Security Protocols

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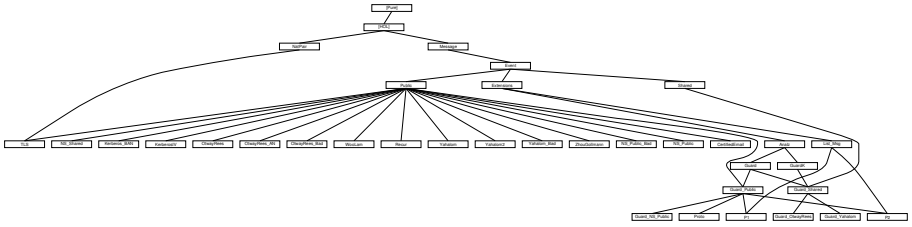

1 Theory of Agents and Messages for Security Protocols

theory *Message* **imports** *Main* **begin**

lemma *[simp] : "A ∪ (B ∪ A) = B ∪ A"*
by *blast*

types
key = nat

consts
all_symmetric :: bool — true if all keys are symmetric
invKey :: "key=>key" — inverse of a symmetric key

specification *(invKey)*
invKey [simp]: "invKey (invKey K) = K"
invKey_symmetric: "all_symmetric --> invKey = id"
by *(rule exI [of _ id], auto)*

The inverse of a symmetric key is itself; that of a public key is the private key and vice versa

constdefs
symKeys :: "key set"
"symKeys == {K. invKey K = K}"

datatype — We allow any number of friendly agents
agent = Server | Friend nat | Spy

datatype
msg = Agent agent — Agent names
| Number nat — Ordinary integers, timestamps, ...
| Nonce nat — Unguessable nonces
| Key key — Crypto keys
| Hash msg — Hashing
| MPair msg msg — Compound messages
| Crypt key msg — Encryption, public- or shared-key

Concrete syntax: messages appear as —A,B,NA—, etc...

syntax
*"@MTuple" :: "['a, args] => 'a * 'b" ("(2{|_,/ _|})")*

syntax *(xsymbols)*
*"@MTuple" :: "['a, args] => 'a * 'b" ("(2⦃_,/ _⦄)")*

translations
"{|x, y, z|}" == "{|x, {|y, z|}|}"
"{|x, y|}" == "MPair x y"

constdefs
HPair :: "[msg,msg] => msg" *("(4Hash[_] /_)" [0, 1000])*

— Message Y paired with a MAC computed with the help of X
"Hash[X] Y == {| Hash{|X,Y|}, Y|}"

keysFor :: "msg set => key set"
— Keys useful to decrypt elements of a message set
"keysFor H == invKey ‘ {K. ∃X. Crypt K X ∈ H}"

1.0.1 Inductive Definition of All Parts" of a Message

consts *parts :: "msg set => msg set"*
inductive *"parts H"*
intros
Inj [intro]: *"X ∈ H ==> X ∈ parts H"*
Fst: *"{|X,Y|} ∈ parts H ==> X ∈ parts H"*
Snd: *"{|X,Y|} ∈ parts H ==> Y ∈ parts H"*
Body: *"Crypt K X ∈ parts H ==> X ∈ parts H"*

Monotonicity

lemma *parts_mono: "G ⊆ H ==> parts(G) ⊆ parts(H)"*
apply *auto*
apply *(erule parts.induct)*
apply *(blast dest: parts.Fst parts.Snd parts.Body)+*
done

Equations hold because constructors are injective.

lemma *Friend_image_eq [simp]: "(Friend x ∈ Friend‘A) = (x:A)"*
by *auto*

lemma *Key_image_eq [simp]: "(Key x ∈ Key‘A) = (x∈A)"*
by *auto*

lemma *Nonce_Key_image_eq [simp]: "(Nonce x ∉ Key‘A)"*
by *auto*

1.0.2 Inverse of keys

lemma *invKey_eq [simp]: "(invKey K = invKey K') = (K=K')"*
apply *safe*
apply *(drule_tac f = invKey* **in** *arg_cong, simp)*
done

1.1 keysFor operator

lemma *keysFor_empty [simp]: "keysFor {} = {}"*
by *(unfold keysFor_def, blast)*

lemma *keysFor_Un [simp]: "keysFor (H ∪ H') = keysFor H ∪ keysFor H'"*
by *(unfold keysFor_def, blast)*

lemma *keysFor_UN [simp]: "keysFor (⋃i∈A. H i) = (⋃i∈A. keysFor (H i))"*
by *(unfold keysFor_def, blast)*

Monotonicity

lemma *keysFor_mono: "G ⊆ H ==> keysFor(G) ⊆ keysFor(H)"*

by *(unfold keysFor_def, blast)*

lemma *keysFor_insert_Agent [simp]: "keysFor (insert (Agent A) H) = keysFor H"*
by *(unfold keysFor_def, auto)*

lemma *keysFor_insert_Nonce [simp]: "keysFor (insert (Nonce N) H) = keysFor H"*
by *(unfold keysFor_def, auto)*

lemma *keysFor_insert_Number [simp]: "keysFor (insert (Number N) H) = keysFor H"*
by *(unfold keysFor_def, auto)*

lemma *keysFor_insert_Key [simp]: "keysFor (insert (Key K) H) = keysFor H"*
by *(unfold keysFor_def, auto)*

lemma *keysFor_insert_Hash [simp]: "keysFor (insert (Hash X) H) = keysFor H"*
by *(unfold keysFor_def, auto)*

lemma *keysFor_insert_MPair [simp]: "keysFor (insert {|X,Y|} H) = keysFor H"*
by *(unfold keysFor_def, auto)*

lemma *keysFor_insert_Crypt [simp]:*
"keysFor (insert (Crypt K X) H) = insert (invKey K) (keysFor H)"
by *(unfold keysFor_def, auto)*

lemma *keysFor_image_Key [simp]: "keysFor (Key`E) = {}"*
by *(unfold keysFor_def, auto)*

lemma *Crypt_imp_invKey_keysFor: "Crypt K X ∈ H ==> invKey K ∈ keysFor H"*
by *(unfold keysFor_def, blast)*

1.2 Inductive relation "parts"

lemma *MPair_parts:*
"[| {|X,Y|} ∈ parts H;
[| X ∈ parts H; Y ∈ parts H |] ==> P |] ==> P"
by *(blast dest: parts.Fst parts.Snd)*

declare *MPair_parts [elim!] parts.Body [dest!]*

NB These two rules are UNSAFE in the formal sense, as they discard the compound message. They work well on THIS FILE. *MPair_parts* is left as SAFE because it speeds up proofs. The Crypt rule is normally kept UNSAFE to avoid breaking up certificates.

lemma *parts_increasing: "H ⊆ parts(H)"*
by *blast*

lemmas *parts_insertI = subset_insertI [THEN parts_mono, THEN subsetD, standard]*

lemma *parts_empty [simp]: "parts{} = {}"*

apply *safe*
apply *(erule parts.induct, blast+)*
done

lemma *parts_emptyE [elim!]: "X∈ parts{} ==> P"*
by *simp*

WARNING: loops if H = Y, therefore must not be repeated!

lemma *parts_singleton: "X∈ parts H ==> ∃Y∈H. X∈ parts {Y}"*
by *(erule parts.induct, blast+)*

1.2.1 Unions

lemma *parts_Un_subset1: "parts(G) ∪ parts(H) ⊆ parts(G ∪ H)"*
by *(intro Un_least parts_mono Un_upper1 Un_upper2)*

lemma *parts_Un_subset2: "parts(G ∪ H) ⊆ parts(G) ∪ parts(H)"*
apply *(rule subsetI)*
apply *(erule parts.induct, blast+)*
done

lemma *parts_Un [simp]: "parts(G ∪ H) = parts(G) ∪ parts(H)"*
by *(intro equalityI parts_Un_subset1 parts_Un_subset2)*

lemma *parts_insert: "parts (insert X H) = parts {X} ∪ parts H"*
apply *(subst insert_is_Un [of _ H])*
apply *(simp only: parts_Un)*
done

TWO inserts to avoid looping. This rewrite is better than nothing. Not suitable for Addsimps: its behaviour can be strange.

lemma *parts_insert2:*
"parts (insert X (insert Y H)) = parts {X} ∪ parts {Y} ∪ parts H"
apply *(simp add: Un_assoc)*
apply *(simp add: parts_insert [symmetric])*
done

lemma *parts_UN_subset1: "(⋃x∈A. parts(H x)) ⊆ parts(⋃x∈A. H x)"*
by *(intro UN_least parts_mono UN_upper)*

lemma *parts_UN_subset2: "parts(⋃x∈A. H x) ⊆ (⋃x∈A. parts(H x))"*
apply *(rule subsetI)*
apply *(erule parts.induct, blast+)*
done

lemma *parts_UN [simp]: "parts(⋃x∈A. H x) = (⋃x∈A. parts(H x))"*
by *(intro equalityI parts_UN_subset1 parts_UN_subset2)*

Added to simplify arguments to parts, analz and synth. NOTE: the UN versions are no longer used!

This allows *blast* to simplify occurrences of *parts (G ∪ H)* in the assumption.

lemmas *in_parts_UnE = parts_Un [THEN equalityD1, THEN subsetD, THEN UnE]*

declare *in_parts_UnE [elim!]*

lemma *parts_insert_subset: "insert X (parts H) ⊆ parts(insert X H)"*
by *(blast intro: parts_mono [THEN [2] rev_subsetD])*

1.2.2 Idempotence and transitivity

lemma *parts_partsD [dest!]: "X∈ parts (parts H) ==> X∈ parts H"*
by *(erule parts.induct, blast+)*

lemma *parts_idem [simp]: "parts (parts H) = parts H"*
by *blast*

lemma *parts_subset_iff [simp]: "(parts G ⊆ parts H) = (G ⊆ parts H)"*
apply *(rule iffI)*
apply *(iprover intro: subset_trans parts_increasing)*
apply *(frule parts_mono, simp)*
done

lemma *parts_trans: "[| X∈ parts G; G ⊆ parts H |] ==> X∈ parts H"*
by *(drule parts_mono, blast)*

Cut

lemma *parts_cut:*
"[| Y∈ parts (insert X G); X∈ parts H |] ==> Y∈ parts (G ∪ H)"
by *(erule parts_trans, auto)*

lemma *parts_cut_eq [simp]: "X∈ parts H ==> parts (insert X H) = parts H"*
by *(force dest!: parts_cut intro: parts_insertI)*

1.2.3 Rewrite rules for pulling out atomic messages

lemmas *parts_insert_eq_I = equalityI [OF subsetI parts_insert_subset]*

lemma *parts_insert_Agent [simp]:*
"parts (insert (Agent agt) H) = insert (Agent agt) (parts H)"
apply *(rule parts_insert_eq_I)*
apply *(erule parts.induct, auto)*
done

lemma *parts_insert_Nonce [simp]:*
"parts (insert (Nonce N) H) = insert (Nonce N) (parts H)"
apply *(rule parts_insert_eq_I)*
apply *(erule parts.induct, auto)*
done

lemma *parts_insert_Number [simp]:*
"parts (insert (Number N) H) = insert (Number N) (parts H)"
apply *(rule parts_insert_eq_I)*
apply *(erule parts.induct, auto)*
done

```
lemma parts_insert_Key [simp]:
     "parts (insert (Key K) H) = insert (Key K) (parts H)"
apply (rule parts_insert_eq_I)
apply (erule parts.induct, auto)
done

lemma parts_insert_Hash [simp]:
     "parts (insert (Hash X) H) = insert (Hash X) (parts H)"
apply (rule parts_insert_eq_I)
apply (erule parts.induct, auto)
done

lemma parts_insert_Crypt [simp]:
     "parts (insert (Crypt K X) H) = insert (Crypt K X) (parts (insert X H))"
apply (rule equalityI)
apply (rule subsetI)
apply (erule parts.induct, auto)
apply (blast intro: parts.Body)
done

lemma parts_insert_MPair [simp]:
     "parts (insert {|X,Y|} H) =
          insert {|X,Y|} (parts (insert X (insert Y H)))"
apply (rule equalityI)
apply (rule subsetI)
apply (erule parts.induct, auto)
apply (blast intro: parts.Fst parts.Snd)+
done

lemma parts_image_Key [simp]: "parts (Key`N) = Key`N"
apply auto
apply (erule parts.induct, auto)
done
```

In any message, there is an upper bound N on its greatest nonce.

```
lemma msg_Nonce_supply: "∃N. ∀n. N≤n --> Nonce n ∉ parts {msg}"
apply (induct_tac "msg")
apply (simp_all (no_asm_simp) add: exI parts_insert2)
```

MPair case: blast works out the necessary sum itself!

```
 prefer 2 apply (blast elim!: add_leE)
```

Nonce case

```
apply (rule_tac x = "N + Suc nat" in exI, auto)
done
```

1.3 Inductive relation "analz"

Inductive definition of "analz" – what can be broken down from a set of messages, including keys. A form of downward closure. Pairs can be taken apart; messages decrypted with known keys.

```
consts  analz   :: "msg set => msg set"
```

```
inductive "analz H"
  intros
    Inj [intro,simp] :    "X ∈ H ==> X ∈ analz H"
    Fst:     "{|X,Y|} ∈ analz H ==> X ∈ analz H"
    Snd:     "{|X,Y|} ∈ analz H ==> Y ∈ analz H"
    Decrypt [dest]:
             "[|Crypt K X ∈ analz H; Key(invKey K): analz H|] ==> X ∈ analz
H"
```

Monotonicity; Lemma 1 of Lowe's paper

```
lemma analz_mono: "G⊆H ==> analz(G) ⊆ analz(H)"
apply auto
apply (erule analz.induct)
apply (auto dest: analz.Fst analz.Snd)
done
```

Making it safe speeds up proofs

```
lemma MPair_analz [elim!]:
     "[| {|X,Y|} ∈ analz H;
             [| X ∈ analz H; Y ∈ analz H |] ==> P
         |] ==> P"
by (blast dest: analz.Fst analz.Snd)

lemma analz_increasing: "H ⊆ analz(H)"
by blast

lemma analz_subset_parts: "analz H ⊆ parts H"
apply (rule subsetI)
apply (erule analz.induct, blast+)
done

lemmas analz_into_parts = analz_subset_parts [THEN subsetD, standard]

lemmas not_parts_not_analz = analz_subset_parts [THEN contra_subsetD, standard]


lemma parts_analz [simp]: "parts (analz H) = parts H"
apply (rule equalityI)
apply (rule analz_subset_parts [THEN parts_mono, THEN subset_trans], simp)
apply (blast intro: analz_increasing [THEN parts_mono, THEN subsetD])
done

lemma analz_parts [simp]: "analz (parts H) = parts H"
apply auto
apply (erule analz.induct, auto)
done

lemmas analz_insertI = subset_insertI [THEN analz_mono, THEN [2] rev_subsetD,
standard]
```

1.3.1 General equational properties

```
lemma analz_empty [simp]: "analz{} = {}"
apply safe
```

apply *(erule analz.induct, blast+)*
done

Converse fails: we can analz more from the union than from the separate parts, as a key in one might decrypt a message in the other

lemma *analz_Un: "analz(G) ∪ analz(H) ⊆ analz(G ∪ H)"*
by *(intro Un_least analz_mono Un_upper1 Un_upper2)*

lemma *analz_insert: "insert X (analz H) ⊆ analz(insert X H)"*
by *(blast intro: analz_mono [THEN [2] rev_subsetD])*

1.3.2 Rewrite rules for pulling out atomic messages

lemmas *analz_insert_eq_I = equalityI [OF subsetI analz_insert]*

lemma *analz_insert_Agent [simp]:*
"analz (insert (Agent agt) H) = insert (Agent agt) (analz H)"
apply *(rule analz_insert_eq_I)*
apply *(erule analz.induct, auto)*
done

lemma *analz_insert_Nonce [simp]:*
"analz (insert (Nonce N) H) = insert (Nonce N) (analz H)"
apply *(rule analz_insert_eq_I)*
apply *(erule analz.induct, auto)*
done

lemma *analz_insert_Number [simp]:*
"analz (insert (Number N) H) = insert (Number N) (analz H)"
apply *(rule analz_insert_eq_I)*
apply *(erule analz.induct, auto)*
done

lemma *analz_insert_Hash [simp]:*
"analz (insert (Hash X) H) = insert (Hash X) (analz H)"
apply *(rule analz_insert_eq_I)*
apply *(erule analz.induct, auto)*
done

Can only pull out Keys if they are not needed to decrypt the rest

lemma *analz_insert_Key [simp]:*
"K ∉ keysFor (analz H) ==>
analz (insert (Key K) H) = insert (Key K) (analz H)"
apply *(unfold keysFor_def)*
apply *(rule analz_insert_eq_I)*
apply *(erule analz.induct, auto)*
done

lemma *analz_insert_MPair [simp]:*
"analz (insert {|X,Y|} H) =
insert {|X,Y|} (analz (insert X (insert Y H)))"
apply *(rule equalityI)*
apply *(rule subsetI)*
apply *(erule analz.induct, auto)*

```
apply (erule analz.induct)
apply (blast intro: analz.Fst analz.Snd)+
done
```

Can pull out enCrypted message if the Key is not known

```
lemma analz_insert_Crypt:
     "Key (invKey K) ∉ analz H
      ==> analz (insert (Crypt K X) H) = insert (Crypt K X) (analz H)"
apply (rule analz_insert_eq_I)
apply (erule analz.induct, auto)

done

lemma lemma1: "Key (invKey K) ∈ analz H ==>
               analz (insert (Crypt K X) H) ⊆
               insert (Crypt K X) (analz (insert X H))"
apply (rule subsetI)
apply (erule_tac xa = x in analz.induct, auto)
done

lemma lemma2: "Key (invKey K) ∈ analz H ==>
               insert (Crypt K X) (analz (insert X H)) ⊆
               analz (insert (Crypt K X) H)"
apply auto
apply (erule_tac xa = x in analz.induct, auto)
apply (blast intro: analz_insertI analz.Decrypt)
done

lemma analz_insert_Decrypt:
     "Key (invKey K) ∈ analz H ==>
               analz (insert (Crypt K X) H) =
               insert (Crypt K X) (analz (insert X H))"
by (intro equalityI lemma1 lemma2)
```

Case analysis: either the message is secure, or it is not! Effective, but can cause subgoals to blow up! Use with `split_if`; apparently `split_tac` does not cope with patterns such as `analz (insert (Crypt K X) H)`

```
lemma analz_Crypt_if [simp]:
     "analz (insert (Crypt K X) H) =
          (if (Key (invKey K) ∈ analz H)
           then insert (Crypt K X) (analz (insert X H))
           else insert (Crypt K X) (analz H))"
by (simp add: analz_insert_Crypt analz_insert_Decrypt)
```

This rule supposes "for the sake of argument" that we have the key.

```
lemma analz_insert_Crypt_subset:
     "analz (insert (Crypt K X) H) ⊆
           insert (Crypt K X) (analz (insert X H))"
apply (rule subsetI)
apply (erule analz.induct, auto)
done
```

```
lemma analz_image_Key [simp]: "analz (Key‘N) = Key‘N"
apply auto
apply (erule analz.induct, auto)
done
```

1.3.3 Idempotence and transitivity

```
lemma analz_analzD [dest!]: "X∈ analz (analz H) ==> X∈ analz H"
by (erule analz.induct, blast+)

lemma analz_idem [simp]: "analz (analz H) = analz H"
by blast

lemma analz_subset_iff [simp]: "(analz G ⊆ analz H) = (G ⊆ analz H)"
apply (rule iffI)
apply (iprover intro: subset_trans analz_increasing)
apply (frule analz_mono, simp)
done

lemma analz_trans: "[| X∈ analz G;  G ⊆ analz H |] ==> X∈ analz H"
by (drule analz_mono, blast)
```

Cut; Lemma 2 of Lowe

```
lemma analz_cut: "[| Y∈ analz (insert X H);  X∈ analz H |] ==> Y∈ analz
H"
by (erule analz_trans, blast)
```

This rewrite rule helps in the simplification of messages that involve the forwarding of unknown components (X). Without it, removing occurrences of X can be very complicated.

```
lemma analz_insert_eq: "X∈ analz H ==> analz (insert X H) = analz H"
by (blast intro: analz_cut analz_insertI)
```

A congruence rule for "analz"

```
lemma analz_subset_cong:
     "[| analz G ⊆ analz G'; analz H ⊆ analz H' |]
      ==> analz (G ∪ H) ⊆ analz (G' ∪ H')"
apply simp
apply (iprover intro: conjI subset_trans analz_mono Un_upper1 Un_upper2)
done

lemma analz_cong:
     "[| analz G = analz G'; analz H = analz H' |]
      ==> analz (G ∪ H) = analz (G' ∪ H')"
by (intro equalityI analz_subset_cong, simp_all)

lemma analz_insert_cong:
     "analz H = analz H' ==> analz(insert X H) = analz(insert X H')"
by (force simp only: insert_def intro!: analz_cong)
```

If there are no pairs or encryptions then analz does nothing

```
lemma analz_trivial:
     "[| ∀X Y. {|X,Y|} ∉ H;  ∀X K. Crypt K X ∉ H |] ==> analz H = H"
```

```
apply safe
apply (erule analz.induct, blast+)
done
```

These two are obsolete (with a single Spy) but cost little to prove...

```
lemma analz_UN_analz_lemma:
     "X∈ analz (⋃i∈A. analz (H i)) ==> X∈ analz (⋃i∈A. H i)"
apply (erule analz.induct)
apply (blast intro: analz_mono [THEN [2] rev_subsetD])+
done

lemma analz_UN_analz [simp]: "analz (⋃i∈A. analz (H i)) = analz (⋃i∈A.
H i)"
by (blast intro: analz_UN_analz_lemma analz_mono [THEN [2] rev_subsetD])
```

1.4 Inductive relation "synth"

Inductive definition of "synth" – what can be built up from a set of messages. A form of upward closure. Pairs can be built, messages encrypted with known keys. Agent names are public domain. Numbers can be guessed, but Nonces cannot be.

```
consts  synth   :: "msg set => msg set"
inductive "synth H"
  intros
    Inj    [intro]:   "X ∈ H ==> X ∈ synth H"
    Agent  [intro]:   "Agent agt ∈ synth H"
    Number [intro]:   "Number n  ∈ synth H"
    Hash   [intro]:   "X ∈ synth H ==> Hash X ∈ synth H"
    MPair  [intro]:   "[|X ∈ synth H;  Y ∈ synth H|] ==> {|X,Y|} ∈ synth
H"
    Crypt  [intro]:   "[|X ∈ synth H;  Key(K) ∈ H|] ==> Crypt K X ∈ synth
H"
```

Monotonicity

```
lemma synth_mono: "G⊆H ==> synth(G) ⊆ synth(H)"
  by (auto, erule synth.induct, auto)
```

NO *Agent_synth*, as any Agent name can be synthesized. The same holds for *Number*

```
inductive_cases Nonce_synth [elim!]: "Nonce n ∈ synth H"
inductive_cases Key_synth   [elim!]: "Key K ∈ synth H"
inductive_cases Hash_synth  [elim!]: "Hash X ∈ synth H"
inductive_cases MPair_synth [elim!]: "{|X,Y|} ∈ synth H"
inductive_cases Crypt_synth [elim!]: "Crypt K X ∈ synth H"


lemma synth_increasing: "H ⊆ synth(H)"
by blast
```

1.4.1 Unions

Converse fails: we can synth more from the union than from the separate parts, building a compound message using elements of each.

lemma *synth_Un: "synth(G) ∪ synth(H) ⊆ synth(G ∪ H)"*
by *(intro Un_least synth_mono Un_upper1 Un_upper2)*

lemma *synth_insert: "insert X (synth H) ⊆ synth(insert X H)"*
by *(blast intro: synth_mono [THEN [2] rev_subsetD])*

1.4.2 Idempotence and transitivity

lemma *synth_synthD [dest!]: "X∈ synth (synth H) ==> X∈ synth H"*
by *(erule synth.induct, blast+)*

lemma *synth_idem: "synth (synth H) = synth H"*
by *blast*

lemma *synth_subset_iff [simp]: "(synth G ⊆ synth H) = (G ⊆ synth H)"*
apply *(rule iffI)*
apply *(iprover intro: subset_trans synth_increasing)*
apply *(frule synth_mono, simp add: synth_idem)*
done

lemma *synth_trans: "[| X∈ synth G; G ⊆ synth H |] ==> X∈ synth H"*
by *(drule synth_mono, blast)*

Cut; Lemma 2 of Lowe

lemma *synth_cut: "[| Y∈ synth (insert X H); X∈ synth H |] ==> Y∈ synth H"*
by *(erule synth_trans, blast)*

lemma *Agent_synth [simp]: "Agent A ∈ synth H"*
by *blast*

lemma *Number_synth [simp]: "Number n ∈ synth H"*
by *blast*

lemma *Nonce_synth_eq [simp]: "(Nonce N ∈ synth H) = (Nonce N ∈ H)"*
by *blast*

lemma *Key_synth_eq [simp]: "(Key K ∈ synth H) = (Key K ∈ H)"*
by *blast*

lemma *Crypt_synth_eq [simp]:*
"Key K ∉ H ==> (Crypt K X ∈ synth H) = (Crypt K X ∈ H)"
by *blast*

lemma *keysFor_synth [simp]:*
"keysFor (synth H) = keysFor H ∪ invKey`{K. Key K ∈ H}"
by *(unfold keysFor_def, blast)*

1.4.3 Combinations of parts, analz and synth

lemma *parts_synth [simp]: "parts (synth H) = parts H ∪ synth H"*
apply *(rule equalityI)*
apply *(rule subsetI)*

```
apply (erule parts.induct)
apply (blast intro: synth_increasing [THEN parts_mono, THEN subsetD]
                    parts.Fst parts.Snd parts.Body)+
done

lemma analz_analz_Un [simp]: "analz (analz G ∪ H) = analz (G ∪ H)"
apply (intro equalityI analz_subset_cong)+
apply simp_all
done

lemma analz_synth_Un [simp]: "analz (synth G ∪ H) = analz (G ∪ H) ∪ synth
G"
apply (rule equalityI)
apply (rule subsetI)
apply (erule analz.induct)
prefer 5 apply (blast intro: analz_mono [THEN [2] rev_subsetD])
apply (blast intro: analz.Fst analz.Snd analz.Decrypt)+
done

lemma analz_synth [simp]: "analz (synth H) = analz H ∪ synth H"
apply (cut_tac H = "{}" in analz_synth_Un)
apply (simp (no_asm_use))
done
```

1.4.4 For reasoning about the Fake rule in traces

```
lemma parts_insert_subset_Un: "X∈ G ==> parts(insert X H) ⊆ parts G ∪ parts
H"
by (rule subset_trans [OF parts_mono parts_Un_subset2], blast)
```

More specifically for Fake. Very occasionally we could do with a version of the form *parts {X} ⊆ synth (analz H) ∪ parts H*

```
lemma Fake_parts_insert:
     "X ∈ synth (analz H) ==>
      parts (insert X H) ⊆ synth (analz H) ∪ parts H"
apply (drule parts_insert_subset_Un)
apply (simp (no_asm_use))
apply blast
done

lemma Fake_parts_insert_in_Un:
     "[|Z ∈ parts (insert X H);  X: synth (analz H)|]
      ==> Z ∈  synth (analz H) ∪ parts H"
by (blast dest: Fake_parts_insert  [THEN subsetD, dest])
```

H is sometimes *Key ‘ KK ∪ spies evs*, so can't put *G = H*.

```
lemma Fake_analz_insert:
     "X∈ synth (analz G) ==>
      analz (insert X H) ⊆ synth (analz G) ∪ analz (G ∪ H)"
apply (rule subsetI)
apply (subgoal_tac "x ∈ analz (synth (analz G) ∪ H) ")
prefer 2 apply (blast intro: analz_mono [THEN [2] rev_subsetD] analz_mono
[THEN synth_mono, THEN [2] rev_subsetD])
apply (simp (no_asm_use))
```

apply *blast*
done

lemma *analz_conj_parts [simp]:*
"(X ∈ analz H & X ∈ parts H) = (X ∈ analz H)"
by *(blast intro: analz_subset_parts [THEN subsetD])*

lemma *analz_disj_parts [simp]:*
"(X ∈ analz H | X ∈ parts H) = (X ∈ parts H)"
by *(blast intro: analz_subset_parts [THEN subsetD])*

Without this equation, other rules for synth and analz would yield redundant cases

lemma *MPair_synth_analz [iff]:*
"({|X,Y|} ∈ synth (analz H)) =
(X ∈ synth (analz H) & Y ∈ synth (analz H))"
by *blast*

lemma *Crypt_synth_analz:*
"[| Key K ∈ analz H; Key (invKey K) ∈ analz H |]
==> (Crypt K X ∈ synth (analz H)) = (X ∈ synth (analz H))"
by *blast*

lemma *Hash_synth_analz [simp]:*
"X ∉ synth (analz H)
==> (Hash{|X,Y|} ∈ synth (analz H)) = (Hash{|X,Y|} ∈ analz H)"
by *blast*

1.5 HPair: a combination of Hash and MPair

1.5.1 Freeness

lemma *Agent_neq_HPair: "Agent A ~= Hash[X] Y"*
by *(unfold HPair_def, simp)*

lemma *Nonce_neq_HPair: "Nonce N ~= Hash[X] Y"*
by *(unfold HPair_def, simp)*

lemma *Number_neq_HPair: "Number N ~= Hash[X] Y"*
by *(unfold HPair_def, simp)*

lemma *Key_neq_HPair: "Key K ~= Hash[X] Y"*
by *(unfold HPair_def, simp)*

lemma *Hash_neq_HPair: "Hash Z ~= Hash[X] Y"*
by *(unfold HPair_def, simp)*

lemma *Crypt_neq_HPair: "Crypt K X' ~= Hash[X] Y"*
by *(unfold HPair_def, simp)*

lemmas *HPair_neqs = Agent_neq_HPair Nonce_neq_HPair Number_neq_HPair*
Key_neq_HPair Hash_neq_HPair Crypt_neq_HPair

```
declare HPair_neqs [iff]
declare HPair_neqs [symmetric, iff]

lemma HPair_eq [iff]: "(Hash[X'] Y' = Hash[X] Y) = (X' = X & Y'=Y)"
by (simp add: HPair_def)

lemma MPair_eq_HPair [iff]:
     "({|X',Y'|} = Hash[X] Y) = (X' = Hash{|X,Y|} & Y'=Y)"
by (simp add: HPair_def)

lemma HPair_eq_MPair [iff]:
     "(Hash[X] Y = {|X',Y'|}) = (X' = Hash{|X,Y|} & Y'=Y)"
by (auto simp add: HPair_def)
```

1.5.2 Specialized laws, proved in terms of those for Hash and MPair

```
lemma keysFor_insert_HPair [simp]: "keysFor (insert (Hash[X] Y) H) = keysFor
H"
by (simp add: HPair_def)

lemma parts_insert_HPair [simp]:
    "parts (insert (Hash[X] Y) H) =
     insert (Hash[X] Y) (insert (Hash{|X,Y|}) (parts (insert Y H)))"
by (simp add: HPair_def)

lemma analz_insert_HPair [simp]:
    "analz (insert (Hash[X] Y) H) =
     insert (Hash[X] Y) (insert (Hash{|X,Y|}) (analz (insert Y H)))"
by (simp add: HPair_def)

lemma HPair_synth_analz [simp]:
     "X ∉ synth (analz H)
    ==> (Hash[X] Y ∈ synth (analz H)) =
        (Hash {|X, Y|} ∈ analz H & Y ∈ synth (analz H))"
by (simp add: HPair_def)
```

We do NOT want Crypt... messages broken up in protocols!!

```
declare parts.Body [rule del]
```

Rewrites to push in Key and Crypt messages, so that other messages can be pulled out using the `analz_insert` rules

ML

```
{*
fun insComm x y = inst "x" x (inst "y" y insert_commute);

bind_thms ("pushKeys",
           map (insComm "Key ?K")
                   ["Agent ?C", "Nonce ?N", "Number ?N",
                    "Hash ?X", "MPair ?X ?Y", "Crypt ?X ?K'"]);

bind_thms ("pushCrypts",
           map (insComm "Crypt ?X ?K")
                     ["Agent ?C", "Nonce ?N", "Number ?N",
                      "Hash ?X'", "MPair ?X' ?Y"]);
```

```
*}
```

Cannot be added with *[simp]* – messages should not always be re-ordered.

lemmas *pushes = pushKeys pushCrypts*

1.6 Tactics useful for many protocol proofs

ML

```
{*
val invKey = thm "invKey"
val keysFor_def = thm "keysFor_def"
val HPair_def = thm "HPair_def"
val symKeys_def = thm "symKeys_def"
val parts_mono = thm "parts_mono";
val analz_mono = thm "analz_mono";
val synth_mono = thm "synth_mono";
val analz_increasing = thm "analz_increasing";

val analz_insertI = thm "analz_insertI";
val analz_subset_parts = thm "analz_subset_parts";
val Fake_parts_insert = thm "Fake_parts_insert";
val Fake_analz_insert = thm "Fake_analz_insert";
val pushes = thms "pushes";


(*Prove base case (subgoal i) and simplify others.  A typical base case
  concerns  Crypt K X ∉ Key‘shrK‘bad  and cannot be proved by rewriting
  alone.*)
fun prove_simple_subgoals_tac i =
    force_tac (claset(), simpset() addsimps [image_eq_UN]) i THEN
    ALLGOALS Asm_simp_tac

(*Analysis of Fake cases.  Also works for messages that forward unknown parts,
  but this application is no longer necessary if analz_insert_eq is used.
  Abstraction over i is ESSENTIAL: it delays the dereferencing of claset
  DEPENDS UPON "X" REFERRING TO THE FRADULENT MESSAGE *)

(*Apply rules to break down assumptions of the form
  Y ∈ parts(insert X H)  and  Y ∈ analz(insert X H)
*)
val Fake_insert_tac =
    dresolve_tac [impOfSubs Fake_analz_insert,
                  impOfSubs Fake_parts_insert] THEN'
    eresolve_tac [asm_rl, thm"synth.Inj"];

fun Fake_insert_simp_tac ss i =
    REPEAT (Fake_insert_tac i) THEN asm_full_simp_tac ss i;

fun atomic_spy_analz_tac (cs,ss) = SELECT_GOAL
    (Fake_insert_simp_tac ss 1
     THEN
     IF_UNSOLVED (Blast.depth_tac
                  (cs addIs [analz_insertI,
                                  impOfSubs analz_subset_parts]) 4 1))
```

```
(*The explicit claset and simpset arguments help it work with Isar*)
fun gen_spy_analz_tac (cs,ss) i =
  DETERM
   (SELECT_GOAL
     (EVERY
      [  (*push in occurrences of X...*)
       (REPEAT o CHANGED)
           (res_inst_tac [("x1","X")] (insert_commute RS ssubst) 1),
       (*...allowing further simplifications*)
       simp_tac ss 1,
       REPEAT (FIRSTGOAL (resolve_tac [allI,impI,notI,conjI,iffI])),
       DEPTH_SOLVE (atomic_spy_analz_tac (cs,ss) 1)]) i)

fun spy_analz_tac i = gen_spy_analz_tac (claset(), simpset()) i
*}
```

By default only `o_apply` is built-in. But in the presence of eta-expansion this means that some terms displayed as $f \circ g$ will be rewritten, and others will not!

declare *o_def [simp]*

lemma *Crypt_notin_image_Key [simp]: "Crypt K X ∉ Key ‘ A"*
by *auto*

lemma *Hash_notin_image_Key [simp] :"Hash X ∉ Key ‘ A"*
by *auto*

lemma *synth_analz_mono: "G⊆H ==> synth (analz(G)) ⊆ synth (analz(H))"*
by *(iprover intro: synth_mono analz_mono)*

lemma *Fake_analz_eq [simp]:*
"X ∈ synth(analz H) ==> synth (analz (insert X H)) = synth (analz H)"
apply *(drule Fake_analz_insert[of _ _ "H"])*
apply *(simp add: synth_increasing[THEN Un_absorb2])*
apply *(drule synth_mono)*
apply *(simp add: synth_idem)*
apply *(rule equalityI)*
apply *(simp add:)*
apply *(rule synth_analz_mono, blast)*
done

Two generalizations of `analz_insert_eq`

lemma *gen_analz_insert_eq [rule_format]:*
"X ∈ analz H ==> ALL G. H ⊆ G --> analz (insert X G) = analz G"
by *(blast intro: analz_cut analz_insertI analz_mono [THEN [2] rev_subsetD])*

lemma *synth_analz_insert_eq [rule_format]:*
"X ∈ synth (analz H)
==> ALL G. H ⊆ G --> (Key K ∈ analz (insert X G)) = (Key K ∈ analz G)"
apply *(erule synth.induct)*

```
apply (simp_all add: gen_analz_insert_eq subset_trans [OF _ subset_insertI])

done

lemma Fake_parts_sing:
     "X ∈ synth (analz H) ==> parts{X} ⊆ synth (analz H) ∪ parts H"
apply (rule subset_trans)
 apply (erule_tac [2] Fake_parts_insert)
apply (rule parts_mono)
apply (blast intro: elim:)
done

lemmas Fake_parts_sing_imp_Un = Fake_parts_sing [THEN [2] rev_subsetD]

method_setup spy_analz = {*
    Method.ctxt_args (fn ctxt =>
        Method.METHOD (fn facts =>
            gen_spy_analz_tac (local_clasimpset_of ctxt) 1)) *}
    "for proving the Fake case when analz is involved"

method_setup atomic_spy_analz = {*
    Method.ctxt_args (fn ctxt =>
        Method.METHOD (fn facts =>
            atomic_spy_analz_tac (local_clasimpset_of ctxt) 1)) *}
    "for debugging spy_analz"

method_setup Fake_insert_simp = {*
    Method.ctxt_args (fn ctxt =>
        Method.METHOD (fn facts =>
            Fake_insert_simp_tac (local_simpset_of ctxt) 1)) *}
    "for debugging spy_analz"


end
```

2 Theory of Events for Security Protocols

```
theory Event imports Message begin

consts
  initState :: "agent => msg set"

datatype
  event = Says  agent agent msg
        | Gets  agent       msg
        | Notes agent       msg

consts
  bad    :: "agent set"
  knows  :: "agent => event list => msg set"
```

