.092
5	0.090	0.092	0.094	0.096	0.098	0.100	0.101	0.103	0.105	0.109
6	0.107	0.109	0.112	0.113	0.115	0.118	0.119	0.120	0.122	0.127
7	0.125	0.127	0.129	0.131	0.133	0.135	0.136	0.138	0.140	0.144
8	0.142	0.144	0.147	0.148	0.150	0.153	0.154	0.155	0.157	0.161
9	0.160	0.161	0.164	0.166	0.168	0.170	0.171	0.173	0.175	0.179
10	0.177	0.179	0.182	0.183	0.185	0.188	0.188	0.190	0.192	0.196
11	0.195	0.196	0.199	0.201	0.202	0.206	0.206	0.208	0.209	0.214
12	0.212	0.214	0.216	0.218	0.220	0.223	0.223	0.225	0.227	0.231
13	0.230	0.231	0.234	0.236	0.237	0.240	0.241	0.243	0.244	0.249
14	0.247	0.249	0.251	0.253	0.255	0.257	0.258	0.260	0.262	0.266
15	0.264	0.266	0.269	0.271	0.272	0.275	0.276	0.278	0.279	0.284
16	0.282	0.284	0.286	0.288	0.290	0.292	0.293	0.295	0.297	0.301
17	0.299	0.301	0.304	0.305	0.307	0.310	0.311	0.312	0.314	0.319
18	0.317	0.319	0.321	0.323	0.325	0.327	0.328	0.330	0.332	0.336
19	0.330	0.336	0.339	0.340	0.342	0.345	0.346	0.347	0.349	0.353
20	0.352	0.353	0.356	0.358	0.360	0.362	0.363	0.365	0.367	0.371
21	0.369	0.371	0.373	0.375	0.377	0.380	0.380	0.382	0.384	0.388
22	0.387	0.388	0.391	0.393	0.394	0.397	0.398	0.400	0 401	0.406
23	0.404	0.406	0.408	0.410	0.412	0.415	0.415	0.417	0 419	0.423
24	0.421	0.423	0.426	0.428	0.429	0.432	0.433	0.435	0.436	0.441
25	0.439	0.441	0.443	0.445	0.447	0.449	0.450	0.452	0.454	0.458
26	0.456	0.458	0.461	0.463	0.464	0.467	0.468	0.469	0.471	0.476
27	0.474	0.478	0.478	0.480	0.482	0.484	0.485	0.487	0.488	0.498
28	0.491	0.493	0.496	0.497	0.499	0.502	0.503	0.504	0.506	0.511
29	0.509	0.511	0.513	0.515	0.517	0.519	0.520	0.522	0.524	0.528
30	0.526	0.528	0.531	0.532	0.534	0.537	0.538	0.539	0.541	0.545

Bending radius, R, mm	2.8	3	3.5	4	4.5	5	6	7	8	9	10
1	—	—	—	—	—	—	—	—	—	—	—
2	0.059	—	—	—	—	—	—	—	—	—	—
3	0.077	0.079	0.082	—	—	—	—	—	—	—	—
4	0.094	0.096	0.100	0.105	0.109	—	—	—	—	—	—
5	0.112	0.113	0.118	0.122	0.127	0.131	—	—	—	—	—
6	0.128	0.131	0.135	0.140	0.144	0.148	0.157	—	—	—	—
7	0.147	0.148	0.153	0.157	0.161	0.168	0.175	0.183	—	—	—
8	0.164	0.166	0.170	0.175	0.179	0.183	0.192	0.201	0.209	—	—
9	0.182	0.183	0.188	0.192	0.196	0.201	0.209	0.218	0.227	0.236	—
10	0.199	0.201	0.205	0.209	0.214	0.218	0.227	0.236	0.244	0.253	0.262
11	0.216	0.218	0.223	0.227	0.231	0.236	0.244	0.253	0.262	0.271	0.279
12	0.234	0.236	0.240	0.244	0.249	0.253	0.262	0.271	0.279	0.288	0.297
13	0.251	0.253	0.257	0.262	0.266	0.271	0.279	0.288	0.297	0.305	0.314
14	0.269	0.271	0.275	0.279	0.284	0.288	0.297	0.305	0.314	0.323	0.332
15	0.286	0.288	0.292	0.297	0.301	0.305	0.314	0.323	0.332	0.340	0.349
16	0.309	0.305	0.310	0.314	0.319	0.323	0.332	0.340	0.349	0.358	0.367
17	0.321	0.323	0.327	0.332	0.336	0.340	0.349	0.358	0.367	0.375	0.384
18	0.339	0.340	0.345	0.349	0.353	0.358	0.367	0.375	0.384	0.393	0.401
19	0.356	0.358	0.362	0.367	0.371	0.375	0.384	0.393	0.401	0.410	0.419
20	0.375	0.375	0.380	0.384	0.388	0.393	0.401	0.410	0.419	0.428	0.436
21	0.391	0.393	0.397	0.401	0.406	0.410	0.419	0.428	0.436	0.446	0.454
22	0.408	0.410	0.415	0.418	0.423	0.428	0.436	0.445	0.454	0.463	0.471
23	0.426	0.428	0.432	0.436	0.441	0.445	0.454	0.463	0.471	0.480	0.489
24	0.443	0.445	0.449	0.454	0.458	0.463	0.471	0.480	0.489	0.497	0.506
25	0.461	0.463	0.467	0.471	0.475	0.480	0.489	0.497	0.506	0.515	0.524
26	0.478	0.480	0.484	0.489	0.493	0.497	0.505	0.515	0.524	0.532	0.541
27	0.496	0.497	0.502	0.506	0.511	0.515	0.524	0.532	0.541	0.550	0.559
28	0.513	0.515	0.519	0.524	0.528	0.532	0.541	0.550	0.559	0.567	0.576
29	0.531	0.532	0.537	0.540	0.545	0.550	0.559	0.567	0.576	0.585	0.593
30	0.548	0.550	0.554	0.559	0.563	0.567	0.576	0.585	0.593	0.602	0.617

4. THE DEVELOPMENT OF A RECTANGULAR BOX

Fig. 69 represents a rectangular box to be made from sheet stock. It is required to calculate the stock dimensions, mark the stock and cut out the development. It is normal to indicate in the drawings the overall (external) dimensions, the thickness of the material and the inside radius of curvature which is all that is necessary for computing the stock dimensions. Fig. 69*a* represents the box in three projections, the following sizes being given:

1. Thickness of the material = 1.5 mm
2. Box width = 180 mm
3. Length = 250 mm
4. Height of sides = 40 mm.

By using the above methods, formulae and tables the stock dimensions (width and length) can be calculated. First determine the width and length of the straight sections:

$$\text{width} = 35.5 + 171 + 35.5 = 242$$
$$\text{length} = 35.5 + 241 + 35.5 = 312$$