The constant "spies" is retained for compatibility's sake

syntax
```
  spies  :: "event list => msg set"
```

translations
```
  "spies"   => "knows Spy"
```

Spy has access to his own key for spoof messages, but Server is secure

specification *(bad)*
```
  Spy_in_bad     [iff]: "Spy ∈ bad"
  Server_not_bad [iff]: "Server ∉ bad"
    by (rule exI [of _ "{Spy}"], simp)
```

primrec
```
  knows_Nil:   "knows A [] = initState A"
  knows_Cons:
    "knows A (ev # evs) =
       (if A = Spy then
        (case ev of
           Says A' B X => insert X (knows Spy evs)
         | Gets A' X => knows Spy evs
         | Notes A' X  =>
             if A' ∈ bad then insert X (knows Spy evs) else knows Spy evs)
        else
        (case ev of
           Says A' B X =>
             if A'=A then insert X (knows A evs) else knows A evs
         | Gets A' X    =>
             if A'=A then insert X (knows A evs) else knows A evs
         | Notes A' X    =>
             if A'=A then insert X (knows A evs) else knows A evs))"
```

consts

```
  used :: "event list => msg set"
```

primrec
```
  used_Nil:   "used []         = (UN B. parts (initState B))"
  used_Cons:  "used (ev # evs) =
                     (case ev of
                        Says A B X => parts {X} ∪ used evs
                      | Gets A X   => used evs
                      | Notes A X  => parts {X} ∪ used evs)"
```

— The case for *Gets* seems anomalous, but *Gets* always follows *Says* in real protocols. Seems difficult to change. See *Gets_correct* in theory *Guard/Extensions.thy*.

lemma *Notes_imp_used [rule_format]: "Notes A X ∈ set evs --> X ∈ used evs"*
apply *(induct_tac evs)*
apply *(auto split: event.split)*
done

lemma *Says_imp_used [rule_format]: "Says A B X ∈ set evs --> X ∈ used evs"*

```
apply (induct_tac evs)
apply (auto split: event.split)
done

lemma MPair_used [rule_format]:
     "MPair X Y ∈ used evs --> X ∈ used evs & Y ∈ used evs"
apply (induct_tac evs)
apply (auto split: event.split)
done
```

2.1 Function *knows*

```
lemmas parts_insert_knows_A = parts_insert [of _ "knows A evs", standard]

lemma knows_Spy_Says [simp]:
     "knows Spy (Says A B X # evs) = insert X (knows Spy evs)"
by simp
```

Letting the Spy see "bad" agents' notes avoids redundant case-splits on whether `A = Spy` and whether `A ∈ bad`

```
lemma knows_Spy_Notes [simp]:
     "knows Spy (Notes A X # evs) =
          (if A:bad then insert X (knows Spy evs) else knows Spy evs)"
by simp

lemma knows_Spy_Gets [simp]: "knows Spy (Gets A X # evs) = knows Spy evs"
by simp

lemma knows_Spy_subset_knows_Spy_Says:
     "knows Spy evs ⊆ knows Spy (Says A B X # evs)"
by (simp add: subset_insertI)

lemma knows_Spy_subset_knows_Spy_Notes:
     "knows Spy evs ⊆ knows Spy (Notes A X # evs)"
by force

lemma knows_Spy_subset_knows_Spy_Gets:
     "knows Spy evs ⊆ knows Spy (Gets A X # evs)"
by (simp add: subset_insertI)
```

Spy sees what is sent on the traffic

```
lemma Says_imp_knows_Spy [rule_format]:
     "Says A B X ∈ set evs --> X ∈ knows Spy evs"
apply (induct_tac "evs")
apply (simp_all (no_asm_simp) split add: event.split)
done

lemma Notes_imp_knows_Spy [rule_format]:
     "Notes A X ∈ set evs --> A: bad --> X ∈ knows Spy evs"
apply (induct_tac "evs")
apply (simp_all (no_asm_simp) split add: event.split)
done
```

Elimination rules: derive contradictions from old Says events containing items

known to be fresh

```
lemmas knows_Spy_partsEs =
    Says_imp_knows_Spy [THEN parts.Inj, THEN revcut_rl, standard]
    parts.Body [THEN revcut_rl, standard]
```

Compatibility for the old "spies" function

```
lemmas spies_partsEs = knows_Spy_partsEs
lemmas Says_imp_spies = Says_imp_knows_Spy
lemmas parts_insert_spies = parts_insert_knows_A [of _ Spy]
```

2.2 Knowledge of Agents

```
lemma knows_Says: "knows A (Says A B X # evs) = insert X (knows A evs)"
by simp

lemma knows_Notes: "knows A (Notes A X # evs) = insert X (knows A evs)"
by simp

lemma knows_Gets:
    "A ≠ Spy --> knows A (Gets A X # evs) = insert X (knows A evs)"
by simp


lemma knows_subset_knows_Says: "knows A evs ⊆ knows A (Says A' B X # evs)"
by (simp add: subset_insertI)

lemma knows_subset_knows_Notes: "knows A evs ⊆ knows A (Notes A' X # evs)"
by (simp add: subset_insertI)

lemma knows_subset_knows_Gets: "knows A evs ⊆ knows A (Gets A' X # evs)"
by (simp add: subset_insertI)
```

Agents know what they say

```
lemma Says_imp_knows [rule_format]: "Says A B X ∈ set evs --> X ∈ knows
A evs"
apply (induct_tac "evs")
apply (simp_all (no_asm_simp) split add: event.split)
apply blast
done
```

Agents know what they note

```
lemma Notes_imp_knows [rule_format]: "Notes A X ∈ set evs --> X ∈ knows
A evs"
apply (induct_tac "evs")
apply (simp_all (no_asm_simp) split add: event.split)
apply blast
done
```

Agents know what they receive

```
lemma Gets_imp_knows_agents [rule_format]:
    "A ≠ Spy --> Gets A X ∈ set evs --> X ∈ knows A evs"
apply (induct_tac "evs")
```

```
apply (simp_all (no_asm_simp) split add: event.split)
done
```

What agents DIFFERENT FROM Spy know was either said, or noted, or got, or known initially

```
lemma knows_imp_Says_Gets_Notes_initState [rule_format]:
     "[| X ∈ knows A evs; A ≠ Spy |] ==> EX B.
  Says A B X ∈ set evs | Gets A X ∈ set evs | Notes A X ∈ set evs | X ∈
initState A"
apply (erule rev_mp)
apply (induct_tac "evs")
apply (simp_all (no_asm_simp) split add: event.split)
apply blast
done
```

What the Spy knows – for the time being – was either said or noted, or known initially

```
lemma knows_Spy_imp_Says_Notes_initState [rule_format]:
     "[| X ∈ knows Spy evs |] ==> EX A B.
  Says A B X ∈ set evs | Notes A X ∈ set evs | X ∈ initState Spy"
apply (erule rev_mp)
apply (induct_tac "evs")
apply (simp_all (no_asm_simp) split add: event.split)
apply blast
done

lemma parts_knows_Spy_subset_used: "parts (knows Spy evs) ⊆ used evs"
apply (induct_tac "evs", force)
apply (simp add: parts_insert_knows_A knows_Cons add: event.split, blast)

done

lemmas usedI = parts_knows_Spy_subset_used [THEN subsetD, intro]

lemma initState_into_used: "X ∈ parts (initState B) ==> X ∈ used evs"
apply (induct_tac "evs")
apply (simp_all add: parts_insert_knows_A split add: event.split, blast)
done

lemma used_Says [simp]: "used (Says A B X # evs) = parts{X} ∪ used evs"
by simp

lemma used_Notes [simp]: "used (Notes A X # evs) = parts{X} ∪ used evs"
by simp

lemma used_Gets [simp]: "used (Gets A X # evs) = used evs"
by simp

lemma used_nil_subset: "used [] ⊆ used evs"
apply simp
apply (blast intro: initState_into_used)
done
```

NOTE REMOVAL–laws above are cleaner, as they don't involve "case"

declare *knows_Cons [simp del]*
used_Nil [simp del] used_Cons [simp del]

For proving theorems of the form *X ∉ analz (knows Spy evs) ⟶ P* New events added by induction to "evs" are discarded. Provided this information isn't needed, the proof will be much shorter, since it will omit complicated reasoning about *analz*.

lemmas *analz_mono_contra =*
knows_Spy_subset_knows_Spy_Says [THEN analz_mono, THEN contra_subsetD]
knows_Spy_subset_knows_Spy_Notes [THEN analz_mono, THEN contra_subsetD]
knows_Spy_subset_knows_Spy_Gets [THEN analz_mono, THEN contra_subsetD]

ML
```
{*
val analz_mono_contra_tac =
  let val analz_impI = inst "P" "?Y ∉ analz (knows Spy ?evs)" impI
  in
    rtac analz_impI THEN'
    REPEAT1 o
      (dresolve_tac (thms"analz_mono_contra"))
    THEN' mp_tac
  end
*}
```

lemma *knows_subset_knows_Cons: "knows A evs ⊆ knows A (e # evs)"*
by *(induct e, auto simp: knows_Cons)*

lemma *initState_subset_knows: "initState A ⊆ knows A evs"*
apply *(induct_tac evs, simp)*
apply *(blast intro: knows_subset_knows_Cons [THEN subsetD])*
done

For proving *new_keys_not_used*

lemma *keysFor_parts_insert:*
"[| K ∈ keysFor (parts (insert X G)); X ∈ synth (analz H) |]
==> K ∈ keysFor (parts (G ∪ H)) | Key (invKey K) ∈ parts H"
by *(force*
dest!: parts_insert_subset_Un [THEN keysFor_mono, THEN [2] rev_subsetD]
analz_subset_parts [THEN keysFor_mono, THEN [2] rev_subsetD]
intro: analz_subset_parts [THEN subsetD] parts_mono [THEN [2] rev_subsetD])

method_setup *analz_mono_contra = {*
Method.no_args
*(Method.METHOD (fn facts => REPEAT_FIRST analz_mono_contra_tac)) *}*
"for proving theorems of the form X ∉ analz (knows Spy evs) --> P"

2.2.1 Useful for case analysis on whether a hash is a spoof or not

ML
```
{*
val knows_Cons     = thm "knows_Cons"
val used_Nil       = thm "used_Nil"
val used_Cons      = thm "used_Cons"
```

```
val Notes_imp_used = thm "Notes_imp_used";
val Says_imp_used = thm "Says_imp_used";
val MPair_used = thm "MPair_used";
val Says_imp_knows_Spy = thm "Says_imp_knows_Spy";
val Notes_imp_knows_Spy = thm "Notes_imp_knows_Spy";
val knows_Spy_partsEs = thms "knows_Spy_partsEs";
val spies_partsEs = thms "spies_partsEs";
val Says_imp_spies = thm "Says_imp_spies";
val parts_insert_spies = thm "parts_insert_spies";
val Says_imp_knows = thm "Says_imp_knows";
val Notes_imp_knows = thm "Notes_imp_knows";
val Gets_imp_knows_agents = thm "Gets_imp_knows_agents";
val knows_imp_Says_Gets_Notes_initState = thm "knows_imp_Says_Gets_Notes_initState";
val knows_Spy_imp_Says_Notes_initState = thm "knows_Spy_imp_Says_Notes_initState";
val usedI = thm "usedI";
val initState_into_used = thm "initState_into_used";
val used_Says = thm "used_Says";
val used_Notes = thm "used_Notes";
val used_Gets = thm "used_Gets";
val used_nil_subset = thm "used_nil_subset";
val analz_mono_contra = thms "analz_mono_contra";
val knows_subset_knows_Cons = thm "knows_subset_knows_Cons";
val initState_subset_knows = thm "initState_subset_knows";
val keysFor_parts_insert = thm "keysFor_parts_insert";


val synth_analz_mono = thm "synth_analz_mono";

val knows_Spy_subset_knows_Spy_Says = thm "knows_Spy_subset_knows_Spy_Says";
val knows_Spy_subset_knows_Spy_Notes = thm "knows_Spy_subset_knows_Spy_Notes";
val knows_Spy_subset_knows_Spy_Gets = thm "knows_Spy_subset_knows_Spy_Gets";


val synth_analz_mono_contra_tac =
  let val syan_impI = inst "P" "?Y ∉ synth (analz (knows Spy ?evs))" impI
  in
    rtac syan_impI THEN'
    REPEAT1 o
      (dresolve_tac
       [knows_Spy_subset_knows_Spy_Says RS synth_analz_mono RS contra_subsetD,
        knows_Spy_subset_knows_Spy_Notes RS synth_analz_mono RS contra_subsetD,
        knows_Spy_subset_knows_Spy_Gets RS synth_analz_mono RS contra_subsetD])
    THEN'
    mp_tac
  end;
*}

method_setup synth_analz_mono_contra = {*
    Method.no_args
      (Method.METHOD (fn facts => REPEAT_FIRST synth_analz_mono_contra_tac))
*}
    "for proving theorems of the form X ∉ synth (analz (knows Spy evs)) -->
P"
```

end

theory *Public* **imports** *Event* **begin**

lemma *invKey_K: "K ∈ symKeys ==> invKey K = K"*
by *(simp add: symKeys_def)*

2.3 Asymmetric Keys

consts

```
  publicKey :: "[bool,agent] => key"
```

syntax

```
  pubEK :: "agent => key"
  pubSK :: "agent => key"

  privateKey :: "[bool,agent] => key"
  priEK :: "agent => key"
  priSK :: "agent => key"
```

translations

```
  "pubEK"  == "publicKey False"
  "pubSK"  == "publicKey True"


  "privateKey b A" == "invKey (publicKey b A)"
  "priEK A"  == "privateKey False A"
  "priSK A"  == "privateKey True A"
```

These translations give backward compatibility. They represent the simple situation where the signature and encryption keys are the same.

syntax

```
  pubK :: "agent => key"
  priK :: "agent => key"
```

translations

```
  "pubK A" == "pubEK A"
  "priK A" == "invKey (pubEK A)"
```

By freeness of agents, no two agents have the same key. Since *True* $\neq$ *False*, no agent has identical signing and encryption keys

specification *(publicKey)*

```
  injective_publicKey:
    "publicKey b A = publicKey c A' ==> b=c & A=A'"
   apply (rule exI [of _ "%b A. 2 * agent_case 0 (λn. n + 2) 1 A + (if b then
1 else 0)"])
   apply (auto simp add: inj_on_def split: agent.split, presburger+)
   done
```

axioms

privateKey_neq_publicKey [iff]: "privateKey b A ≠ publicKey c A'"

declare *privateKey_neq_publicKey [THEN not_sym, iff]*

2.4 Basic properties of *pubK* and *priK*

lemma *[iff]: "(publicKey b A = publicKey c A') = (b=c & A=A')"*
by *(blast dest!: injective_publicKey)*

lemma *not_symKeys_pubK [iff]: "publicKey b A ∉ symKeys"*
by *(simp add: symKeys_def)*

lemma *not_symKeys_priK [iff]: "privateKey b A ∉ symKeys"*
by *(simp add: symKeys_def)*

lemma *symKey_neq_priEK: "K ∈ symKeys ==> K ≠ priEK A"*
by *auto*

lemma *symKeys_neq_imp_neq: "(K ∈ symKeys) ≠ (K' ∈ symKeys) ==> K ≠ K'"*
by *blast*

lemma *symKeys_invKey_iff [iff]: "(invKey K ∈ symKeys) = (K ∈ symKeys)"*
by *(unfold symKeys_def, auto)*

lemma *analz_symKeys_Decrypt:*
"[| Crypt K X ∈ analz H; K ∈ symKeys; Key K ∈ analz H |]
==> X ∈ analz H"
by *(auto simp add: symKeys_def)*

2.5 "Image" equations that hold for injective functions

lemma *invKey_image_eq [simp]: "(invKey x ∈ invKey‘A) = (x ∈ A)"*
by *auto*

lemma *publicKey_image_eq [simp]:*
"(publicKey b x ∈ publicKey c ‘ AA) = (b=c & x ∈ AA)"
by *auto*

lemma *privateKey_notin_image_publicKey [simp]: "privateKey b x ∉ publicKey c ‘ AA"*
by *auto*

lemma *privateKey_image_eq [simp]:*
"(privateKey b A ∈ invKey ‘ publicKey c ‘ AS) = (b=c & A∈AS)"
by *auto*

lemma *publicKey_notin_image_privateKey [simp]: "publicKey b A ∉ invKey ‘ publicKey c ‘ AS"*
by *auto*

2.6 Symmetric Keys

For some protocols, it is convenient to equip agents with symmetric as well as asymmetric keys. The theory *Shared* assumes that all keys are symmetric.

consts
shrK :: "agent => key" — long-term shared keys

specification *(shrK)*
inj_shrK: "inj shrK"
— No two agents have the same long-term key
apply *(rule exI [of _ "agent_case 0 (λn. n + 2) 1"])*
apply *(simp add: inj_on_def split: agent.split)*
done

axioms
sym_shrK [iff]: "shrK X ∈ symKeys" — All shared keys are symmetric

declare *inj_shrK [THEN inj_eq, iff]*

lemma *invKey_shrK [simp]: "invKey (shrK A) = shrK A"*
by *(simp add: invKey_K)*

lemma *analz_shrK_Decrypt:*
"[| Crypt (shrK A) X ∈ analz H; Key(shrK A) ∈ analz H |] ==> X ∈ analz H"
by *auto*

lemma *analz_Decrypt':*
"[| Crypt K X ∈ analz H; K ∈ symKeys; Key K ∈ analz H |] ==> X ∈ analz H"
by *(auto simp add: invKey_K)*

lemma *priK_neq_shrK [iff]: "shrK A ≠ privateKey b C"*
by *(simp add: symKeys_neq_imp_neq)*

declare *priK_neq_shrK [THEN not_sym, simp]*

lemma *pubK_neq_shrK [iff]: "shrK A ≠ publicKey b C"*
by *(simp add: symKeys_neq_imp_neq)*

declare *pubK_neq_shrK [THEN not_sym, simp]*

lemma *priEK_noteq_shrK [simp]: "priEK A ≠ shrK B"*
by *auto*

lemma *publicKey_notin_image_shrK [simp]: "publicKey b x ∉ shrK ‘ AA"*
by *auto*

lemma *privateKey_notin_image_shrK [simp]: "privateKey b x ∉ shrK ‘ AA"*
by *auto*

lemma *shrK_notin_image_publicKey [simp]: "shrK x ∉ publicKey b ‘ AA"*
by *auto*

```
lemma shrK_notin_image_privateKey [simp]: "shrK x ∉ invKey ‘ publicKey b
‘ AA"
by auto

lemma shrK_image_eq [simp]: "(shrK x ∈ shrK ‘ AA) = (x ∈ AA)"
by auto
```

For some reason, moving this up can make some proofs loop!

```
declare invKey_K [simp]
```

2.7 Initial States of Agents

Note: for all practical purposes, all that matters is the initial knowledge of the Spy. All other agents are automata, merely following the protocol.

```
primrec

  initState_Server:
    "initState Server     =
       {Key (priEK Server), Key (priSK Server)} ∪
       (Key ‘ range pubEK) ∪ (Key ‘ range pubSK) ∪ (Key ‘ range shrK)"

  initState_Friend:
    "initState (Friend i) =
       {Key (priEK(Friend i)), Key (priSK(Friend i)), Key (shrK(Friend i))}
∪
       (Key ‘ range pubEK) ∪ (Key ‘ range pubSK)"

  initState_Spy:
    "initState Spy        =
       (Key ‘ invKey ‘ pubEK ‘ bad) ∪ (Key ‘ invKey ‘ pubSK ‘ bad) ∪
       (Key ‘ shrK ‘ bad) ∪
       (Key ‘ range pubEK) ∪ (Key ‘ range pubSK)"
```

These lemmas allow reasoning about `used evs` rather than `knows Spy evs`, which is useful when there are private Notes. Because they depend upon the definition of `initState`, they cannot be moved up.

```
lemma used_parts_subset_parts [rule_format]:
     "∀X ∈ used evs. parts {X} ⊆ used evs"
apply (induct evs)
 prefer 2
 apply (simp add: used_Cons)
 apply (rule ballI)
 apply (case_tac a, auto)
apply (auto dest!: parts_cut)
```

Base case

```
apply (simp add: used_Nil)
done

lemma MPair_used_D: "{|X,Y|} ∈ used H ==> X ∈ used H & Y ∈ used H"
by (drule used_parts_subset_parts, simp, blast)
```

```
lemma MPair_used [elim!]:
     "[| {|X,Y|} ∈ used H;
         [| X ∈ used H; Y ∈ used H |] ==> P |]
      ==> P"
by (blast dest: MPair_used_D)
```

Rewrites should not refer to `initState (Friend i)` because that expression is not in normal form.

```
lemma keysFor_parts_initState [simp]: "keysFor (parts (initState C)) = {}"
apply (unfold keysFor_def)
apply (induct_tac "C")
apply (auto intro: range_eqI)
done

lemma Crypt_notin_initState: "Crypt K X ∉ parts (initState B)"
by (induct B, auto)

lemma Crypt_notin_used_empty [simp]: "Crypt K X ∉ used []"
by (simp add: Crypt_notin_initState used_Nil)



lemma shrK_in_initState [iff]: "Key (shrK A) ∈ initState A"
by (induct_tac "A", auto)

lemma shrK_in_knows [iff]: "Key (shrK A) ∈ knows A evs"
by (simp add: initState_subset_knows [THEN subsetD])

lemma shrK_in_used [iff]: "Key (shrK A) ∈ used evs"
by (rule initState_into_used, blast)



lemma Key_not_used [simp]: "Key K ∉ used evs ==> K ∉ range shrK"
by blast

lemma shrK_neq: "Key K ∉ used evs ==> shrK B ≠ K"
by blast

declare shrK_neq [THEN not_sym, simp]
```

2.8 Function `knows Spy`

Agents see their own private keys!

```
lemma priK_in_initState [iff]: "Key (privateKey b A) ∈ initState A"
by (induct_tac "A", auto)
```

Agents see all public keys!

```
lemma publicKey_in_initState [iff]: "Key (publicKey b A) ∈ initState B"
```

by *(case_tac "B", auto)*

All public keys are visible

lemma *spies_pubK [iff]: "Key (publicKey b A) ∈ spies evs"*
apply *(induct_tac "evs")*
apply *(simp_all add: imageI knows_Cons split add: event.split)*
done

declare *spies_pubK [THEN analz.Inj, iff]*

Spy sees private keys of bad agents!

lemma *Spy_spies_bad_privateKey [intro!]:*
"A ∈ bad ==> Key (privateKey b A) ∈ spies evs"
apply *(induct_tac "evs")*
apply *(simp_all add: imageI knows_Cons split add: event.split)*
done

Spy sees long-term shared keys of bad agents!

lemma *Spy_spies_bad_shrK [intro!]:*
"A ∈ bad ==> Key (shrK A) ∈ spies evs"
apply *(induct_tac "evs")*
apply *(simp_all add: imageI knows_Cons split add: event.split)*
done

lemma *publicKey_into_used [iff] :"Key (publicKey b A) ∈ used evs"*
apply *(rule initState_into_used)*
apply *(rule publicKey_in_initState [THEN parts.Inj])*
done

lemma *privateKey_into_used [iff]: "Key (privateKey b A) ∈ used evs"*
apply*(rule initState_into_used)*
apply*(rule priK_in_initState [THEN parts.Inj])*
done

lemma *Crypt_Spy_analz_bad:*
"[| Crypt (shrK A) X ∈ analz (knows Spy evs); A ∈ bad |]
==> X ∈ analz (knows Spy evs)"
by *force*

2.9 Fresh Nonces

lemma *Nonce_notin_initState [iff]: "Nonce N ∉ parts (initState B)"*
by *(induct_tac "B", auto)*

lemma *Nonce_notin_used_empty [simp]: "Nonce N ∉ used []"*
by *(simp add: used_Nil)*

2.10 Supply fresh nonces for possibility theorems

In any trace, there is an upper bound N on the greatest nonce in use

lemma *Nonce_supply_lemma: "EX N. ALL n. N<=n --> Nonce n ∉ used evs"*
apply *(induct_tac "evs")*

apply *(rule_tac x = 0* **in** *exI)*
apply *(simp_all (no_asm_simp) add: used_Cons split add: event.split)*
apply *safe*
apply *(rule msg_Nonce_supply [THEN exE], blast elim!: add_leE)+*
done

lemma *Nonce_supply1: "EX N. Nonce N ∉ used evs"*
by *(rule Nonce_supply_lemma [THEN exE], blast)*

lemma *Nonce_supply: "Nonce (@ N. Nonce N ∉ used evs) ∉ used evs"*
apply *(rule Nonce_supply_lemma [THEN exE])*
apply *(rule someI, fast)*
done

2.11 Specialized Rewriting for Theorems About *analz* and Image

lemma *insert_Key_singleton: "insert (Key K) H = Key ‘ {K} Un H"*
by *blast*

lemma *insert_Key_image: "insert (Key K) (Key‘KK ∪ C) = Key ‘ (insert K KK) ∪ C"*
by *blast*

ML
```
{*
val Key_not_used = thm "Key_not_used";
val insert_Key_singleton = thm "insert_Key_singleton";
val insert_Key_image = thm "insert_Key_image";
*}
```

lemma *Crypt_imp_keysFor :"[|Crypt K X ∈ H; K ∈ symKeys|] ==> K ∈ keysFor H"*
by *(drule Crypt_imp_invKey_keysFor, simp)*

Lemma for the trivial direction of the if-and-only-if of the Session Key Compromise Theorem

lemma *analz_image_freshK_lemma:*
```
"(Key K ∈ analz (Key‘nE ∪ H)) --> (K ∈ nE | Key K ∈ analz H)  ==>
    (Key K ∈ analz (Key‘nE ∪ H)) = (K ∈ nE | Key K ∈ analz H)"
```
by *(blast intro: analz_mono [THEN [2] rev_subsetD])*

lemmas *analz_image_freshK_simps =*
simp_thms mem_simps — these two allow its use with *only:*
disj_comms
image_insert [THEN sym] image_Un [THEN sym] empty_subsetI insert_subset
analz_insert_eq Un_upper2 [THEN analz_mono, THEN subsetD]
insert_Key_singleton
Key_not_used insert_Key_image Un_assoc [THEN sym]

ML
```
{*
val analz_image_freshK_lemma = thm "analz_image_freshK_lemma";
```

```
val analz_image_freshK_simps = thms "analz_image_freshK_simps";

val analz_image_freshK_ss =
     simpset() delsimps [image_insert, image_Un]
               delsimps [imp_disjL]    (*reduces blow-up*)
               addsimps thms "analz_image_freshK_simps"
*}

method_setup analz_freshK = {*
    Method.no_args
     (Method.METHOD
      (fn facts => EVERY [REPEAT_FIRST (resolve_tac [allI, ballI, impI]),
                          REPEAT_FIRST (rtac analz_image_freshK_lemma),
                          ALLGOALS (asm_simp_tac analz_image_freshK_ss)]))
*}
    "for proving the Session Key Compromise theorem"
```

2.12 Specialized Methods for Possibility Theorems

ML

```
{*
val Nonce_supply = thm "Nonce_supply";

(*Tactic for possibility theorems (Isar interface)*)
fun gen_possibility_tac ss state = state |>
    REPEAT (*omit used_Says so that Nonces start from different traces!*)
    (ALLGOALS (simp_tac (ss delsimps [used_Says]))
     THEN
     REPEAT_FIRST (eq_assume_tac ORELSE'
                   resolve_tac [refl, conjI, Nonce_supply]))

(*Tactic for possibility theorems (ML script version)*)
fun possibility_tac state = gen_possibility_tac (simpset()) state

(*For harder protocols (such as Recur) where we have to set up some
  nonces and keys initially*)
fun basic_possibility_tac st = st |>
    REPEAT
    (ALLGOALS (asm_simp_tac (simpset() setSolver safe_solver))
     THEN
     REPEAT_FIRST (resolve_tac [refl, conjI]))
*}

method_setup possibility = {*
    Method.ctxt_args (fn ctxt =>
        Method.METHOD (fn facts =>
            gen_possibility_tac (local_simpset_of ctxt))) *}
    "for proving possibility theorems"

end
```

3 The Needham-Schroeder Shared-Key Protocol

theory *NS_Shared* **imports** *Public* **begin**

From page 247 of Burrows, Abadi and Needham (1989). A Logic of Authentication. Proc. Royal Soc. 426

consts *ns_shared :: "event list set"*
inductive *"ns_shared"*
intros

Nil: *"[] ∈ ns_shared"*

Fake: *"⟦evsf ∈ ns_shared; X ∈ synth (analz (spies evsf))⟧*
⟹ Says Spy B X # evsf ∈ ns_shared"

NS1: *"⟦evs1 ∈ ns_shared; Nonce NA ∉ used evs1⟧*
⟹ Says A Server {|Agent A, Agent B, Nonce NA|} # evs1 ∈ ns_shared"

NS2: *"⟦evs2 ∈ ns_shared; Key KAB ∉ used evs2; KAB ∈ symKeys;*
Says A' Server {|Agent A, Agent B, Nonce NA|} ∈ set evs2⟧
⟹ Says Server A
(Crypt (shrK A)
{|Nonce NA, Agent B, Key KAB,
(Crypt (shrK B) {|Key KAB, Agent A|})|})
 # evs2 ∈ ns_shared"

NS3: *"⟦evs3 ∈ ns_shared; A ≠ Server;*
Says S A (Crypt (shrK A) {|Nonce NA, Agent B, Key K, X|}) ∈ set evs3;
Says A Server {|Agent A, Agent B, Nonce NA|} ∈ set evs3⟧
⟹ Says A B X # evs3 ∈ ns_shared"

NS4: *"⟦evs4 ∈ ns_shared; Nonce NB ∉ used evs4; K ∈ symKeys;*
Says A' B (Crypt (shrK B) {|Key K, Agent A|}) ∈ set evs4⟧
⟹ Says B A (Crypt K (Nonce NB)) # evs4 ∈ ns_shared"

NS5: *"⟦evs5 ∈ ns_shared; K ∈ symKeys;*
Says B' A (Crypt K (Nonce NB)) ∈ set evs5;
Says S A (Crypt (shrK A) {|Nonce NA, Agent B, Key K, X|})
∈ set evs5⟧
⟹ Says A B (Crypt K {|Nonce NB, Nonce NB|}) # evs5 ∈ ns_shared"

Oops: *"⟦evso ∈ ns_shared; Says B A (Crypt K (Nonce NB)) ∈ set evso;*
Says Server A (Crypt (shrK A) {|Nonce NA, Agent B, Key K, X|})
∈ set evso⟧
⟹ Notes Spy {|Nonce NA, Nonce NB, Key K|} # evso ∈ ns_shared"

```
declare Says_imp_knows_Spy [THEN parts.Inj, dest]
declare parts.Body  [dest]
declare Fake_parts_insert_in_Un  [dest]
declare analz_into_parts [dest]
declare image_eq_UN [simp]
```

A "possibility property": there are traces that reach the end

```
lemma "[| A ≠ Server; Key K ∉ used []; K ∈ symKeys |]
       ==> ∃N. ∃evs ∈ ns_shared.
                     Says A B (Crypt K {|Nonce N, Nonce N|}) ∈ set evs"
apply (intro exI bexI)
apply (rule_tac [2] ns_shared.Nil
       [THEN ns_shared.NS1, THEN ns_shared.NS2, THEN ns_shared.NS3,
        THEN ns_shared.NS4, THEN ns_shared.NS5])
apply (possibility, simp add: used_Cons)
done
```

3.1 Inductive proofs about ns_shared

3.1.1 Forwarding lemmas, to aid simplification

For reasoning about the encrypted portion of message NS3

```
lemma NS3_msg_in_parts_spies:
     "Says S A (Crypt KA {|N, B, K, X|}) ∈ set evs ⟹ X ∈ parts (spies evs)"
by blast
```

For reasoning about the Oops message

```
lemma Oops_parts_spies:
     "Says Server A (Crypt (shrK A) {|NA, B, K, X|}) ∈ set evs
           ⟹ K ∈ parts (spies evs)"
by blast
```

Theorems of the form `X ∉ parts (knows Spy evs)` imply that NOBODY sends messages containing `X`

Spy never sees another agent's shared key! (unless it's bad at start)

```
lemma Spy_see_shrK [simp]:
     "evs ∈ ns_shared ⟹ (Key (shrK A) ∈ parts (spies evs)) = (A ∈ bad)"
apply (erule ns_shared.induct, force, drule_tac [4] NS3_msg_in_parts_spies,
simp_all, blast+)
done

lemma Spy_analz_shrK [simp]:
     "evs ∈ ns_shared ⟹ (Key (shrK A) ∈ analz (spies evs)) = (A ∈ bad)"
by auto
```

Nobody can have used non-existent keys!

```
lemma new_keys_not_used [simp]:
    "[|Key K ∉ used evs; K ∈ symKeys; evs ∈ ns_shared|]
     ==> K ∉ keysFor (parts (spies evs))"
apply (erule rev_mp)
```

apply *(erule ns_shared.induct, force, drule_tac [4] NS3_msg_in_parts_spies, simp_all)*

Fake, NS2, NS4, NS5

apply *(force dest!: keysFor_parts_insert, blast+)*
done

3.1.2 Lemmas concerning the form of items passed in messages

Describes the form of K, X and K' when the Server sends this message.

lemma *Says_Server_message_form:*
```
"⟦Says Server A (Crypt K' {|N, Agent B, Key K, X|}) ∈ set evs;
  evs ∈ ns_shared⟧
 ⟹ K ∉ range shrK ∧
    X = (Crypt (shrK B) {|Key K, Agent A|}) ∧
    K' = shrK A"
```
by *(erule rev_mp, erule ns_shared.induct, auto)*

If the encrypted message appears then it originated with the Server

lemma *A_trusts_NS2:*
```
"⟦Crypt (shrK A) {|NA, Agent B, Key K, X|} ∈ parts (spies evs);
  A ∉ bad;  evs ∈ ns_shared⟧
 ⟹ Says Server A (Crypt (shrK A) {|NA, Agent B, Key K, X|}) ∈ set evs"
```
apply *(erule rev_mp)*
apply *(erule ns_shared.induct, force, drule_tac [4] NS3_msg_in_parts_spies, auto)*
done

lemma *cert_A_form:*
```
"⟦Crypt (shrK A) {|NA, Agent B, Key K, X|} ∈ parts (spies evs);
  A ∉ bad;  evs ∈ ns_shared⟧
 ⟹ K ∉ range shrK ∧  X = (Crypt (shrK B) {|Key K, Agent A|})"
```
by *(blast dest!: A_trusts_NS2 Says_Server_message_form)*

EITHER describes the form of X when the following message is sent, OR reduces it to the Fake case. Use *Says_Server_message_form* if applicable.

lemma *Says_S_message_form:*
```
"⟦Says S A (Crypt (shrK A) {|Nonce NA, Agent B, Key K, X|}) ∈ set evs;
  evs ∈ ns_shared⟧
 ⟹ (K ∉ range shrK ∧ X = (Crypt (shrK B) {|Key K, Agent A|}))
    ∨ X ∈ analz (spies evs)"
```
by *(blast dest: Says_imp_knows_Spy analz_shrK_Decrypt cert_A_form analz.Inj)*

NOT useful in this form, but it says that session keys are not used to encrypt messages containing other keys, in the actual protocol. We require that agents should behave like this subsequently also.

lemma
```
"⟦evs ∈ ns_shared;  Kab ∉ range shrK⟧ ⟹
 (Crypt KAB X) ∈ parts (spies evs) ∧
 Key K ∈ parts {X} ⟶ Key K ∈ parts (spies evs)"
```
apply *(erule ns_shared.induct, force, drule_tac [4] NS3_msg_in_parts_spies, simp_all)*

Fake

apply *(blast dest: parts_insert_subset_Un)*

Base, NS4 and NS5

apply *auto*
done

3.1.3 Session keys are not used to encrypt other session keys

The equality makes the induction hypothesis easier to apply

lemma *analz_image_freshK [rule_format]:*
"evs ∈ ns_shared ⟹
∀K KK. KK ⊆ - (range shrK) ⟶
(Key K ∈ analz (Key‘KK ∪ (spies evs))) =
(K ∈ KK ∨ Key K ∈ analz (spies evs))"
apply *(erule ns_shared.induct)*
apply *(drule_tac [8] Says_Server_message_form)*
apply *(erule_tac [5] Says_S_message_form [THEN disjE], analz_freshK, spy_analz)*

NS2, NS3

apply *blast+*
done

lemma *analz_insert_freshK:*
"⟦evs ∈ ns_shared; KAB ∉ range shrK⟧ ⟹
(Key K ∈ analz (insert (Key KAB) (spies evs))) =
(K = KAB ∨ Key K ∈ analz (spies evs))"
by *(simp only: analz_image_freshK analz_image_freshK_simps)*

3.1.4 The session key K uniquely identifies the message

In messages of this form, the session key uniquely identifies the rest

lemma *unique_session_keys:*
"⟦Says Server A (Crypt (shrK A) {|NA, Agent B, Key K, X|}) ∈ set evs;
Says Server A' (Crypt (shrK A') {|NA', Agent B', Key K, X'|}) ∈ set evs;
evs ∈ ns_shared⟧ ⟹ A=A' ∧ NA=NA' ∧ B=B' ∧ X = X'"
apply *(erule rev_mp, erule rev_mp, erule ns_shared.induct, simp_all, blast+)*
done

3.1.5 Crucial secrecy property: Spy does not see the keys sent in msg NS2

Beware of *[rule_format]* and the universal quantifier!

lemma *secrecy_lemma:*
"⟦Says Server A (Crypt (shrK A) {|NA, Agent B, Key K,
Crypt (shrK B) {|Key K, Agent A|}|})
∈ set evs;
A ∉ bad; B ∉ bad; evs ∈ ns_shared⟧
⟹ (∀NB. Notes Spy {|NA, NB, Key K|} ∉ set evs) ⟶
Key K ∉ analz (spies evs)"
apply *(erule rev_mp)*

```
apply (erule ns_shared.induct, force)
apply (frule_tac [7] Says_Server_message_form)
apply (frule_tac [4] Says_S_message_form)
apply (erule_tac [5] disjE)
apply (simp_all add: analz_insert_eq analz_insert_freshK pushes split_ifs,
spy_analz)
```

NS2

```
apply blast
```

NS3, Server sub-case

```
apply (blast dest!: Crypt_Spy_analz_bad A_trusts_NS2
            dest:  Says_imp_knows_Spy analz.Inj unique_session_keys)
```

NS3, Spy sub-case; also Oops

```
apply (blast dest: unique_session_keys)+
done
```

Final version: Server's message in the most abstract form

```
lemma Spy_not_see_encrypted_key:
     "⟦Says Server A (Crypt K' {|NA, Agent B, Key K, X|}) ∈ set evs;
       ∀NB. Notes Spy {|NA, NB, Key K|} ∉ set evs;
       A ∉ bad;  B ∉ bad;  evs ∈ ns_shared⟧
      ⟹ Key K ∉ analz (spies evs)"
by (blast dest: Says_Server_message_form secrecy_lemma)
```

3.2 Guarantees available at various stages of protocol

If the encrypted message appears then it originated with the Server

```
lemma B_trusts_NS3:
     "⟦Crypt (shrK B) {|Key K, Agent A|} ∈ parts (spies evs);
       B ∉ bad;  evs ∈ ns_shared⟧
      ⟹ ∃NA. Says Server A
                (Crypt (shrK A) {|NA, Agent B, Key K,
                                  Crypt (shrK B) {|Key K, Agent A|}|})
               ∈ set evs"
apply (erule rev_mp)
apply (erule ns_shared.induct, force, drule_tac [4] NS3_msg_in_parts_spies,
auto)
done


lemma A_trusts_NS4_lemma [rule_format]:
   "evs ∈ ns_shared ⟹
      Key K ∉ analz (spies evs) ⟶
      Says Server A (Crypt (shrK A) {|NA, Agent B, Key K, X|}) ∈ set evs ⟶
      Crypt K (Nonce NB) ∈ parts (spies evs) ⟶
      Says B A (Crypt K (Nonce NB)) ∈ set evs"
apply (erule ns_shared.induct, force, drule_tac [4] NS3_msg_in_parts_spies)
apply (analz_mono_contra, simp_all, blast)
```

NS2: contradiction from the assumptions *Key K ∉ used evs2* and *Crypt K (Nonce NB) ∈ parts (knows Spy evs2)*

apply *(force dest!: Crypt_imp_keysFor)*

NS3

apply *blast*

NS4

apply *(blast dest: B_trusts_NS3*
Says_imp_knows_Spy [THEN analz.Inj]
Crypt_Spy_analz_bad unique_session_keys)
done

This version no longer assumes that K is secure

lemma *A_trusts_NS4:*
"⟦Crypt K (Nonce NB) ∈ parts (spies evs);
Crypt (shrK A) {|NA, Agent B, Key K, X|} ∈ parts (spies evs);
∀NB. Notes Spy {|NA, NB, Key K|} ∉ set evs;
A ∉ bad; B ∉ bad; evs ∈ ns_shared⟧
⟹ Says B A (Crypt K (Nonce NB)) ∈ set evs"
by *(blast intro: A_trusts_NS4_lemma*
dest: A_trusts_NS2 Spy_not_see_encrypted_key)

If the session key has been used in NS4 then somebody has forwarded component X in some instance of NS4. Perhaps an interesting property, but not needed (after all) for the proofs below.

theorem *NS4_implies_NS3 [rule_format]:*
"evs ∈ ns_shared ⟹
Key K ∉ analz (spies evs) ⟶
Says Server A (Crypt (shrK A) {|NA, Agent B, Key K, X|}) ∈ set evs ⟶
Crypt K (Nonce NB) ∈ parts (spies evs) ⟶
(∃A'. Says A' B X ∈ set evs)"
apply *(erule ns_shared.induct, force, drule_tac [4] NS3_msg_in_parts_spies,*
analz_mono_contra)
apply *(simp_all add: ex_disj_distrib, blast)*

NS2

apply *(blast dest!: new_keys_not_used Crypt_imp_keysFor)*

NS3

apply *blast*

NS4

apply *(blast dest: B_trusts_NS3*
dest: Says_imp_knows_Spy [THEN analz.Inj]
unique_session_keys Crypt_Spy_analz_bad)
done

lemma *B_trusts_NS5_lemma [rule_format]:*
"⟦B ∉ bad; evs ∈ ns_shared⟧ ⟹
Key K ∉ analz (spies evs) ⟶
Says Server A
(Crypt (shrK A) {|NA, Agent B, Key K,

```
                          Crypt (shrK B) {|Key K, Agent A|}) ∈ set evs ⟶
     Crypt K {|Nonce NB, Nonce NB|} ∈ parts (spies evs) ⟶
     Says A B (Crypt K {|Nonce NB, Nonce NB|}) ∈ set evs"
apply (erule ns_shared.induct, force, drule_tac [4] NS3_msg_in_parts_spies,
analz_mono_contra, simp_all, blast)
```

NS2

```
apply (blast dest!: new_keys_not_used Crypt_imp_keysFor)
```

NS3

```
apply (blast dest!: cert_A_form)
```

NS5

```
apply (blast dest!: A_trusts_NS2
             dest: Says_imp_knows_Spy [THEN analz.Inj]
                   unique_session_keys Crypt_Spy_analz_bad)
done
```

Very strong Oops condition reveals protocol's weakness

```
lemma B_trusts_NS5:
     "⟦Crypt K {|Nonce NB, Nonce NB|} ∈ parts (spies evs);
       Crypt (shrK B) {|Key K, Agent A|} ∈ parts (spies evs);
       ∀NA NB. Notes Spy {|NA, NB, Key K|} ∉ set evs;
       A ∉ bad;  B ∉ bad;  evs ∈ ns_shared⟧
      ⟹ Says A B (Crypt K {|Nonce NB, Nonce NB|}) ∈ set evs"
by (blast intro: B_trusts_NS5_lemma
          dest: B_trusts_NS3 Spy_not_see_encrypted_key)

end
```

4 The Kerberos Protocol, BAN Version

```
theory Kerberos_BAN imports Public begin
```

From page 251 of Burrows, Abadi and Needham (1989). A Logic of Authentication. Proc. Royal Soc. 426

Confidentiality (secrecy) and authentication properties rely on temporal checks: strong guarantees in a little abstracted - but very realistic - model.

```
syntax
    CT :: "event list=>nat"
    Expired :: "[nat, event list] => bool"
    RecentAuth :: "[nat, event list] => bool"

consts


    SesKeyLife   :: nat


    AutLife :: nat
```

The ticket should remain fresh for two journeys on the network at least

```
specification (SesKeyLife)
  SesKeyLife_LB [iff]: "2 ≤ SesKeyLife"
    by blast
```

The authenticator only for one journey

```
specification (AutLife)
  AutLife_LB [iff]:    "Suc 0 ≤ AutLife"
    by blast


translations
   "CT" == "length "

   "Expired T evs" == "SesKeyLife + T < CT evs"

   "RecentAuth T evs" == "CT evs ≤ AutLife + T"

consts  kerberos_ban   :: "event list set"
inductive "kerberos_ban"
 intros

   Nil:  "[] ∈ kerberos_ban"

   Fake: "[| evsf ∈ kerberos_ban;  X ∈ synth (analz (spies evsf)) |]
          ==> Says Spy B X # evsf ∈ kerberos_ban"


   Kb1:  "[| evs1 ∈ kerberos_ban |]
          ==> Says A Server {|Agent A, Agent B|} # evs1
                ∈  kerberos_ban"


   Kb2:  "[| evs2 ∈ kerberos_ban;  Key KAB ∉ used evs2;  KAB ∈ symKeys;
              Says A' Server {|Agent A, Agent B|} ∈ set evs2 |]
          ==> Says Server A
                (Crypt (shrK A)
                   {|Number (CT evs2), Agent B, Key KAB,
                     (Crypt (shrK B) {|Number (CT evs2), Agent A, Key KAB|})|})
                # evs2 ∈ kerberos_ban"


   Kb3:  "[| evs3 ∈ kerberos_ban;
              Says S A (Crypt (shrK A) {|Number Ts, Agent B, Key K, X|})
                ∈ set evs3;
              Says A Server {|Agent A, Agent B|} ∈ set evs3;
              ~ Expired Ts evs3 |]
          ==> Says A B {|X, Crypt K {|Agent A, Number (CT evs3)|} |}
                # evs3 ∈ kerberos_ban"


   Kb4:  "[| evs4 ∈ kerberos_ban;
              Says A' B {|(Crypt (shrK B) {|Number Ts, Agent A, Key K|}),
                          (Crypt K {|Agent A, Number Ta|}) |}: set evs4;
```

```
           ~ Expired Ts evs4;  RecentAuth Ta evs4 |]
        ==> Says B A (Crypt K (Number Ta)) # evs4
              ∈ kerberos_ban"


  Oops: "[| evso ∈ kerberos_ban;
           Says Server A (Crypt (shrK A) {|Number Ts, Agent B, Key K, X|})
             ∈ set evso;
           Expired Ts evso |]
        ==> Notes Spy {|Number Ts, Key K|} # evso ∈ kerberos_ban"


declare Says_imp_knows_Spy [THEN parts.Inj, dest]
declare parts.Body [dest]
declare analz_into_parts [dest]
declare Fake_parts_insert_in_Un [dest]
```

A "possibility property": there are traces that reach the end.

```
lemma "[|Key K ∉ used []; K ∈ symKeys|]
      ==> ∃Timestamp. ∃evs ∈ kerberos_ban.
            Says B A (Crypt K (Number Timestamp))
                 ∈ set evs"
apply (cut_tac SesKeyLife_LB)
apply (intro exI bexI)
apply (rule_tac [2]
          kerberos_ban.Nil [THEN kerberos_ban.Kb1, THEN kerberos_ban.Kb2,
                            THEN kerberos_ban.Kb3, THEN kerberos_ban.Kb4])
apply (possibility, simp_all (no_asm_simp) add: used_Cons)
done
```

Forwarding Lemma for reasoning about the encrypted portion of message Kb3

```
lemma Kb3_msg_in_parts_spies:
     "Says S A (Crypt KA {|Timestamp, B, K, X|}) ∈ set evs
      ==> X ∈ parts (spies evs)"
by blast

lemma Oops_parts_spies:
     "Says Server A (Crypt (shrK A) {|Timestamp, B, K, X|}) ∈ set evs
      ==> K ∈ parts (spies evs)"
by blast
```

Spy never sees another agent's shared key! (unless it's bad at start)

```
lemma Spy_see_shrK [simp]:
     "evs ∈ kerberos_ban ==> (Key (shrK A) ∈ parts (spies evs)) = (A ∈ bad)"
apply (erule kerberos_ban.induct)
apply (frule_tac [7] Oops_parts_spies)
apply (frule_tac [5] Kb3_msg_in_parts_spies, simp_all, blast+)
done


lemma Spy_analz_shrK [simp]:
     "evs ∈ kerberos_ban ==> (Key (shrK A) ∈ analz (spies evs)) = (A ∈ bad)"
by auto
```

```
lemma Spy_see_shrK_D [dest!]:
     "[| Key (shrK A) ∈ parts (spies evs);
                 evs ∈ kerberos_ban |] ==> A:bad"
by (blast dest: Spy_see_shrK)

lemmas Spy_analz_shrK_D = analz_subset_parts [THEN subsetD, THEN Spy_see_shrK_D,
dest!]
```

Nobody can have used non-existent keys!

```
lemma new_keys_not_used [simp]:
    "[|Key K ∉ used evs; K ∈ symKeys; evs ∈ kerberos_ban|]
     ==> K ∉ keysFor (parts (spies evs))"
apply (erule rev_mp)
apply (erule kerberos_ban.induct)
apply (frule_tac [7] Oops_parts_spies)
apply (frule_tac [5] Kb3_msg_in_parts_spies, simp_all)
```

Fake

```
apply (force dest!: keysFor_parts_insert)
```

Kb2, Kb3, Kb4

```
apply (force dest!: analz_shrK_Decrypt)+
done
```

4.1 Lemmas concerning the form of items passed in messages

Describes the form of K, X and K' when the Server sends this message.

```
lemma Says_Server_message_form:
     "[| Says Server A (Crypt K' {|Number Ts, Agent B, Key K, X|})
         ∈ set evs; evs ∈ kerberos_ban |]
      ==> K ∉ range shrK &
          X = (Crypt (shrK B) {|Number Ts, Agent A, Key K|}) &
          K' = shrK A"
apply (erule rev_mp)
apply (erule kerberos_ban.induct, auto)
done
```

If the encrypted message appears then it originated with the Server PROVIDED that A is NOT compromised!

This shows implicitly the FRESHNESS OF THE SESSION KEY to A

```
lemma A_trusts_K_by_Kb2:
     "[| Crypt (shrK A) {|Number Ts, Agent B, Key K, X|}
           ∈ parts (spies evs);
         A ∉ bad;  evs ∈ kerberos_ban |]
       ==> Says Server A (Crypt (shrK A) {|Number Ts, Agent B, Key K, X|})
             ∈ set evs"
apply (erule rev_mp)
apply (erule kerberos_ban.induct)
apply (frule_tac [7] Oops_parts_spies)
apply (frule_tac [5] Kb3_msg_in_parts_spies, simp_all, blast)
```

done

If the TICKET appears then it originated with the Server

FRESHNESS OF THE SESSION KEY to B

```
lemma B_trusts_K_by_Kb3:
     "[| Crypt (shrK B) {|Number Ts, Agent A, Key K|} ∈ parts (spies evs);
         B ∉ bad;  evs ∈ kerberos_ban |]
       ==> Says Server A
            (Crypt (shrK A) {|Number Ts, Agent B, Key K,
                              Crypt (shrK B) {|Number Ts, Agent A, Key K|}|})
           ∈ set evs"
apply (erule rev_mp)
apply (erule kerberos_ban.induct)
apply (frule_tac [7] Oops_parts_spies)
apply (frule_tac [5] Kb3_msg_in_parts_spies, simp_all, blast)
done
```

EITHER describes the form of X when the following message is sent, OR reduces it to the Fake case. Use *Says_Server_message_form* if applicable.

```
lemma Says_S_message_form:
     "[| Says S A (Crypt (shrK A) {|Number Ts, Agent B, Key K, X|})
           ∈ set evs;
         evs ∈ kerberos_ban |]
 ==> (K ∉ range shrK & X = (Crypt (shrK B) {|Number Ts, Agent A, Key K|}))
          | X ∈ analz (spies evs)"
apply (case_tac "A ∈ bad")
apply (force dest!: Says_imp_spies [THEN analz.Inj])
apply (frule Says_imp_spies [THEN parts.Inj])
apply (blast dest!: A_trusts_K_by_Kb2 Says_Server_message_form)
done
```

Session keys are not used to encrypt other session keys

```
lemma analz_image_freshK [rule_format (no_asm)]:
     "evs ∈ kerberos_ban ==>
   ∀K KK. KK ⊆ - (range shrK) -->
          (Key K ∈ analz (Key‘KK Un (spies evs))) =
          (K ∈ KK | Key K ∈ analz (spies evs))"
apply (erule kerberos_ban.induct)
apply (drule_tac [7] Says_Server_message_form)
apply (erule_tac [5] Says_S_message_form [THEN disjE], analz_freshK, spy_analz,
auto)
done


lemma analz_insert_freshK:
     "[| evs ∈ kerberos_ban;  KAB ∉ range shrK |] ==>
      (Key K ∈ analz (insert (Key KAB) (spies evs))) =
      (K = KAB | Key K ∈ analz (spies evs))"
by (simp only: analz_image_freshK analz_image_freshK_simps)
```

The session key K uniquely identifies the message

```
lemma unique_session_keys:
```

```
     "[| Says Server A
           (Crypt (shrK A) {|Number Ts, Agent B, Key K, X|}) ∈ set evs;
         Says Server A'
          (Crypt (shrK A') {|Number Ts', Agent B', Key K, X'|}) ∈ set evs;
         evs ∈ kerberos_ban |] ==> A=A' & Ts=Ts' & B=B' & X = X'"
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule kerberos_ban.induct)
apply (frule_tac [7] Oops_parts_spies)
apply (frule_tac [5] Kb3_msg_in_parts_spies, simp_all)
```

Kb2: it can't be a new key

```
apply blast
done
```

Lemma: the session key sent in msg Kb2 would be EXPIRED if the spy could see it!

```
lemma lemma2 [rule_format (no_asm)]:
     "[| A ∉ bad;  B ∉ bad;  evs ∈ kerberos_ban |]
  ==> Says Server A
          (Crypt (shrK A) {|Number Ts, Agent B, Key K,
                            Crypt (shrK B) {|Number Ts, Agent A, Key K|}|})
         ∈ set evs -->
      Key K ∈ analz (spies evs) --> Expired Ts evs"
apply (erule kerberos_ban.induct)
apply (frule_tac [7] Says_Server_message_form)
apply (frule_tac [5] Says_S_message_form [THEN disjE])
apply (simp_all (no_asm_simp) add: less_SucI analz_insert_eq analz_insert_freshK
pushes)
```

Fake

```
apply spy_analz
```

Kb2

```
apply (blast intro: parts_insertI less_SucI)
```

Kb3

```
apply (case_tac "Aa ∈ bad")
 prefer 2 apply (blast dest: A_trusts_K_by_Kb2 unique_session_keys)
apply (blast dest: Says_imp_spies [THEN analz.Inj] Crypt_Spy_analz_bad elim!:
MPair_analz intro: less_SucI)
```

Oops: PROOF FAILED if addIs below

```
apply (blast dest: unique_session_keys intro!: less_SucI)
done
```

Confidentiality for the Server: Spy does not see the keys sent in msg Kb2 as long as they have not expired.

```
lemma Confidentiality_S:
     "[| Says Server A
           (Crypt K' {|Number T, Agent B, Key K, X|}) ∈ set evs;
         ~ Expired T evs;
```

```
        A ∉ bad;  B ∉ bad;  evs ∈ kerberos_ban
     |] ==> Key K ∉ analz (spies evs)"
apply (frule Says_Server_message_form, assumption)
apply (blast intro: lemma2)
done
```

Confidentiality for Alice

```
lemma Confidentiality_A:
     "[| Crypt (shrK A) {|Number T, Agent B, Key K, X|} ∈ parts (spies evs);
         ~ Expired T evs;
         A ∉ bad;  B ∉ bad;  evs ∈ kerberos_ban
      |] ==> Key K ∉ analz (spies evs)"
by (blast dest!: A_trusts_K_by_Kb2 Confidentiality_S)
```

Confidentiality for Bob

```
lemma Confidentiality_B:
     "[| Crypt (shrK B) {|Number Tk, Agent A, Key K|}
          ∈ parts (spies evs);
        ~ Expired Tk evs;
       A ∉ bad;  B ∉ bad;  evs ∈ kerberos_ban
      |] ==> Key K ∉ analz (spies evs)"
by (blast dest!: B_trusts_K_by_Kb3 Confidentiality_S)


lemma lemma_B [rule_format]:
     "[| B ∉ bad;  evs ∈ kerberos_ban |]
      ==> Key K ∉ analz (spies evs) -->
          Says Server A (Crypt (shrK A) {|Number Ts, Agent B, Key K, X|})
          ∈ set evs -->
          Crypt K (Number Ta) ∈ parts (spies evs) -->
          Says B A (Crypt K (Number Ta)) ∈ set evs"
apply (erule kerberos_ban.induct)
apply (frule_tac [7] Oops_parts_spies)
apply (frule_tac [5] Says_S_message_form)
apply (drule_tac [6] Kb3_msg_in_parts_spies, analz_mono_contra)
apply (simp_all (no_asm_simp) add: all_conj_distrib)
```

Fake

```
apply blast
```

Kb2

```
apply (force dest: Crypt_imp_invKey_keysFor)
```

Kb4

```
apply (blast dest: B_trusts_K_by_Kb3 unique_session_keys
                   Says_imp_spies [THEN analz.Inj] Crypt_Spy_analz_bad)
done
```

Authentication of B to A

```
lemma Authentication_B:
     "[| Crypt K (Number Ta) ∈ parts (spies evs);
         Crypt (shrK A) {|Number Ts, Agent B, Key K, X|}
         ∈ parts (spies evs);
```

```
         ~ Expired Ts evs;
         A ∉ bad;  B ∉ bad;  evs ∈ kerberos_ban |]
      ==> Says B A (Crypt K (Number Ta)) ∈ set evs"
by (blast dest!: A_trusts_K_by_Kb2
          intro!: lemma_B elim!: Confidentiality_S [THEN [2] rev_notE])

lemma lemma_A [rule_format]:
     "[| A ∉ bad; B ∉ bad; evs ∈ kerberos_ban |]
      ==>
         Key K ∉ analz (spies evs) -->
         Says Server A (Crypt (shrK A) {|Number Ts, Agent B, Key K, X|})
         ∈ set evs -->
          Crypt K {|Agent A, Number Ta|} ∈ parts (spies evs) -->
         Says A B {|X, Crypt K {|Agent A, Number Ta|}|}
             ∈ set evs"
apply (erule kerberos_ban.induct)
apply (frule_tac [7] Oops_parts_spies)
apply (frule_tac [5] Says_S_message_form)
apply (frule_tac [6] Kb3_msg_in_parts_spies, analz_mono_contra)
apply (simp_all (no_asm_simp) add: all_conj_distrib)
```

Fake

```
apply blast
```

Kb2

```
apply (force dest: Crypt_imp_invKey_keysFor)
```

Kb3

```
apply (blast dest: A_trusts_K_by_Kb2 unique_session_keys)
done
```

Authentication of A to B

```
lemma Authentication_A:
     "[| Crypt K {|Agent A, Number Ta|} ∈ parts (spies evs);
         Crypt (shrK B) {|Number Ts, Agent A, Key K|}
         ∈ parts (spies evs);
         ~ Expired Ts evs;
         A ∉ bad;  B ∉ bad;  evs ∈ kerberos_ban |]
      ==> Says A B {|Crypt (shrK B) {|Number Ts, Agent A, Key K|},
                     Crypt K {|Agent A, Number Ta|}|} ∈ set evs"
by (blast dest!: B_trusts_K_by_Kb3
          intro!: lemma_A
          elim!: Confidentiality_S [THEN [2] rev_notE])

end
```

5 The Kerberos Protocol, Version IV

```
theory KerberosIV imports Public begin
```

syntax

```
  Kas :: agent
  Tgs :: agent
```
— the two servers are translations...

translations

```
  "Kas"       == "Server "
  "Tgs"       == "Friend 0"
```

axioms

```
  Tgs_not_bad [iff]: "Tgs ∉ bad"
```
— Tgs is secure — we already know that Kas is secure

syntax

```
    CT :: "event list=>nat"

    ExpirAuth :: "[nat, event list] => bool"

    ExpirServ :: "[nat, event list] => bool"

    ExpirAutc :: "[nat, event list] => bool"

    RecentResp :: "[nat, nat] => bool"
```

constdefs

```
    AuthKeys :: "event list => key set"
    "AuthKeys evs == {AuthKey. ∃A Peer Tk. Says Kas A
                         (Crypt (shrK A) {|Key AuthKey, Agent Peer, Tk,
                      (Crypt (shrK Peer) {|Agent A, Agent Peer, Key AuthKey, Tk|})
                     |}) ∈ set evs}"


  Issues :: "[agent, agent, msg, event list] => bool"
             ("_ Issues _ with _ on _")
   "A Issues B with X on evs ==
      ∃Y. Says A B Y ∈ set evs & X ∈ parts {Y} &
      X ∉ parts (spies (takeWhile (% z. z  ≠ Says A B Y) (rev evs)))"
```

consts

```
    AuthLife   :: nat


    ServLife   :: nat


    AutcLife   :: nat


    RespLife   :: nat
```

```
specification (AuthLife)
  AuthLife_LB [iff]: "2 ≤ AuthLife"
    by blast

specification (ServLife)
  ServLife_LB [iff]: "2 ≤ ServLife"
    by blast

specification (AutcLife)
  AutcLife_LB [iff]: "Suc 0 ≤ AutcLife"
    by blast

specification (RespLife)
  RespLife_LB [iff]: "Suc 0 ≤ RespLife"
    by blast

translations
   "CT" == "length "

   "ExpirAuth T evs" == "AuthLife + T < CT evs"

   "ExpirServ T evs" == "ServLife + T < CT evs"

   "ExpirAutc T evs" == "AutcLife + T < CT evs"

   "RecentResp T1 T2" == "T1 <= RespLife + T2"




constdefs
  KeyCryptKey :: "[key, key, event list] => bool"
  "KeyCryptKey AuthKey ServKey evs ==
     ∃A B tt.
       Says Tgs A (Crypt AuthKey
                     {|Key ServKey, Agent B, tt,
                       Crypt (shrK B) {|Agent A, Agent B, Key ServKey, tt|}
|})
         ∈ set evs"

consts

kerberos   :: "event list set"
inductive "kerberos"
  intros

   Nil:  "[] ∈ kerberos"

   Fake: "[| evsf ∈ kerberos;  X ∈ synth (analz (spies evsf)) |]
          ==> Says Spy B X  # evsf ∈ kerberos"
```

```
K1:  "[| evs1 ∈ kerberos |]
      ==> Says A Kas {|Agent A, Agent Tgs, Number (CT evs1)|} # evs1
      ∈ kerberos"
```

```
K2:  "[| evs2 ∈ kerberos; Key AuthKey ∉ used evs2; AuthKey ∈ symKeys;
         Says A' Kas {|Agent A, Agent Tgs, Number Ta|} ∈ set evs2 |]
      ==> Says Kas A
            (Crypt (shrK A) {|Key AuthKey, Agent Tgs, Number (CT evs2),
                  (Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key AuthKey,
                      Number (CT evs2)|})|}) # evs2 ∈ kerberos"
```

```
K3:  "[| evs3 ∈ kerberos;
         Says A Kas {|Agent A, Agent Tgs, Number Ta|} ∈ set evs3;
         Says Kas' A (Crypt (shrK A) {|Key AuthKey, Agent Tgs, Number Tk,
           AuthTicket|}) ∈ set evs3;
         RecentResp Tk Ta
       |]
      ==> Says A Tgs {|AuthTicket,
                       (Crypt AuthKey {|Agent A, Number (CT evs3)|}),
                       Agent B|} # evs3 ∈ kerberos"
```

```
K4:  "[| evs4 ∈ kerberos; Key ServKey ∉ used evs4; ServKey ∈ symKeys;
         B ≠ Tgs;  AuthKey ∈ symKeys;
         Says A' Tgs {|
          (Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key AuthKey,
                              Number Tk|}),
          (Crypt AuthKey {|Agent A, Number Ta1|}), Agent B|}
            ∈ set evs4;
         ~ ExpirAuth Tk evs4;
         ~ ExpirAutc Ta1 evs4;
         ServLife + (CT evs4) <= AuthLife + Tk
       |]
      ==> Says Tgs A
            (Crypt AuthKey {|Key ServKey, Agent B, Number (CT evs4),
                            Crypt (shrK B) {|Agent A, Agent B, Key ServKey,
                                             Number (CT evs4)|} |})
            # evs4 ∈ kerberos"
```

```
K5:  "[| evs5 ∈ kerberos; AuthKey ∈ symKeys; ServKey ∈ symKeys;
         Says A Tgs
             {|AuthTicket, Crypt AuthKey {|Agent A, Number Ta1|},
               Agent B|}
           ∈ set evs5;
         Says Tgs' A
          (Crypt AuthKey {|Key ServKey, Agent B, Number Tt, ServTicket|})
            ∈ set evs5;
         RecentResp Tt Ta1 |]
       ==> Says A B {|ServTicket,
                      Crypt ServKey {|Agent A, Number (CT evs5)|} |}
            # evs5 ∈ kerberos"


 K6:  "[| evs6 ∈ kerberos;
         Says A' B {|
           (Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Number Tt|}),
           (Crypt ServKey {|Agent A, Number Ta2|})|}
         ∈ set evs6;
         ~ ExpirServ Tt evs6;
         ~ ExpirAutc Ta2 evs6
      |]
       ==> Says B A (Crypt ServKey (Number Ta2))
            # evs6 ∈ kerberos"


Oops1: "[| evsO1 ∈ kerberos;  A ≠ Spy;
           Says Kas A
             (Crypt (shrK A) {|Key AuthKey, Agent Tgs, Number Tk,
                               AuthTicket|})  ∈ set evsO1;
           ExpirAuth Tk evsO1 |]
       ==> Says A Spy {|Agent A, Agent Tgs, Number Tk, Key AuthKey|}
            # evsO1 ∈ kerberos"


Oops2: "[| evsO2 ∈ kerberos;  A ≠ Spy;
           Says Tgs A
             (Crypt AuthKey {|Key ServKey, Agent B, Number Tt, ServTicket|})
                ∈ set evsO2;
           ExpirServ Tt evsO2 |]
       ==> Says A Spy {|Agent A, Agent B, Number Tt, Key ServKey|}
            # evsO2 ∈ kerberos"
```

```
declare Says_imp_knows_Spy [THEN parts.Inj, dest]
declare parts.Body [dest]
declare analz_into_parts [dest]
declare Fake_parts_insert_in_Un [dest]
```

5.1 Lemmas about Lists

```
lemma spies_Says_rev: "spies (evs @ [Says A B X]) = insert X (spies evs)"
apply (induct_tac "evs")
apply (induct_tac [2] "a", auto)
done

lemma spies_Gets_rev: "spies (evs @ [Gets A X]) = spies evs"
apply (induct_tac "evs")
apply (induct_tac [2] "a", auto)
done

lemma spies_Notes_rev: "spies (evs @ [Notes A X]) =
          (if A:bad then insert X (spies evs) else spies evs)"
apply (induct_tac "evs")
apply (induct_tac [2] "a", auto)
done

lemma spies_evs_rev: "spies evs = spies (rev evs)"
apply (induct_tac "evs")
apply (induct_tac [2] "a")
apply (simp_all (no_asm_simp) add: spies_Says_rev spies_Gets_rev spies_Notes_rev)
done

lemmas parts_spies_evs_revD2 = spies_evs_rev [THEN equalityD2, THEN parts_mono]

lemma spies_takeWhile: "spies (takeWhile P evs) <=  spies evs"
apply (induct_tac "evs")
apply (induct_tac [2] "a", auto)
```

Resembles `used_subset_append` in theory Event.

```
done

lemmas parts_spies_takeWhile_mono = spies_takeWhile [THEN parts_mono]
```

5.2 Lemmas about `AuthKeys`

```
lemma AuthKeys_empty: "AuthKeys [] = {}"
apply (unfold AuthKeys_def)
apply (simp (no_asm))
done

lemma AuthKeys_not_insert:
 "(∀A Tk akey Peer.
   ev ≠ Says Kas A (Crypt (shrK A) {|akey, Agent Peer, Tk,
              (Crypt (shrK Peer) {|Agent A, Agent Peer, akey, Tk|})|}))
       ==> AuthKeys (ev # evs) = AuthKeys evs"
by (unfold AuthKeys_def, auto)
```

```
lemma AuthKeys_insert:
  "AuthKeys
     (Says Kas A (Crypt (shrK A) {|Key K, Agent Peer, Number Tk,
      (Crypt (shrK Peer) {|Agent A, Agent Peer, Key K, Number Tk|})|}) # evs)
       = insert K (AuthKeys evs)"
by (unfold AuthKeys_def, auto)

lemma AuthKeys_simp:
   "K ∈ AuthKeys
    (Says Kas A (Crypt (shrK A) {|Key K', Agent Peer, Number Tk,
     (Crypt (shrK Peer) {|Agent A, Agent Peer, Key K', Number Tk|})|}) # evs)
        ==> K = K' | K ∈ AuthKeys evs"
by (unfold AuthKeys_def, auto)

lemma AuthKeysI:
   "Says Kas A (Crypt (shrK A) {|Key K, Agent Tgs, Number Tk,
     (Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key K, Number Tk|})|}) ∈ set
evs
        ==> K ∈ AuthKeys evs"
by (unfold AuthKeys_def, auto)

lemma AuthKeys_used: "K ∈ AuthKeys evs ==> Key K ∈ used evs"
by (simp add: AuthKeys_def, blast)
```

5.3 Forwarding Lemmas

–For reasoning about the encrypted portion of message K3–

```
lemma K3_msg_in_parts_spies:
     "Says Kas' A (Crypt KeyA {|AuthKey, Peer, Tk, AuthTicket|})
                ∈ set evs ==> AuthTicket ∈ parts (spies evs)"
by blast

lemma Oops_range_spies1:
     "[| Says Kas A (Crypt KeyA {|Key AuthKey, Peer, Tk, AuthTicket|})
           ∈ set evs ;
         evs ∈ kerberos |] ==> AuthKey ∉ range shrK & AuthKey ∈ symKeys"
apply (erule rev_mp)
apply (erule kerberos.induct, auto)
done
```

–For reasoning about the encrypted portion of message K5–

```
lemma K5_msg_in_parts_spies:
     "Says Tgs' A (Crypt AuthKey {|ServKey, Agent B, Tt, ServTicket|})
                ∈ set evs ==> ServTicket ∈ parts (spies evs)"
by blast

lemma Oops_range_spies2:
     "[| Says Tgs A (Crypt AuthKey {|Key ServKey, Agent B, Tt, ServTicket|})
           ∈ set evs ;
         evs ∈ kerberos |] ==> ServKey ∉ range shrK & ServKey ∈ symKeys"
apply (erule rev_mp)
apply (erule kerberos.induct, auto)
```

done

lemma *Says_ticket_in_parts_spies:*
```
     "Says S A (Crypt K {|SesKey, B, TimeStamp, Ticket|}) ∈ set evs
      ==> Ticket ∈ parts (spies evs)"
```
by *blast*

lemma *Spy_see_shrK [simp]:*
```
     "evs ∈ kerberos ==> (Key (shrK A) ∈ parts (spies evs)) = (A ∈ bad)"
```
apply *(erule kerberos.induct)*
apply *(frule_tac [7] K5_msg_in_parts_spies)*
apply *(frule_tac [5] K3_msg_in_parts_spies, simp_all)*
apply *(blast+)*
done

lemma *Spy_analz_shrK [simp]:*
```
     "evs ∈ kerberos ==> (Key (shrK A) ∈ analz (spies evs)) = (A ∈ bad)"
```
by *auto*

lemma *Spy_see_shrK_D [dest!]:*
```
     "[| Key (shrK A) ∈ parts (spies evs);  evs ∈ kerberos |] ==> A:bad"
```
by *(blast dest: Spy_see_shrK)*
lemmas *Spy_analz_shrK_D = analz_subset_parts [THEN subsetD, THEN Spy_see_shrK_D, dest!]*

Nobody can have used non-existent keys!

lemma *new_keys_not_used [simp]:*
```
    "[|Key K ∉ used evs; K ∈ symKeys; evs ∈ kerberos|]
     ==> K ∉ keysFor (parts (spies evs))"
```
apply *(erule rev_mp)*
apply *(erule kerberos.induct)*
apply *(frule_tac [7] K5_msg_in_parts_spies)*
apply *(frule_tac [5] K3_msg_in_parts_spies, simp_all)*

Fake

apply *(force dest!: keysFor_parts_insert)*

Others

apply *(force dest!: analz_shrK_Decrypt)+*
done

lemma *new_keys_not_analzd:*
```
 "[|evs ∈ kerberos; K ∈ symKeys; Key K ∉ used evs|]
  ==> K ∉ keysFor (analz (spies evs))"
```
by *(blast dest: new_keys_not_used intro: keysFor_mono [THEN subsetD])*

5.4 Regularity Lemmas

These concern the form of items passed in messages

Describes the form of AuthKey, AuthTicket, and K sent by Kas

```
lemma Says_Kas_message_form:
     "[| Says Kas A (Crypt K {|Key AuthKey, Agent Peer, Tk, AuthTicket|})
           ∈ set evs;
         evs ∈ kerberos |]
      ==> AuthKey ∉ range shrK & AuthKey ∈ AuthKeys evs & AuthKey ∈ symKeys
&
  AuthTicket = (Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key AuthKey, Tk|}) &
             K = shrK A  & Peer = Tgs"
apply (erule rev_mp)
apply (erule kerberos.induct)
apply (simp_all (no_asm) add: AuthKeys_def AuthKeys_insert)
apply (blast+)
done


lemma SesKey_is_session_key:
     "[| Crypt (shrK Tgs_B) {|Agent A, Agent Tgs_B, Key SesKey, Number T|}
           ∈ parts (spies evs); Tgs_B ∉ bad;
         evs ∈ kerberos |]
      ==> SesKey ∉ range shrK"
apply (erule rev_mp)
apply (erule kerberos.induct)
apply (frule_tac [7] K5_msg_in_parts_spies)
apply (frule_tac [5] K3_msg_in_parts_spies, simp_all, blast)
done

lemma A_trusts_AuthTicket:
     "[| Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key AuthKey, Tk|}
           ∈ parts (spies evs);
         evs ∈ kerberos |]
      ==> Says Kas A (Crypt (shrK A) {|Key AuthKey, Agent Tgs, Tk,
                Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key AuthKey, Tk|}|})
           ∈ set evs"
apply (erule rev_mp)
apply (erule kerberos.induct)
apply (frule_tac [7] K5_msg_in_parts_spies)
apply (frule_tac [5] K3_msg_in_parts_spies, simp_all)
```

Fake, K4

```
apply (blast+)
done

lemma AuthTicket_crypt_AuthKey:
     "[| Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key AuthKey, Number Tk|}
           ∈ parts (spies evs);
         evs ∈ kerberos |]
      ==> AuthKey ∈ AuthKeys evs"
apply (frule A_trusts_AuthTicket, assumption)
apply (simp (no_asm) add: AuthKeys_def)
apply blast
done
```

Describes the form of ServKey, ServTicket and AuthKey sent by Tgs

```
lemma Says_Tgs_message_form:
```

```
     "[| Says Tgs A (Crypt AuthKey {|Key ServKey, Agent B, Tt, ServTicket|})
           ∈ set evs;
         evs ∈ kerberos |]
   ==> B ≠ Tgs &
       ServKey ∉ range shrK & ServKey ∉ AuthKeys evs & ServKey ∈ symKeys
&
       ServTicket = (Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Tt|})
&
       AuthKey ∉ range shrK & AuthKey ∈ AuthKeys evs & AuthKey ∈ symKeys"
apply (erule rev_mp)
apply (erule kerberos.induct)
apply (simp_all add: AuthKeys_insert AuthKeys_not_insert AuthKeys_empty AuthKeys_simp,
blast, auto)
```