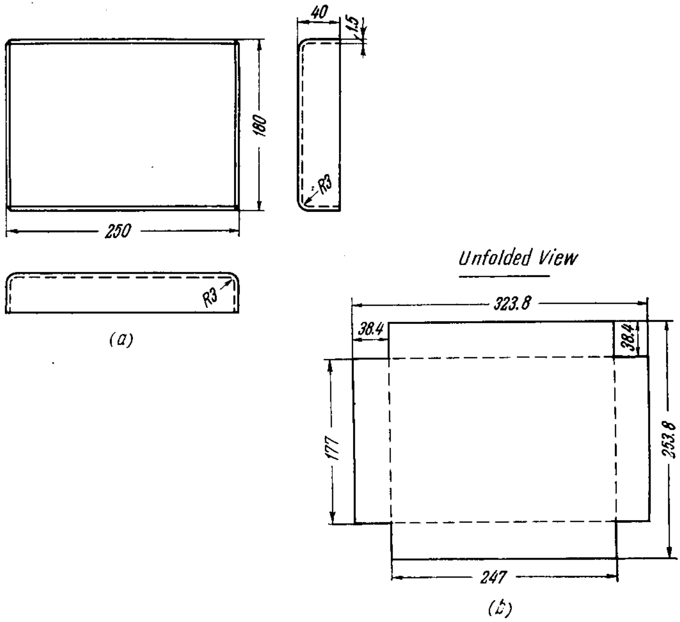


Fig. 69. Box:
(a) drawing of the box in three projections; (b) box development

Add the length of the neutral lines. The neutral line radius is 3.75 mm, hence the length of the neutral lines for two radii will be $3.75 \times 3.14 = 11.8$ mm; therefore

stock width $= 242 + 11.8 = 253.8$

stock length $= 312 + 11.8 = 323.8$

On calculating the stock dimensions, start laying them out in imagination. The development configuration can be visualized by mentally unbending the sides, turning the part into a flat figure and drawing it according to the dimensions. The development of a rectangular box is given in Fig. 69*b*.

How to Find the Size of a Pulley.— In order to run any farm machine at its proper speed, that is the speed which the manufacturer recommends as the best, you must be able to calculate the size of the pulley needed on the machine.

The pulley on your tractor is, of course, of a fixed size and if you want to run, say, an ensilage cutter at a given speed, a *ratio* between the sizes of the tractor pulley and the ensilage cutter pulley must be had. You can easily find the size needed by using the following formula:

$$\text{Diameter of Machine Pulley} = \frac{\text{Diam. of Tractor Pulley} \times \text{R.P.M. of Tractor Pulley}}{\text{R.P.M. of the Machine Pulley}}$$

Now let us take an example. An ensilage cutter, or other machine, is to be run at a speed of 800 r. p. m., by a tractor whose pulley is 16 inches in diameter and which is driven at a speed of 400 r. p. m. What must be the size of the pulley on the ensilage cutter or other machine?

Substituting now the known figures for the formula above we have

$$\text{Diameter of Machine Pulley} = \frac{16 \times 400}{800}$$

or worked out

$$\text{Diameter of Machine Pulley} = \frac{6400}{800} = 8 \text{ inches}$$

and 8 inches is the diameter of the pulley you want on your ensilage cutter or other machine.

How to Find the Belt Speed.— The belt speed of your tractor is the rate of travel of any one point on the belt and it is measured in feet per minute. You can find it by the following formula:

$$\text{Belt Speed} = \frac{(\text{Diameter of Tractor Pulley} \times 3.1416) \times \text{R.P.M. of Tractor Pulley}}{12 \text{ inches}}$$

where the diameter of the tractor pulley $\times$ 3.1416 = the circumference of the tractor pulley in inches.

As an example, suppose the diameter of the tractor pulley is 18 inches and its speed is 550 r. p. m. Substituting these figures in the formula we have,

$$\text{Belt Speed} = \frac{18 \times 3.1416 \times 550}{12}$$

or,

$$\text{Belt Speed} = \frac{31{,}200}{12} = 2{,}600 \text{ feet,}$$

and 2,600 feet is the speed of the belt in feet per minute.

PULLEY SPEEDS. The principle applied to gearing in regard to the ratio between the speeds of two shafts may be directly applied to pulleys, with the only difference that the number of inches to the diameter of the pulley should be substituted for the number of teeth in the gear. (See GEAR TRAINS.)

Assume that a shaft is required to make 300 revolutions per minute, and that it is driven from a lineshaft making 180 revolutions per minute, as indicated in Fig. 1. The pulley on the lineshaft is 15 inches in diameter. What should the diameter of the pulley on the shaft making 300 revolutions per minute be made? As the belt on the two pulleys runs at the same speed as the periphery (circumference) of either of the pulleys, it is clear that the peripheries of both pulleys run at the same speed, providing there is no slip between the belt and the pulleys. The pulley running a smaller number of revolutions must be larger in order that its periphery may run at the same speed as the periphery of the pulley making a greater number of revolutions. The circumference of a circle (and, therefore, also the circumference of a pulley) equals the diameter × 3.1416. Therefore, the circumference of the pulley making 180 revolutions and having a diameter of 15 inches, passes, in one minute, through a distance equal to 180 times its circumference, or 180 × 15 × 3.1416.

The circumference of the pulley making 300 revolutions must pass through the same distance in one minute; therefore, for *each* revolution, this pulley must pass through the distance 180 × 15 × 3.1416 divided by 300. This, then, would equal the circumference of the smaller pulley; but the circumference also equals the *diameter* × 3.1416. Therefore:

$$\frac{180 \times 15 \times 3.1416}{300} = \text{diameter of smaller pulley} \times 3.1416.$$

As 3.1416 enters as a factor on both sides of the equals sign, it can be canceled. Then:

$$\frac{180 \times 15}{300} = \text{diameter of smaller pulley}.$$

From this, the following rule for the relation between the sizes of pulleys and the number of revolutions of two shafts can be formulated:

The number of revolutions of one shaft multiplied by the diameter of the pulley on the same shaft, divided by the number of revolutions of the second shaft, gives the diameter of the pulley on the second shaft.

This rule can be expressed as a formula:

$$\text{Diameter of pulley on second shaft} = \frac{\text{revolutions of first shaft} \times \text{diam. of pulley on first shaft}}{\text{revolutions of second shaft}}.$$

If one pulley makes 200 revolutions while another pulley makes 100 revolutions, the *speed ratio* between the two pulleys is 2 to 1. If one pulley makes 200 revolutions while another makes 50 revolutions, the speed ratio is 4 to 1, because one shaft makes four times as many revolutions as the other. If one shaft runs at 200 revolutions and another at 140 revolutions, the speed ratio would be 200 to 140. By canceling equal factors in 200 and 140, reduce the ratio so as to express it with smaller numbers. In this case the ratio would be 10 to 7, because $\frac{200}{140} = \frac{20}{14} = \frac{10}{7}$.