Three subcases of Message 4

```
apply (blast dest!: AuthKeys_used Says_Kas_message_form)
apply (blast dest!: SesKey_is_session_key)
apply (blast dest: AuthTicket_crypt_AuthKey)
done
```

Authenticity of AuthKey for A: If a certain encrypted message appears then it originated with Kas

```
lemma A_trusts_AuthKey:
     "[| Crypt (shrK A) {|Key AuthKey, Peer, Tk, AuthTicket|}
           ∈ parts (spies evs);
         A ∉ bad;  evs ∈ kerberos |]
      ==> Says Kas A (Crypt (shrK A) {|Key AuthKey, Peer, Tk, AuthTicket|})
            ∈ set evs"
apply (erule rev_mp)
apply (erule kerberos.induct)
apply (frule_tac [7] K5_msg_in_parts_spies)
apply (frule_tac [5] K3_msg_in_parts_spies, simp_all)
```

Fake

```
apply blast
```

K4

```
apply (blast dest!: A_trusts_AuthTicket [THEN Says_Kas_message_form])
done
```

If a certain encrypted message appears then it originated with Tgs

```
lemma A_trusts_K4:
     "[| Crypt AuthKey {|Key ServKey, Agent B, Tt, ServTicket|}
           ∈ parts (spies evs);
         Key AuthKey ∉ analz (spies evs);
         AuthKey ∉ range shrK;
         evs ∈ kerberos |]
 ==> ∃A. Says Tgs A (Crypt AuthKey {|Key ServKey, Agent B, Tt, ServTicket|})
       ∈ set evs"
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule kerberos.induct, analz_mono_contra)
apply (frule_tac [7] K5_msg_in_parts_spies)
```

apply *(frule_tac [5] K3_msg_in_parts_spies, simp_all)*

Fake

apply *blast*

K2

apply *blast*

K4

apply *auto*
done

lemma *AuthTicket_form:*
```
     "[| Crypt (shrK A) {|Key AuthKey, Agent Tgs, Tk, AuthTicket|}
           ∈ parts (spies evs);
         A ∉ bad;
         evs ∈ kerberos |]
    ==> AuthKey ∉ range shrK & AuthKey ∈ symKeys &
        AuthTicket = Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key AuthKey, Tk|}"
```
apply *(erule rev_mp)*
apply *(erule kerberos.induct)*
apply *(frule_tac [7] K5_msg_in_parts_spies)*
apply *(frule_tac [5] K3_msg_in_parts_spies, simp_all)*
apply *(blast+)*
done

This form holds also over an AuthTicket, but is not needed below.

lemma *ServTicket_form:*
```
     "[| Crypt AuthKey {|Key ServKey, Agent B, Tt, ServTicket|}
             ∈ parts (spies evs);
           Key AuthKey ∉ analz (spies evs);
           evs ∈ kerberos |]
         ==> ServKey ∉ range shrK & ServKey ∈ symKeys &
    (∃A. ServTicket = Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Tt|})"
```
apply *(erule rev_mp)*
apply *(erule rev_mp)*
apply *(erule kerberos.induct, analz_mono_contra)*
apply *(frule_tac [7] K5_msg_in_parts_spies)*
apply *(frule_tac [5] K3_msg_in_parts_spies, simp_all, blast)*
done

Essentially the same as *AuthTicket_form*

lemma *Says_kas_message_form:*
```
     "[| Says Kas' A (Crypt (shrK A)
              {|Key AuthKey, Agent Tgs, Tk, AuthTicket|}) ∈ set evs;
         evs ∈ kerberos |]
      ==> AuthKey ∉ range shrK & AuthKey ∈ symKeys &
          AuthTicket =
                 Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key AuthKey, Tk|}
          | AuthTicket ∈ analz (spies evs)"
```
by *(blast dest: analz_shrK_Decrypt AuthTicket_form*
Says_imp_spies [THEN analz.Inj])

```
lemma Says_tgs_message_form:
 "[| Says Tgs' A (Crypt AuthKey {|Key ServKey, Agent B, Tt, ServTicket|})
       ∈ set evs;  AuthKey ∈ symKeys;
     evs ∈ kerberos |]
  ==> ServKey ∉ range shrK &
      (∃A. ServTicket =
             Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Tt|})
       | ServTicket ∈ analz (spies evs)"
apply (frule Says_imp_spies [THEN analz.Inj], auto)
 apply (force dest!: ServTicket_form)
apply (frule analz_into_parts)
apply (frule ServTicket_form, auto)
done
```

5.5 Unicity Theorems

The session key, if secure, uniquely identifies the Ticket whether AuthTicket or ServTicket. As a matter of fact, one can read also Tgs in the place of B.

```
lemma unique_CryptKey:
     "[| Crypt (shrK B)  {|Agent A,  Agent B,  Key SesKey, T|}
           ∈ parts (spies evs);
         Crypt (shrK B') {|Agent A', Agent B', Key SesKey, T'|}
           ∈ parts (spies evs);  Key SesKey ∉ analz (spies evs);
         evs ∈ kerberos |]
      ==> A=A' & B=B' & T=T'"
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule kerberos.induct, analz_mono_contra)
apply (frule_tac [7] K5_msg_in_parts_spies)
apply (frule_tac [5] K3_msg_in_parts_spies, simp_all)
```

Fake, K2, K4

```
apply (blast+)
done
```

An AuthKey is encrypted by one and only one Shared key. A ServKey is encrypted by one and only one AuthKey.

```
lemma Key_unique_SesKey:
     "[| Crypt K  {|Key SesKey,  Agent B, T, Ticket|}
           ∈ parts (spies evs);
         Crypt K' {|Key SesKey,  Agent B', T', Ticket'|}
           ∈ parts (spies evs);  Key SesKey ∉ analz (spies evs);
         evs ∈ kerberos |]
      ==> K=K' & B=B' & T=T' & Ticket=Ticket'"
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule kerberos.induct, analz_mono_contra)
apply (frule_tac [7] K5_msg_in_parts_spies)
apply (frule_tac [5] K3_msg_in_parts_spies, simp_all)
```

Fake, K2, K4

```
apply (blast+)
done
```

```
lemma unique_AuthKeys:
     "[| Says Kas A
              (Crypt Ka {|Key AuthKey, Agent Tgs, Tk, X|}) ∈ set evs;
         Says Kas A'
              (Crypt Ka' {|Key AuthKey, Agent Tgs, Tk', X'|}) ∈ set evs;
         evs ∈ kerberos |] ==> A=A' & Ka=Ka' & Tk=Tk' & X=X'"
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule kerberos.induct)
apply (frule_tac [7] K5_msg_in_parts_spies)
apply (frule_tac [5] K3_msg_in_parts_spies, simp_all)
```

K2

```
apply blast
done
```

ServKey uniquely identifies the message from Tgs

```
lemma unique_ServKeys:
     "[| Says Tgs A
              (Crypt K {|Key ServKey, Agent B, Tt, X|}) ∈ set evs;
         Says Tgs A'
              (Crypt K' {|Key ServKey, Agent B', Tt', X'|}) ∈ set evs;
         evs ∈ kerberos |] ==> A=A' & B=B' & K=K' & Tt=Tt' & X=X'"
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule kerberos.induct)
apply (frule_tac [7] K5_msg_in_parts_spies)
apply (frule_tac [5] K3_msg_in_parts_spies, simp_all)
```

K4

```
apply blast
done
```

5.6 Lemmas About the Predicate *KeyCryptKey*

```
lemma not_KeyCryptKey_Nil [iff]: "~ KeyCryptKey AuthKey ServKey []"
by (simp add: KeyCryptKey_def)
```

```
lemma KeyCryptKeyI:
 "[| Says Tgs A (Crypt AuthKey {|Key ServKey, Agent B, tt, X |}) ∈ set evs;
     evs ∈ kerberos |] ==> KeyCryptKey AuthKey ServKey evs"
apply (unfold KeyCryptKey_def)
apply (blast dest: Says_Tgs_message_form)
done
```

```
lemma KeyCryptKey_Says [simp]:
```

```
   "KeyCryptKey AuthKey ServKey (Says S A X # evs) =
     (Tgs = S &
      (∃B tt. X = Crypt AuthKey
                {|Key ServKey, Agent B, tt,
                  Crypt (shrK B) {|Agent A, Agent B, Key ServKey, tt|} |})
     | KeyCryptKey AuthKey ServKey evs)"
apply (unfold KeyCryptKey_def)
apply (simp (no_asm))
apply blast
done


lemma Auth_fresh_not_KeyCryptKey:
     "[| Key AuthKey ∉ used evs; evs ∈ kerberos |]
      ==> ~ KeyCryptKey AuthKey ServKey evs"
apply (unfold KeyCryptKey_def)
apply (erule rev_mp)
apply (erule kerberos.induct)
apply (frule_tac [7] K5_msg_in_parts_spies)
apply (frule_tac [5] K3_msg_in_parts_spies, simp_all, blast)
done


lemma Serv_fresh_not_KeyCryptKey:
 "Key ServKey ∉ used evs ==> ~ KeyCryptKey AuthKey ServKey evs"
apply (unfold KeyCryptKey_def, blast)
done

lemma AuthKey_not_KeyCryptKey:
     "[| Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key AuthKey, tk|}
           ∈ parts (spies evs);  evs ∈ kerberos |]
      ==> ~ KeyCryptKey K AuthKey evs"
apply (erule rev_mp)
apply (erule kerberos.induct)
apply (frule_tac [7] K5_msg_in_parts_spies)
apply (frule_tac [5] K3_msg_in_parts_spies, simp_all)
```

Fake

```
apply blast
```

K2: by freshness

```
apply (simp add: KeyCryptKey_def)
```

K4

```
apply (blast+)
done
```

A secure serverkey cannot have been used to encrypt others

```
lemma ServKey_not_KeyCryptKey:
 "[| Crypt (shrK B) {|Agent A, Agent B, Key SK, tt|} ∈ parts (spies evs);
     Key SK ∉ analz (spies evs);  SK ∈ symKeys;
     B ≠ Tgs;  evs ∈ kerberos |]
  ==> ~ KeyCryptKey SK K evs"
```

```
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule kerberos.induct, analz_mono_contra)
apply (frule_tac [7] K5_msg_in_parts_spies)
apply (frule_tac [5] K3_msg_in_parts_spies, simp_all, blast)
```

K4 splits into distinct subcases

```
apply auto
```

ServKey can't have been enclosed in two certificates

```
 prefer 2 apply (blast dest: unique_CryptKey)
```

ServKey is fresh and so could not have been used, by *new_keys_not_used*

```
apply (force dest!: Crypt_imp_invKey_keysFor simp add: KeyCryptKey_def)
done
```

Long term keys are not issued as ServKeys

```
lemma shrK_not_KeyCryptKey:
     "evs ∈ kerberos ==> ~ KeyCryptKey K (shrK A) evs"
apply (unfold KeyCryptKey_def)
apply (erule kerberos.induct)
apply (frule_tac [7] K5_msg_in_parts_spies)
apply (frule_tac [5] K3_msg_in_parts_spies, auto)
done
```

The Tgs message associates ServKey with AuthKey and therefore not with any other key AuthKey.

```
lemma Says_Tgs_KeyCryptKey:
     "[| Says Tgs A (Crypt AuthKey {|Key ServKey, Agent B, tt, X |})
           ∈ set evs;
         AuthKey' ≠ AuthKey;  evs ∈ kerberos |]
      ==> ~ KeyCryptKey AuthKey' ServKey evs"
apply (unfold KeyCryptKey_def)
apply (blast dest: unique_ServKeys)
done

lemma KeyCryptKey_not_KeyCryptKey:
     "[| KeyCryptKey AuthKey ServKey evs;  evs ∈ kerberos |]
      ==> ~ KeyCryptKey ServKey K evs"
apply (erule rev_mp)
apply (erule kerberos.induct)
apply (frule_tac [7] K5_msg_in_parts_spies)
apply (frule_tac [5] K3_msg_in_parts_spies, simp_all, safe)
```

K4 splits into subcases

```
apply simp_all
prefer 4 apply (blast dest!: AuthKey_not_KeyCryptKey)
```

ServKey is fresh and so could not have been used, by *new_keys_not_used*

```
 prefer 2
 apply (force dest!: Crypt_imp_invKey_keysFor simp add: KeyCryptKey_def)
```

Others by freshness

```
apply (blast+)
done
```

The only session keys that can be found with the help of session keys are those sent by Tgs in step K4.

We take some pains to express the property as a logical equivalence so that the simplifier can apply it.

```
lemma Key_analz_image_Key_lemma:
     "P --> (Key K ∈ analz (Key‘KK Un H)) --> (K:KK | Key K ∈ analz H)
      ==>
      P --> (Key K ∈ analz (Key‘KK Un H)) = (K:KK | Key K ∈ analz H)"
by (blast intro: analz_mono [THEN subsetD])


lemma KeyCryptKey_analz_insert:
     "[| KeyCryptKey K K' evs; K ∈ symKeys; evs ∈ kerberos |]
      ==> Key K' ∈ analz (insert (Key K) (spies evs))"
apply (simp add: KeyCryptKey_def, clarify)
apply (drule Says_imp_spies [THEN analz.Inj, THEN analz_insertI], auto)
done

lemma AuthKeys_are_not_KeyCryptKey:
     "[| K ∈ AuthKeys evs Un range shrK;  evs ∈ kerberos |]
      ==> ∀SK. ~ KeyCryptKey SK K evs"
apply (simp add: KeyCryptKey_def)
apply (blast dest: Says_Tgs_message_form)
done

lemma not_AuthKeys_not_KeyCryptKey:
     "[| K ∉ AuthKeys evs;
         K ∉ range shrK; evs ∈ kerberos |]
      ==> ∀SK. ~ KeyCryptKey K SK evs"
apply (simp add: KeyCryptKey_def)
apply (blast dest: Says_Tgs_message_form)
done
```

5.7 Secrecy Theorems

For the Oops2 case of the next theorem

```
lemma Oops2_not_KeyCryptKey:
     "[| evs ∈ kerberos;
         Says Tgs A (Crypt AuthKey
                     {|Key ServKey, Agent B, Number Tt, ServTicket|})
           ∈ set evs |]
      ==> ~ KeyCryptKey ServKey SK evs"
apply (blast dest: KeyCryptKeyI KeyCryptKey_not_KeyCryptKey)
done
```

Big simplification law for keys SK that are not crypted by keys in KK It helps prove three, otherwise hard, facts about keys. These facts are exploited as simplification laws for analz, and also "limit the damage" in case of loss of a key to the spy. See ESORICS98. [simplified by LCP]

lemma *Key_analz_image_Key [rule_format (no_asm)]:*
```
     "evs ∈ kerberos ==>
      (∀SK KK. SK ∈ symKeys & KK <= -(range shrK) -->
       (∀K ∈ KK. ~ KeyCryptKey K SK evs)   -->
       (Key SK ∈ analz (Key‘KK Un (spies evs))) =
       (SK ∈ KK | Key SK ∈ analz (spies evs)))"
```
apply *(erule kerberos.induct)*
apply *(frule_tac [10] Oops_range_spies2)*
apply *(frule_tac [9] Oops_range_spies1)*
apply *(frule_tac [7] Says_tgs_message_form)*
apply *(frule_tac [5] Says_kas_message_form)*
apply *(safe del: impI intro!: Key_analz_image_Key_lemma [THEN impI])*

Case-splits for Oops1 and message 5: the negated case simplifies using the induction hypothesis

apply *(case_tac [11] "KeyCryptKey AuthKey SK evsO1")*
apply *(case_tac [8] "KeyCryptKey ServKey SK evs5")*
apply *(simp_all del: image_insert*
add: analz_image_freshK_simps KeyCryptKey_Says)

Fake

apply *spy_analz*
apply *(simp_all del: image_insert*
add: shrK_not_KeyCryptKey
Oops2_not_KeyCryptKey Auth_fresh_not_KeyCryptKey
Serv_fresh_not_KeyCryptKey Says_Tgs_KeyCryptKey Spy_analz_shrK)
— Splitting the *simp_all* into two parts makes it faster.

K2

apply *blast*

K3

apply *blast*

K4

apply *(blast dest!: AuthKey_not_KeyCryptKey)*

K5

apply *(case_tac "Key ServKey ∈ analz (spies evs5) ")*

If ServKey is compromised then the result follows directly...

apply *(simp (no_asm_simp) add: analz_insert_eq Un_upper2 [THEN analz_mono, THEN subsetD])*

...therefore ServKey is uncompromised.

The KeyCryptKey ServKey SK evs5 case leads to a contradiction.

apply *(blast elim!: ServKey_not_KeyCryptKey [THEN [2] rev_notE] del: allE ballE)*

Another K5 case

apply *blast*

Oops1

```
apply simp
apply (blast dest!: KeyCryptKey_analz_insert)
done
```

First simplification law for analz: no session keys encrypt authentication keys or shared keys.

```
lemma analz_insert_freshK1:
     "[| evs ∈ kerberos;  K ∈ (AuthKeys evs) Un range shrK;
         K ∈ symKeys;
         SesKey ∉ range shrK |]
      ==> (Key K ∈ analz (insert (Key SesKey) (spies evs))) =
          (K = SesKey | Key K ∈ analz (spies evs))"
apply (frule AuthKeys_are_not_KeyCryptKey, assumption)
apply (simp del: image_insert
            add: analz_image_freshK_simps add: Key_analz_image_Key)
done
```

Second simplification law for analz: no service keys encrypt any other keys.

```
lemma analz_insert_freshK2:
     "[| evs ∈ kerberos;  ServKey ∉ (AuthKeys evs); ServKey ∉ range shrK;
         K ∈ symKeys|]
      ==> (Key K ∈ analz (insert (Key ServKey) (spies evs))) =
          (K = ServKey | Key K ∈ analz (spies evs))"
apply (frule not_AuthKeys_not_KeyCryptKey, assumption, assumption)
apply (simp del: image_insert
            add: analz_image_freshK_simps add: Key_analz_image_Key)
done
```

Third simplification law for analz: only one authentication key encrypts a certain service key.

```
lemma analz_insert_freshK3:
 "[| Says Tgs A
             (Crypt AuthKey {|Key ServKey, Agent B, Number Tt, ServTicket|})
        ∈ set evs;  ServKey ∈ symKeys;
     AuthKey ≠ AuthKey'; AuthKey' ∉ range shrK; evs ∈ kerberos |]
        ==> (Key ServKey ∈ analz (insert (Key AuthKey') (spies evs))) =
                (ServKey = AuthKey' | Key ServKey ∈ analz (spies evs))"
apply (drule_tac AuthKey' = AuthKey' in Says_Tgs_KeyCryptKey, blast, assumption)
apply (simp del: image_insert
            add: analz_image_freshK_simps add: Key_analz_image_Key)
done
```

a weakness of the protocol

```
lemma AuthKey_compromises_ServKey:
     "[| Says Tgs A
              (Crypt AuthKey {|Key ServKey, Agent B, Number Tt, ServTicket|})
           ∈ set evs;  AuthKey ∈ symKeys;
         Key AuthKey ∈ analz (spies evs); evs ∈ kerberos |]
      ==> Key ServKey ∈ analz (spies evs)"
by (force dest: Says_imp_spies [THEN analz.Inj, THEN analz.Decrypt, THEN analz.Fst])
```

5.8 Guarantees for Kas

lemma *ServKey_notin_AuthKeysD:*
```
"[| Crypt AuthKey {|Key ServKey, Agent B, Tt,
                    Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Tt|}|}
      ∈ parts (spies evs);
   Key ServKey ∉ analz (spies evs);
   B ≠ Tgs; evs ∈ kerberos |]
 ==> ServKey ∉ AuthKeys evs"
```
apply *(erule rev_mp)*
apply *(erule rev_mp)*
apply *(simp add: AuthKeys_def)*
apply *(erule kerberos.induct, analz_mono_contra)*
apply *(frule_tac [7] K5_msg_in_parts_spies)*
apply *(frule_tac [5] K3_msg_in_parts_spies, simp_all)*
apply *(blast+)*
done

If Spy sees the Authentication Key sent in msg K2, then the Key has expired.

lemma *Confidentiality_Kas_lemma [rule_format]:*
```
"[| AuthKey ∈ symKeys; A ∉ bad;  evs ∈ kerberos |]
 ==> Says Kas A
          (Crypt (shrK A)
             {|Key AuthKey, Agent Tgs, Number Tk,
       Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key AuthKey, Number Tk|}|})
         ∈ set evs -->
       Key AuthKey ∈ analz (spies evs) -->
       ExpirAuth Tk evs"
```
apply *(erule kerberos.induct)*
apply *(frule_tac [10] Oops_range_spies2)*
apply *(frule_tac [9] Oops_range_spies1)*
apply *(frule_tac [7] Says_tgs_message_form)*
apply *(frule_tac [5] Says_kas_message_form)*
apply *(safe del: impI conjI impCE)*
apply *(simp_all (no_asm_simp) add: Says_Kas_message_form less_SucI analz_insert_eq not_parts_not_analz analz_insert_freshK1 pushes)*

Fake

apply *spy_analz*

K2

apply *blast*

K4

apply *blast*

Level 8: K5

apply *(blast dest: ServKey_notin_AuthKeysD Says_Kas_message_form intro: less_SucI)*

Oops1

apply *(blast dest!: unique_AuthKeys intro: less_SucI)*

Oops2

apply *(blast dest: Says_Tgs_message_form Says_Kas_message_form)*
done

lemma *Confidentiality_Kas:*
```
     "[| Says Kas A
              (Crypt Ka {|Key AuthKey, Agent Tgs, Number Tk, AuthTicket|})
           ∈ set evs;
         ~ ExpirAuth Tk evs;
         A ∉ bad;  evs ∈ kerberos |]
      ==> Key AuthKey ∉ analz (spies evs)"
```
by *(blast dest: Says_Kas_message_form Confidentiality_Kas_lemma)*

5.9 Guarantees for Tgs

If Spy sees the Service Key sent in msg K4, then the Key has expired.

lemma *Confidentiality_lemma [rule_format]:*
```
     "[| Says Tgs A
            (Crypt AuthKey
               {|Key ServKey, Agent B, Number Tt,
                 Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Number Tt|}|})
          ∈ set evs;
        Key AuthKey ∉ analz (spies evs);
        ServKey ∈ symKeys;
        A ∉ bad;  B ∉ bad; B ≠ Tgs; evs ∈ kerberos |]
      ==> Key ServKey ∈ analz (spies evs) -->
          ExpirServ Tt evs"
```
apply *(erule rev_mp)*
apply *(erule rev_mp)*
apply *(erule kerberos.induct)*
apply *(rule_tac [9] impI)+*

— The Oops1 case is unusual: must simplify *Authkey ∉ analz (knows Spy (ev # evs))*, not letting *analz_mono_contra* weaken it to *Authkey ∉ analz (knows Spy evs)*, for we then conclude *AuthKey ≠ AuthKeya*.

apply *analz_mono_contra*
apply *(frule_tac [10] Oops_range_spies2)*
apply *(frule_tac [9] Oops_range_spies1)*
apply *(frule_tac [7] Says_tgs_message_form)*
apply *(frule_tac [5] Says_kas_message_form)*
apply *(safe del: impI conjI impCE)*
apply *(simp_all add: less_SucI new_keys_not_analzd Says_Kas_message_form Says_Tgs_message_form analz_insert_eq not_parts_not_analz analz_insert_freshK1 analz_insert_freshK2 analz_insert_freshK3 pushes)*

Fake

apply *spy_analz*

K2

apply *(blast intro: parts_insertI less_SucI)*

K4

apply *(blast dest: A_trusts_AuthTicket Confidentiality_Kas)*

Oops2

```
  prefer 3
  apply (blast dest: Says_imp_spies [THEN parts.Inj] Key_unique_SesKey intro:
less_SucI)
```

Oops1

```
 prefer 2
 apply (blast dest: Says_Kas_message_form Says_Tgs_message_form intro: less_SucI)
```

K5. Not clear how this step could be integrated with the main simplification step.

```
apply clarify
apply (erule_tac V = "Says Aa Tgs ?X ∈ set ?evs" in thin_rl)
apply (frule Says_imp_spies [THEN parts.Inj, THEN ServKey_notin_AuthKeysD])
apply (assumption, blast, assumption)
apply (simp add: analz_insert_freshK2)
apply (blast dest: Says_imp_spies [THEN parts.Inj] Key_unique_SesKey intro:
less_SucI)
done
```

In the real world Tgs can't check wheter AuthKey is secure!

```
lemma Confidentiality_Tgs1:
     "[| Says Tgs A
              (Crypt AuthKey {|Key ServKey, Agent B, Number Tt, ServTicket|})
           ∈ set evs;
         Key AuthKey ∉ analz (spies evs);
         ~ ExpirServ Tt evs;
         A ∉ bad;  B ∉ bad; B ≠ Tgs; evs ∈ kerberos |]
      ==> Key ServKey ∉ analz (spies evs)"
apply (blast dest: Says_Tgs_message_form Confidentiality_lemma)
done
```

In the real world Tgs CAN check what Kas sends!

```
lemma Confidentiality_Tgs2:
     "[| Says Kas A
               (Crypt Ka {|Key AuthKey, Agent Tgs, Number Tk, AuthTicket|})
           ∈ set evs;
         Says Tgs A
              (Crypt AuthKey {|Key ServKey, Agent B, Number Tt, ServTicket|})
           ∈ set evs;
         ~ ExpirAuth Tk evs; ~ ExpirServ Tt evs;
         A ∉ bad;  B ∉ bad; B ≠ Tgs; evs ∈ kerberos |]
      ==> Key ServKey ∉ analz (spies evs)"
apply (blast dest!: Confidentiality_Kas Confidentiality_Tgs1)
done
```

Most general form

```
lemmas Confidentiality_Tgs3 = A_trusts_AuthTicket [THEN Confidentiality_Tgs2]
```

5.10 Guarantees for Alice

```
lemmas Confidentiality_Auth_A = A_trusts_AuthKey [THEN Confidentiality_Kas]
```

```
lemma A_trusts_K4_bis:
 "[| Says Kas A
```

```
       (Crypt (shrK A) {|Key AuthKey, Agent Tgs, Tk, AuthTicket|}) ∈ set evs;
     Crypt AuthKey {|Key ServKey, Agent B, Tt, ServTicket|}
       ∈ parts (spies evs);
     Key AuthKey ∉ analz (spies evs);
     evs ∈ kerberos |]
 ==> Says Tgs A (Crypt AuthKey {|Key ServKey, Agent B, Tt, ServTicket|})
       ∈ set evs"
apply (frule Says_Kas_message_form, assumption)
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule kerberos.induct, analz_mono_contra)
apply (frule_tac [7] K5_msg_in_parts_spies)
apply (frule_tac [5] K3_msg_in_parts_spies, simp_all, blast)
```

K2 and K4 remain

```
prefer 2 apply (blast dest!: unique_CryptKey)
apply (blast dest!: A_trusts_K4 Says_Tgs_message_form AuthKeys_used)
done


lemma Confidentiality_Serv_A:
     "[| Crypt (shrK A) {|Key AuthKey, Agent Tgs, Number Tk, AuthTicket|}
           ∈ parts (spies evs);
         Crypt AuthKey {|Key ServKey, Agent B, Number Tt, ServTicket|}
           ∈ parts (spies evs);
         ~ ExpirAuth Tk evs; ~ ExpirServ Tt evs;
         A ∉ bad;  B ∉ bad; B ≠ Tgs; evs ∈ kerberos |]
      ==> Key ServKey ∉ analz (spies evs)"
apply (drule A_trusts_AuthKey, assumption, assumption)
apply (blast dest: Confidentiality_Kas Says_Kas_message_form A_trusts_K4_bis
Confidentiality_Tgs2)
done
```

5.11 Guarantees for Bob

Theorems for the refined model have suffix "refined"

```
lemma K4_imp_K2:
"[| Says Tgs A (Crypt AuthKey {|Key ServKey, Agent B, Number Tt, ServTicket|})
      ∈ set evs;  evs ∈ kerberos|]
   ==> ∃Tk. Says Kas A
        (Crypt (shrK A)
         {|Key AuthKey, Agent Tgs, Number Tk,
           Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key AuthKey, Number Tk|}|})
        ∈ set evs"
apply (erule rev_mp)
apply (erule kerberos.induct)
apply (frule_tac [7] K5_msg_in_parts_spies)
apply (frule_tac [5] K3_msg_in_parts_spies, simp_all, auto)
apply (blast dest!: Says_imp_spies [THEN parts.Inj, THEN parts.Fst, THEN A_trusts_AuthTicket])
done

lemma K4_imp_K2_refined:
```

```
"[| Says Tgs A (Crypt AuthKey {|Key ServKey, Agent B, Number Tt, ServTicket|})
      ∈ set evs; evs ∈ kerberos|]
   ==> ∃Tk. (Says Kas A (Crypt (shrK A) {|Key AuthKey, Agent Tgs, Number Tk,
          Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key AuthKey, Number Tk|}|})
            ∈ set evs
         & ServLife + Tt <= AuthLife + Tk)"
apply (erule rev_mp)
apply (erule kerberos.induct)
apply (frule_tac [7] K5_msg_in_parts_spies)
apply (frule_tac [5] K3_msg_in_parts_spies, simp_all, auto)
apply (blast dest!: Says_imp_spies [THEN parts.Inj, THEN parts.Fst, THEN A_trusts_AuthTicket])
done
```

Authenticity of ServKey for B

```
lemma B_trusts_ServKey:
     "[| Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Tt|}
           ∈ parts (spies evs);  B ≠ Tgs;  B ∉ bad;
         evs ∈ kerberos |]
 ==> ∃AuthKey.
       Says Tgs A (Crypt AuthKey {|Key ServKey, Agent B, Tt,
                     Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Tt|}|})
       ∈ set evs"
apply (erule rev_mp)
apply (erule kerberos.induct)
apply (frule_tac [7] K5_msg_in_parts_spies)
apply (frule_tac [5] K3_msg_in_parts_spies, simp_all)
apply (blast+)
done

lemma B_trusts_ServTicket_Kas:
     "[| Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Number Tt|}
           ∈ parts (spies evs);  B ≠ Tgs;  B ∉ bad;
         evs ∈ kerberos |]
  ==> ∃AuthKey Tk.
       Says Kas A
         (Crypt (shrK A) {|Key AuthKey, Agent Tgs, Number Tk,
            Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key AuthKey, Number Tk|}|})
        ∈ set evs"
by (blast dest!: B_trusts_ServKey K4_imp_K2)

lemma B_trusts_ServTicket_Kas_refined:
     "[| Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Number Tt|}
           ∈ parts (spies evs);  B ≠ Tgs;  B ∉ bad;
         evs ∈ kerberos |]
  ==> ∃AuthKey Tk. Says Kas A (Crypt(shrK A) {|Key AuthKey, Agent Tgs, Number
Tk,
          Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key AuthKey, Number Tk|}|})
            ∈ set evs
          & ServLife + Tt <= AuthLife + Tk"
by (blast dest!: B_trusts_ServKey K4_imp_K2_refined)

lemma B_trusts_ServTicket:
     "[| Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Number Tt|}
           ∈ parts (spies evs);  B ≠ Tgs;  B ∉ bad;
```

```
        evs ∈ kerberos |]
 ==> ∃Tk AuthKey.
     Says Kas A (Crypt (shrK A) {|Key AuthKey, Agent Tgs, Number Tk,
                   Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key AuthKey, Number
Tk|}|})
       ∈ set evs
     & Says Tgs A (Crypt AuthKey {|Key ServKey, Agent B, Number Tt,
                   Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Number Tt|}|})
       ∈ set evs"
by (blast dest: B_trusts_ServKey K4_imp_K2)

lemma B_trusts_ServTicket_refined:
     "[| Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Number Tt|}
           ∈ parts (spies evs);  B ≠ Tgs;  B ∉ bad;
         evs ∈ kerberos |]
 ==> ∃Tk AuthKey.
     (Says Kas A (Crypt (shrK A) {|Key AuthKey, Agent Tgs, Number Tk,
                   Crypt (shrK Tgs) {|Agent A, Agent Tgs, Key AuthKey, Number
Tk|}|})
       ∈ set evs
     & Says Tgs A (Crypt AuthKey {|Key ServKey, Agent B, Number Tt,
                   Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Number Tt|}|})
       ∈ set evs
     & ServLife + Tt <= AuthLife + Tk)"
by (blast dest: B_trusts_ServKey K4_imp_K2_refined)


lemma NotExpirServ_NotExpirAuth_refined:
     "[| ~ ExpirServ Tt evs; ServLife + Tt <= AuthLife + Tk |]
      ==> ~ ExpirAuth Tk evs"
by (blast dest: leI le_trans dest: leD)


lemma Confidentiality_B:
     "[| Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Number Tt|}
           ∈ parts (spies evs);
         Crypt AuthKey {|Key ServKey, Agent B, Number Tt, ServTicket|}
           ∈ parts (spies evs);
         Crypt (shrK A) {|Key AuthKey, Agent Tgs, Number Tk, AuthTicket|}
           ∈ parts (spies evs);
         ~ ExpirServ Tt evs; ~ ExpirAuth Tk evs;
         A ∉ bad;  B ∉ bad; B ≠ Tgs; evs ∈ kerberos |]
      ==> Key ServKey ∉ analz (spies evs)"
apply (frule A_trusts_AuthKey)
apply (frule_tac [3] Confidentiality_Kas)
apply (frule_tac [6] B_trusts_ServTicket, auto)
apply (blast dest!: Confidentiality_Tgs2 dest: Says_Kas_message_form A_trusts_K4
unique_ServKeys unique_AuthKeys)
done
```

Most general form – only for refined model!

```
lemma Confidentiality_B_refined:
     "[| Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Number Tt|}
           ∈ parts (spies evs);
```

```
        ~ ExpirServ Tt evs;
        A ∉ bad;  B ∉ bad; B ≠ Tgs; evs ∈ kerberos |]
     ==> Key ServKey ∉ analz (spies evs)"
apply (blast dest: B_trusts_ServTicket_refined NotExpirServ_NotExpirAuth_refined
Confidentiality_Tgs2)
done
```

5.12 Authenticity theorems

1. Session Keys authenticity: they originated with servers.

Authenticity of ServKey for A

```
lemma A_trusts_ServKey:
     "[| Crypt (shrK A) {|Key AuthKey, Agent Tgs, Number Tk, AuthTicket|}
           ∈ parts (spies evs);
         Crypt AuthKey {|Key ServKey, Agent B, Number Tt, ServTicket|}
           ∈ parts (spies evs);
         ~ ExpirAuth Tk evs; A ∉ bad; evs ∈ kerberos |]
 ==>Says Tgs A (Crypt AuthKey {|Key ServKey, Agent B, Number Tt, ServTicket|})
       ∈ set evs"
by (blast dest: A_trusts_AuthKey Confidentiality_Auth_A A_trusts_K4_bis)
```

Note: requires a temporal check

B checks authenticity of A by theorems `A_Authenticity` and `A_authenticity_refined`

```
lemma Says_Auth:
     "[| Crypt ServKey {|Agent A, Number Ta|} ∈ parts (spies evs);
         Says Tgs A (Crypt AuthKey {|Key ServKey, Agent B, Number Tt,
                                     ServTicket|}) ∈ set evs;
         Key ServKey ∉ analz (spies evs);
         A ∉ bad; B ∉ bad; evs ∈ kerberos |]
 ==> Says A B {|ServTicket, Crypt ServKey {|Agent A, Number Ta|}|} ∈ set evs"
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule kerberos.induct, analz_mono_contra)
apply (frule_tac [5] Says_ticket_in_parts_spies)
apply (frule_tac [7] Says_ticket_in_parts_spies)
apply (simp_all (no_asm_simp) add: all_conj_distrib)
apply blast
```

K3

```
apply (blast dest: A_trusts_AuthKey Says_Kas_message_form Says_Tgs_message_form)
```

K4

```
apply (force dest!: Crypt_imp_keysFor)
```

K5

```
apply (blast dest: Key_unique_SesKey)
done
```

The second assumption tells B what kind of key ServKey is.

```
lemma A_Authenticity:
```

```
"[| Crypt ServKey {|Agent A, Number Ta|} ∈ parts (spies evs);
    Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Number Tt|}
      ∈ parts (spies evs);
    Crypt AuthKey {|Key ServKey, Agent B, Number Tt, ServTicket|}
      ∈ parts (spies evs);
    Crypt (shrK A) {|Key AuthKey, Agent Tgs, Number Tk, AuthTicket|}
      ∈ parts (spies evs);
    ~ ExpirServ Tt evs; ~ ExpirAuth Tk evs;
    B ≠ Tgs; A ∉ bad;  B ∉ bad;  evs ∈ kerberos |]
 ==> Says A B {|Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Number Tt|},
                Crypt ServKey {|Agent A, Number Ta|} |} ∈ set evs"
by (blast intro: Says_Auth dest: Confidentiality_B Key_unique_SesKey B_trusts_ServKey)
```

Stronger form in the refined model

```
lemma A_Authenticity_refined:
    "[| Crypt ServKey {|Agent A, Number Ta2|} ∈ parts (spies evs);
        Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Number Tt|}
          ∈ parts (spies evs);
        ~ ExpirServ Tt evs;
        B ≠ Tgs; A ∉ bad;  B ∉ bad;  evs ∈ kerberos |]
 ==> Says A B {|Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Number Tt|},
                Crypt ServKey {|Agent A, Number Ta2|} |} ∈ set evs"
by (blast dest: Confidentiality_B_refined B_trusts_ServKey Key_unique_SesKey
intro: Says_Auth)
```

A checks authenticity of B by theorem `B_authenticity`

```
lemma Says_K6:
    "[| Crypt ServKey (Number Ta) ∈ parts (spies evs);
        Says Tgs A (Crypt AuthKey {|Key ServKey, Agent B, Number Tt,
                                    ServTicket|}) ∈ set evs;
        Key ServKey ∉ analz (spies evs);
        A ∉ bad; B ∉ bad; evs ∈ kerberos |]
     ==> Says B A (Crypt ServKey (Number Ta)) ∈ set evs"
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule kerberos.induct, analz_mono_contra)
apply (frule_tac [5] Says_ticket_in_parts_spies)
apply (frule_tac [7] Says_ticket_in_parts_spies)
apply (simp_all (no_asm_simp))
apply blast
apply (force dest!: Crypt_imp_keysFor, clarify)
apply (frule Says_Tgs_message_form, assumption, clarify)
apply (blast dest: unique_CryptKey)
done

lemma K4_trustworthy:
    "[| Crypt AuthKey {|Key ServKey, Agent B, T, ServTicket|}
          ∈ parts (spies evs);
        Key AuthKey ∉ analz (spies evs);  AuthKey ∉ range shrK;
        evs ∈ kerberos |]
  ==> ∃A. Says Tgs A (Crypt AuthKey {|Key ServKey, Agent B, T, ServTicket|})
            ∈ set evs"
apply (erule rev_mp)
```

```
apply (erule rev_mp)
apply (erule kerberos.induct, analz_mono_contra)
apply (frule_tac [7] K5_msg_in_parts_spies)
apply (frule_tac [5] K3_msg_in_parts_spies, simp_all)
apply (blast+)
done

lemma B_Authenticity:
     "[| Crypt ServKey (Number Ta) ∈ parts (spies evs);
         Crypt AuthKey {|Key ServKey, Agent B, Number Tt, ServTicket|}
           ∈ parts (spies evs);
         Crypt (shrK A) {|Key AuthKey, Agent Tgs, Number Tk, AuthTicket|}
           ∈ parts (spies evs);
         ~ ExpirAuth Tk evs; ~ ExpirServ Tt evs;
         A ∉ bad;  B ∉ bad; B ≠ Tgs; evs ∈ kerberos |]
      ==> Says B A (Crypt ServKey (Number Ta)) ∈ set evs"
apply (frule A_trusts_AuthKey)
apply (frule_tac [3] Says_Kas_message_form)
apply (frule_tac [4] Confidentiality_Kas)
apply (frule_tac [7] K4_trustworthy)
prefer 8 apply blast
apply (erule_tac [9] exE)
apply (frule_tac [9] K4_imp_K2)
```

Yes the proof's a mess, but I don't know how to improve it.

```
apply assumption+
apply (blast dest: Key_unique_SesKey intro!: Says_K6 dest: Confidentiality_Tgs1
)
done



lemma B_Knows_B_Knows_ServKey_lemma:
     "[| Says B A (Crypt ServKey (Number Ta)) ∈ set evs;
         Key ServKey ∉ analz (spies evs);
         A ∉ bad;  B ∉ bad; B ≠ Tgs; evs ∈ kerberos |]
      ==> B Issues A with (Crypt ServKey (Number Ta)) on evs"
apply (simp (no_asm) add: Issues_def)
apply (rule exI)
apply (rule conjI, assumption)
apply (simp (no_asm))
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule kerberos.induct, analz_mono_contra)
apply (frule_tac [5] Says_ticket_in_parts_spies)
apply (frule_tac [7] Says_ticket_in_parts_spies)
apply (simp_all (no_asm_simp) add: all_conj_distrib)
apply blast
```

K6 requires numerous lemmas

```
apply (simp add: takeWhile_tail)
apply (blast dest: B_trusts_ServTicket parts_spies_takeWhile_mono [THEN subsetD]
parts_spies_evs_revD2 [THEN subsetD] intro: Says_K6)
```

done

lemma *B_Knows_B_Knows_ServKey:*
```
"[| Says B A (Crypt ServKey (Number Ta)) ∈ set evs;
    Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Number Tt|}
      ∈ parts (spies evs);
    Crypt AuthKey {|Key ServKey, Agent B, Number Tt, ServTicket|}
      ∈ parts (spies evs);
    Crypt (shrK A) {|Key AuthKey, Agent Tgs, Number Tk, AuthTicket|}
      ∈ parts (spies evs);
    ~ ExpirServ Tt evs; ~ ExpirAuth Tk evs;
    A ∉ bad;  B ∉ bad; B ≠ Tgs; evs ∈ kerberos |]
  ==> B Issues A with (Crypt ServKey (Number Ta)) on evs"
```
by *(blast dest!: Confidentiality_B B_Knows_B_Knows_ServKey_lemma)*

lemma *B_Knows_B_Knows_ServKey_refined:*
```
"[| Says B A (Crypt ServKey (Number Ta)) ∈ set evs;
    Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Number Tt|}
      ∈ parts (spies evs);
    ~ ExpirServ Tt evs;
    A ∉ bad;  B ∉ bad; B ≠ Tgs; evs ∈ kerberos |]
  ==> B Issues A with (Crypt ServKey (Number Ta)) on evs"
```
by *(blast dest!: Confidentiality_B_refined B_Knows_B_Knows_ServKey_lemma)*

lemma *A_Knows_B_Knows_ServKey:*
```
"[| Crypt ServKey (Number Ta) ∈ parts (spies evs);
    Crypt AuthKey {|Key ServKey, Agent B, Number Tt, ServTicket|}
      ∈ parts (spies evs);
    Crypt (shrK A) {|Key AuthKey, Agent Tgs, Number Tk, AuthTicket|}
      ∈ parts (spies evs);
    ~ ExpirAuth Tk evs; ~ ExpirServ Tt evs;
    A ∉ bad;  B ∉ bad; B ≠ Tgs; evs ∈ kerberos |]
  ==> B Issues A with (Crypt ServKey (Number Ta)) on evs"
```
by *(blast dest!: B_Authenticity Confidentiality_Serv_A B_Knows_B_Knows_ServKey_lemma)*

lemma *K3_imp_K2:*
```
"[| Says A Tgs
        {|AuthTicket, Crypt AuthKey {|Agent A, Number Ta|}, Agent B|}
      ∈ set evs;
    A ∉ bad;  evs ∈ kerberos |]
  ==> ∃Tk. Says Kas A (Crypt (shrK A)
                 {|Key AuthKey, Agent Tgs, Tk, AuthTicket|})
               ∈ set evs"
```
apply *(erule rev_mp)*
apply *(erule kerberos.induct)*
apply *(frule_tac [7] K5_msg_in_parts_spies)*
apply *(frule_tac [5] K3_msg_in_parts_spies, simp_all, blast, blast)*
apply *(blast dest: Says_imp_spies [THEN parts.Inj, THEN A_trusts_AuthKey])*
done

lemma *K4_trustworthy':*
```
"[| Crypt AuthKey {|Key ServKey, Agent B, Number Tt, ServTicket|}
      ∈ parts (spies evs);
```

```
        Says Kas A (Crypt (shrK A)
                    {|Key AuthKey, Agent Tgs, Tk, AuthTicket|})
        ∈ set evs;
        Key AuthKey ∉ analz (spies evs);
        B ≠ Tgs; A ∉ bad;  B ∉ bad;  evs ∈ kerberos |]
  ==> Says Tgs A (Crypt AuthKey
                    {|Key ServKey, Agent B, Number Tt, ServTicket|})
        ∈ set evs"
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule kerberos.induct, analz_mono_contra)
apply (frule_tac [7] K5_msg_in_parts_spies)
apply (frule_tac [5] K3_msg_in_parts_spies, simp_all, blast)
apply (force dest!: Crypt_imp_keysFor)
apply (blast dest: Says_imp_spies [THEN parts.Inj, THEN parts.Fst, THEN A_trusts_AuthTicket]
unique_AuthKeys)
done

lemma A_Knows_A_Knows_ServKey_lemma:
     "[| Says A B {|ServTicket, Crypt ServKey {|Agent A, Number Ta|}|}
           ∈ set evs;
        Key ServKey ∉ analz (spies evs);
        B ≠ Tgs; A ∉ bad;  B ∉ bad;  evs ∈ kerberos |]
  ==> A Issues B with (Crypt ServKey {|Agent A, Number Ta|}) on evs"
apply (simp (no_asm) add: Issues_def)
apply (rule exI)
apply (rule conjI, assumption)
apply (simp (no_asm))
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule kerberos.induct, analz_mono_contra)
apply (frule_tac [5] Says_ticket_in_parts_spies)
apply (frule_tac [7] Says_ticket_in_parts_spies)
apply (simp_all (no_asm_simp))
apply clarify
```

K5

```
apply auto
apply (simp add: takeWhile_tail)
```

Level 15: case study necessary because the assumption doesn't state the form of ServTicket. The guarantee becomes stronger.

```
apply (blast dest: Says_imp_spies [THEN analz.Inj, THEN analz_Decrypt']
                   K3_imp_K2 K4_trustworthy'
                   parts_spies_takeWhile_mono [THEN subsetD]
                   parts_spies_evs_revD2 [THEN subsetD]
              intro: Says_Auth)
apply (simp add: takeWhile_tail)
done

lemma A_Knows_A_Knows_ServKey:
     "[| Says A B {|ServTicket, Crypt ServKey {|Agent A, Number Ta|}|}
           ∈ set evs;
```

```
        Crypt (shrK A) {|Key AuthKey, Agent Tgs, Number Tk, AuthTicket|}
          ∈ parts (spies evs);
        Crypt AuthKey {|Key ServKey, Agent B, Number Tt, ServTicket|}
          ∈ parts (spies evs);
        ~ ExpirAuth Tk evs; ~ ExpirServ Tt evs;
        B ≠ Tgs; A ∉ bad;  B ∉ bad;  evs ∈ kerberos |]
   ==> A Issues B with (Crypt ServKey {|Agent A, Number Ta|}) on evs"
by (blast dest!: Confidentiality_Serv_A A_Knows_A_Knows_ServKey_lemma)

lemma B_Knows_A_Knows_ServKey:
     "[| Crypt ServKey {|Agent A, Number Ta|} ∈ parts (spies evs);
         Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Number Tt|}
           ∈ parts (spies evs);
         Crypt AuthKey {|Key ServKey, Agent B, Number Tt, ServTicket|}
           ∈ parts (spies evs);
         Crypt (shrK A) {|Key AuthKey, Agent Tgs, Number Tk, AuthTicket|}
           ∈ parts (spies evs);
         ~ ExpirServ Tt evs; ~ ExpirAuth Tk evs;
         B ≠ Tgs; A ∉ bad;  B ∉ bad;  evs ∈ kerberos |]
   ==> A Issues B with (Crypt ServKey {|Agent A, Number Ta|}) on evs"
by (blast dest: A_Authenticity Confidentiality_B A_Knows_A_Knows_ServKey_lemma)


lemma B_Knows_A_Knows_ServKey_refined:
     "[| Crypt ServKey {|Agent A, Number Ta|} ∈ parts (spies evs);
         Crypt (shrK B) {|Agent A, Agent B, Key ServKey, Number Tt|}
           ∈ parts (spies evs);
         ~ ExpirServ Tt evs;
         B ≠ Tgs; A ∉ bad;  B ∉ bad;  evs ∈ kerberos |]
   ==> A Issues B with (Crypt ServKey {|Agent A, Number Ta|}) on evs"
by (blast dest: A_Authenticity_refined Confidentiality_B_refined A_Knows_A_Knows_ServKey_lemma

end
```

6 The Original Otway-Rees Protocol

```
theory OtwayRees imports Public begin
```

From page 244 of Burrows, Abadi and Needham (1989). A Logic of Authentication. Proc. Royal Soc. 426

This is the original version, which encrypts Nonce NB.

```
consts  otway   :: "event list set"
inductive "otway"
  intros

   Nil:  "[] ∈ otway"


   Fake: "[| evsf ∈ otway;  X ∈ synth (analz (knows Spy evsf)) |]
          ==> Says Spy B X  # evsf ∈ otway"
```

```
Reception: "[| evsr ∈ otway;  Says A B X ∈set evsr |]
            ==> Gets B X # evsr ∈ otway"


OR1:  "[| evs1 ∈ otway;  Nonce NA ∉ used evs1 |]
       ==> Says A B {|Nonce NA, Agent A, Agent B,
                      Crypt (shrK A) {|Nonce NA, Agent A, Agent B|} |}
             # evs1 : otway"


OR2:  "[| evs2 ∈ otway;  Nonce NB ∉ used evs2;
          Gets B {|Nonce NA, Agent A, Agent B, X|} : set evs2 |]
       ==> Says B Server
               {|Nonce NA, Agent A, Agent B, X,
                 Crypt (shrK B)
                   {|Nonce NA, Nonce NB, Agent A, Agent B|}|}
             # evs2 : otway"


OR3:  "[| evs3 ∈ otway;  Key KAB ∉ used evs3;
          Gets Server
               {|Nonce NA, Agent A, Agent B,
                 Crypt (shrK A) {|Nonce NA, Agent A, Agent B|},
                 Crypt (shrK B) {|Nonce NA, Nonce NB, Agent A, Agent B|}|}
            : set evs3 |]
       ==> Says Server B
               {|Nonce NA,
                 Crypt (shrK A) {|Nonce NA, Key KAB|},
                 Crypt (shrK B) {|Nonce NB, Key KAB|}|}
             # evs3 : otway"


OR4:  "[| evs4 ∈ otway;  B ≠ Server;
          Says B Server {|Nonce NA, Agent A, Agent B, X',
                          Crypt (shrK B)
                                {|Nonce NA, Nonce NB, Agent A, Agent B|}|}
            : set evs4;
          Gets B {|Nonce NA, X, Crypt (shrK B) {|Nonce NB, Key K|}|}
            : set evs4 |]
       ==> Says B A {|Nonce NA, X|} # evs4 : otway"


Oops: "[| evso ∈ otway;
          Says Server B {|Nonce NA, X, Crypt (shrK B) {|Nonce NB, Key K|}|}
            : set evso |]
       ==> Notes Spy {|Nonce NA, Nonce NB, Key K|} # evso : otway"
```

declare *Says_imp_knows_Spy [THEN analz.Inj, dest]*
declare *parts.Body [dest]*
declare *analz_into_parts [dest]*
declare *Fake_parts_insert_in_Un [dest]*

A "possibility property": there are traces that reach the end

```
lemma "[| B ≠ Server; Key K ∉ used [] |]
      ==> ∃evs ∈ otway.
            Says B A {|Nonce NA, Crypt (shrK A) {|Nonce NA, Key K|}|}
              ∈ set evs"
apply (intro exI bexI)
apply (rule_tac [2] otway.Nil
                  [THEN otway.OR1, THEN otway.Reception,
                   THEN otway.OR2, THEN otway.Reception,
                   THEN otway.OR3, THEN otway.Reception, THEN otway.OR4])

apply (possibility, simp add: used_Cons)
done

lemma Gets_imp_Says [dest!]:
     "[| Gets B X ∈ set evs; evs ∈ otway |] ==> ∃A. Says A B X ∈ set evs"
apply (erule rev_mp)
apply (erule otway.induct, auto)
done




lemma OR2_analz_knows_Spy:
     "[| Gets B {|N, Agent A, Agent B, X|} ∈ set evs;  evs ∈ otway |]
      ==> X ∈ analz (knows Spy evs)"
by blast

lemma OR4_analz_knows_Spy:
     "[| Gets B {|N, X, Crypt (shrK B) X'|} ∈ set evs;  evs ∈ otway |]
      ==> X ∈ analz (knows Spy evs)"
by blast


lemmas OR2_parts_knows_Spy =
    OR2_analz_knows_Spy [THEN analz_into_parts, standard]
```

Theorems of the form `X ∉ parts (knows Spy evs)` imply that NOBODY sends messages containing X!

Spy never sees a good agent's shared key!

```
lemma Spy_see_shrK [simp]:
     "evs ∈ otway ==> (Key (shrK A) ∈ parts (knows Spy evs)) = (A ∈ bad)"
by (erule otway.induct, force,
    drule_tac [4] OR2_parts_knows_Spy, simp_all, blast+)


lemma Spy_analz_shrK [simp]:
     "evs ∈ otway ==> (Key (shrK A) ∈ analz (knows Spy evs)) = (A ∈ bad)"
by auto

lemma Spy_see_shrK_D [dest!]:
     "[|Key (shrK A) ∈ parts (knows Spy evs);  evs ∈ otway|] ==> A ∈ bad"
by (blast dest: Spy_see_shrK)
```

6.1 Towards Secrecy: Proofs Involving analz

```
lemma Says_Server_message_form:
     "[| Says Server B {|NA, X, Crypt (shrK B) {|NB, Key K|}|} ∈ set evs;
         evs ∈ otway |]
      ==> K ∉ range shrK & (∃i. NA = Nonce i) & (∃j. NB = Nonce j)"
by (erule rev_mp, erule otway.induct, simp_all, blast)
```

Session keys are not used to encrypt other session keys

The equality makes the induction hypothesis easier to apply

```
lemma analz_image_freshK [rule_format]:
 "evs ∈ otway ==>
   ∀K KK. KK <= -(range shrK) -->
          (Key K ∈ analz (Key‘KK Un (knows Spy evs))) =
          (K ∈ KK | Key K ∈ analz (knows Spy evs))"
apply (erule otway.induct)
apply (frule_tac [8] Says_Server_message_form)
apply (drule_tac [7] OR4_analz_knows_Spy)
apply (drule_tac [5] OR2_analz_knows_Spy, analz_freshK, spy_analz, auto)
done

lemma analz_insert_freshK:
  "[| evs ∈ otway;  KAB ∉ range shrK |] ==>
      (Key K ∈ analz (insert (Key KAB) (knows Spy evs))) =
      (K = KAB | Key K ∈ analz (knows Spy evs))"
by (simp only: analz_image_freshK analz_image_freshK_simps)
```

The Key K uniquely identifies the Server's message.

```
lemma unique_session_keys:
     "[| Says Server B {|NA, X, Crypt (shrK B) {|NB, K|}|}   ∈ set evs;
         Says Server B' {|NA',X',Crypt (shrK B') {|NB',K|}|} ∈ set evs;
         evs ∈ otway |] ==> X=X' & B=B' & NA=NA' & NB=NB'"
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule otway.induct, simp_all)
apply blast+  — OR3 and OR4
done
```

6.2 Authenticity properties relating to NA

Only OR1 can have caused such a part of a message to appear.

```
lemma Crypt_imp_OR1 [rule_format]:
 "[| A ∉ bad;  evs ∈ otway |]
  ==> Crypt (shrK A) {|NA, Agent A, Agent B|} ∈ parts (knows Spy evs) -->
      Says A B {|NA, Agent A, Agent B,
                 Crypt (shrK A) {|NA, Agent A, Agent B|}|}
        ∈ set evs"
by (erule otway.induct, force,
    drule_tac [4] OR2_parts_knows_Spy, simp_all, blast+)

lemma Crypt_imp_OR1_Gets:
     "[| Gets B {|NA, Agent A, Agent B,
```

```
                Crypt (shrK A) {|NA, Agent A, Agent B|}|} ∈ set evs;
        A ∉ bad; evs ∈ otway |]
      ==> Says A B {|NA, Agent A, Agent B,
                     Crypt (shrK A) {|NA, Agent A, Agent B|}|}
            ∈ set evs"
by (blast dest: Crypt_imp_OR1)
```

The Nonce NA uniquely identifies A's message

```
lemma unique_NA:
     "[| Crypt (shrK A) {|NA, Agent A, Agent B|} ∈ parts (knows Spy evs);
         Crypt (shrK A) {|NA, Agent A, Agent C|} ∈ parts (knows Spy evs);
         evs ∈ otway;  A ∉ bad |]
      ==> B = C"
apply (erule rev_mp, erule rev_mp)
apply (erule otway.induct, force,
       drule_tac [4] OR2_parts_knows_Spy, simp_all, blast+)
done
```

It is impossible to re-use a nonce in both OR1 and OR2. This holds because OR2 encrypts Nonce NB. It prevents the attack that can occur in the over-simplified version of this protocol: see *OtwayRees_Bad*.

```
lemma no_nonce_OR1_OR2:
   "[| Crypt (shrK A) {|NA, Agent A, Agent B|} ∈ parts (knows Spy evs);
       A ∉ bad;  evs ∈ otway |]
    ==> Crypt (shrK A) {|NA', NA, Agent A', Agent A|} ∉ parts (knows Spy evs)"
apply (erule rev_mp)
apply (erule otway.induct, force,
       drule_tac [4] OR2_parts_knows_Spy, simp_all, blast+)
done
```

Crucial property: If the encrypted message appears, and A has used NA to start a run, then it originated with the Server!

```
lemma NA_Crypt_imp_Server_msg [rule_format]:
     "[| A ∉ bad;  evs ∈ otway |]
      ==> Says A B {|NA, Agent A, Agent B,
                     Crypt (shrK A) {|NA, Agent A, Agent B|}|} ∈ set evs -->
          Crypt (shrK A) {|NA, Key K|} ∈ parts (knows Spy evs)
          --> (∃NB. Says Server B
                        {|NA,
                          Crypt (shrK A) {|NA, Key K|},
                          Crypt (shrK B) {|NB, Key K|}|} ∈ set evs)"
apply (erule otway.induct, force,
       drule_tac [4] OR2_parts_knows_Spy, simp_all, blast)
apply blast  — OR1: by freshness
apply (blast dest!: no_nonce_OR1_OR2 intro: unique_NA)  — OR3
apply (blast intro!: Crypt_imp_OR1)  — OR4
done
```

Corollary: if A receives B's OR4 message and the nonce NA agrees then the key really did come from the Server! CANNOT prove this of the bad form of this protocol, even though we can prove *Spy_not_see_encrypted_key*

```
lemma A_trusts_OR4:
     "[| Says A  B {|NA, Agent A, Agent B,
```

```
                    Crypt (shrK A) {|NA, Agent A, Agent B|}|} ∈ set evs;
         Says B' A {|NA, Crypt (shrK A) {|NA, Key K|}|} ∈ set evs;
     A ∉ bad;  evs ∈ otway |]
  ==> ∃NB. Says Server B
               {|NA,
                 Crypt (shrK A) {|NA, Key K|},
                 Crypt (shrK B) {|NB, Key K|}|}
                 ∈ set evs"
by (blast intro!: NA_Crypt_imp_Server_msg)
```

Crucial secrecy property: Spy does not see the keys sent in msg OR3 Does not in itself guarantee security: an attack could violate the premises, e.g. by having `A = Spy`

```
lemma secrecy_lemma:
 "[| A ∉ bad;  B ∉ bad;  evs ∈ otway |]
  ==> Says Server B
        {|NA, Crypt (shrK A) {|NA, Key K|},
          Crypt (shrK B) {|NB, Key K|}|} ∈ set evs -->
      Notes Spy {|NA, NB, Key K|} ∉ set evs -->
      Key K ∉ analz (knows Spy evs)"
apply (erule otway.induct, force)
apply (frule_tac [7] Says_Server_message_form)
apply (drule_tac [6] OR4_analz_knows_Spy)
apply (drule_tac [4] OR2_analz_knows_Spy)
apply (simp_all add: analz_insert_eq analz_insert_freshK pushes)
apply spy_analz  — Fake
apply (blast dest: unique_session_keys)+  — OR3, OR4, Oops
done

theorem Spy_not_see_encrypted_key:
     "[| Says Server B
          {|NA, Crypt (shrK A) {|NA, Key K|},
                Crypt (shrK B) {|NB, Key K|}|} ∈ set evs;
         Notes Spy {|NA, NB, Key K|} ∉ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ otway |]
      ==> Key K ∉ analz (knows Spy evs)"
by (blast dest: Says_Server_message_form secrecy_lemma)
```

This form is an immediate consequence of the previous result. It is similar to the assertions established by other methods. It is equivalent to the previous result in that the Spy already has `analz` and `synth` at his disposal. However, the conclusion `Key K ∉ knows Spy evs` appears not to be inductive: all the cases other than Fake are trivial, while Fake requires `Key K ∉ analz (knows Spy evs)`.

```
lemma Spy_not_know_encrypted_key:
     "[| Says Server B
          {|NA, Crypt (shrK A) {|NA, Key K|},
                Crypt (shrK B) {|NB, Key K|}|} ∈ set evs;
         Notes Spy {|NA, NB, Key K|} ∉ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ otway |]
      ==> Key K ∉ knows Spy evs"
by (blast dest: Spy_not_see_encrypted_key)
```

A's guarantee. The Oops premise quantifies over NB because A cannot know what it is.

```
lemma A_gets_good_key:
     "[| Says A  B {|NA, Agent A, Agent B,
                    Crypt (shrK A) {|NA, Agent A, Agent B|}|} ∈ set evs;
         Says B' A {|NA, Crypt (shrK A) {|NA, Key K|}|} ∈ set evs;
         ∀NB. Notes Spy {|NA, NB, Key K|} ∉ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ otway |]
      ==> Key K ∉ analz (knows Spy evs)"
by (blast dest!: A_trusts_OR4 Spy_not_see_encrypted_key)
```

6.3 Authenticity properties relating to NB

Only OR2 can have caused such a part of a message to appear. We do not know anything about X: it does NOT have to have the right form.

```
lemma Crypt_imp_OR2:
     "[| Crypt (shrK B) {|NA, NB, Agent A, Agent B|} ∈ parts (knows Spy evs);
         B ∉ bad;  evs ∈ otway |]
      ==> ∃X. Says B Server
                 {|NA, Agent A, Agent B, X,
                   Crypt (shrK B) {|NA, NB, Agent A, Agent B|}|}
                 ∈ set evs"
apply (erule rev_mp)
apply (erule otway.induct, force,
       drule_tac [4] OR2_parts_knows_Spy, simp_all, blast+)
done
```

The Nonce NB uniquely identifies B's message

```
lemma unique_NB:
     "[| Crypt (shrK B) {|NA, NB, Agent A, Agent B|} ∈ parts(knows Spy evs);
         Crypt (shrK B) {|NC, NB, Agent C, Agent B|} ∈ parts(knows Spy evs);
           evs ∈ otway;  B ∉ bad |]
         ==> NC = NA & C = A"
apply (erule rev_mp, erule rev_mp)
apply (erule otway.induct, force,
       drule_tac [4] OR2_parts_knows_Spy, simp_all)
apply blast+  — Fake, OR2
done
```

If the encrypted message appears, and B has used Nonce NB, then it originated with the Server! Quite messy proof.

```
lemma NB_Crypt_imp_Server_msg [rule_format]:
 "[| B ∉ bad;  evs ∈ otway |]
  ==> Crypt (shrK B) {|NB, Key K|} ∈ parts (knows Spy evs)
      --> (∀X'. Says B Server
                     {|NA, Agent A, Agent B, X',
                       Crypt (shrK B) {|NA, NB, Agent A, Agent B|}|}
           ∈ set evs
           --> Says Server B
                {|NA, Crypt (shrK A) {|NA, Key K|},
                      Crypt (shrK B) {|NB, Key K|}|}
               ∈ set evs)"
apply simp
apply (erule otway.induct, force,
       drule_tac [4] OR2_parts_knows_Spy, simp_all)
```

```
apply blast  — Fake
apply blast  — OR2
apply (blast dest: unique_NB dest!: no_nonce_OR1_OR2)  — OR3
apply (blast dest!: Crypt_imp_OR2)  — OR4
done
```

Guarantee for B: if it gets a message with matching NB then the Server has sent the correct message.

```
theorem B_trusts_OR3:
     "[| Says B Server {|NA, Agent A, Agent B, X',
                         Crypt (shrK B) {|NA, NB, Agent A, Agent B|} |}
           ∈ set evs;
         Gets B {|NA, X, Crypt (shrK B) {|NB, Key K|}|} ∈ set evs;
         B ∉ bad;  evs ∈ otway |]
      ==> Says Server B
               {|NA,
                 Crypt (shrK A) {|NA, Key K|},
                 Crypt (shrK B) {|NB, Key K|}|}
                 ∈ set evs"
by (blast intro!: NB_Crypt_imp_Server_msg)
```

The obvious combination of *B_trusts_OR3* with *Spy_not_see_encrypted_key*

```
lemma B_gets_good_key:
     "[| Says B Server {|NA, Agent A, Agent B, X',
                         Crypt (shrK B) {|NA, NB, Agent A, Agent B|} |}
           ∈ set evs;
         Gets B {|NA, X, Crypt (shrK B) {|NB, Key K|}|} ∈ set evs;
         Notes Spy {|NA, NB, Key K|} ∉ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ otway |]
      ==> Key K ∉ analz (knows Spy evs)"
by (blast dest!: B_trusts_OR3 Spy_not_see_encrypted_key)


lemma OR3_imp_OR2:
     "[| Says Server B
              {|NA, Crypt (shrK A) {|NA, Key K|},
                Crypt (shrK B) {|NB, Key K|}|} ∈ set evs;
         B ∉ bad;  evs ∈ otway |]
  ==> ∃X. Says B Server {|NA, Agent A, Agent B, X,
                          Crypt (shrK B) {|NA, NB, Agent A, Agent B|} |}
              ∈ set evs"
apply (erule rev_mp)
apply (erule otway.induct, simp_all)
apply (blast dest!: Crypt_imp_OR2)+
done
```

After getting and checking OR4, agent A can trust that B has been active. We could probably prove that X has the expected form, but that is not strictly necessary for authentication.

```
theorem A_auths_B:
     "[| Says B' A {|NA, Crypt (shrK A) {|NA, Key K|}|} ∈ set evs;
         Says A  B {|NA, Agent A, Agent B,
                     Crypt (shrK A) {|NA, Agent A, Agent B|}|} ∈ set evs;
```

```
        A ∉ bad;  B ∉ bad;  evs ∈ otway |]
  ==> ∃NB X. Says B Server {|NA, Agent A, Agent B, X,
                             Crypt (shrK B)  {|NA, NB, Agent A, Agent B|}
|}
                ∈ set evs"
by (blast dest!: A_trusts_OR4 OR3_imp_OR2)
```

end

7 The Otway-Rees Protocol as Modified by Abadi and Needham

theory *OtwayRees_AN* **imports** *Public* **begin**

This simplified version has minimal encryption and explicit messages.

Note that the formalization does not even assume that nonces are fresh. This is because the protocol does not rely on uniqueness of nonces for security, only for freshness, and the proof script does not prove freshness properties.

From page 11 of Abadi and Needham (1996). Prudent Engineering Practice for Cryptographic Protocols. IEEE Trans. SE 22 (1)

```
consts  otway   :: "event list set"
inductive "otway"
  intros
   Nil: — The empty trace
        "[] ∈ otway"
```

Fake: — The Spy may say anything he can say. The sender field is correct, but agents don't use that information.

```
        "[| evsf ∈ otway;  X ∈ synth (analz (knows Spy evsf)) |]
         ==> Says Spy B X  # evsf ∈ otway"
```

Reception: — A message that has been sent can be received by the intended recipient.

```
             "[| evsr ∈ otway;  Says A B X ∈set evsr |]
              ==> Gets B X # evsr ∈ otway"
```

OR1: — Alice initiates a protocol run

```
        "evs1 ∈ otway
         ==> Says A B {|Agent A, Agent B, Nonce NA|} # evs1 ∈ otway"
```

OR2: — Bob's response to Alice's message.

```
        "[| evs2 ∈ otway;
            Gets B {|Agent A, Agent B, Nonce NA|} ∈set evs2 |]
         ==> Says B Server {|Agent A, Agent B, Nonce NA, Nonce NB|}
               # evs2 ∈ otway"
```

OR3: — The Server receives Bob's message. Then he sends a new session key to Bob with a packet for forwarding to Alice.

```
        "[| evs3 ∈ otway;  Key KAB ∉ used evs3;
            Gets Server {|Agent A, Agent B, Nonce NA, Nonce NB|}
```

```
         ∈set evs3 |]
      ==> Says Server B
           {|Crypt (shrK A) {|Nonce NA, Agent A, Agent B, Key KAB|},
             Crypt (shrK B) {|Nonce NB, Agent A, Agent B, Key KAB|}|}
          # evs3 ∈ otway"
```

OR4: — Bob receives the Server's (?) message and compares the Nonces with those in the message he previously sent the Server. Need *B* ≠ *Server* because we allow messages to self.

```
"[| evs4 ∈ otway;  B ≠ Server;
    Says B Server {|Agent A, Agent B, Nonce NA, Nonce NB|} ∈set evs4;
    Gets B {|X, Crypt(shrK B){|Nonce NB,Agent A,Agent B,Key K|}|}
      ∈set evs4 |]
 ==> Says B A X # evs4 ∈ otway"
```

Oops: — This message models possible leaks of session keys. The nonces identify the protocol run.

```
"[| evso ∈ otway;
    Says Server B
            {|Crypt (shrK A) {|Nonce NA, Agent A, Agent B, Key K|},
              Crypt (shrK B) {|Nonce NB, Agent A, Agent B, Key K|}|}
      ∈set evso |]
 ==> Notes Spy {|Nonce NA, Nonce NB, Key K|} # evso ∈ otway"
```

declare *Says_imp_knows_Spy [THEN analz.Inj, dest]*
declare *parts.Body [dest]*
declare *analz_into_parts [dest]*
declare *Fake_parts_insert_in_Un [dest]*

A "possibility property": there are traces that reach the end

```
lemma "[| B ≠ Server; Key K ∉ used [] |]
      ==> ∃evs ∈ otway.
           Says B A (Crypt (shrK A) {|Nonce NA, Agent A, Agent B, Key K|})
             ∈ set evs"
apply (intro exI bexI)
apply (rule_tac [2] otway.Nil
                    [THEN otway.OR1, THEN otway.Reception,
                     THEN otway.OR2, THEN otway.Reception,
                     THEN otway.OR3, THEN otway.Reception, THEN otway.OR4])
apply (possibility, simp add: used_Cons)
done
```

```
lemma Gets_imp_Says [dest!]:
     "[| Gets B X ∈ set evs; evs ∈ otway |] ==> ∃A. Says A B X ∈ set evs"
by (erule rev_mp, erule otway.induct, auto)
```

For reasoning about the encrypted portion of messages

```
lemma OR4_analz_knows_Spy:
     "[| Gets B {|X, Crypt(shrK B) X'|} ∈ set evs;  evs ∈ otway |]
      ==> X ∈ analz (knows Spy evs)"
by blast
```

Theorems of the form *X* ∉ *parts (knows Spy evs)* imply that NOBODY sends

messages containing X!