If very accurate results are required, one thickness of the belt should be added to the diameter of the pulley itself, and the dimension thus obtained should be used in the preceding formulas instead of the diameter of the pulley rim. If the pulley is 5 inches in diameter and the belt $\frac{1}{8}$ inch thick, the diameter to be used in the formulas should be $5\frac{1}{8}$ inches. The results obtained in this manner will be very accurate provided there is no slipping of the belt on either of the pulleys. For ordinary practical purposes, however, it is customary to figure with the diameter of the pulley rim, taking no account of the thickness of the belt.

The following rules apply to the practical questions relating to pulley speeds and diameters that are most frequently met with in shop installations.

Speed of Driven Pulley Required. — Diameter and speed of driving pulley, and diameter of driven pulley are known. *Rule:* Multiply the diameter of the driving pulley by its speed in revolutions per minute, and divide the product by the diameter of the driven pulley.

Example: — If the diameter of the driving pulley is 15 inches and its speed, 180 revolutions per minute, and the

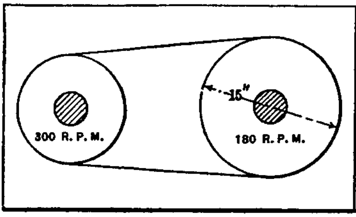


Fig. 1

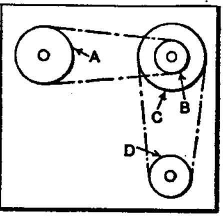


Fig. 2

diameter of the driven pulley, 9 inches, then the speed of the driven pulley $= \frac{15 \times 180}{9} = 300$ revolutions per minute.

Diameter of Driven Pulley Required. — Diameter and speed of driving pulley, and revolutions per minute of driven pulley are known. *Rule:* Multiply the diameter of the driving pulley by its speed in revolutions per minute, and divide the product by the required speed of the driven pulley.

Example: — If the diameter of the driving pulley is 24 inches and its speed, 100 revolutions per minute, and the driven pulley is to rotate 600 revolutions per minute, then the diameter of the driven pulley $= \frac{24 \times 100}{600} = 4$ inches.

Diameter of Driving Pulley Required. — Diameter and speed of driven pulley, and speed of driving pulley are known. *Rule:* Multiply the diameter of the driven pulley by its speed in revolutions per minute, and divide the product by the speed of the driving pulley.

Example: — If the diameter of the driven pulley is 36 inches and its required speed, 150 revolutions per minute, and the speed of the driving pulley is 600 revolutions per minute, then the diameter of the driving pulley

$$= \frac{36 \times 150}{600} = 9 \text{ inches.}$$

Speed of Driving Pulley Required. — Diameters of driving and driven pulleys, and speed of driven pulley are known. *Rule:* Multiply the diameter of the driven pulley by its speed, and divide the product by the diameter of the driving pulley.

Example:—If the diameter of the driven pulley is 4 inches, its required speed, 800 revolutions per minute, and the diameter of the driver, 26 inches, then the required speed of the driver $= \frac{4 \times 800}{26} = 123$ revolutions per minute, approximately.

Speed of Driven Pulley in Compound Drive Required.—Diameters of pulleys A, B, C, and D (see Fig. 2), and speed of pulley A are known; find speed of pulley D. *Rule:* Divide product of diameters of driving pulleys by product of diameters of driven pulleys, and multiply quotient by speed of first driving pulley.

Example:—If the diameters of the driving pulleys A and C are 18 and 24 inches; the diameters of the driven pulleys B and D, 12 and 13 inches; and the speed of the driver A, 260 revolutions per minute; then the speed of the driven pulley D equals:

$$\frac{18 \times 24}{12 \times 13} \times 260 = 720 \text{ revolutions per minute.}$$

Pulley Diameters in Compound Drive Required.—Speeds of driving and driven pulleys are known; find diameters of the four pulleys A, B, C, and D. *Rule:* Place the speed of the driving pulley as the numerator of a fraction, and the speed of driven pulley as the denominator, and reduce this fraction to its lowest terms; then resolve both the numerator and denominator into two factors, and multiply each "pair" of factors (a pair being one factor in the numerator and one in the denominator) by a trial number which will give pulleys of suitable diameters.

Example:—If the speed of pulley A is 260 revolutions per minute, and the required speed of pulley D is 720 revolutions per minute, find the diameters of the four pulleys. The fraction $\frac{260}{720}$ reduced to its lowest terms is $\frac{13}{36}$, which represents the required speed ratio. Resolve $\frac{13}{36}$ into two factors; $\frac{13}{36} = \frac{1 \times 13}{2 \times 18}$. Multiply by trial numbers 12 and 1:

$$\frac{(1 \times 12) \times (13 \times 1)}{(2 \times 12) \times (18 \times 1)} = \frac{12 \times 13}{24 \times 18}$$

The values 12 and 13 in the numerator represent the diameters of the *driven* pulleys B and D, and values 24 and 18 in the denominator, the diameters of the *driving* pulleys.

Length of Belt on Pulleys.—A simple rule, which can be used with fair accuracy when the pulley diameters are nearly equal, is as follows: Add the diameters of the two pulleys, divide the sum by 2, and multiply the quotient by $3\frac{1}{7}$; then add to the product twice the distance between the centers of the shafts. Expressing this rule as a formula, in which D = diameter of large pulley; d = diameter of small pulley; C = center distance between shafts; L = belt length:

$$L = \frac{D + d}{2} \times 3\tfrac{1}{7} + 2\,C.$$

Example:—Diameter of large pulley equals 15 inches; diameter of small pulley equals 13 inches. The center distance is 92 inches. Find length of belt.

$$L = \frac{15 + 13}{2} \times 3\tfrac{1}{7} + 2 \times 92 = 228 \text{ inches.}$$

SIMPLE INTEREST TABLE

Amount and Time		4%	5%	6%	7%	8%
$1.00	1 month	$ .003	$ .004	$ .005	$ .005	$ .006
$1.00	2 months	.007	.008	.010	.011	.013
$1.00	3 months	.011	.013	.015	.017	.020
$1.00	6 months	.020	.025	.030	.035	.040
$1.00	12 months	.040	.050	.060	.070	.080
$100.00	1 day	.011	.013	.016	.019	.022
$100.00	2 days	.022	.027	.032	.038	.044
$100.00	3 days	.034	.041	.050	.058	.067
$100.00	4 days	.045	.053	.066	.077	.089
$100.00	5 days	.056	.069	.082	.097	.111
$100.00	6 days	.067	.083	.100	.116	.133
$100.00	1 month	.334	.416	.500	.583	.667
$100.00	2 months	.667	.832	1.000	1.166	1.333
$100.00	3 months	1.000	1.250	1.500	1.750	2.000
$100.00	6 months	2.000	2.500	3.000	3.500	4.000
$100.00	12 months	4.000	5.000	6.000	7.000	8.000

TO FIND THE INTEREST ON ANY SUM FOR ANY TIME

Point off two places from the right of the principal and multiply it by the number of months. One-half the result is the interest at 6 percent. Deduct one-sixth for 5 percent; one-third for 4 percent; add one-sixth for 7 percent; one-tbird for 8 percent, etc.