Spy never sees a good agent's shared key!

```
lemma Spy_see_shrK [simp]:
     "evs ∈ otway ==> (Key (shrK A) ∈ parts (knows Spy evs)) = (A ∈ bad)"
by (erule otway.induct, simp_all, blast+)

lemma Spy_analz_shrK [simp]:
     "evs ∈ otway ==> (Key (shrK A) ∈ analz (knows Spy evs)) = (A ∈ bad)"
by auto

lemma Spy_see_shrK_D [dest!]:
     "[|Key (shrK A) ∈ parts (knows Spy evs);  evs ∈ otway|] ==> A ∈ bad"
by (blast dest: Spy_see_shrK)
```

7.1 Proofs involving analz

Describes the form of K and NA when the Server sends this message.

```
lemma Says_Server_message_form:
     "[| Says Server B
            {|Crypt (shrK A) {|NA, Agent A, Agent B, Key K|},
              Crypt (shrK B) {|NB, Agent A, Agent B, Key K|}|}
           ∈ set evs;
         evs ∈ otway |]
      ==> K ∉ range shrK & (∃i. NA = Nonce i) & (∃j. NB = Nonce j)"
apply (erule rev_mp)
apply (erule otway.induct, auto)
done
```

Session keys are not used to encrypt other session keys

The equality makes the induction hypothesis easier to apply

```
lemma analz_image_freshK [rule_format]:
 "evs ∈ otway ==>
   ∀K KK. KK <= -(range shrK) -->
          (Key K ∈ analz (Key`KK Un (knows Spy evs))) =
          (K ∈ KK | Key K ∈ analz (knows Spy evs))"
apply (erule otway.induct)
apply (frule_tac [8] Says_Server_message_form)
apply (drule_tac [7] OR4_analz_knows_Spy, analz_freshK, spy_analz, auto)
done

lemma analz_insert_freshK:
  "[| evs ∈ otway;  KAB ∉ range shrK |] ==>
      (Key K ∈ analz (insert (Key KAB) (knows Spy evs))) =
      (K = KAB | Key K ∈ analz (knows Spy evs))"
by (simp only: analz_image_freshK analz_image_freshK_simps)
```

The Key K uniquely identifies the Server's message.

```
lemma unique_session_keys:
     "[| Says Server B
           {|Crypt (shrK A) {|NA, Agent A, Agent B, K|},
             Crypt (shrK B) {|NB, Agent A, Agent B, K|}|}
```

```
         ∈ set evs;
        Says Server B'
          {|Crypt (shrK A') {|NA', Agent A', Agent B', K|},
            Crypt (shrK B') {|NB', Agent A', Agent B', K|}|}
         ∈ set evs;
        evs ∈ otway |]
     ==> A=A' & B=B' & NA=NA' & NB=NB'"
apply (erule rev_mp, erule rev_mp, erule otway.induct, simp_all)
apply blast+  — OR3 and OR4
done
```

7.2 Authenticity properties relating to NA

If the encrypted message appears then it originated with the Server!

```
lemma NA_Crypt_imp_Server_msg [rule_format]:
    "[| A ∉ bad;  A ≠ B;  evs ∈ otway |]
     ==> Crypt (shrK A) {|NA, Agent A, Agent B, Key K|} ∈ parts (knows Spy
evs)
       --> (∃NB. Says Server B
                    {|Crypt (shrK A) {|NA, Agent A, Agent B, Key K|},
                      Crypt (shrK B) {|NB, Agent A, Agent B, Key K|}|}
                    ∈ set evs)"
apply (erule otway.induct, force)
apply (simp_all add: ex_disj_distrib)
apply blast+  — Fake, OR3
done
```

Corollary: if A receives B's OR4 message then it originated with the Server. Freshness may be inferred from nonce NA.

```
lemma A_trusts_OR4:
     "[| Says B' A (Crypt (shrK A) {|NA, Agent A, Agent B, Key K|}) ∈ set
evs;
         A ∉ bad;  A ≠ B;  evs ∈ otway |]
      ==> ∃NB. Says Server B
                  {|Crypt (shrK A) {|NA, Agent A, Agent B, Key K|},
                    Crypt (shrK B) {|NB, Agent A, Agent B, Key K|}|}
                ∈ set evs"
by (blast intro!: NA_Crypt_imp_Server_msg)
```

Crucial secrecy property: Spy does not see the keys sent in msg OR3 Does not in itself guarantee security: an attack could violate the premises, e.g. by having `A = Spy`

```
lemma secrecy_lemma:
     "[| A ∉ bad;  B ∉ bad;  evs ∈ otway |]
      ==> Says Server B
            {|Crypt (shrK A) {|NA, Agent A, Agent B, Key K|},
              Crypt (shrK B) {|NB, Agent A, Agent B, Key K|}|}
           ∈ set evs -->
           Notes Spy {|NA, NB, Key K|} ∉ set evs -->
           Key K ∉ analz (knows Spy evs)"
apply (erule otway.induct, force)
apply (frule_tac [7] Says_Server_message_form)
apply (drule_tac [6] OR4_analz_knows_Spy)
```

```
apply (simp_all add: analz_insert_eq analz_insert_freshK pushes)
apply spy_analz  — Fake
apply (blast dest: unique_session_keys)+  — OR3, OR4, Oops
done
```

```
lemma Spy_not_see_encrypted_key:
     "[| Says Server B
           {|Crypt (shrK A) {|NA, Agent A, Agent B, Key K|},
             Crypt (shrK B) {|NB, Agent A, Agent B, Key K|}|}
         ∈ set evs;
        Notes Spy {|NA, NB, Key K|} ∉ set evs;
        A ∉ bad;  B ∉ bad;  evs ∈ otway |]
      ==> Key K ∉ analz (knows Spy evs)"
by (blast dest: Says_Server_message_form secrecy_lemma)
```

A's guarantee. The Oops premise quantifies over NB because A cannot know what it is.

```
lemma A_gets_good_key:
     "[| Says B' A (Crypt (shrK A) {|NA, Agent A, Agent B, Key K|}) ∈ set
evs;
         ∀NB. Notes Spy {|NA, NB, Key K|} ∉ set evs;
         A ∉ bad;  B ∉ bad;  A ≠ B;  evs ∈ otway |]
      ==> Key K ∉ analz (knows Spy evs)"
by (blast dest!: A_trusts_OR4 Spy_not_see_encrypted_key)
```

7.3 Authenticity properties relating to NB

If the encrypted message appears then it originated with the Server!

```
lemma NB_Crypt_imp_Server_msg [rule_format]:
 "[| B ∉ bad;  A ≠ B;  evs ∈ otway |]
  ==> Crypt (shrK B) {|NB, Agent A, Agent B, Key K|} ∈ parts (knows Spy evs)
      --> (∃NA. Says Server B
                  {|Crypt (shrK A) {|NA, Agent A, Agent B, Key K|},
                    Crypt (shrK B) {|NB, Agent A, Agent B, Key K|}|}
                  ∈ set evs)"
apply (erule otway.induct, force, simp_all add: ex_disj_distrib)
apply blast+  — Fake, OR3
done
```

Guarantee for B: if it gets a well-formed certificate then the Server has sent the correct message in round 3.

```
lemma B_trusts_OR3:
     "[| Says S B {|X, Crypt (shrK B) {|NB, Agent A, Agent B, Key K|}|}
           ∈ set evs;
        B ∉ bad;  A ≠ B;  evs ∈ otway |]
      ==> ∃NA. Says Server B
                  {|Crypt (shrK A) {|NA, Agent A, Agent B, Key K|},
                    Crypt (shrK B) {|NB, Agent A, Agent B, Key K|}|}
                  ∈ set evs"
by (blast intro!: NB_Crypt_imp_Server_msg)
```

The obvious combination of *B_trusts_OR3* with *Spy_not_see_encrypted_key*

lemma *B_gets_good_key:*
```
"[| Gets B {|X, Crypt (shrK B) {|NB, Agent A, Agent B, Key K|}|}
     ∈ set evs;
    ∀NA. Notes Spy {|NA, NB, Key K|} ∉ set evs;
    A ∉ bad;  B ∉ bad;  A ≠ B;  evs ∈ otway |]
 ==> Key K ∉ analz (knows Spy evs)"
```
by *(blast dest: B_trusts_OR3 Spy_not_see_encrypted_key)*

end

8 The Otway-Rees Protocol: The Faulty BAN Version

theory *OtwayRees_Bad* **imports** *Public* **begin**

The FAULTY version omitting encryption of Nonce NB, as suggested on page 247 of Burrows, Abadi and Needham (1988). A Logic of Authentication. Proc. Royal Soc. 426

This file illustrates the consequences of such errors. We can still prove impressive-looking properties such as *Spy_not_see_encrypted_key*, yet the protocol is open to a middleperson attack. Attempting to prove some key lemmas indicates the possibility of this attack.

consts *otway :: "event list set"*
inductive *"otway"*
intros
Nil: — The empty trace
```
"[] ∈ otway"
```

Fake: — The Spy may say anything he can say. The sender field is correct, but agents don't use that information.
```
"[| evsf ∈ otway;  X ∈ synth (analz (knows Spy evsf)) |]
 ==> Says Spy B X  # evsf ∈ otway"
```

Reception: — A message that has been sent can be received by the intended recipient.
```
"[| evsr ∈ otway;  Says A B X ∈set evsr |]
 ==> Gets B X # evsr ∈ otway"
```

OR1: — Alice initiates a protocol run
```
"[| evs1 ∈ otway;  Nonce NA ∉ used evs1 |]
 ==> Says A B {|Nonce NA, Agent A, Agent B,
                Crypt (shrK A) {|Nonce NA, Agent A, Agent B|} |}
       # evs1 ∈ otway"
```

OR2: — Bob's response to Alice's message. This variant of the protocol does NOT encrypt NB.
```
"[| evs2 ∈ otway;  Nonce NB ∉ used evs2;
   Gets B {|Nonce NA, Agent A, Agent B, X|} ∈ set evs2 |]
 ==> Says B Server
        {|Nonce NA, Agent A, Agent B, X, Nonce NB,
```

```
                Crypt (shrK B) {|Nonce NA, Agent A, Agent B|}|}
              # evs2 ∈ otway"
```

OR3: — The Server receives Bob's message and checks that the three NAs match. Then he sends a new session key to Bob with a packet for forwarding to Alice.

```
"[| evs3 ∈ otway;  Key KAB ∉ used evs3;
    Gets Server
         {|Nonce NA, Agent A, Agent B,
           Crypt (shrK A) {|Nonce NA, Agent A, Agent B|},
           Nonce NB,
           Crypt (shrK B) {|Nonce NA, Agent A, Agent B|}|}
      ∈ set evs3 |]
 ==> Says Server B
         {|Nonce NA,
           Crypt (shrK A) {|Nonce NA, Key KAB|},
           Crypt (shrK B) {|Nonce NB, Key KAB|}|}
        # evs3 ∈ otway"
```

OR4: — Bob receives the Server's (?) message and compares the Nonces with those in the message he previously sent the Server. Need *B ≠ Server* because we allow messages to self.

```
"[| evs4 ∈ otway;  B ≠ Server;
    Says B Server {|Nonce NA, Agent A, Agent B, X', Nonce NB,
                    Crypt (shrK B) {|Nonce NA, Agent A, Agent B|}|}
      ∈ set evs4;
    Gets B {|Nonce NA, X, Crypt (shrK B) {|Nonce NB, Key K|}|}
      ∈ set evs4 |]
 ==> Says B A {|Nonce NA, X|} # evs4 ∈ otway"
```

Oops: — This message models possible leaks of session keys. The nonces identify the protocol run.

```
"[| evso ∈ otway;
    Says Server B {|Nonce NA, X, Crypt (shrK B) {|Nonce NB, Key K|}|}
      ∈ set evso |]
 ==> Notes Spy {|Nonce NA, Nonce NB, Key K|} # evso ∈ otway"
```

declare *Says_imp_knows_Spy [THEN analz.Inj, dest]*
declare *parts.Body [dest]*
declare *analz_into_parts [dest]*
declare *Fake_parts_insert_in_Un [dest]*

A "possibility property": there are traces that reach the end

```
lemma "[| B ≠ Server; Key K ∉ used [] |]
      ==> ∃NA. ∃evs ∈ otway.
            Says B A {|Nonce NA, Crypt (shrK A) {|Nonce NA, Key K|}|}
              ∈ set evs"
apply (intro exI bexI)
apply (rule_tac [2] otway.Nil
                    [THEN otway.OR1, THEN otway.Reception,
                     THEN otway.OR2, THEN otway.Reception,
                     THEN otway.OR3, THEN otway.Reception, THEN otway.OR4])
apply (possibility, simp add: used_Cons)
done
```

```
lemma Gets_imp_Says [dest!]:
     "[| Gets B X ∈ set evs; evs ∈ otway |] ==> ∃A. Says A B X ∈ set evs"
apply (erule rev_mp)
apply (erule otway.induct, auto)
done
```

8.1 For reasoning about the encrypted portion of messages

```
lemma OR2_analz_knows_Spy:
     "[| Gets B {|N, Agent A, Agent B, X|} ∈ set evs;  evs ∈ otway |]
      ==> X ∈ analz (knows Spy evs)"
by blast

lemma OR4_analz_knows_Spy:
     "[| Gets B {|N, X, Crypt (shrK B) X'|} ∈ set evs;  evs ∈ otway |]
      ==> X ∈ analz (knows Spy evs)"
by blast

lemma Oops_parts_knows_Spy:
     "Says Server B {|NA, X, Crypt K' {|NB,K|}|} ∈ set evs
      ==> K ∈ parts (knows Spy evs)"
by blast
```

Forwarding lemma: see comments in OtwayRees.thy

```
lemmas OR2_parts_knows_Spy =
    OR2_analz_knows_Spy [THEN analz_into_parts, standard]
```

Theorems of the form `X ∉ parts (knows Spy evs)` imply that NOBODY sends messages containing X!

Spy never sees a good agent's shared key!

```
lemma Spy_see_shrK [simp]:
     "evs ∈ otway ==> (Key (shrK A) ∈ parts (knows Spy evs)) = (A ∈ bad)"
by (erule otway.induct, force,
    drule_tac [4] OR2_parts_knows_Spy, simp_all, blast+)


lemma Spy_analz_shrK [simp]:
     "evs ∈ otway ==> (Key (shrK A) ∈ analz (knows Spy evs)) = (A ∈ bad)"
by auto

lemma Spy_see_shrK_D [dest!]:
     "[|Key (shrK A) ∈ parts (knows Spy evs);  evs ∈ otway|] ==> A ∈ bad"
by (blast dest: Spy_see_shrK)
```

8.2 Proofs involving analz

Describes the form of K and NA when the Server sends this message. Also for Oops case.

```
lemma Says_Server_message_form:
     "[| Says Server B {|NA, X, Crypt (shrK B) {|NB, Key K|}|} ∈ set evs;
```

```
        evs ∈ otway |]
     ==> K ∉ range shrK & (∃i. NA = Nonce i) & (∃j. NB = Nonce j)"
apply (erule rev_mp)
apply (erule otway.induct, simp_all, blast)
done
```

Session keys are not used to encrypt other session keys

The equality makes the induction hypothesis easier to apply

```
lemma analz_image_freshK [rule_format]:
 "evs ∈ otway ==>
   ∀K KK. KK <= -(range shrK) -->
          (Key K ∈ analz (Key`KK Un (knows Spy evs))) =
          (K ∈ KK | Key K ∈ analz (knows Spy evs))"
apply (erule otway.induct)
apply (frule_tac [8] Says_Server_message_form)
apply (drule_tac [7] OR4_analz_knows_Spy)
apply (drule_tac [5] OR2_analz_knows_Spy, analz_freshK, spy_analz, auto)
done

lemma analz_insert_freshK:
  "[| evs ∈ otway;  KAB ∉ range shrK |] ==>
      (Key K ∈ analz (insert (Key KAB) (knows Spy evs))) =
      (K = KAB | Key K ∈ analz (knows Spy evs))"
by (simp only: analz_image_freshK analz_image_freshK_simps)
```

The Key K uniquely identifies the Server's message.

```
lemma unique_session_keys:
     "[| Says Server B {|NA, X, Crypt (shrK B) {|NB, K|}|}   ∈ set evs;
         Says Server B' {|NA',X',Crypt (shrK B') {|NB',K|}|} ∈ set evs;
         evs ∈ otway |] ==> X=X' & B=B' & NA=NA' & NB=NB'"
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule otway.induct, simp_all)
apply blast+  — OR3 and OR4
done
```

Crucial secrecy property: Spy does not see the keys sent in msg OR3 Does not in itself guarantee security: an attack could violate the premises, e.g. by having `A = Spy`

```
lemma secrecy_lemma:
 "[| A ∉ bad;  B ∉ bad;  evs ∈ otway |]
  ==> Says Server B
        {|NA, Crypt (shrK A) {|NA, Key K|},
          Crypt (shrK B) {|NB, Key K|}|} ∈ set evs -->
      Notes Spy {|NA, NB, Key K|} ∉ set evs -->
      Key K ∉ analz (knows Spy evs)"
apply (erule otway.induct, force)
apply (frule_tac [7] Says_Server_message_form)
apply (drule_tac [6] OR4_analz_knows_Spy)
apply (drule_tac [4] OR2_analz_knows_Spy)
apply (simp_all add: analz_insert_eq analz_insert_freshK pushes)
apply spy_analz  — Fake
```

```
apply (blast dest: unique_session_keys)+  — OR3, OR4, Oops
done


lemma Spy_not_see_encrypted_key:
     "[| Says Server B
          {|NA, Crypt (shrK A) {|NA, Key K|},
                Crypt (shrK B) {|NB, Key K|}|} ∈ set evs;
         Notes Spy {|NA, NB, Key K|} ∉ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ otway |]
      ==> Key K ∉ analz (knows Spy evs)"
by (blast dest: Says_Server_message_form secrecy_lemma)
```

8.3 Attempting to prove stronger properties

Only OR1 can have caused such a part of a message to appear. The premise `A ≠ B` prevents OR2's similar-looking cryptogram from being picked up. Original Otway-Rees doesn't need it.

```
lemma Crypt_imp_OR1 [rule_format]:
     "[| A ∉ bad;  A ≠ B;  evs ∈ otway |]
      ==> Crypt (shrK A) {|NA, Agent A, Agent B|} ∈ parts (knows Spy evs)
-->
          Says A B {|NA, Agent A, Agent B,
                     Crypt (shrK A) {|NA, Agent A, Agent B|}|}  ∈ set evs"
by (erule otway.induct, force,
    drule_tac [4] OR2_parts_knows_Spy, simp_all, blast+)
```

Crucial property: If the encrypted message appears, and A has used NA to start a run, then it originated with the Server! The premise `A ≠ B` allows use of `Crypt_imp_OR1`

Only it is FALSE. Somebody could make a fake message to Server substituting some other nonce NA' for NB.

```
lemma "[| A ∉ bad;  A ≠ B;  evs ∈ otway |]
       ==> Crypt (shrK A) {|NA, Key K|} ∈ parts (knows Spy evs) -->
           Says A B {|NA, Agent A, Agent B,
                      Crypt (shrK A) {|NA, Agent A, Agent B|}|}
            ∈ set evs -->
           (∃B NB. Says Server B
                {|NA,
                  Crypt (shrK A) {|NA, Key K|},
                  Crypt (shrK B) {|NB, Key K|}|}  ∈ set evs)"
apply (erule otway.induct, force,
       drule_tac [4] OR2_parts_knows_Spy, simp_all)
apply blast  — Fake
apply blast  — OR1: it cannot be a new Nonce, contradiction.
```

OR3 and OR4

```
apply (simp_all add: ex_disj_distrib)
 prefer 2 apply (blast intro!: Crypt_imp_OR1)  — OR4
```

OR3

```
apply clarify
```

oops

end

9 The Woo-Lam Protocol

theory *WooLam* **imports** *Public* **begin**

Simplified version from page 11 of Abadi and Needham (1996). Prudent Engineering Practice for Cryptographic Protocols. IEEE Trans. S.E. 22(1), pages 6-15.

Note: this differs from the Woo-Lam protocol discussed by Lowe (1996): Some New Attacks upon Security Protocols. Computer Security Foundations Workshop

```
consts  woolam  :: "event list set"
inductive woolam
  intros

   Nil:  "[] ∈ woolam"


   Fake: "[| evsf ∈ woolam;  X ∈ synth (analz (spies evsf)) |]
          ==> Says Spy B X  # evsf ∈ woolam"


   WL1:  "evs1 ∈ woolam ==> Says A B (Agent A) # evs1 ∈ woolam"


   WL2:  "[| evs2 ∈ woolam;  Says A' B (Agent A) ∈ set evs2 |]
          ==> Says B A (Nonce NB) # evs2 ∈ woolam"


   WL3:  "[| evs3 ∈ woolam;
              Says A  B (Agent A)  ∈ set evs3;
              Says B' A (Nonce NB) ∈ set evs3 |]
          ==> Says A B (Crypt (shrK A) (Nonce NB)) # evs3 ∈ woolam"


   WL4:  "[| evs4 ∈ woolam;
              Says A'  B X         ∈ set evs4;
              Says A'' B (Agent A) ∈ set evs4 |]
          ==> Says B Server {|Agent A, Agent B, X|} # evs4 ∈ woolam"


   WL5:  "[| evs5 ∈ woolam;
              Says B' Server {|Agent A, Agent B, Crypt (shrK A) (Nonce NB)|}
```

```
          ∈ set evs5 |]
       ==> Says Server B (Crypt (shrK B) {|Agent A, Nonce NB|})
             # evs5 ∈ woolam"


declare Says_imp_knows_Spy [THEN analz.Inj, dest]
declare parts.Body  [dest]
declare analz_into_parts [dest]
declare Fake_parts_insert_in_Un  [dest]



lemma "∃NB. ∃evs ∈ woolam.
           Says Server B (Crypt (shrK B) {|Agent A, Nonce NB|}) ∈ set evs"
apply (intro exI bexI)
apply (rule_tac [2] woolam.Nil
                  [THEN woolam.WL1, THEN woolam.WL2, THEN woolam.WL3,
                   THEN woolam.WL4, THEN woolam.WL5], possibility)
done




lemma Spy_see_shrK [simp]:
     "evs ∈ woolam ==> (Key (shrK A) ∈ parts (spies evs)) = (A ∈ bad)"
by (erule woolam.induct, force, simp_all, blast+)

lemma Spy_analz_shrK [simp]:
     "evs ∈ woolam ==> (Key (shrK A) ∈ analz (spies evs)) = (A ∈ bad)"
by auto

lemma Spy_see_shrK_D [dest!]:
     "[|Key (shrK A) ∈ parts (knows Spy evs);  evs ∈ woolam|] ==> A ∈ bad"
by (blast dest: Spy_see_shrK)




lemma NB_Crypt_imp_Alice_msg:
     "[| Crypt (shrK A) (Nonce NB) ∈ parts (spies evs);
         A ∉ bad;  evs ∈ woolam |]
      ==> ∃B. Says A B (Crypt (shrK A) (Nonce NB)) ∈ set evs"
by (erule rev_mp, erule woolam.induct, force, simp_all, blast+)


lemma Server_trusts_WL4 [dest]:
     "[| Says B' Server {|Agent A, Agent B, Crypt (shrK A) (Nonce NB)|}
```

```
          ∈ set evs;
        A ∉ bad;  evs ∈ woolam |]
     ==> ∃B. Says A B (Crypt (shrK A) (Nonce NB)) ∈ set evs"
by (blast intro!: NB_Crypt_imp_Alice_msg)
```

```
lemma Server_sent_WL5 [dest]:
     "[| Says Server B (Crypt (shrK B) {|Agent A, NB|}) ∈ set evs;
         evs ∈ woolam |]
      ==> ∃B'. Says B' Server {|Agent A, Agent B, Crypt (shrK A) NB|}
            ∈ set evs"
by (erule rev_mp, erule woolam.induct, force, simp_all, blast+)
```

```
lemma NB_Crypt_imp_Server_msg [rule_format]:
     "[| Crypt (shrK B) {|Agent A, NB|} ∈ parts (spies evs);
         B ∉ bad;  evs ∈ woolam |]
      ==> Says Server B (Crypt (shrK B) {|Agent A, NB|}) ∈ set evs"
by (erule rev_mp, erule woolam.induct, force, simp_all, blast+)
```

```
lemma B_trusts_WL5:
     "[| Says S B (Crypt (shrK B) {|Agent A, Nonce NB|}): set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ woolam  |]
      ==> ∃B. Says A B (Crypt (shrK A) (Nonce NB)) ∈ set evs"
by (blast dest!: NB_Crypt_imp_Server_msg)
```

```
lemma B_said_WL2:
     "[| Says B A (Nonce NB) ∈ set evs;  B ≠ Spy;  evs ∈ woolam |]
      ==> ∃A'. Says A' B (Agent A) ∈ set evs"
by (erule rev_mp, erule woolam.induct, force, simp_all, blast+)
```

```
lemma "[| A ∉ bad;  B ≠ Spy;  evs ∈ woolam |]
  ==> Crypt (shrK A) (Nonce NB) ∈ parts (spies evs) &
      Says B A (Nonce NB) ∈ set evs
      --> Says A B (Crypt (shrK A) (Nonce NB)) ∈ set evs"
apply (erule rev_mp, erule woolam.induct, force, simp_all, blast, auto)
oops

end
```

10 The Otway-Bull Recursive Authentication Protocol

```
theory Recur imports Public begin
```

End marker for message bundles

```
syntax       END :: "msg"
translations "END" == "Number 0"


consts     respond :: "event list => (msg*msg*key)set"
inductive "respond evs"
  intros
   One:  "Key KAB ∉ used evs
          ==> (Hash[Key(shrK A)] {|Agent A, Agent B, Nonce NA, END|},
               {|Crypt (shrK A) {|Key KAB, Agent B, Nonce NA|}, END|},
               KAB)   ∈ respond evs"


   Cons: "[| (PA, RA, KAB) ∈ respond evs;
             Key KBC ∉ used evs;  Key KBC ∉ parts {RA};
             PA = Hash[Key(shrK A)] {|Agent A, Agent B, Nonce NA, P|} |]
          ==> (Hash[Key(shrK B)] {|Agent B, Agent C, Nonce NB, PA|},
               {|Crypt (shrK B) {|Key KBC, Agent C, Nonce NB|},
                 Crypt (shrK B) {|Key KAB, Agent A, Nonce NB|},
                 RA|},
               KBC)
              ∈ respond evs"



consts     responses :: "event list => msg set"
inductive "responses evs"
  intros

   Nil:  "END ∈ responses evs"

   Cons: "[| RA ∈ responses evs;  Key KAB ∉ used evs |]
          ==> {|Crypt (shrK B) {|Key KAB, Agent A, Nonce NB|},
                RA|}  ∈ responses evs"


consts     recur   :: "event list set"
inductive "recur"
  intros

   Nil:  "[] ∈ recur"


   Fake: "[| evsf ∈ recur;  X ∈ synth (analz (knows Spy evsf)) |]
          ==> Says Spy B X  # evsf ∈ recur"


   RA1:  "[| evs1 ∈ recur;  Nonce NA ∉ used evs1 |]
          ==> Says A B (Hash[Key(shrK A)] {|Agent A, Agent B, Nonce NA, END|})
              # evs1 ∈ recur"


   RA2:  "[| evs2 ∈ recur;  Nonce NB ∉ used evs2;
```

```
           Says A' B PA ∈ set evs2 |]
        ==> Says B C (Hash[Key(shrK B)] {|Agent B, Agent C, Nonce NB, PA|})
            # evs2 ∈ recur"


  RA3:  "[| evs3 ∈ recur;  Says B' Server PB ∈ set evs3;
           (PB,RB,K) ∈ respond evs3 |]
        ==> Says Server B RB # evs3 ∈ recur"


  RA4:  "[| evs4 ∈ recur;
           Says B  C {|XH, Agent B, Agent C, Nonce NB,
                       XA, Agent A, Agent B, Nonce NA, P|} ∈ set evs4;
           Says C' B {|Crypt (shrK B) {|Key KBC, Agent C, Nonce NB|},
                       Crypt (shrK B) {|Key KAB, Agent A, Nonce NB|},
                       RA|} ∈ set evs4 |]
        ==> Says B A RA # evs4 ∈ recur"
```

```
declare Says_imp_knows_Spy [THEN analz.Inj, dest]
declare parts.Body  [dest]
declare analz_into_parts [dest]
declare Fake_parts_insert_in_Un  [dest]
```

Simplest case: Alice goes directly to the server

```
lemma "Key K ∉ used []
      ==> ∃NA. ∃evs ∈ recur.
            Says Server A {|Crypt (shrK A) {|Key K, Agent Server, Nonce NA|},
                  END|}  ∈ set evs"
apply (intro exI bexI)
apply (rule_tac [2] recur.Nil [THEN recur.RA1,
                               THEN recur.RA3 [OF _ _ respond.One]])
apply (possibility, simp add: used_Cons)
done
```

Case two: Alice, Bob and the server

```
lemma "[| Key K ∉ used []; Key K' ∉ used []; K ≠ K';
         Nonce NA ∉ used []; Nonce NB ∉ used []; NA < NB |]
      ==> ∃NA. ∃evs ∈ recur.
       Says B A {|Crypt (shrK A) {|Key K, Agent B, Nonce NA|},
                  END|}  ∈ set evs"
apply (intro exI bexI)
apply (rule_tac [2]
         recur.Nil
          [THEN recur.RA1 [of _ NA],
           THEN recur.RA2 [of _ NB],
           THEN recur.RA3 [OF _ _ respond.One
                                  [THEN respond.Cons [of _ _ K _ K']]],
           THEN recur.RA4], possibility)
apply (auto simp add: used_Cons)
done
```

```
lemma "[| Key K ∉ used []; Key K' ∉ used [];
          Key K'' ∉ used []; K ≠ K'; K' ≠ K''; K ≠ K'';
          Nonce NA ∉ used []; Nonce NB ∉ used []; Nonce NC ∉ used [];
          NA < NB; NB < NC |]
       ==> ∃K. ∃NA. ∃evs ∈ recur.
             Says B A {|Crypt (shrK A) {|Key K, Agent B, Nonce NA|},
                        END|}  ∈ set evs"
apply (intro exI bexI)
apply (rule_tac [2]
          recur.Nil [THEN recur.RA1,
                     THEN recur.RA2, THEN recur.RA2,
                     THEN recur.RA3
                          [OF _ _ respond.One
                                  [THEN respond.Cons, THEN respond.Cons]],
                     THEN recur.RA4, THEN recur.RA4])
apply (tactic "basic_possibility_tac")
apply (tactic "DEPTH_SOLVE (swap_res_tac [refl, conjI, disjCI] 1)")
done


lemma respond_imp_not_used: "(PA,RB,KAB) ∈ respond evs ==> Key KAB ∉ used
evs"
by (erule respond.induct, simp_all)

lemma Key_in_parts_respond [rule_format]:
   "[| Key K ∈ parts {RB};  (PB,RB,K') ∈ respond evs |] ==> Key K ∉ used
evs"
apply (erule rev_mp, erule respond.induct)
apply (auto dest: Key_not_used respond_imp_not_used)
done
```

Simple inductive reasoning about responses

```
lemma respond_imp_responses:
     "(PA,RB,KAB) ∈ respond evs ==> RB ∈ responses evs"
apply (erule respond.induct)
apply (blast intro!: respond_imp_not_used responses.intros)+
done




lemmas RA2_analz_spies = Says_imp_spies [THEN analz.Inj]

lemma RA4_analz_spies:
     "Says C' B {|Crypt K X, X', RA|} ∈ set evs ==> RA ∈ analz (spies evs)"
by blast




lemmas RA2_parts_spies =  RA2_analz_spies [THEN analz_into_parts]
lemmas RA4_parts_spies =  RA4_analz_spies [THEN analz_into_parts]
```

lemma *Spy_see_shrK [simp]:*
```
"evs ∈ recur ==> (Key (shrK A) ∈ parts (spies evs)) = (A ∈ bad)"
```
apply *(erule recur.induct, auto)*

RA3. It's ugly to call auto twice, but it seems necessary.

apply *(auto dest: Key_in_parts_respond simp add: parts_insert_spies)*
done

lemma *Spy_analz_shrK [simp]:*
```
"evs ∈ recur ==> (Key (shrK A) ∈ analz (spies evs)) = (A ∈ bad)"
```
by *auto*

lemma *Spy_see_shrK_D [dest!]:*
```
"[|Key (shrK A) ∈ parts (knows Spy evs);  evs ∈ recur|] ==> A ∈ bad"
```
by *(blast dest: Spy_see_shrK)*

lemma *resp_analz_image_freshK_lemma:*
```
"[| RB ∈ responses evs;
    ∀K KK. KK ⊆ - (range shrK) -->
              (Key K ∈ analz (Key‘KK Un H)) =
              (K ∈ KK | Key K ∈ analz H) |]
==> ∀K KK. KK ⊆ - (range shrK) -->
              (Key K ∈ analz (insert RB (Key‘KK Un H))) =
              (K ∈ KK | Key K ∈ analz (insert RB H))"
```
apply *(erule responses.induct)*
apply *(simp_all del: image_insert
            add: analz_image_freshK_simps, auto)*
done

Version for the protocol. Proof is easy, thanks to the lemma.

lemma *raw_analz_image_freshK:*
```
"evs ∈ recur ==>
  ∀K KK. KK ⊆ - (range shrK) -->
        (Key K ∈ analz (Key‘KK Un (spies evs))) =
        (K ∈ KK | Key K ∈ analz (spies evs))"
```
apply *(erule recur.induct)*
apply *(drule_tac [4] RA2_analz_spies,*
drule_tac [5] respond_imp_responses,
drule_tac [6] RA4_analz_spies, analz_freshK, spy_analz)

RA3

apply *(simp_all add: resp_analz_image_freshK_lemma)*
done

```
lemmas resp_analz_image_freshK =
       resp_analz_image_freshK_lemma [OF _ raw_analz_image_freshK]

lemma analz_insert_freshK:
     "[| evs ∈ recur;  KAB ∉ range shrK |]
      ==> (Key K ∈ analz (insert (Key KAB) (spies evs))) =
          (K = KAB | Key K ∈ analz (spies evs))"
by (simp del: image_insert
         add: analz_image_freshK_simps raw_analz_image_freshK)
```

Everything that's hashed is already in past traffic.

```
lemma Hash_imp_body:
     "[| Hash {|Key(shrK A), X|} ∈ parts (spies evs);
         evs ∈ recur;  A ∉ bad |] ==> X ∈ parts (spies evs)"
apply (erule rev_mp)
apply (erule recur.induct,
       drule_tac [6] RA4_parts_spies,
       drule_tac [5] respond_imp_responses,
       drule_tac [4] RA2_parts_spies)
```

RA3 requires a further induction

```
apply (erule_tac [5] responses.induct, simp_all)
```

Fake

```
apply (blast intro: parts_insertI)
done
```

```
lemma unique_NA:
  "[| Hash {|Key(shrK A), Agent A, B, NA, P|} ∈ parts (spies evs);
      Hash {|Key(shrK A), Agent A, B',NA, P'|} ∈ parts (spies evs);
      evs ∈ recur;  A ∉ bad |]
    ==> B=B' & P=P'"
apply (erule rev_mp, erule rev_mp)
apply (erule recur.induct,
       drule_tac [5] respond_imp_responses)
apply (force, simp_all)
```

Fake

```
apply blast
apply (erule_tac [3] responses.induct)
```

RA1,2: creation of new Nonce

```
apply simp_all
apply (blast dest!: Hash_imp_body)+
done
```

```
lemma shrK_in_analz_respond [simp]:
     "[| RB ∈ responses evs;  evs ∈ recur |]
  ==> (Key (shrK B) ∈ analz (insert RB (spies evs))) = (B:bad)"
apply (erule responses.induct)
apply (simp_all del: image_insert
                add: analz_image_freshK_simps resp_analz_image_freshK, auto)

done


lemma resp_analz_insert_lemma:
     "[| Key K ∈ analz (insert RB H);
         ∀K KK. KK ⊆ - (range shrK) -->
                   (Key K ∈ analz (Key‘KK Un H)) =
                   (K ∈ KK | Key K ∈ analz H);
         RB ∈ responses evs |]
     ==> (Key K ∈ parts{RB} | Key K ∈ analz H)"
apply (erule rev_mp, erule responses.induct)
apply (simp_all del: image_insert
             add: analz_image_freshK_simps resp_analz_image_freshK_lemma)
```

Simplification using two distinct treatments of "image"

```
apply (simp add: parts_insert2, blast)
done

lemmas resp_analz_insert =
       resp_analz_insert_lemma [OF _ raw_analz_image_freshK]
```

The last key returned by respond indeed appears in a certificate

```
lemma respond_certificate:
     "(Hash[Key(shrK A)] {|Agent A, B, NA, P|}, RA, K) ∈ respond evs
      ==> Crypt (shrK A) {|Key K, B, NA|} ∈ parts {RA}"
apply (ind_cases "(X, RA, K) ∈ respond evs")
apply simp_all
done


lemma unique_lemma [rule_format]:
     "(PB,RB,KXY) ∈ respond evs ==>
      ∀A B N. Crypt (shrK A) {|Key K, Agent B, N|} ∈ parts {RB} -->
      (∀A' B' N'. Crypt (shrK A') {|Key K, Agent B', N'|} ∈ parts {RB} -->
      (A'=A & B'=B) | (A'=B & B'=A))"
apply (erule respond.induct)
apply (simp_all add: all_conj_distrib)
apply (blast dest: respond_certificate)
done

lemma unique_session_keys:
     "[| Crypt (shrK A) {|Key K, Agent B, N|} ∈ parts {RB};
         Crypt (shrK A') {|Key K, Agent B', N'|} ∈ parts {RB};
         (PB,RB,KXY) ∈ respond evs |]
      ==> (A'=A & B'=B) | (A'=B & B'=A)"
```

by *(rule unique_lemma, auto)*

lemma *respond_Spy_not_see_session_key [rule_format]:*
```
     "[| (PB,RB,KAB) ∈ respond evs;  evs ∈ recur |]
      ==> ∀A A' N. A ∉ bad & A' ∉ bad -->
          Crypt (shrK A) {|Key K, Agent A', N|} ∈ parts{RB} -->
          Key K ∉ analz (insert RB (spies evs))"
```
apply *(erule respond.induct)*
apply *(frule_tac [2] respond_imp_responses)*
apply *(frule_tac [2] respond_imp_not_used)*
apply *(simp_all del: image_insert*
add: analz_image_freshK_simps split_ifs shrK_in_analz_respond
resp_analz_image_freshK parts_insert2)

Base case of respond

apply *blast*

Inductive step of respond

apply *(intro allI conjI impI, simp_all)*

by unicity, either `B = Aa` or `B = A'`, a contradiction if `B ∈ bad`

apply *(blast dest: unique_session_keys respond_certificate)*
apply *(blast dest!: respond_certificate)*
apply *(blast dest!: resp_analz_insert)*
done

lemma *Spy_not_see_session_key:*
```
     "[| Crypt (shrK A) {|Key K, Agent A', N|} ∈ parts (spies evs);
         A ∉ bad;  A' ∉ bad;  evs ∈ recur |]
      ==> Key K ∉ analz (spies evs)"
```
apply *(erule rev_mp)*
apply *(erule recur.induct)*
apply *(drule_tac [4] RA2_analz_spies,*
frule_tac [5] respond_imp_responses,
drule_tac [6] RA4_analz_spies,
simp_all add: split_ifs analz_insert_eq analz_insert_freshK)

Fake

apply *spy_analz*

RA2

apply *blast*

RA3 remains

apply *(simp add: parts_insert_spies)*

Now we split into two cases. A single blast could do it, but it would take a CPU minute.

apply *(safe del: impCE)*

RA3, case 1: use lemma previously proved by induction

apply *(blast elim: rev_notE [OF _ respond_Spy_not_see_session_key])*

RA3, case 2: K is an old key

apply *(blast dest: resp_analz_insert dest: Key_in_parts_respond)*

RA4

apply *blast*
done

The response never contains Hashes

lemma *Hash_in_parts_respond:*
```
     "[| Hash {|Key (shrK B), M|} ∈ parts (insert RB H);
         (PB,RB,K) ∈ respond evs |]
      ==> Hash {|Key (shrK B), M|} ∈ parts H"
```
apply *(erule rev_mp)*
apply *(erule respond_imp_responses [THEN responses.induct], auto)*
done

Only RA1 or RA2 can have caused such a part of a message to appear. This result is of no use to B, who cannot verify the Hash. Moreover, it can say nothing about how recent A's message is. It might later be used to prove B's presence to A at the run's conclusion.

lemma *Hash_auth_sender [rule_format]:*
```
     "[| Hash {|Key(shrK A), Agent A, Agent B, NA, P|} ∈ parts(spies evs);
         A ∉ bad;  evs ∈ recur |]
      ==> Says A B (Hash[Key(shrK A)] {|Agent A, Agent B, NA, P|}) ∈ set evs"
```
apply *(unfold HPair_def)*
apply *(erule rev_mp)*
apply *(erule recur.induct,*
drule_tac [6] RA4_parts_spies,
drule_tac [4] RA2_parts_spies,
simp_all)

Fake, RA3

apply *(blast dest: Hash_in_parts_respond)+*
done

Certificates can only originate with the Server.

lemma *Cert_imp_Server_msg:*
```
     "[| Crypt (shrK A) Y ∈ parts (spies evs);
         A ∉ bad;  evs ∈ recur |]
      ==> ∃C RC. Says Server C RC ∈ set evs  &
                 Crypt (shrK A) Y ∈ parts {RC}"
```
apply *(erule rev_mp, erule recur.induct, simp_all)*

Fake

apply *blast*

RA1

apply *blast*

RA2: it cannot be a new Nonce, contradiction.

apply *blast*

RA3. Pity that the proof is so brittle: this step requires the rewriting, which however would break all other steps.

apply *(simp add: parts_insert_spies, blast)*

RA4

apply *blast*
done

end

11 The Yahalom Protocol

theory *Yahalom* **imports** *Public* **begin**

From page 257 of Burrows, Abadi and Needham (1989). A Logic of Authentication. Proc. Royal Soc. 426

This theory has the prototypical example of a secrecy relation, KeyCryptNonce.

```
consts  yahalom   :: "event list set"
inductive "yahalom"
  intros

   Nil:  "[] ∈ yahalom"


   Fake: "[| evsf ∈ yahalom;  X ∈ synth (analz (knows Spy evsf)) |]
          ==> Says Spy B X  # evsf ∈ yahalom"


   Reception: "[| evsr ∈ yahalom;  Says A B X ∈ set evsr |]
               ==> Gets B X # evsr ∈ yahalom"


   YM1:  "[| evs1 ∈ yahalom;  Nonce NA ∉ used evs1 |]
          ==> Says A B {|Agent A, Nonce NA|} # evs1 ∈ yahalom"


   YM2:  "[| evs2 ∈ yahalom;  Nonce NB ∉ used evs2;
             Gets B {|Agent A, Nonce NA|} ∈ set evs2 |]
          ==> Says B Server
                  {|Agent B, Crypt (shrK B) {|Agent A, Nonce NA, Nonce NB|}|}
                # evs2 ∈ yahalom"


   YM3:  "[| evs3 ∈ yahalom;  Key KAB ∉ used evs3;  KAB ∈ symKeys;
             Gets Server
                  {|Agent B, Crypt (shrK B) {|Agent A, Nonce NA, Nonce NB|}|}
               ∈ set evs3 |]
          ==> Says Server A
```

```
                {|Crypt (shrK A) {|Agent B, Key KAB, Nonce NA, Nonce NB|},
                  Crypt (shrK B) {|Agent A, Key KAB|}|}
             # evs3 ∈ yahalom"
```

YM4:
— Alice receives the Server's (?) message, checks her Nonce, and uses the new session key to send Bob his Nonce. The premise `A ≠ Server` is needed for *Says_Server_not_range*. Alice can check that K is symmetric by its length.

```
        "[| evs4 ∈ yahalom;  A ≠ Server;  K ∈ symKeys;
            Gets A {|Crypt(shrK A) {|Agent B, Key K, Nonce NA, Nonce NB|},
X|}
               ∈ set evs4;
            Says A B {|Agent A, Nonce NA|} ∈ set evs4 |]
         ==> Says A B {|X, Crypt K (Nonce NB)|} # evs4 ∈ yahalom"
```

```
  Oops: "[| evso ∈ yahalom;
            Says Server A {|Crypt (shrK A)
                                  {|Agent B, Key K, Nonce NA, Nonce NB|},
                            X|}  ∈ set evso |]
         ==> Notes Spy {|Nonce NA, Nonce NB, Key K|} # evso ∈ yahalom"
```

constdefs

```
  KeyWithNonce :: "[key, nat, event list] => bool"
  "KeyWithNonce K NB evs ==
     ∃A B na X.
       Says Server A {|Crypt (shrK A) {|Agent B, Key K, na, Nonce NB|}, X|}

         ∈ set evs"
```

declare *Says_imp_knows_Spy [THEN analz.Inj, dest]*
declare *parts.Body [dest]*
declare *Fake_parts_insert_in_Un [dest]*
declare *analz_into_parts [dest]*

A "possibility property": there are traces that reach the end

lemma *"[| A ≠ Server; K ∈ symKeys; Key K ∉ used [] |]*
*   ==> ∃X NB. ∃evs ∈ yahalom.*
*   Says A B {|X, Crypt K (Nonce NB)|} ∈ set evs"*
apply *(intro exI bexI)*
apply *(rule_tac [2] yahalom.Nil*
*   [THEN yahalom.YM1, THEN yahalom.Reception,*
*   THEN yahalom.YM2, THEN yahalom.Reception,*
*   THEN yahalom.YM3, THEN yahalom.Reception,*
*   THEN yahalom.YM4])*
apply *(possibility, simp add: used_Cons)*
done

11.1 Regularity Lemmas for Yahalom

lemma *Gets_imp_Says:*
*   "[| Gets B X ∈ set evs; evs ∈ yahalom |] ==> ∃A. Says A B X ∈ set evs"*

```
by (erule rev_mp, erule yahalom.induct, auto)
```

Must be proved separately for each protocol

```
lemma Gets_imp_knows_Spy:
     "[| Gets B X ∈ set evs; evs ∈ yahalom |]  ==> X ∈ knows Spy evs"
by (blast dest!: Gets_imp_Says Says_imp_knows_Spy)

declare Gets_imp_knows_Spy [THEN analz.Inj, dest]
```

Lets us treat YM4 using a similar argument as for the Fake case.

```
lemma YM4_analz_knows_Spy:
     "[| Gets A {|Crypt (shrK A) Y, X|} ∈ set evs;  evs ∈ yahalom |]
      ==> X ∈ analz (knows Spy evs)"
by blast

lemmas YM4_parts_knows_Spy =
       YM4_analz_knows_Spy [THEN analz_into_parts, standard]
```

For Oops

```
lemma YM4_Key_parts_knows_Spy:
     "Says Server A {|Crypt (shrK A) {|B,K,NA,NB|}, X|} ∈ set evs
      ==> K ∈ parts (knows Spy evs)"
by (blast dest!: parts.Body Says_imp_knows_Spy [THEN parts.Inj])
```

Theorems of the form `X ∉ parts (knows Spy evs)` imply that NOBODY sends messages containing X!

Spy never sees a good agent's shared key!

```
lemma Spy_see_shrK [simp]:
     "evs ∈ yahalom ==> (Key (shrK A) ∈ parts (knows Spy evs)) = (A ∈ bad)"
by (erule yahalom.induct, force,
    drule_tac [6] YM4_parts_knows_Spy, simp_all, blast+)

lemma Spy_analz_shrK [simp]:
     "evs ∈ yahalom ==> (Key (shrK A) ∈ analz (knows Spy evs)) = (A ∈ bad)"
by auto

lemma Spy_see_shrK_D [dest!]:
     "[|Key (shrK A) ∈ parts (knows Spy evs);  evs ∈ yahalom|] ==> A ∈ bad"
by (blast dest: Spy_see_shrK)
```

Nobody can have used non-existent keys! Needed to apply `analz_insert_Key`

```
lemma new_keys_not_used [simp]:
    "[|Key K ∉ used evs; K ∈ symKeys; evs ∈ yahalom|]
     ==> K ∉ keysFor (parts (spies evs))"
apply (erule rev_mp)
apply (erule yahalom.induct, force,
       frule_tac [6] YM4_parts_knows_Spy, simp_all)
```

Fake

```
apply (force dest!: keysFor_parts_insert, auto)
done
```

Earlier, all protocol proofs declared this theorem. But only a few proofs need it, e.g. Yahalom and Kerberos IV.

```
lemma new_keys_not_analzd:
 "[|K ∈ symKeys; evs ∈ yahalom; Key K ∉ used evs|]
  ==> K ∉ keysFor (analz (knows Spy evs))"
by (blast dest: new_keys_not_used intro: keysFor_mono [THEN subsetD])
```

Describes the form of K when the Server sends this message. Useful for Oops as well as main secrecy property.

```
lemma Says_Server_not_range [simp]:
     "[| Says Server A {|Crypt (shrK A) {|Agent B, Key K, na, nb|}, X|}
           ∈ set evs;   evs ∈ yahalom |]
      ==> K ∉ range shrK"
by (erule rev_mp, erule yahalom.induct, simp_all, blast)
```

11.2 Secrecy Theorems

Session keys are not used to encrypt other session keys

```
lemma analz_image_freshK [rule_format]:
 "evs ∈ yahalom ==>
   ∀K KK. KK <= - (range shrK) -->
          (Key K ∈ analz (Key‘KK Un (knows Spy evs))) =
          (K ∈ KK | Key K ∈ analz (knows Spy evs))"
apply (erule yahalom.induct,
       drule_tac [7] YM4_analz_knows_Spy, analz_freshK, spy_analz, blast)
apply (simp only: Says_Server_not_range analz_image_freshK_simps)
done

lemma analz_insert_freshK:
     "[| evs ∈ yahalom;  KAB ∉ range shrK |] ==>
      (Key K ∈ analz (insert (Key KAB) (knows Spy evs))) =
      (K = KAB | Key K ∈ analz (knows Spy evs))"
by (simp only: analz_image_freshK analz_image_freshK_simps)
```

The Key K uniquely identifies the Server's message.

```
lemma unique_session_keys:
     "[| Says Server A
           {|Crypt (shrK A) {|Agent B, Key K, na, nb|}, X|} ∈ set evs;
         Says Server A'
           {|Crypt (shrK A') {|Agent B', Key K, na', nb'|}, X'|} ∈ set evs;
         evs ∈ yahalom |]
     ==> A=A' & B=B' & na=na' & nb=nb'"
apply (erule rev_mp, erule rev_mp)
apply (erule yahalom.induct, simp_all)
```

YM3, by freshness, and YM4

```
apply blast+
done
```

Crucial secrecy property: Spy does not see the keys sent in msg YM3

```
lemma secrecy_lemma:
     "[| A ∉ bad;  B ∉ bad;  evs ∈ yahalom |]
```

```
      ==> Says Server A
            {|Crypt (shrK A) {|Agent B, Key K, na, nb|},
              Crypt (shrK B) {|Agent A, Key K|}|}
           ∈ set evs -->
          Notes Spy {|na, nb, Key K|} ∉ set evs -->
          Key K ∉ analz (knows Spy evs)"
apply (erule yahalom.induct, force,
       drule_tac [6] YM4_analz_knows_Spy)
apply (simp_all add: pushes analz_insert_eq analz_insert_freshK, spy_analz)
```
— Fake
```
apply (blast dest: unique_session_keys)+  — YM3, Oops
done
```

Final version

```
lemma Spy_not_see_encrypted_key:
     "[| Says Server A
           {|Crypt (shrK A) {|Agent B, Key K, na, nb|},
             Crypt (shrK B) {|Agent A, Key K|}|}
          ∈ set evs;
         Notes Spy {|na, nb, Key K|} ∉ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom |]
      ==> Key K ∉ analz (knows Spy evs)"
by (blast dest: secrecy_lemma)
```

11.2.1 Security Guarantee for A upon receiving YM3

If the encrypted message appears then it originated with the Server

```
lemma A_trusts_YM3:
     "[| Crypt (shrK A) {|Agent B, Key K, na, nb|} ∈ parts (knows Spy evs);
         A ∉ bad;  evs ∈ yahalom |]
       ==> Says Server A
            {|Crypt (shrK A) {|Agent B, Key K, na, nb|},
              Crypt (shrK B) {|Agent A, Key K|}|}
           ∈ set evs"
apply (erule rev_mp)
apply (erule yahalom.induct, force,
       frule_tac [6] YM4_parts_knows_Spy, simp_all)
```

Fake, YM3

```
apply blast+
done
```

The obvious combination of *A_trusts_YM3* with *Spy_not_see_encrypted_key*

```
lemma A_gets_good_key:
     "[| Crypt (shrK A) {|Agent B, Key K, na, nb|} ∈ parts (knows Spy evs);
         Notes Spy {|na, nb, Key K|} ∉ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom |]
      ==> Key K ∉ analz (knows Spy evs)"
by (blast dest!: A_trusts_YM3 Spy_not_see_encrypted_key)
```

11.2.2 Security Guarantees for B upon receiving YM4

B knows, by the first part of A's message, that the Server distributed the key for A and B. But this part says nothing about nonces.

```
lemma B_trusts_YM4_shrK:
     "[| Crypt (shrK B) {|Agent A, Key K|} ∈ parts (knows Spy evs);
         B ∉ bad;  evs ∈ yahalom |]
      ==> ∃NA NB. Says Server A
                     {|Crypt (shrK A) {|Agent B, Key K,
                                        Nonce NA, Nonce NB|},
                       Crypt (shrK B) {|Agent A, Key K|}|}
                    ∈ set evs"
apply (erule rev_mp)
apply (erule yahalom.induct, force,
       frule_tac [6] YM4_parts_knows_Spy, simp_all)
```

Fake, YM3

```
apply blast+
done
```

B knows, by the second part of A's message, that the Server distributed the key quoting nonce NB. This part says nothing about agent names. Secrecy of NB is crucial. Note that `Nonce NB ∉ analz (knows Spy evs)` must be the FIRST antecedent of the induction formula.

```
lemma B_trusts_YM4_newK [rule_format]:
     "[|Crypt K (Nonce NB) ∈ parts (knows Spy evs);
        Nonce NB ∉ analz (knows Spy evs);  evs ∈ yahalom|]
      ==> ∃A B NA. Says Server A
                     {|Crypt (shrK A) {|Agent B, Key K, Nonce NA, Nonce NB|},
                       Crypt (shrK B) {|Agent A, Key K|}|}
                    ∈ set evs"
apply (erule rev_mp, erule rev_mp)
apply (erule yahalom.induct, force,
       frule_tac [6] YM4_parts_knows_Spy)
apply (analz_mono_contra, simp_all)
```

Fake, YM3

```
apply blast
apply blast
```

YM4. A is uncompromised because NB is secure A's certificate guarantees the existence of the Server message

```
apply (blast dest!: Gets_imp_Says Crypt_Spy_analz_bad
             dest: Says_imp_spies
                   parts.Inj [THEN parts.Fst, THEN A_trusts_YM3])
done
```

11.2.3 Towards proving secrecy of Nonce NB

Lemmas about the predicate KeyWithNonce

```
lemma KeyWithNonceI:
 "Says Server A
          {|Crypt (shrK A) {|Agent B, Key K, na, Nonce NB|}, X|}
        ∈ set evs ==> KeyWithNonce K NB evs"
by (unfold KeyWithNonce_def, blast)
```

```
lemma KeyWithNonce_Says [simp]:
   "KeyWithNonce K NB (Says S A X # evs) =
      (Server = S &
       (∃B n X'. X = {|Crypt (shrK A) {|Agent B, Key K, n, Nonce NB|}, X'|})
      | KeyWithNonce K NB evs)"
by (simp add: KeyWithNonce_def, blast)


lemma KeyWithNonce_Notes [simp]:
   "KeyWithNonce K NB (Notes A X # evs) = KeyWithNonce K NB evs"
by (simp add: KeyWithNonce_def)

lemma KeyWithNonce_Gets [simp]:
   "KeyWithNonce K NB (Gets A X # evs) = KeyWithNonce K NB evs"
by (simp add: KeyWithNonce_def)
```

A fresh key cannot be associated with any nonce (with respect to a given trace).

```
lemma fresh_not_KeyWithNonce:
     "Key K ∉ used evs ==> ~ KeyWithNonce K NB evs"
by (unfold KeyWithNonce_def, blast)
```

The Server message associates K with NB' and therefore not with any other nonce NB.

```
lemma Says_Server_KeyWithNonce:
 "[| Says Server A {|Crypt (shrK A) {|Agent B, Key K, na, Nonce NB'|}, X|}
       ∈ set evs;
     NB ≠ NB';  evs ∈ yahalom |]
  ==> ~ KeyWithNonce K NB evs"
by (unfold KeyWithNonce_def, blast dest: unique_session_keys)
```

The only nonces that can be found with the help of session keys are those distributed as nonce NB by the Server. The form of the theorem recalls *analz_image_freshK*, but it is much more complicated.

As with *analz_image_freshK*, we take some pains to express the property as a logical equivalence so that the simplifier can apply it.

```
lemma Nonce_secrecy_lemma:
     "P --> (X ∈ analz (G Un H)) --> (X ∈ analz H)  ==>
      P --> (X ∈ analz (G Un H)) = (X ∈ analz H)"
by (blast intro: analz_mono [THEN subsetD])


lemma Nonce_secrecy:
     "evs ∈ yahalom ==>
      (∀KK. KK <= - (range shrK) -->
           (∀K ∈ KK. K ∈ symKeys --> ~ KeyWithNonce K NB evs)   -->
           (Nonce NB ∈ analz (Key‘KK Un (knows Spy evs))) =
           (Nonce NB ∈ analz (knows Spy evs)))"
apply (erule yahalom.induct,
       frule_tac [7] YM4_analz_knows_Spy)
apply (safe del: allI impI intro!: Nonce_secrecy_lemma [THEN impI, THEN allI])
apply (simp_all del: image_insert image_Un
       add: analz_image_freshK_simps split_ifs
            all_conj_distrib ball_conj_distrib
```

```
          analz_image_freshK fresh_not_KeyWithNonce
          imp_disj_not1
          Says_Server_KeyWithNonce)
```

For Oops, simplification proves *NBa* $\neq$ *NB*. By *Says_Server_KeyWithNonce*, we get $\neg$ *KeyWithNonce K NB evs*; then simplification can apply the induction hypothesis with *KK = {K}*.

Fake

apply *spy_analz*

YM2

apply *blast*

YM3

apply *blast*

YM4

apply *(erule_tac V = "∀KK. ?P KK"* **in** *thin_rl, clarify)*

If *A* $\in$ *bad* then *NBa* is known, therefore *NBa* $\neq$ *NB*. Previous two steps make the next step faster.

```
apply (blast dest!: Gets_imp_Says Says_imp_spies Crypt_Spy_analz_bad
         dest: analz.Inj
           parts.Inj [THEN parts.Fst, THEN A_trusts_YM3, THEN KeyWithNonceI])
done
```

Version required below: if NB can be decrypted using a session key then it was distributed with that key. The more general form above is required for the induction to carry through.

```
lemma single_Nonce_secrecy:
     "[| Says Server A
          {|Crypt (shrK A) {|Agent B, Key KAB, na, Nonce NB'|}, X|}
         ∈ set evs;
         NB ≠ NB';  KAB ∉ range shrK;  evs ∈ yahalom |]
      ==> (Nonce NB ∈ analz (insert (Key KAB) (knows Spy evs))) =
          (Nonce NB ∈ analz (knows Spy evs))"
by (simp_all del: image_insert image_Un imp_disjL
             add: analz_image_freshK_simps split_ifs
                  Nonce_secrecy Says_Server_KeyWithNonce)
```

11.2.4 The Nonce NB uniquely identifies B's message.

```
lemma unique_NB:
     "[| Crypt (shrK B) {|Agent A, Nonce NA, nb|} ∈ parts (knows Spy evs);
         Crypt (shrK B') {|Agent A', Nonce NA', nb|} ∈ parts (knows Spy evs);
        evs ∈ yahalom;  B ∉ bad;  B' ∉ bad |]
      ==> NA' = NA & A' = A & B' = B"
apply (erule rev_mp, erule rev_mp)
apply (erule yahalom.induct, force,
       frule_tac [6] YM4_parts_knows_Spy, simp_all)
```

Fake, and YM2 by freshness

```
apply blast+
done
```

Variant useful for proving secrecy of NB. Because nb is assumed to be secret, we no longer must assume B, B' not bad.

```
lemma Says_unique_NB:
     "[| Says C S   {|X,  Crypt (shrK B) {|Agent A, Nonce NA, nb|}|}
           ∈ set evs;
         Gets S' {|X', Crypt (shrK B') {|Agent A', Nonce NA', nb|}|}
           ∈ set evs;
         nb ∉ analz (knows Spy evs);  evs ∈ yahalom |]
      ==> NA' = NA & A' = A & B' = B"
by (blast dest!: Gets_imp_Says Crypt_Spy_analz_bad
          dest: Says_imp_spies unique_NB parts.Inj analz.Inj)
```

11.2.5 A nonce value is never used both as NA and as NB

```
lemma no_nonce_YM1_YM2:
     "[|Crypt (shrK B') {|Agent A', Nonce NB, nb'|} ∈ parts(knows Spy evs);
        Nonce NB ∉ analz (knows Spy evs);  evs ∈ yahalom|]
  ==> Crypt (shrK B)  {|Agent A, na, Nonce NB|} ∉ parts(knows Spy evs)"
apply (erule rev_mp, erule rev_mp)
apply (erule yahalom.induct, force,
       frule_tac [6] YM4_parts_knows_Spy)
apply (analz_mono_contra, simp_all)
```

Fake, YM2

```
apply blast+
done
```

The Server sends YM3 only in response to YM2.

```
lemma Says_Server_imp_YM2:
     "[| Says Server A {|Crypt (shrK A) {|Agent B, k, na, nb|}, X|} ∈ set
evs;
         evs ∈ yahalom |]
      ==> Gets Server {| Agent B, Crypt (shrK B) {|Agent A, na, nb|} |}
            ∈ set evs"
by (erule rev_mp, erule yahalom.induct, auto)
```

A vital theorem for B, that nonce NB remains secure from the Spy.

```
lemma Spy_not_see_NB :
     "[| Says B Server
                {|Agent B, Crypt (shrK B) {|Agent A, Nonce NA, Nonce NB|}|}
           ∈ set evs;
         (∀k. Notes Spy {|Nonce NA, Nonce NB, k|} ∉ set evs);
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom |]
      ==> Nonce NB ∉ analz (knows Spy evs)"
apply (erule rev_mp, erule rev_mp)
apply (erule yahalom.induct, force,
       frule_tac [6] YM4_analz_knows_Spy)
apply (simp_all add: split_ifs pushes new_keys_not_analzd analz_insert_eq
                     analz_insert_freshK)
```

Fake

apply *spy_analz*

YM1: NB=NA is impossible anyway, but NA is secret because it is fresh!

apply *blast*

YM2

apply *blast*

Prove YM3 by showing that no NB can also be an NA

apply *(blast dest!: no_nonce_YM1_YM2 dest: Gets_imp_Says Says_unique_NB)*

LEVEL 7: YM4 and Oops remain

apply *(clarify, simp add: all_conj_distrib)*

YM4: key K is visible to Spy, contradicting session key secrecy theorem

Case analysis on Aa:bad; PROOF FAILED problems use *Says_unique_NB* to identify message components: *Aa = A*, *Ba = B*

```
apply (blast dest!: Says_unique_NB analz_shrK_Decrypt
                    parts.Inj [THEN parts.Fst, THEN A_trusts_YM3]
             dest: Gets_imp_Says Says_imp_spies Says_Server_imp_YM2
                   Spy_not_see_encrypted_key)
```

Oops case: if the nonce is betrayed now, show that the Oops event is covered by the quantified Oops assumption.

```
apply (clarify, simp add: all_conj_distrib)
apply (frule Says_Server_imp_YM2, assumption)
apply (case_tac "NB = NBa")
```

If NB=NBa then all other components of the Oops message agree

apply *(blast dest: Says_unique_NB)*

case *NB* $\neq$ *NBa*

```
apply (simp add: single_Nonce_secrecy)
apply (blast dest!: no_nonce_YM1_YM2 )
done
```

B's session key guarantee from YM4. The two certificates contribute to a single conclusion about the Server's message. Note that the "Notes Spy" assumption must quantify over $\forall$ POSSIBLE keys instead of our particular K. If this run is broken and the spy substitutes a certificate containing an old key, B has no means of telling.

```
lemma B_trusts_YM4:
     "[| Gets B {|Crypt (shrK B) {|Agent A, Key K|},
                  Crypt K (Nonce NB)|} ∈ set evs;
         Says B Server
           {|Agent B, Crypt (shrK B) {|Agent A, Nonce NA, Nonce NB|}|}
           ∈ set evs;
         ∀k. Notes Spy {|Nonce NA, Nonce NB, k|} ∉ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom |]
       ==> Says Server A
                  {|Crypt (shrK A) {|Agent B, Key K,
```

```
                          Nonce NA, Nonce NB|},
                    Crypt (shrK B) {|Agent A, Key K|}|}
           ∈ set evs"
by (blast dest: Spy_not_see_NB Says_unique_NB
                Says_Server_imp_YM2 B_trusts_YM4_newK)
```

The obvious combination of *B_trusts_YM4* with *Spy_not_see_encrypted_key*

```
lemma B_gets_good_key:
     "[| Gets B {|Crypt (shrK B) {|Agent A, Key K|},
                  Crypt K (Nonce NB)|} ∈ set evs;
         Says B Server
           {|Agent B, Crypt (shrK B) {|Agent A, Nonce NA, Nonce NB|}|}
           ∈ set evs;
         ∀k. Notes Spy {|Nonce NA, Nonce NB, k|} ∉ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom |]
      ==> Key K ∉ analz (knows Spy evs)"
by (blast dest!: B_trusts_YM4 Spy_not_see_encrypted_key)
```

11.3 Authenticating B to A

The encryption in message YM2 tells us it cannot be faked.

```
lemma B_Said_YM2 [rule_format]:
     "[|Crypt (shrK B) {|Agent A, Nonce NA, nb|} ∈ parts (knows Spy evs);
        evs ∈ yahalom|]
      ==> B ∉ bad -->
          Says B Server {|Agent B, Crypt (shrK B) {|Agent A, Nonce NA, nb|}|}
            ∈ set evs"
apply (erule rev_mp, erule yahalom.induct, force,
       frule_tac [6] YM4_parts_knows_Spy, simp_all)
```

Fake

```
apply blast
done
```

If the server sends YM3 then B sent YM2

```
lemma YM3_auth_B_to_A_lemma:
     "[|Says Server A {|Crypt (shrK A) {|Agent B, Key K, Nonce NA, nb|}, X|}
       ∈ set evs;  evs ∈ yahalom|]
      ==> B ∉ bad -->
          Says B Server {|Agent B, Crypt (shrK B) {|Agent A, Nonce NA, nb|}|}
            ∈ set evs"
apply (erule rev_mp, erule yahalom.induct, simp_all)
```

YM3, YM4

```
apply (blast dest!: B_Said_YM2)+
done
```

If A receives YM3 then B has used nonce NA (and therefore is alive)

```
lemma YM3_auth_B_to_A:
     "[| Gets A {|Crypt (shrK A) {|Agent B, Key K, Nonce NA, nb|}, X|}
           ∈ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom |]
```

```
     ==> Says B Server {|Agent B, Crypt (shrK B) {|Agent A, Nonce NA, nb|}|}
      ∈ set evs"
by (blast dest!: A_trusts_YM3 YM3_auth_B_to_A_lemma elim: knows_Spy_partsEs)
```

11.4 Authenticating A to B using the certificate *Crypt K (Nonce NB)*

Assuming the session key is secure, if both certificates are present then A has said NB. We can't be sure about the rest of A's message, but only NB matters for freshness.

```
lemma A_Said_YM3_lemma [rule_format]:
     "evs ∈ yahalom
      ==> Key K ∉ analz (knows Spy evs) -->
          Crypt K (Nonce NB) ∈ parts (knows Spy evs) -->
          Crypt (shrK B) {|Agent A, Key K|} ∈ parts (knows Spy evs) -->
          B ∉ bad -->
          (∃X. Says A B {|X, Crypt K (Nonce NB)|} ∈ set evs)"
apply (erule yahalom.induct, force,
       frule_tac [6] YM4_parts_knows_Spy)
apply (analz_mono_contra, simp_all)
```

Fake

```
apply blast
```

YM3: by *new_keys_not_used*, the message *Crypt K (Nonce NB)* could not exist

```
apply (force dest!: Crypt_imp_keysFor)
```

YM4: was *Crypt K (Nonce NB)* the very last message? If not, use the induction hypothesis

```
apply (simp add: ex_disj_distrib)
```

yes: apply unicity of session keys

```
apply (blast dest!: Gets_imp_Says A_trusts_YM3 B_trusts_YM4_shrK
                    Crypt_Spy_analz_bad
             dest: Says_imp_knows_Spy [THEN parts.Inj] unique_session_keys)
done
```

If B receives YM4 then A has used nonce NB (and therefore is alive). Moreover, A associates K with NB (thus is talking about the same run). Other premises guarantee secrecy of K.

```
lemma YM4_imp_A_Said_YM3 [rule_format]:
     "[| Gets B {|Crypt (shrK B) {|Agent A, Key K|},
                  Crypt K (Nonce NB)|} ∈ set evs;
         Says B Server
           {|Agent B, Crypt (shrK B) {|Agent A, Nonce NA, Nonce NB|}|}
           ∈ set evs;
         (∀NA k. Notes Spy {|Nonce NA, Nonce NB, k|} ∉ set evs);
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom |]
      ==> ∃X. Says A B {|X, Crypt K (Nonce NB)|} ∈ set evs"
by (blast intro!: A_Said_YM3_lemma
          dest: Spy_not_see_encrypted_key B_trusts_YM4 Gets_imp_Says)
```

end

12 The Yahalom Protocol, Variant 2

theory *Yahalom2* **imports** *Public* **begin**

This version trades encryption of NB for additional explicitness in YM3. Also in YM3, care is taken to make the two certificates distinct.

From page 259 of Burrows, Abadi and Needham (1989). A Logic of Authentication. Proc. Royal Soc. 426

This theory has the prototypical example of a secrecy relation, KeyCryptNonce.

```
consts  yahalom   :: "event list set"
inductive "yahalom"
  intros

   Nil:  "[] ∈ yahalom"


   Fake: "[| evsf ∈ yahalom;  X ∈ synth (analz (knows Spy evsf)) |]
          ==> Says Spy B X  # evsf ∈ yahalom"


   Reception: "[| evsr ∈ yahalom;  Says A B X ∈ set evsr |]
               ==> Gets B X # evsr ∈ yahalom"


   YM1:  "[| evs1 ∈ yahalom;  Nonce NA ∉ used evs1 |]
          ==> Says A B {|Agent A, Nonce NA|} # evs1 ∈ yahalom"


   YM2:  "[| evs2 ∈ yahalom;  Nonce NB ∉ used evs2;
             Gets B {|Agent A, Nonce NA|} ∈ set evs2 |]
          ==> Says B Server
                  {|Agent B, Nonce NB, Crypt (shrK B) {|Agent A, Nonce NA|}|}
           # evs2 ∈ yahalom"


   YM3:  "[| evs3 ∈ yahalom;  Key KAB ∉ used evs3;
             Gets Server {|Agent B, Nonce NB,
                           Crypt (shrK B) {|Agent A, Nonce NA|}|}
           ∈ set evs3 |]
          ==> Says Server A
           {|Nonce NB,
             Crypt (shrK A) {|Agent B, Key KAB, Nonce NA|},
             Crypt (shrK B) {|Agent A, Agent B, Key KAB, Nonce NB|}|}
             # evs3 ∈ yahalom"


   YM4:  "[| evs4 ∈ yahalom;
             Gets A {|Nonce NB, Crypt (shrK A) {|Agent B, Key K, Nonce NA|},
                      X|}  ∈ set evs4;
```

```
           Says A B {|Agent A, Nonce NA|} ∈ set evs4 |]
        ==> Says A B {|X, Crypt K (Nonce NB)|} # evs4 ∈ yahalom"


  Oops: "[| evso ∈ yahalom;
           Says Server A {|Nonce NB,
                           Crypt (shrK A) {|Agent B, Key K, Nonce NA|},
                           X|}  ∈ set evso |]
        ==> Notes Spy {|Nonce NA, Nonce NB, Key K|} # evso ∈ yahalom"


declare Says_imp_knows_Spy [THEN analz.Inj, dest]
declare parts.Body  [dest]
declare Fake_parts_insert_in_Un  [dest]
declare analz_into_parts [dest]
```

A "possibility property": there are traces that reach the end

```
lemma "Key K ∉ used []
      ==> ∃X NB. ∃evs ∈ yahalom.
            Says A B {|X, Crypt K (Nonce NB)|} ∈ set evs"
apply (intro exI bexI)
apply (rule_tac [2] yahalom.Nil
                  [THEN yahalom.YM1, THEN yahalom.Reception,
                   THEN yahalom.YM2, THEN yahalom.Reception,
                   THEN yahalom.YM3, THEN yahalom.Reception,
                   THEN yahalom.YM4])
apply (possibility, simp add: used_Cons)
done

lemma Gets_imp_Says:
     "[| Gets B X ∈ set evs; evs ∈ yahalom |] ==> ∃A. Says A B X ∈ set evs"
by (erule rev_mp, erule yahalom.induct, auto)
```

Must be proved separately for each protocol

```
lemma Gets_imp_knows_Spy:
     "[| Gets B X ∈ set evs; evs ∈ yahalom |]  ==> X ∈ knows Spy evs"
by (blast dest!: Gets_imp_Says Says_imp_knows_Spy)

declare Gets_imp_knows_Spy [THEN analz.Inj, dest]
```

12.1 Inductive Proofs

Result for reasoning about the encrypted portion of messages. Lets us treat YM4 using a similar argument as for the Fake case.

```
lemma YM4_analz_knows_Spy:
     "[| Gets A {|NB, Crypt (shrK A) Y, X|} ∈ set evs;  evs ∈ yahalom |]
      ==> X ∈ analz (knows Spy evs)"
by blast

lemmas YM4_parts_knows_Spy =
       YM4_analz_knows_Spy [THEN analz_into_parts, standard]
```

Spy never sees a good agent's shared key!

```
lemma Spy_see_shrK [simp]:
     "evs ∈ yahalom ==> (Key (shrK A) ∈ parts (knows Spy evs)) = (A ∈ bad)"
by (erule yahalom.induct, force,
    drule_tac [6] YM4_parts_knows_Spy, simp_all, blast+)

lemma Spy_analz_shrK [simp]:
     "evs ∈ yahalom ==> (Key (shrK A) ∈ analz (knows Spy evs)) = (A ∈ bad)"
by auto

lemma Spy_see_shrK_D [dest!]:
     "[|Key (shrK A) ∈ parts (knows Spy evs);  evs ∈ yahalom|] ==> A ∈ bad"
by (blast dest: Spy_see_shrK)
```

Nobody can have used non-existent keys! Needed to apply *analz_insert_Key*

```
lemma new_keys_not_used [simp]:
    "[|Key K ∉ used evs; K ∈ symKeys; evs ∈ yahalom|]
     ==> K ∉ keysFor (parts (spies evs))"
apply (erule rev_mp)
apply (erule yahalom.induct, force,
       frule_tac [6] YM4_parts_knows_Spy, simp_all)
```

Fake

```
apply (force dest!: keysFor_parts_insert)
```

YM3

```
apply blast
```

YM4

```
apply auto
apply (blast dest!: Gets_imp_knows_Spy [THEN parts.Inj])
done
```

Describes the form of K when the Server sends this message. Useful for Oops as well as main secrecy property.

```
lemma Says_Server_message_form:
     "[| Says Server A {|nb', Crypt (shrK A) {|Agent B, Key K, na|}, X|}
         ∈ set evs;  evs ∈ yahalom |]
      ==> K ∉ range shrK"
by (erule rev_mp, erule yahalom.induct, simp_all)
```

```
lemma analz_image_freshK [rule_format]:
 "evs ∈ yahalom ==>
   ∀K KK. KK <= - (range shrK) -->
          (Key K ∈ analz (Key‘KK Un (knows Spy evs))) =
          (K ∈ KK | Key K ∈ analz (knows Spy evs))"
apply (erule yahalom.induct)
apply (frule_tac [8] Says_Server_message_form)
apply (drule_tac [7] YM4_analz_knows_Spy, analz_freshK, spy_analz, blast)
```

done

```
lemma analz_insert_freshK:
     "[| evs ∈ yahalom;  KAB ∉ range shrK |] ==>
      (Key K ∈ analz (insert (Key KAB) (knows Spy evs))) =
      (K = KAB | Key K ∈ analz (knows Spy evs))"
by (simp only: analz_image_freshK analz_image_freshK_simps)
```

The Key K uniquely identifies the Server's message

```
lemma unique_session_keys:
     "[| Says Server A
           {|nb, Crypt (shrK A) {|Agent B, Key K, na|}, X|} ∈ set evs;
         Says Server A'
           {|nb', Crypt (shrK A') {|Agent B', Key K, na'|}, X'|} ∈ set evs;
         evs ∈ yahalom |]
      ==> A=A' & B=B' & na=na' & nb=nb'"
apply (erule rev_mp, erule rev_mp)
apply (erule yahalom.induct, simp_all)
```

YM3, by freshness

```
apply blast
done
```

12.2 Crucial Secrecy Property: Spy Does Not See Key *KAB*

```
lemma secrecy_lemma:
     "[| A ∉ bad;  B ∉ bad;  evs ∈ yahalom |]
      ==> Says Server A
            {|nb, Crypt (shrK A) {|Agent B, Key K, na|},
                  Crypt (shrK B) {|Agent A, Agent B, Key K, nb|}|}
           ∈ set evs -->
          Notes Spy {|na, nb, Key K|} ∉ set evs -->
          Key K ∉ analz (knows Spy evs)"
apply (erule yahalom.induct, force, frule_tac [7] Says_Server_message_form,
       drule_tac [6] YM4_analz_knows_Spy)
apply (simp_all add: pushes analz_insert_eq analz_insert_freshK, spy_analz)
apply (blast dest: unique_session_keys)+
done
```

Final version

```
lemma Spy_not_see_encrypted_key:
     "[| Says Server A
           {|nb, Crypt (shrK A) {|Agent B, Key K, na|},
                 Crypt (shrK B) {|Agent A, Agent B, Key K, nb|}|}
         ∈ set evs;
         Notes Spy {|na, nb, Key K|} ∉ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom |]
      ==> Key K ∉ analz (knows Spy evs)"
by (blast dest: secrecy_lemma Says_Server_message_form)
```

This form is an immediate consequence of the previous result. It is similar to the assertions established by other methods. It is equivalent to the previous result in that the Spy already has *analz* and *synth* at his disposal. However,

the conclusion *Key K ∉ knows Spy evs* appears not to be inductive: all the cases other than Fake are trivial, while Fake requires *Key K ∉ analz (knows Spy evs)*.

```
lemma Spy_not_know_encrypted_key:
     "[| Says Server A
           {|nb, Crypt (shrK A) {|Agent B, Key K, na|},
                 Crypt (shrK B) {|Agent A, Agent B, Key K, nb|}|}
        ∈ set evs;
        Notes Spy {|na, nb, Key K|} ∉ set evs;
        A ∉ bad;  B ∉ bad;  evs ∈ yahalom |]
      ==> Key K ∉ knows Spy evs"
by (blast dest: Spy_not_see_encrypted_key)
```

12.3 Security Guarantee for A upon receiving YM3

If the encrypted message appears then it originated with the Server. May now apply *Spy_not_see_encrypted_key*, subject to its conditions.

```
lemma A_trusts_YM3:
     "[| Crypt (shrK A) {|Agent B, Key K, na|} ∈ parts (knows Spy evs);
         A ∉ bad;  evs ∈ yahalom |]
      ==> ∃nb. Says Server A
                    {|nb, Crypt (shrK A) {|Agent B, Key K, na|},
                          Crypt (shrK B) {|Agent A, Agent B, Key K, nb|}|}
                  ∈ set evs"
apply (erule rev_mp)
apply (erule yahalom.induct, force,
       frule_tac [6] YM4_parts_knows_Spy, simp_all)
```

Fake, YM3

```
apply blast+
done
```

The obvious combination of *A_trusts_YM3* with *Spy_not_see_encrypted_key*

```
theorem A_gets_good_key:
     "[| Crypt (shrK A) {|Agent B, Key K, na|} ∈ parts (knows Spy evs);
         ∀nb. Notes Spy {|na, nb, Key K|} ∉ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom |]
      ==> Key K ∉ analz (knows Spy evs)"
by (blast dest!: A_trusts_YM3 Spy_not_see_encrypted_key)
```

12.4 Security Guarantee for B upon receiving YM4

B knows, by the first part of A's message, that the Server distributed the key for A and B, and has associated it with NB.

```
lemma B_trusts_YM4_shrK:
     "[| Crypt (shrK B) {|Agent A, Agent B, Key K, Nonce NB|}
           ∈ parts (knows Spy evs);
         B ∉ bad;  evs ∈ yahalom |]
  ==> ∃NA. Says Server A
              {|Nonce NB,
                Crypt (shrK A) {|Agent B, Key K, Nonce NA|},
                Crypt (shrK B) {|Agent A, Agent B, Key K, Nonce NB|}|}
```

```
            ∈ set evs"
apply (erule rev_mp)
apply (erule yahalom.induct, force,
       frule_tac [6] YM4_parts_knows_Spy, simp_all)
```

Fake, YM3

```
apply blast+
done
```

With this protocol variant, we don't need the 2nd part of YM4 at all: Nonce NB is available in the first part.