VI. Compound Interest

Compound Interest: $(1 + r)^n$

AMOUNT OF ONE DOLLAR PRINCIPAL AT COMPOUND INTEREST AFTER *n* YEARS

n	2 %	2½ %	3 %	3½ %	4 %	4½ %	5 %	6 %	7 %
1	1.0200	1.0250	1.0300	1.0350	1.0400	1.0450	1.0500	1.0600	1.0700
2	1.0404	1.0506	1.0609	1.0712	1.0816	1.0920	1.1025	1.1236	1.1449
3	1.0612	1.0769	1.0927	1.1087	1.1249	1.1412	1.1576	1.1910	1.2250
4	1.0824	1.1038	1.1255	1.1475	1.1699	1.1925	1.2155	1.2625	1.3108
5	1.1041	1.1314	1.1593	1.1877	1.2167	1.2462	1.2763	1.3382	1.4026
6	1.1262	1.1597	1.1941	1.2293	1.2653	1.3023	1.3401	1.4185	1.5007
7	1.1487	1.1887	1.2299	1.2723	1.3159	1.3609	1.4071	1.5036	1.6058
8	1.1717	1.2184	1.2668	1.3168	1.3686	1.4221	1.4775	1.5938	1.7182
9	1.1951	1.2489	1.3048	1.3629	1.4233	1.4861	1.5513	1.6895	1.8385
10	1.2190	1.2801	1.3439	1.4106	1.4802	1.5530	1.6289	1.7908	1.9672
11	1.2434	1.3121	1.3842	1.4600	1.5395	1.6229	1.7103	1.8983	2.1049
12	1.2682	1.3449	1.4258	1.5111	1.6010	1.6959	1.7959	2.0122	2.2522
13	1.2936	1.3785	1.4685	1.5640	1.6651	1.7722	1.8856	2.1329	2.4098
14	1.3195	1.4130	1.5126	1.6187	1.7317	1.8519	1.9799	2.2609	2.5785
15	1.3459	1.4483	1.5580	1.6753	1.8009	1.9353	2.0789	2.3966	2.7590
16	1.3728	1.4845	1.6047	1.7340	1.8730	2.0224	2.1829	2.5404	2.9522
17	1.4002	1.5216	1.6528	1.7947	1.9479	2.1134	2.2920	2.6928	3.1588
18	1.4282	1.5597	1.7024	1.8575	2.0258	2.2085	2.4066	2.8543	3.3799
19	1.4568	1.5987	1.7535	1.9225	2.1068	2.3079	2.5270	3.0256	3.6165
20	1.4859	1.6386	1.8061	1.9898	2.1911	2.4117	2.6533	3.2071	3.8697
21	1.5157	1.6796	1.8603	2.0594	2.2788	2.5202	2.7860	3.3996	4.1406
22	1.5460	1.7216	1.9161	2.1315	2.3699	2.6337	2.9253	3.6035	4.4304
23	1.5769	1.7646	1.9736	2.2061	2.4647	2.7522	3.0715	3.8197	4.7405
24	1.6084	1.8087	2.0328	2.2833	2.5633	2.8760	3.2251	4.0489	5.0724
25	1.6406	1.8539	2.0938	2.3632	2.6658	3.0054	3.3864	4.2919	5.4274
26	1.6734	1.9003	2.1566	2.4460	2.7725	3.1407	3.5557	4.5494	5.8074
27	1.7069	1.9478	2.2213	2.5316	2.8834	3.2820	3.7335	4.8223	6.2139
28	1.7410	1.9965	2.2879	2.6202	2.9987	3.4297	3.9201	5.1117	6.6488
29	1.7758	2.0464	2.3566	2.7119	3.1187	3.5840	4.1161	5.4184	7.1143
30	1.8114	2.0976	2.4273	2.8068	3.2434	3.7453	4.3219	5.7435	7.6123
31	1.8476	2.1500	2.5001	2.9050	3.3731	3.9139	4.5380	6.0881	8.1451
32	1.8845	2.2038	2.5751	3.0067	3.5081	4.0900	4.7649	6.4534	8.7153
33	1.9222	2.2589	2.6523	3.1119	3.6484	4.2740	5.0032	6.8406	9.3253
34	1.9607	2.3153	2.7319	3.2209	3.7943	4.4664	5.2533	7.2510	9.9781
35	1.9999	2.3732	2.8139	3.3336	3.9461	4.6673	5.5160	7.6861	10.6766
36	2.0399	2.4325	2.8983	3.4503	4.1039	4.8774	5.7918	8.1473	11.4239
37	2.0807	2.4933	2.9852	3.5710	4.2681	5.0969	6.0814	8.6361	12.2236
38	2.1223	2.5557	3.0748	3.6960	4.4388	5.3262	6.3855	9.1543	13.0793
39	2.1647	2.6196	3.1670	3.8254	4.6164	5.5659	6.7048	9.7035	13.9948
40	2.2080	2.6851	3.2620	3.9593	4.8010	5.8164	7.0400	10.2857	14.9745
41	2.2522	2.7522	3.3599	4.0978	4.9931	6.0781	7.3920	10.9029	16.0227
42	2.2972	2.8210	3.4607	4.2413	5.1928	6.3516	7.7616	11.5570	17.1443
43	2.3432	2.8915	3.5645	4.3897	5.4005	6.6374	8.1497	12.2505	18.3444
44	2.3901	2.9638	3.6715	4.5433	5.6165	6.9361	8.5572	12.9855	19.6285
45	2.4379	3.0379	3.7816	4.7024	5.8412	7.2482	8.9850	13.7646	21.0025
46	2.4866	3.1139	3.8950	4.8669	6.0748	7.5744	9.4343	14.5905	22.4726
47	2.5363	3.1917	4.0119	5.0373	6.3178	7.9153	9.9060	15.4659	24.0457
48	2.5871	3.2715	4.1323	5.2136	6.5705	8.2715	10.4013	16.3939	25.7289
49	2.6388	3.3533	4.2562	5.3961	6.8333	8.6437	10.9213	17.3775	27.5299
50	2.6916	3.4371	4.3839	5.5849	7.1067	9.0326	11.4674	18.4202	29.4570

Investment appraisal

Appraisal for investment usually means considering the rates of return for various projects and selecting the one with the 'best' return. Methods of appraisal for investment depend first on the estimation of the net cash flow from an investment, where all taxes, subsidies, grants, etc., have been allowed for in estimating the cash flow. Methods differ according to the way in which the flows of cash over time are related to the initial investment:

Rate of return = the total of the estimated cash flows over the life of the investment, divided by the number of years life, and expressed as a percentage of the investment. Where a unit of 100 is invested to give the following net cash flows over 10 years: 1, 10.0; 2, 11.0; 3, 12.1; 4, 13.3; 5, 14.6; 6, 16.1; 7, 17.7; 8, 19.5; 9, 21.4; 10, 23.6; then:
the crude rate of return is the total of those flows = 159.3, divided by 10 = 15.93, which is 15.93% as a percentage of the 100 units originally invested.

Pay back = the number of years after which the original investment would have been recouped. In the above example, this would be after year 8, when a cumulative amount of 114.3 would have been received net.

Discounted cash flow

The above two methods make no allowance for the time factor in the cash flows; as any sum could be earning interest in an alternative investment, it is more realistic to 'discount' the cash flows to be received in future years by an expected rate of interest. Tables for the amount accumulated by 1 unit at different rates of interest and for different periods are included on pages 208–211. The 'discount factor', giving the present value of any future amount allowing for the rate of interest concerned, is the reciprocal of the figure in the accumulation tables. A special table of discount factors is included on the facing page.

The following table shows the effect of 'discounting' the cash flows in the above example at interest rates of 5% and 10%; this follows the convention of regarding the first year as not subject to discounting, so that year 2 is discounted at the rate for 1 year as shown in the discount factor table. Further, the convention of regarding all receipts and payments as taking place on one day of each year is adopted.

Period	Net cash flow	Interest at 5% Discount factor	Present value	Interest at 10% Discount factor	Present value
1	10.0	1.0000	10.0	1.0000	10.0
2	11.0	0.9524	10.5	0.9091	10.0
3	12.1	0.9070	11.0	0.8264	10.0
4	13.3	0.8638	11.5	0.7513	10.0
5	14.6	0.8227	12.0	0.6830	10.0
6	16.1	0.7835	12.6	0.6209	10.0
7	17.7	0.7462	13.2	0.5645	10.0
8	19.5	0.7107	13.9	0.5132	10.0
9	21.4	0.6768	14.5	0.4665	10.0
10	23.6	0.6446	15.2	0.4241	10.0

The total cash flow at present value is 124.4 discounted at 5%, and 100 discounted at 10%, compared with the crude total flow of 159.3. Assuming an interest rate of 5% the investment of 100 units will give a 'profit' of 24.4, and assuming an interest rate of 10%, the investment of 100 gives a return of exactly 100, so it yields no profit at that rate of interest (this is because the net cash flows used in the example are the same as the amount accumulated from 10 at 10% p.a., as indicated in the table on page 210).

Where the present value of the flow of cash, discounted at a certain rate, exactly equals the initial amount of investment, that rate is sometimes called the DCF 'solution' rate, since it is the rate of return given by the investment made. In the above example, 10% is the DCF solution rate. DCF solution rates are usually calculated by interpolation from rates which are approximately known.

Where the investment is also spread over more than one period, allowance is also made for discounting the investment payments. In the following example the effect of spreading a 100 unit investment over 3 periods is illustrated, discounting taking place at the DCF solution rate which differs accordingly.

Period	Investment in 1 period		Investment in 3 periods	
	Net cash flow	Present value	Net cash flow	Present value
Investment				
1	100	100	25	25.0
2			50	37.8
3			25	14.3
Returns				
1	20	20.0	20	20.0
2	20	16.6	20	15.1
3	20	13.8	20	11.5
4	20	11.5	20	8.7
5	20	9.6	20	6.6
6	20	8.0	20	5.0
7	20	6.6	20	3.8
8	20	5.5	20	2.8
9	20	4.6	20	2.2
10	20	3.8	20	1.6

The total discounted amount or present value is 100 for the 1 period investment example, with a DCF solution rate of 20.3%; for the example with investment over 3 periods, the total discounted amount is 77.2 units (since investment flows are also discounted) with a DCF solution rate of 32.2%. The higher rate is obtained because the investment does not need to be made immediately, but the returns are nevertheless the same.

The basic formula for computing a present value is:

$$\frac{B_t}{(1+r)^t}$$

where B_t is the benefit in year t and r is the discount rate. The same formula would hold for costs – simply substituting C for B. The general formula for computing a present value of a set of benefits *and* costs that occur through time, known as a **net present value** (NPV) would be:

$$\Sigma_t \frac{B_t - C_t}{(1+r)^t}$$

The CBA rule then, is that for any policy or project, the NPV should be positive.

To illustrate the above rule, consider a project that has the following sequence of costs and benefits:

	Year 1	Year 2	Year 3	Year 4	Year 5
Cost	30	10	0	0	0
Benefit	0	5	15	15	15
Net benefit	−30	−5	15	15	15

Note that costs appear as minuses and benefits as pluses. Suppose the discount rate, r, is 10 per cent (which is written as 0.1). Then the computation is:

$$-30/1.1 - 5/(1.1)^2 + 15/(1.1)^3 + 15/(1.1)^4 + 15(1.1)^5$$

Typically this would be done with a discounted cash flow computer program, but in this case the calculations are simple:

$$-27.3 - 4.1 + 11.3 + 10.3 + 9.3 = -0.5$$

The NPV is negative and therefore the project is not worthwhile. Notice that without the discounting procedure, benefits of 45 exceed costs of 35. Discounting can therefore make a big difference to the ultimate decision to accept or reject a project.

The choice faced by a decision-maker may be a simple 'accept/reject' decision like the one above, but it may also be one of choosing between competing alternatives, e.g. a hydroelectric power plant or a coal-fired plant or a nuclear power station. If each option has a positive NPV, the choice should be made on the basis of the highest NPV. Yet another decision context arises where a number of projects can be chosen but the budget available is limited. The rule then is to *rank* the projects according to the *ratio* of the PV of benefits to the PV of costs (the 'benefit–cost ratio') and work down the ranked list until the budget is exhausted.

The practice of *discounting* arises because individuals attach less weight to a benefit or cost in the future than they do to a benefit or cost now. Impatience, or 'time preference', is one reason why the present is preferred to the future. The second reason is that, since capital is productive, £1's worth of resources now will generate more than a £1's worth of goods and services in the future. Hence an entrepreneur would be willing to pay more than £1 in the future to acquire £1's worth of these resources now. This argument for discounting is referred to as the 'marginal productivity of capital' argument, the use of the word 'marginal' indicating that it is the productivity of additional units of capital that is relevant.

Discount factors

Years / Rate of interest: %	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Years
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.9009	0.8929	0.8850	0.8772	0.8696	1
2	0.9803	0.9612	0.9426	0.9246	0.9070	0.8900	0.8734	0.8573	0.8417	0.8264	0.8116	0.7972	0.7831	0.7695	0.7561	2
3	0.9706	0.9423	0.9151	0.8890	0.8638	0.8396	0.8163	0.7938	0.7722	0.7513	0.7312	0.7118	0.6931	0.6750	0.6575	3
4	0.9610	0.9238	0.8885	0.8548	0.8227	0.7921	0.7629	0.7350	0.7084	0.6830	0.6587	0.6355	0.6133	0.5921	0.5718	4
5	0.9515	0.9057	0.8626	0.8219	0.7835	0.7473	0.7130	0.6806	0.6499	0.6209	0.5935	0.5674	0.5428	0.5194	0.4972	5
6	0.9420	0.8880	0.8375	0.7903	0.7462	0.7050	0.6663	0.6302	0.5963	0.5645	0.5346	0.5066	0.4803	0.4556	0.4323	6
7	0.9327	0.8706	0.8131	0.7599	0.7107	0.6651	0.6227	0.5835	0.5470	0.5132	0.4817	0.4523	0.4251	0.3996	0.3759	7
8	0.9235	0.8535	0.7894	0.7307	0.6768	0.6274	0.5820	0.5403	0.5019	0.4665	0.4339	0.4039	0.3762	0.3506	0.3269	8
9	0.9143	0.8368	0.7664	0.7026	0.6446	0.5919	0.5439	0.5002	0.4604	0.4241	0.3909	0.3606	0.3329	0.3075	0.2843	9
10	0.9053	0.8203	0.7441	0.6756	0.6139	0.5584	0.5083	0.4632	0.4224	0.3855	0.3522	0.3220	0.2946	0.2697	0.2472	10
11	0.8963	0.8043	0.7224	0.6496	0.5847	0.5268	0.4751	0.4289	0.3875	0.3505	0.3173	0.2875	0.2607	0.2366	0.2149	11
12	0.8874	0.7885	0.7014	0.6246	0.5568	0.4970	0.4440	0.3971	0.3555	0.3186	0.2858	0.2567	0.2307	0.2076	0.1869	12
13	0.8787	0.7730	0.6810	0.6006	0.5303	0.4688	0.4150	0.3677	0.3262	0.2897	0.2575	0.2292	0.2042	0.1821	0.1625	13
14	0.8700	0.7579	0.6611	0.5775	0.5051	0.4423	0.3878	0.3405	0.2992	0.2633	0.2320	0.2046	0.1807	0.1597	0.1413	14
15	0.8613	0.7430	0.6419	0.5553	0.4810	0.4173	0.3624	0.3152	0.2745	0.2394	0.2090	0.1827	0.1599	0.1401	0.1229	15
16	0.8528	0.7284	0.6232	0.5339	0.4581	0.3936	0.3387	0.2919	0.2519	0.2176	0.1883	0.1631	0.1415	0.1229	0.1069	16
17	0.8444	0.7142	0.6050	0.5134	0.4363	0.3714	0.3166	0.2703	0.2311	0.1978	0.1696	0.1456	0.1252	0.1078	0.0929	17
18	0.8360	0.7002	0.5874	0.4936	0.4155	0.3503	0.2959	0.2502	0.2120	0.1799	0.1528	0.1300	0.1108	0.0946	0.0808	18
19	0.8277	0.6864	0.5703	0.4746	0.3957	0.3305	0.2765	0.2317	0.1945	0.1635	0.1377	0.1161	0.0981	0.0829	0.0703	19
20	0.8195	0.6730	0.5537	0.4564	0.3769	0.3118	0.2584	0.2145	0.1784	0.1486	0.1240	0.1037	0.0868	0.0728	0.0611	20
21	0.8114	0.6598	0.5375	0.4388	0.3589	0.2942	0.2415	0.1987	0.1637	0.1351	0.1117	0.0926	0.0768	0.0638	0.0531	21
22	0.8034	0.6468	0.5219	0.4220	0.3418	0.2775	0.2257	0.1839	0.1502	0.1228	0.1007	0.0826	0.0680	0.0560	0.0462	22
23	0.7954	0.6342	0.5067	0.4057	0.3256	0.2618	0.2109	0.1703	0.1378	0.1117	0.0907	0.0738	0.0601	0.0491	0.0402	23
24	0.7876	0.6217	0.4919	0.3901	0.3101	0.2470	0.1971	0.1577	0.1264	0.1015	0.0817	0.0659	0.0532	0.0431	0.0349	24
25	0.7798	0.6095	0.4776	0.3751	0.2953	0.2330	0.1842	0.1460	0.1160	0.0923	0.0736	0.0588	0.0471	0.0378	0.0304	25
30	0.7419	0.5521	0.4120	0.3083	0.2314	0.1741	0.1314	0.0994	0.0754	0.0573	0.0437	0.0334	0.0256	0.0196	0.0151	30
40	0.6717	0.4529	0.3066	0.2083	0.1420	0.0972	0.0668	0.0460	0.0318	0.0221	0.0154	0.0107	0.0075	0.0053	0.0037	40
50	0.6080	0.3715	0.2281	0.1407	0.0872	0.0543	0.0339	0.0213	0.0134	0.0085	0.0054	0.0035	0.0022	0.0014	0.0009	50

Years / Rate of interest: %	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	Years
1	0.8621	0.8547	0.8475	0.8403	0.8333	0.8264	0.8197	0.8130	0.8065	0.8000	0.7937	0.7874	0.7812	0.7752	0.7692	1
2	0.7432	0.7305	0.7182	0.7062	0.6944	0.6830	0.6719	0.6610	0.6504	0.6400	0.6299	0.6200	0.6104	0.6009	0.5917	2
3	0.6407	0.6244	0.6086	0.5934	0.5787	0.5645	0.5507	0.5374	0.5245	0.5120	0.4999	0.4882	0.4768	0.4658	0.4552	3
4	0.5523	0.5337	0.5158	0.4987	0.4823	0.4665	0.4514	0.4369	0.4230	0.4096	0.3968	0.3844	0.3725	0.3611	0.3501	4
5	0.4761	0.4561	0.4371	0.4190	0.4019	0.3855	0.3700	0.3552	0.3411	0.3277	0.3149	0.3027	0.2910	0.2799	0.2693	5
6	0.4104	0.3898	0.3704	0.3521	0.3349	0.3186	0.3033	0.2888	0.2751	0.2621	0.2499	0.2383	0.2274	0.2170	0.2072	6
7	0.3538	0.3332	0.3139	0.2959	0.2791	0.2633	0.2486	0.2348	0.2218	0.2097	0.1983	0.1877	0.1776	0.1682	0.1594	7
8	0.3050	0.2848	0.2660	0.2487	0.2326	0.2176	0.2038	0.1909	0.1789	0.1678	0.1574	0.1478	0.1388	0.1304	0.1226	8
9	0.2630	0.2434	0.2255	0.2090	0.1938	0.1799	0.1670	0.1552	0.1443	0.1342	0.1249	0.1164	0.1084	0.1011	0.0943	9
10	0.2267	0.2080	0.1911	0.1756	0.1615	0.1486	0.1369	0.1262	0.1164	0.1074	0.0992	0.0916	0.0847	0.0784	0.0725	10
11	0.1954	0.1778	0.1619	0.1476	0.1346	0.1228	0.1122	0.1026	0.0938	0.0859	0.0787	0.0721	0.0662	0.0607	0.0558	11
12	0.1685	0.1520	0.1372	0.1240	0.1122	0.1015	0.0920	0.0834	0.0757	0.0687	0.0625	0.0568	0.0517	0.0471	0.0429	12
13	0.1452	0.1299	0.1163	0.1042	0.0935	0.0839	0.0754	0.0678	0.0610	0.0550	0.0496	0.0447	0.0404	0.0365	0.0330	13
14	0.1252	0.1110	0.0985	0.0876	0.0779	0.0693	0.0618	0.0551	0.0492	0.0440	0.0393	0.0352	0.0316	0.0283	0.0254	14
15	0.1079	0.0949	0.0835	0.0736	0.0649	0.0573	0.0507	0.0448	0.0397	0.0352	0.0312	0.0277	0.0247	0.0219	0.0195	15
16	0.0930	0.0811	0.0708	0.0618	0.0541	0.0474	0.0415	0.0364	0.0320	0.0281	0.0248	0.0218	0.0193	0.0170	0.0150	16
17	0.0802	0.0693	0.0600	0.0520	0.0451	0.0391	0.0340	0.0296	0.0258	0.0225	0.0197	0.0172	0.0150	0.0132	0.0116	17
18	0.0691	0.0592	0.0508	0.0437	0.0376	0.0323	0.0279	0.0241	0.0208	0.0180	0.0156	0.0135	0.0118	0.0102	0.0089	18
19	0.0596	0.0506	0.0431	0.0367	0.0313	0.0267	0.0229	0.0196	0.0168	0.0144	0.0124	0.0107	0.0092	0.0079	0.0068	19
20	0.0514	0.0433	0.0365	0.0308	0.0261	0.0221	0.0187	0.0159	0.0135	0.0115	0.0098	0.0084	0.0072	0.0061	0.0053	20
21	0.0443	0.0370	0.0309	0.0259	0.0217	0.0183	0.0154	0.0129	0.0109	0.0092	0.0078	0.0066	0.0056	0.0048	0.0040	21
22	0.0382	0.0316	0.0262	0.0218	0.0181	0.0151	0.0126	0.0105	0.0088	0.0074	0.0062	0.0052	0.0044	0.0037	0.0031	22
23	0.0329	0.0270	0.0222	0.0183	0.0151	0.0125	0.0103	0.0086	0.0071	0.0059	0.0049	0.0041	0.0034	0.0029	0.0024	23
24	0.0284	0.0231	0.0188	0.0154	0.0126	0.0103	0.0085	0.0070	0.0057	0.0047	0.0039	0.0032	0.0027	0.0022	0.0018	24
25	0.0245	0.0197	0.0160	0.0129	0.0105	0.0085	0.0069	0.0057	0.0046	0.0038	0.0031	0.0025	0.0021	0.0017	0.0014	25
30	0.0116	0.0090	0.0070	0.0054	0.0042	0.0033	0.0026	0.0020	0.0016	0.0012	0.0010	0.0008	0.0006	0.0005	0.0004	30
40	0.0026	0.0019	0.0013	0.0010	0.0007	0.0005	0.0004	0.0003	0.0002	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	40
50	0.0006	0.0004	0.0003	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	50

Step 6.
Determining Cost Effectiveness*

The important economic consideration when designing a passive solar heated building is the trade-off between the cost of extra thermal mass and movable insulation (less the installed cost of the conventional construction it replaces) and the future cost of the fuel saved by the system over its lifetime. Operating and maintenance cost must also be included; however, for most passive systems this cost is negligible. The cost of solar heat can be estimated by the following formula:

$$\text{cost of solar heat} = \frac{\left(\begin{array}{c}\text{solar}\\ \text{system}\\ \text{cost}\end{array} \times \begin{array}{c}\text{capital recovery}\\ \text{factor}\end{array}\right) + \begin{array}{c}\text{annual operating}\\ \text{and maintenance}\\ \text{cost}\end{array}}{\text{annual solar heating contribution } (Q_{c\ year})}$$

*Adapted from Los Alamos Scientific Laboratory, *Pacific Regional Solar Heating Handbook*. ERDA, San Francisco, California, 1976.

The *capital recovery factor* is determined from bankers' tables or formulas. It is defined as the value of capital to the individual. It may be the interest rate that your money would earn if you invested it, or the annual cost of a loan made to finance the extra cost of the passive system. For example, the capital recovery factor of a 10% 30-year loan is 0.106.

To illustrate the use of the formula, if we assume, for example, that

—the passive solar heating system costs $5,000 above installed conventional construction costs,

—the capital recovery factor is 0.106 for a 30-year loan at 10% interest,

—the operating and maintenance cost for the system is $25 a year, and

—the annual solar heating contribution is 100 million Btu's,

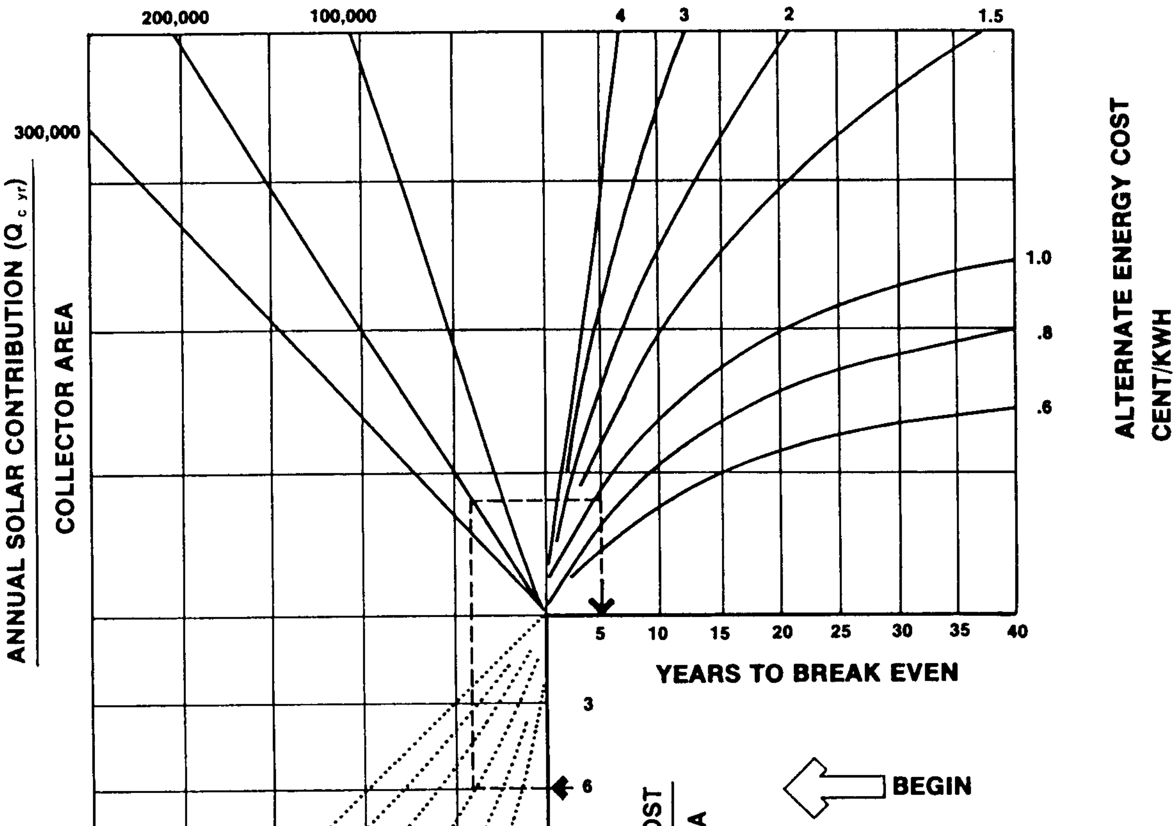
from the formula, the cost of solar heat is then:

$$\text{cost of solar heat} = \frac{(\$5{,}000 \times 0.106) + \$25}{100 \text{ million Btu's}}$$

$$= \$5.55 \text{ per million Btu's.}$$

This figure does not take into account considerations that would make the cost less expensive, such as tax incentives, deduction of interest payments and business depreciation, or considerations that can make it more expensive, such as property tax evaluation increases and fuel cost deductions (business expense).

Another method for calculating the cost effectiveness of a system is the nomograph in figure 1-9. This method allows for the increase in future annual fuel costs to be included in the procedure. By plotting the cost of the system, the annual projected increase in energy costs, the annual solar heating contribution and the cost of conventional fuel, the nomograph computes the break-even time on the system's initial cost.



1. Begin with installed total solar system cost, expressed in terms of $/sq ft of collector. (Example: $6/sq ft)
2. Intersect line representing estimated average annual increase in cost of alternate fuels. (Example: 10%)
3. Intersect line representing the amount of collected solar energy which is *utilized* in the building, expressed in Btu/sq ft of collector/year. (Example: 200,000)
4. Intersect line representing the cost of alternate fuels expressed in cents/kwh. (Example: Gas at 1.0 cent/kwh)
5. Read the number of years to break even on the initial investment. (Example: 5 years)

Based on 8% interest, 1% maintenance/year.

Fig. 1-9: Solar system cost nomograph.

Source: Adapted from GSA, "Energy Conservation Design Guidelines for New Office Buildings," as quoted by P.D. Maycock in "Solar Energy: The Outlook for Widespread Commercialization of Solar Heating and Cooling," ERDA.

Table 1-4 Capital Recovery Factors

Years	Interest Rate						
	5½%	6%	7%	8%	10%	12%	15%
1	1.055 00	1.060 00	1.070 00	1.080 00	1.100 00	1.120 00	1.150 00
2	0.541 62	0.545 44	0.553 09	0.560 77	0.576 19	0.591 70	0.615 12
3	0.370 65	0.374 11	0.381 05	0.388 03	0.402 11	0.416 35	0.437 98
4	0.285 29	0.288 59	0.295 23	0.301 92	0.315 47	0.329 23	0.350 27
5	0.234 18	0.237 40	0.243 89	0.250 46	0.263 80	0.277 41	0.298 32
6	0.200 18	0.203 36	0.209 80	0.216 32	0.229 61	0.243 23	0.264 24
7	0.175 96	0.179 14	0.185 55	0.192 07	0.205 41	0.219 12	0.240 36
8	0.157 86	0.161 04	0.167 47	0.174 01	0.187 44	0.201 30	0.222 85
9	0.143 84	0.147 02	0.153 49	0.160 08	0.173 64	0.187 68	0.209 57
10	0.132 67	0.135 87	0.142 38	0.149 03	0.162 75	0.176 98	0.199 25
11	0.123 57	0.126 79	0.133 36	0.140 08	0.153 96	0.168 42	0.191 07
12	0.116 03	0.119 28	0.125 90	0.132 70	0.146 76	0.161 44	0.184 48
13	0.109 68	0.112 96	0.119 65	0.126 52	0.140 78	0.155 68	0.179 11
14	0.104 28	0.107 58	0.114 34	0.121 30	0.135 75	0.150 87	0.174 69
15	0.099 63	0.102 96	0.109 79	0.116 83	0.131 47	0.146 82	0.171 02
16	0.095 58	0.098 95	0.105 86	0.112 98	0.127 82	0.143 39	0.167 95
17	0.092 04	0.095 44	0.102 43	0.109 63	0.124 66	0.140 46	0.165 37
18	0.088 92	0.092 36	0.099 41	0.106 70	0.121 93	0.137 94	0.163 19
19	0.086 15	0.089 62	0.096 75	0.104 13	0.119 55	0.135 76	0.161 34
20	0.083 68	0.087 18	0.094 39	0.101 85	0.117 46	0.133 88	0.159 76
21	0.081 46	0.085 00	0.092 29	0.099 83	0.115 62	0.132 24	0.158 42
22	0.079 47	0.083 05	0.090 41	0.098 03	0.114 01	0.130 81	0.157 27
23	0.077 67	0.081 28	0.088 71	0.096 42	0.112 57	0.129 56	0.156 28
24	0.076 04	0.079 68	0.087 19	0.094 98	0.111 30	0.128 46	0.155 43
25	0.074 55	0.078 23	0.085 81	0.093 68	0.110 17	0.127 50	0.154 70
26	0.073 19	0.076 90	0.084 56	0.092 51	0.109 16	0.126 65	0.154 07
27	0.071 95	0.075 70	0.083 43	0.091 45	0.108 26	0.125 90	0.153 53
28	0.070 81	0.074 59	0.082 39	0.090 49	0.107 45	0.125 24	0.153 06
29	0.069 77	0.073 58	0.081 45	0.089 62	0.106 73	0.124 66	0.152 65
30	0.068 81	0.072 65	0.080 59	0.088 83	0.106 08	0.124 14	0.152 30
31	0.067 92	0.071 79	0.079 80	0.088 11	0.105 50	1.123 69	0.152 00
32	0.067 10	0.071 00	0.079 07	0.087 45	0.104 97	0.123 28	0.151 73
33	0.066 33	0.070 27	0.078 41	0.086 85	0.104 50	0.122 92	0.151 50
34	0.065 63	0.069 60	0.077 80	0.086 30	0.104 07	0.122 60	0.151 31
35	0.064 97	0.068 97	0.077 23	0.085 80	0.103 69	0.122 32	0.151 13