What can B deduce from receipt of YM4? Stronger and simpler than Yahalom because we do not have to show that NB is secret.

```
lemma B_trusts_YM4:
     "[| Gets B {|Crypt (shrK B) {|Agent A, Agent B, Key K, Nonce NB|},  X|}
           ∈ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom |]
  ==> ∃NA. Says Server A
             {|Nonce NB,
               Crypt (shrK A) {|Agent B, Key K, Nonce NA|},
               Crypt (shrK B) {|Agent A, Agent B, Key K, Nonce NB|}|}
            ∈ set evs"
by (blast dest!: B_trusts_YM4_shrK)
```

The obvious combination of *B_trusts_YM4* with *Spy_not_see_encrypted_key*

```
theorem B_gets_good_key:
     "[| Gets B {|Crypt (shrK B) {|Agent A, Agent B, Key K, Nonce NB|}, X|}
           ∈ set evs;
         ∀na. Notes Spy {|na, Nonce NB, Key K|} ∉ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom |]
      ==> Key K ∉ analz (knows Spy evs)"
by (blast dest!: B_trusts_YM4 Spy_not_see_encrypted_key)
```

12.5 Authenticating B to A

The encryption in message YM2 tells us it cannot be faked.

```
lemma B_Said_YM2:
     "[| Crypt (shrK B) {|Agent A, Nonce NA|} ∈ parts (knows Spy evs);
         B ∉ bad;  evs ∈ yahalom |]
      ==> ∃NB. Says B Server {|Agent B, Nonce NB,
                               Crypt (shrK B) {|Agent A, Nonce NA|}|}
                     ∈ set evs"
apply (erule rev_mp)
apply (erule yahalom.induct, force,
       frule_tac [6] YM4_parts_knows_Spy, simp_all)
```

Fake, YM2

```
apply blast+
done
```

If the server sends YM3 then B sent YM2, perhaps with a different NB

```
lemma YM3_auth_B_to_A_lemma:
     "[| Says Server A {|nb, Crypt (shrK A) {|Agent B, Key K, Nonce NA|}, X|}
           ∈ set evs;
         B ∉ bad;  evs ∈ yahalom |]
      ==> ∃nb'. Says B Server {|Agent B, nb',
                                 Crypt (shrK B) {|Agent A, Nonce NA|}|}
                     ∈ set evs"
apply (erule rev_mp)
apply (erule yahalom.induct, simp_all)
```

Fake, YM2, YM3

```
apply (blast dest!: B_Said_YM2)+
done
```

If A receives YM3 then B has used nonce NA (and therefore is alive)

```
theorem YM3_auth_B_to_A:
     "[| Gets A {|nb, Crypt (shrK A) {|Agent B, Key K, Nonce NA|}, X|}
           ∈ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom |]
 ==> ∃nb'. Says B Server
                 {|Agent B, nb', Crypt (shrK B) {|Agent A, Nonce NA|}|}
               ∈ set evs"
by (blast dest!: A_trusts_YM3 YM3_auth_B_to_A_lemma)
```

12.6 Authenticating A to B

using the certificate `Crypt K (Nonce NB)`

Assuming the session key is secure, if both certificates are present then A has said NB. We can't be sure about the rest of A's message, but only NB matters for freshness. Note that `Key K ∉ analz (knows Spy evs)` must be the FIRST antecedent of the induction formula.

This lemma allows a use of `unique_session_keys` in the next proof, which otherwise is extremely slow.

```
lemma secure_unique_session_keys:
     "[| Crypt (shrK A) {|Agent B, Key K, na|} ∈ analz (spies evs);
         Crypt (shrK A') {|Agent B', Key K, na'|} ∈ analz (spies evs);
         Key K ∉ analz (knows Spy evs);  evs ∈ yahalom |]
     ==> A=A' & B=B'"
by (blast dest!: A_trusts_YM3 dest: unique_session_keys Crypt_Spy_analz_bad)


lemma Auth_A_to_B_lemma [rule_format]:
     "evs ∈ yahalom
      ==> Key K ∉ analz (knows Spy evs) -->
          K ∈ symKeys -->
          Crypt K (Nonce NB) ∈ parts (knows Spy evs) -->
          Crypt (shrK B) {|Agent A, Agent B, Key K, Nonce NB|}
            ∈ parts (knows Spy evs) -->
          B ∉ bad -->
          (∃X. Says A B {|X, Crypt K (Nonce NB)|} ∈ set evs)"
apply (erule yahalom.induct, force,
```

```
      frule_tac [6] YM4_parts_knows_Spy)
apply (analz_mono_contra, simp_all)
```

Fake

apply *blast*

YM3: by *new_keys_not_used*, the message *Crypt K (Nonce NB)* could not exist

apply *(force dest!: Crypt_imp_keysFor)*

YM4: was *Crypt K (Nonce NB)* the very last message? If so, apply unicity of session keys; if not, use the induction hypothesis

apply *(blast dest!: B_trusts_YM4_shrK dest: secure_unique_session_keys)*
done

If B receives YM4 then A has used nonce NB (and therefore is alive). Moreover, A associates K with NB (thus is talking about the same run). Other premises guarantee secrecy of K.

```
theorem YM4_imp_A_Said_YM3 [rule_format]:
     "[| Gets B {|Crypt (shrK B) {|Agent A, Agent B, Key K, Nonce NB|},
                  Crypt K (Nonce NB)|} ∈ set evs;
         (∀NA. Notes Spy {|Nonce NA, Nonce NB, Key K|} ∉ set evs);
         K ∈ symKeys;  A ∉ bad;  B ∉ bad;  evs ∈ yahalom |]
      ==> ∃X. Says A B {|X, Crypt K (Nonce NB)|} ∈ set evs"
by (blast intro: Auth_A_to_B_lemma
          dest: Spy_not_see_encrypted_key B_trusts_YM4_shrK)
```

end

13 The Yahalom Protocol: A Flawed Version

theory *Yahalom_Bad* **imports** *Public* **begin**

Demonstrates of why Oops is necessary. This protocol can be attacked because it doesn't keep NB secret, but without Oops it can be "verified" anyway. The issues are discussed in lcp's LICS 2000 invited lecture.

```
consts  yahalom   :: "event list set"
inductive "yahalom"
  intros

   Nil:  "[] ∈ yahalom"


   Fake: "[| evsf ∈ yahalom;  X ∈ synth (analz (knows Spy evsf)) |]
          ==> Says Spy B X  # evsf ∈ yahalom"


   Reception: "[| evsr ∈ yahalom;  Says A B X ∈ set evsr |]
               ==> Gets B X # evsr ∈ yahalom"


   YM1:  "[| evs1 ∈ yahalom;  Nonce NA ∉ used evs1 |]
```

```
         ==> Says A B {|Agent A, Nonce NA|} # evs1 ∈ yahalom"


   YM2:  "[| evs2 ∈ yahalom;  Nonce NB ∉ used evs2;
             Gets B {|Agent A, Nonce NA|} ∈ set evs2 |]
          ==> Says B Server
                 {|Agent B, Nonce NB, Crypt (shrK B) {|Agent A, Nonce NA|}|}
               # evs2 ∈ yahalom"


   YM3:  "[| evs3 ∈ yahalom;  Key KAB ∉ used evs3;  KAB ∈ symKeys;
             Gets Server
                  {|Agent B, Nonce NB, Crypt (shrK B) {|Agent A, Nonce NA|}|}
               ∈ set evs3 |]
          ==> Says Server A
                   {|Crypt (shrK A) {|Agent B, Key KAB, Nonce NA, Nonce NB|},
                     Crypt (shrK B) {|Agent A, Key KAB|}|}
               # evs3 ∈ yahalom"


   YM4:  "[| evs4 ∈ yahalom;  A ≠ Server;  K ∈ symKeys;
             Gets A {|Crypt(shrK A) {|Agent B, Key K, Nonce NA, Nonce NB|},
X|}
                ∈ set evs4;
             Says A B {|Agent A, Nonce NA|} ∈ set evs4 |]
          ==> Says A B {|X, Crypt K (Nonce NB)|} # evs4 ∈ yahalom"


declare Says_imp_knows_Spy [THEN analz.Inj, dest]
declare parts.Body  [dest]
declare Fake_parts_insert_in_Un  [dest]
declare analz_into_parts [dest]
```

A "possibility property": there are traces that reach the end

```
lemma "[| A ≠ Server; Key K ∉ used []; K ∈ symKeys |]
       ==> ∃X NB. ∃evs ∈ yahalom.
             Says A B {|X, Crypt K (Nonce NB)|} ∈ set evs"
apply (intro exI bexI)
apply (rule_tac [2] yahalom.Nil
                    [THEN yahalom.YM1, THEN yahalom.Reception,
                     THEN yahalom.YM2, THEN yahalom.Reception,
                     THEN yahalom.YM3, THEN yahalom.Reception,
                     THEN yahalom.YM4])
apply (possibility, simp add: used_Cons)
done
```

13.1 Regularity Lemmas for Yahalom

```
lemma Gets_imp_Says:
     "[| Gets B X ∈ set evs; evs ∈ yahalom |] ==> ∃A. Says A B X ∈ set evs"
by (erule rev_mp, erule yahalom.induct, auto)


lemma Gets_imp_knows_Spy:
```

```
    "[| Gets B X ∈ set evs; evs ∈ yahalom |]  ==> X ∈ knows Spy evs"
by (blast dest!: Gets_imp_Says Says_imp_knows_Spy)

declare Gets_imp_knows_Spy [THEN analz.Inj, dest]
```

13.2 For reasoning about the encrypted portion of messages

Lets us treat YM4 using a similar argument as for the Fake case.

```
lemma YM4_analz_knows_Spy:
     "[| Gets A {|Crypt (shrK A) Y, X|} ∈ set evs;  evs ∈ yahalom |]
      ==> X ∈ analz (knows Spy evs)"
by blast

lemmas YM4_parts_knows_Spy =
       YM4_analz_knows_Spy [THEN analz_into_parts, standard]
```

Theorems of the form `X ∉ parts (knows Spy evs)` imply that NOBODY sends messages containing X!

Spy never sees a good agent's shared key!

```
lemma Spy_see_shrK [simp]:
     "evs ∈ yahalom ==> (Key (shrK A) ∈ parts (knows Spy evs)) = (A ∈ bad)"
apply (erule yahalom.induct, force,
       drule_tac [6] YM4_parts_knows_Spy, simp_all, blast+)
done

lemma Spy_analz_shrK [simp]:
     "evs ∈ yahalom ==> (Key (shrK A) ∈ analz (knows Spy evs)) = (A ∈ bad)"
by auto

lemma Spy_see_shrK_D [dest!]:
     "[|Key (shrK A) ∈ parts (knows Spy evs);  evs ∈ yahalom|] ==> A ∈ bad"
by (blast dest: Spy_see_shrK)
```

Nobody can have used non-existent keys! Needed to apply `analz_insert_Key`

```
lemma new_keys_not_used [simp]:
    "[|Key K ∉ used evs; K ∈ symKeys; evs ∈ yahalom|]
     ==> K ∉ keysFor (parts (spies evs))"
apply (erule rev_mp)
apply (erule yahalom.induct, force,
       frule_tac [6] YM4_parts_knows_Spy, simp_all)
```

Fake

```
apply (force dest!: keysFor_parts_insert, auto)
done
```

13.3 Secrecy Theorems

13.4 Session keys are not used to encrypt other session keys

```
lemma analz_image_freshK [rule_format]:
```

```
 "evs ∈ yahalom ==>
   ∀K KK. KK <= - (range shrK) -->
          (Key K ∈ analz (Key‘KK Un (knows Spy evs))) =
          (K ∈ KK | Key K ∈ analz (knows Spy evs))"
by (erule yahalom.induct,
    drule_tac [7] YM4_analz_knows_Spy, analz_freshK, spy_analz, blast)

lemma analz_insert_freshK:
     "[| evs ∈ yahalom;  KAB ∉ range shrK |] ==>
      (Key K ∈ analz (insert (Key KAB) (knows Spy evs))) =
      (K = KAB | Key K ∈ analz (knows Spy evs))"
by (simp only: analz_image_freshK analz_image_freshK_simps)
```

The Key K uniquely identifies the Server's message.

```
lemma unique_session_keys:
     "[| Says Server A
          {|Crypt (shrK A) {|Agent B, Key K, na, nb|}, X|} ∈ set evs;
         Says Server A'
          {|Crypt (shrK A') {|Agent B', Key K, na', nb'|}, X'|} ∈ set evs;
         evs ∈ yahalom |]
      ==> A=A' & B=B' & na=na' & nb=nb'"
apply (erule rev_mp, erule rev_mp)
apply (erule yahalom.induct, simp_all)
```

YM3, by freshness, and YM4

```
apply blast+
done
```

Crucial secrecy property: Spy does not see the keys sent in msg YM3

```
lemma secrecy_lemma:
     "[| A ∉ bad;  B ∉ bad;  evs ∈ yahalom |]
      ==> Says Server A
            {|Crypt (shrK A) {|Agent B, Key K, na, nb|},
              Crypt (shrK B) {|Agent A, Key K|}|}
           ∈ set evs -->
          Key K ∉ analz (knows Spy evs)"
apply (erule yahalom.induct, force, drule_tac [6] YM4_analz_knows_Spy)
apply (simp_all add: pushes analz_insert_eq analz_insert_freshK, spy_analz)

apply (blast dest: unique_session_keys)
done
```

Final version

```
lemma Spy_not_see_encrypted_key:
     "[| Says Server A
           {|Crypt (shrK A) {|Agent B, Key K, na, nb|},
             Crypt (shrK B) {|Agent A, Key K|}|}
          ∈ set evs;
        A ∉ bad;  B ∉ bad;  evs ∈ yahalom |]
      ==> Key K ∉ analz (knows Spy evs)"
by (blast dest: secrecy_lemma)
```

13.5 Security Guarantee for A upon receiving YM3

If the encrypted message appears then it originated with the Server

```
lemma A_trusts_YM3:
     "[| Crypt (shrK A) {|Agent B, Key K, na, nb|} ∈ parts (knows Spy evs);
         A ∉ bad;  evs ∈ yahalom |]
       ==> Says Server A
            {|Crypt (shrK A) {|Agent B, Key K, na, nb|},
              Crypt (shrK B) {|Agent A, Key K|}|}
           ∈ set evs"
apply (erule rev_mp)
apply (erule yahalom.induct, force,
       frule_tac [6] YM4_parts_knows_Spy, simp_all)
```

Fake, YM3

```
apply blast+
done
```

The obvious combination of *A_trusts_YM3* with *Spy_not_see_encrypted_key*

```
lemma A_gets_good_key:
     "[| Crypt (shrK A) {|Agent B, Key K, na, nb|} ∈ parts (knows Spy evs);
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom |]
      ==> Key K ∉ analz (knows Spy evs)"
by (blast dest!: A_trusts_YM3 Spy_not_see_encrypted_key)
```

13.6 Security Guarantees for B upon receiving YM4

B knows, by the first part of A's message, that the Server distributed the key for A and B. But this part says nothing about nonces.

```
lemma B_trusts_YM4_shrK:
     "[| Crypt (shrK B) {|Agent A, Key K|} ∈ parts (knows Spy evs);
         B ∉ bad;  evs ∈ yahalom |]
      ==> ∃NA NB. Says Server A
                     {|Crypt (shrK A) {|Agent B, Key K, Nonce NA, Nonce NB|},
                       Crypt (shrK B) {|Agent A, Key K|}|}
                    ∈ set evs"
apply (erule rev_mp)
apply (erule yahalom.induct, force,
       frule_tac [6] YM4_parts_knows_Spy, simp_all)
```

Fake, YM3

```
apply blast+
done
```

13.7 The Flaw in the Model

Up to now, the reasoning is similar to standard Yahalom. Now the doubtful reasoning occurs. We should not be assuming that an unknown key is secure, but the model allows us to: there is no Oops rule to let session keys become compromised.

B knows, by the second part of A's message, that the Server distributed the key quoting nonce NB. This part says nothing about agent names. Secrecy of K is assumed; the valid Yahalom proof uses (and later proves) the secrecy of NB.

```
lemma B_trusts_YM4_newK [rule_format]:
     "[|Key K ∉ analz (knows Spy evs);  evs ∈ yahalom|]
      ==> Crypt K (Nonce NB) ∈ parts (knows Spy evs) -->
          (∃A B NA. Says Server A
                      {|Crypt (shrK A) {|Agent B, Key K,
                                Nonce NA, Nonce NB|},
                        Crypt (shrK B) {|Agent A, Key K|}|}
                    ∈ set evs)"
apply (erule rev_mp)
apply (erule yahalom.induct, force,
       frule_tac [6] YM4_parts_knows_Spy)
apply (analz_mono_contra, simp_all)
```

Fake

```
apply blast
```

YM3

```
apply blast
```

A is uncompromised because NB is secure A's certificate guarantees the existence of the Server message

```
apply (blast dest!: Gets_imp_Says Crypt_Spy_analz_bad
             dest: Says_imp_spies
                   parts.Inj [THEN parts.Fst, THEN A_trusts_YM3])
done
```

B's session key guarantee from YM4. The two certificates contribute to a single conclusion about the Server's message.

```
lemma B_trusts_YM4:
     "[| Gets B {|Crypt (shrK B) {|Agent A, Key K|},
                  Crypt K (Nonce NB)|} ∈ set evs;
         Says B Server
           {|Agent B, Nonce NB, Crypt (shrK B) {|Agent A, Nonce NA|}|}
           ∈ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom |]
       ==> ∃na nb. Says Server A
                   {|Crypt (shrK A) {|Agent B, Key K, na, nb|},
                     Crypt (shrK B) {|Agent A, Key K|}|}
             ∈ set evs"
by (blast dest: B_trusts_YM4_newK B_trusts_YM4_shrK Spy_not_see_encrypted_key
                unique_session_keys)
```

The obvious combination of *B_trusts_YM4* with *Spy_not_see_encrypted_key*

```
lemma B_gets_good_key:
     "[| Gets B {|Crypt (shrK B) {|Agent A, Key K|},
                  Crypt K (Nonce NB)|} ∈ set evs;
         Says B Server
           {|Agent B, Nonce NB, Crypt (shrK B) {|Agent A, Nonce NA|}|}
           ∈ set evs;
```

```
        A ∉ bad;  B ∉ bad;  evs ∈ yahalom |]
     ==> Key K ∉ analz (knows Spy evs)"
by (blast dest!: B_trusts_YM4 Spy_not_see_encrypted_key)
```

Assuming the session key is secure, if both certificates are present then A has said NB. We can't be sure about the rest of A's message, but only NB matters for freshness.

```
lemma A_Said_YM3_lemma [rule_format]:
     "evs ∈ yahalom
      ==> Key K ∉ analz (knows Spy evs) -->
          Crypt K (Nonce NB) ∈ parts (knows Spy evs) -->
          Crypt (shrK B) {|Agent A, Key K|} ∈ parts (knows Spy evs) -->
          B ∉ bad -->
          (∃X. Says A B {|X, Crypt K (Nonce NB)|} ∈ set evs)"
apply (erule yahalom.induct, force,
       frule_tac [6] YM4_parts_knows_Spy)
apply (analz_mono_contra, simp_all)
```

Fake

```
apply blast
```

YM3: by *new_keys_not_used*, the message *Crypt K (Nonce NB)* could not exist

```
apply (force dest!: Crypt_imp_keysFor)
```

YM4: was *Crypt K (Nonce NB)* the very last message? If not, use the induction hypothesis

```
apply (simp add: ex_disj_distrib)
```

yes: apply unicity of session keys

```
apply (blast dest!: Gets_imp_Says A_trusts_YM3 B_trusts_YM4_shrK
                    Crypt_Spy_analz_bad
             dest: Says_imp_knows_Spy [THEN parts.Inj] unique_session_keys)
done
```

If B receives YM4 then A has used nonce NB (and therefore is alive). Moreover, A associates K with NB (thus is talking about the same run). Other premises guarantee secrecy of K.

```
lemma YM4_imp_A_Said_YM3 [rule_format]:
     "[| Gets B {|Crypt (shrK B) {|Agent A, Key K|},
                  Crypt K (Nonce NB)|} ∈ set evs;
         Says B Server
           {|Agent B, Nonce NB, Crypt (shrK B) {|Agent A, Nonce NA|}|}
           ∈ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ yahalom |]
      ==> ∃X. Says A B {|X, Crypt K (Nonce NB)|} ∈ set evs"
by (blast intro!: A_Said_YM3_lemma
          dest: Spy_not_see_encrypted_key B_trusts_YM4 Gets_imp_Says)
```

end

theory *ZhouGollmann* **imports** *Public* **begin**

syntax
TTP :: agent

translations
"TTP" == " Server "

syntax
f_sub :: nat
f_nro :: nat
f_nrr :: nat
f_con :: nat

translations
"f_sub" == " 5 "
"f_nro" == " 2 "
"f_nrr" == " 3 "
"f_con" == " 4 "

constdefs
broken :: "agent set"
— the compromised honest agents; TTP is included as it's not allowed to use the protocol
"broken == bad - {Spy}"

declare *broken_def [simp]*

consts *zg :: "event list set"*

inductive *zg*
intros

Nil: "[] ∈ zg"

Fake: "[| evsf ∈ zg; X ∈ synth (analz (spies evsf)) |]
==> Says Spy B X # evsf ∈ zg"

Reception: "[| evsr ∈ zg; Says A B X ∈ set evsr |] ==> Gets B X # evsr ∈ zg"

ZG1: "[| evs1 ∈ zg; Nonce L ∉ used evs1; C = Crypt K (Number m);
K ∈ symKeys;
NRO = Crypt (priK A) {|Number f_nro, Agent B, Nonce L, C|}|]
==> Says A B {|Number f_nro, Agent B, Nonce L, C, NRO|} # evs1 ∈ zg"

ZG2: "[| evs2 ∈ zg;
Gets B {|Number f_nro, Agent B, Nonce L, C, NRO|} ∈ set evs2;
NRO = Crypt (priK A) {|Number f_nro, Agent B, Nonce L, C|};
NRR = Crypt (priK B) {|Number f_nrr, Agent A, Nonce L, C|}|]
==> Says B A {|Number f_nrr, Agent A, Nonce L, NRR|} # evs2 ∈ zg"

```
  ZG3: "[| evs3 ∈ zg; C = Crypt K M; K ∈ symKeys;
           Says A B {|Number f_nro, Agent B, Nonce L, C, NRO|} ∈ set evs3;
           Gets A {|Number f_nrr, Agent A, Nonce L, NRR|} ∈ set evs3;
           NRR = Crypt (priK B) {|Number f_nrr, Agent A, Nonce L, C|};
           sub_K = Crypt (priK A) {|Number f_sub, Agent B, Nonce L, Key K|}|]
        ==> Says A TTP {|Number f_sub, Agent B, Nonce L, Key K, sub_K|}
              # evs3 ∈ zg"


  ZG4: "[| evs4 ∈ zg; K ∈ symKeys;
           Gets TTP {|Number f_sub, Agent B, Nonce L, Key K, sub_K|}
             ∈ set evs4;
           sub_K = Crypt (priK A) {|Number f_sub, Agent B, Nonce L, Key K|};
           con_K = Crypt (priK TTP) {|Number f_con, Agent A, Agent B,
                                      Nonce L, Key K|}|]
        ==> Says TTP Spy con_K
           #
           Notes TTP {|Number f_con, Agent A, Agent B, Nonce L, Key K, con_K|}
           # evs4 ∈ zg"


declare Says_imp_knows_Spy [THEN analz.Inj, dest]
declare Fake_parts_insert_in_Un  [dest]
declare analz_into_parts [dest]

declare symKey_neq_priEK [simp]
declare symKey_neq_priEK [THEN not_sym, simp]
```

A "possibility property": there are traces that reach the end

```
lemma "[|A ≠ B; TTP ≠ A; TTP ≠ B; K ∈ symKeys|] ==>
     ∃L. ∃evs ∈ zg.
           Notes TTP {|Number f_con, Agent A, Agent B, Nonce L, Key K,
               Crypt (priK TTP) {|Number f_con, Agent A, Agent B, Nonce L,
Key K|} |}
               ∈ set evs"
apply (intro exI bexI)
apply (rule_tac [2] zg.Nil
                    [THEN zg.ZG1, THEN zg.Reception [of _ A B],
                     THEN zg.ZG2, THEN zg.Reception [of _ B A],
                     THEN zg.ZG3, THEN zg.Reception [of _ A TTP],
                     THEN zg.ZG4])
apply (possibility, auto)
done
```

13.8 Basic Lemmas

```
lemma Gets_imp_Says:
     "[| Gets B X ∈ set evs; evs ∈ zg |] ==> ∃A. Says A B X ∈ set evs"
apply (erule rev_mp)
apply (erule zg.induct, auto)
done
```

```
lemma Gets_imp_knows_Spy:
     "[| Gets B X ∈ set evs; evs ∈ zg |]  ==> X ∈ spies evs"
by (blast dest!: Gets_imp_Says Says_imp_knows_Spy)
```

Lets us replace proofs about *used evs* by simpler proofs about *parts (knows Spy evs)*.

```
lemma Crypt_used_imp_spies:
     "[| Crypt K X ∈ used evs; evs ∈ zg |]
      ==> Crypt K X ∈ parts (spies evs)"
apply (erule rev_mp)
apply (erule zg.induct)
apply (simp_all add: parts_insert_knows_A)
done

lemma Notes_TTP_imp_Gets:
     "[|Notes TTP {|Number f_con, Agent A, Agent B, Nonce L, Key K, con_K |}
           ∈ set evs;
        sub_K = Crypt (priK A) {|Number f_sub, Agent B, Nonce L, Key K|};
        evs ∈ zg|]
    ==> Gets TTP {|Number f_sub, Agent B, Nonce L, Key K, sub_K|} ∈ set evs"
apply (erule rev_mp)
apply (erule zg.induct, auto)
done
```

For reasoning about C, which is encrypted in message ZG2

```
lemma ZG2_msg_in_parts_spies:
     "[|Gets B {|F, B', L, C, X|} ∈ set evs; evs ∈ zg|]
      ==> C ∈ parts (spies evs)"
by (blast dest: Gets_imp_Says)


lemma Spy_see_priK [simp]:
     "evs ∈ zg ==> (Key (priK A) ∈ parts (spies evs)) = (A ∈ bad)"
apply (erule zg.induct)
apply (frule_tac [5] ZG2_msg_in_parts_spies, auto)
done
```

So that blast can use it too

```
declare  Spy_see_priK [THEN [2] rev_iffD1, dest!]

lemma Spy_analz_priK [simp]:
     "evs ∈ zg ==> (Key (priK A) ∈ analz (spies evs)) = (A ∈ bad)"
by auto
```

13.9 About NRO: Validity for B

Below we prove that if *NRO* exists then *A* definitely sent it, provided *A* is not broken.

Strong conclusion for a good agent

```
lemma NRO_validity_good:
     "[|NRO = Crypt (priK A) {|Number f_nro, Agent B, Nonce L, C|};
```

```
        NRO ∈ parts (spies evs);
        A ∉ bad;  evs ∈ zg |]
     ==> Says A B {|Number f_nro, Agent B, Nonce L, C, NRO|} ∈ set evs"
apply clarify
apply (erule rev_mp)
apply (erule zg.induct)
apply (frule_tac [5] ZG2_msg_in_parts_spies, auto)
done

lemma NRO_sender:
     "[|Says A' B {|n, b, l, C, Crypt (priK A) X|} ∈ set evs; evs ∈ zg|]
    ==> A' ∈ {A,Spy}"
apply (erule rev_mp)
apply (erule zg.induct, simp_all)
done
```

Holds also for `A = Spy`!

```
theorem NRO_validity:
     "[|Gets B {|Number f_nro, Agent B, Nonce L, C, NRO|} ∈ set evs;
        NRO = Crypt (priK A) {|Number f_nro, Agent B, Nonce L, C|};
        A ∉ broken;  evs ∈ zg |]
     ==> Says A B {|Number f_nro, Agent B, Nonce L, C, NRO|} ∈ set evs"
apply (drule Gets_imp_Says, assumption)
apply clarify
apply (frule NRO_sender, auto)
```

We are left with the case where the sender is `Spy` and not equal to `A`, because `A ∉ bad`. Thus theorem `NRO_validity_good` applies.

```
apply (blast dest: NRO_validity_good [OF refl])
done
```

13.10 About NRR: Validity for `A`

Below we prove that if `NRR` exists then `B` definitely sent it, provided `B` is not broken.

Strong conclusion for a good agent

```
lemma NRR_validity_good:
     "[|NRR = Crypt (priK B) {|Number f_nrr, Agent A, Nonce L, C|};
        NRR ∈ parts (spies evs);
        B ∉ bad;  evs ∈ zg |]
     ==> Says B A {|Number f_nrr, Agent A, Nonce L, NRR|} ∈ set evs"
apply clarify
apply (erule rev_mp)
apply (erule zg.induct)
apply (frule_tac [5] ZG2_msg_in_parts_spies, auto)
done

lemma NRR_sender:
     "[|Says B' A {|n, a, l, Crypt (priK B) X|} ∈ set evs; evs ∈ zg|]
    ==> B' ∈ {B,Spy}"
apply (erule rev_mp)
apply (erule zg.induct, simp_all)
```

done

Holds also for *B = Spy*!

theorem *NRR_validity:*
```
     "[|Says B' A {|Number f_nrr, Agent A, Nonce L, NRR|} ∈ set evs;
        NRR = Crypt (priK B) {|Number f_nrr, Agent A, Nonce L, C|};
        B ∉ broken; evs ∈ zg|]
    ==> Says B A {|Number f_nrr, Agent A, Nonce L, NRR|} ∈ set evs"
```
apply *clarify*
apply *(frule NRR_sender, auto)*

We are left with the case where *B' = Spy* and *B' ≠ B*, i.e. *B ∉ bad*, when we can apply *NRR_validity_good*.

apply *(blast dest: NRR_validity_good [OF refl])*
done

13.11 Proofs About *sub_K*

Below we prove that if *sub_K* exists then *A* definitely sent it, provided *A* is not broken.

Strong conclusion for a good agent

lemma *sub_K_validity_good:*
```
     "[|sub_K = Crypt (priK A) {|Number f_sub, Agent B, Nonce L, Key K|};
        sub_K ∈ parts (spies evs);
        A ∉ bad;  evs ∈ zg |]
     ==> Says A TTP {|Number f_sub, Agent B, Nonce L, Key K, sub_K|} ∈ set
evs"
```
apply *clarify*
apply *(erule rev_mp)*
apply *(erule zg.induct)*
apply *(frule_tac [5] ZG2_msg_in_parts_spies, simp_all)*

Fake

apply *(blast dest!: Fake_parts_sing_imp_Un)*
done

lemma *sub_K_sender:*
```
     "[|Says A' TTP {|n, b, l, k, Crypt (priK A) X|} ∈ set evs;  evs ∈ zg|]
    ==> A' ∈ {A,Spy}"
```
apply *(erule rev_mp)*
apply *(erule zg.induct, simp_all)*
done

Holds also for *A = Spy*!

theorem *sub_K_validity:*
```
     "[|Gets TTP {|Number f_sub, Agent B, Nonce L, Key K, sub_K|} ∈ set evs;
        sub_K = Crypt (priK A) {|Number f_sub, Agent B, Nonce L, Key K|};
        A ∉ broken;  evs ∈ zg |]
     ==> Says A TTP {|Number f_sub, Agent B, Nonce L, Key K, sub_K|} ∈ set
evs"
```
apply *(drule Gets_imp_Says, assumption)*

```
apply clarify
apply (frule sub_K_sender, auto)
```

We are left with the case where the sender is *Spy* and not equal to *A*, because *A* $\notin$ *bad*. Thus theorem *sub_K_validity_good* applies.

```
apply (blast dest: sub_K_validity_good [OF refl])
done
```

13.12 Proofs About *con_K*

Below we prove that if *con_K* exists, then *TTP* has it, and therefore *A* and *B*) can get it too. Moreover, we know that *A* sent *sub_K*

```
lemma con_K_validity:
     "[|con_K ∈ used evs;
        con_K = Crypt (priK TTP)
                  {|Number f_con, Agent A, Agent B, Nonce L, Key K|};
        evs ∈ zg |]
    ==> Notes TTP {|Number f_con, Agent A, Agent B, Nonce L, Key K, con_K|}
          ∈ set evs"
apply clarify
apply (erule rev_mp)
apply (erule zg.induct)
apply (frule_tac [5] ZG2_msg_in_parts_spies, simp_all)
```

Fake

```
apply (blast dest!: Fake_parts_sing_imp_Un)
```

ZG2

```
apply (blast dest: parts_cut)
done
```

If *TTP* holds *con_K* then *A* sent *sub_K*. We assume that *A* is not broken. Importantly, nothing needs to be assumed about the form of *con_K*!

```
lemma Notes_TTP_imp_Says_A:
     "[|Notes TTP {|Number f_con, Agent A, Agent B, Nonce L, Key K, con_K|}
           ∈ set evs;
        sub_K = Crypt (priK A) {|Number f_sub, Agent B, Nonce L, Key K|};
        A ∉ broken; evs ∈ zg|]
     ==> Says A TTP {|Number f_sub, Agent B, Nonce L, Key K, sub_K|} ∈ set
evs"
apply clarify
apply (erule rev_mp)
apply (erule zg.induct)
apply (frule_tac [5] ZG2_msg_in_parts_spies, simp_all)
```

ZG4

```
apply clarify
apply (rule sub_K_validity, auto)
done
```

If *con_K* exists, then *A* sent *sub_K*. We again assume that *A* is not broken.

```
theorem B_sub_K_validity:
```

```
     "[|con_K ∈ used evs;
        con_K = Crypt (priK TTP) {|Number f_con, Agent A, Agent B,
                                   Nonce L, Key K|};
        sub_K = Crypt (priK A) {|Number f_sub, Agent B, Nonce L, Key K|};
        A ∉ broken; evs ∈ zg|]
     ==> Says A TTP {|Number f_sub, Agent B, Nonce L, Key K, sub_K|} ∈ set
evs"
by (blast dest: con_K_validity Notes_TTP_imp_Says_A)
```

13.13 Proving fairness

Cannot prove that, if `B` has NRO, then `A` has her NRR. It would appear that `B` has a small advantage, though it is useless to win disputes: `B` needs to present `con_K` as well.

Strange: unicity of the label protects `A`?

```
lemma A_unicity:
     "[|NRO = Crypt (priK A) {|Number f_nro, Agent B, Nonce L, Crypt K M|};
        NRO ∈ parts (spies evs);
        Says A B {|Number f_nro, Agent B, Nonce L, Crypt K M', NRO'|}
          ∈ set evs;
        A ∉ bad; evs ∈ zg |]
     ==> M'=M"
apply clarify
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule zg.induct)
apply (frule_tac [5] ZG2_msg_in_parts_spies, auto)
```

ZG1: freshness

```
apply (blast dest: parts.Body)
done
```

Fairness lemma: if `sub_K` exists, then `A` holds NRR. Relies on unicity of labels.

```
lemma sub_K_implies_NRR:
     "[| NRO = Crypt (priK A) {|Number f_nro, Agent B, Nonce L, Crypt K M|};
         NRR = Crypt (priK B) {|Number f_nrr, Agent A, Nonce L, Crypt K M|};
         sub_K ∈ parts (spies evs);
         NRO ∈ parts (spies evs);
         sub_K = Crypt (priK A) {|Number f_sub, Agent B, Nonce L, Key K|};
         A ∉ bad;  evs ∈ zg |]
     ==> Gets A {|Number f_nrr, Agent A, Nonce L, NRR|} ∈ set evs"
apply clarify
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule zg.induct)
apply (frule_tac [5] ZG2_msg_in_parts_spies, simp_all)
```

Fake

```
apply blast
```

ZG1: freshness

```
apply (blast dest: parts.Body)
```

ZG3

```
apply (blast dest: A_unicity [OF refl])
done
```

```
lemma Crypt_used_imp_L_used:
     "[| Crypt (priK TTP) {|F, A, B, L, K|} ∈ used evs; evs ∈ zg |]
      ==> L ∈ used evs"
apply (erule rev_mp)
apply (erule zg.induct, auto)
```

Fake

```
apply (blast dest!: Fake_parts_sing_imp_Un)
```

ZG2: freshness

```
apply (blast dest: parts.Body)
done
```

Fairness for `A`: if `con_K` and `NRO` exist, then `A` holds NRR. `A` must be uncompromised, but there is no assumption about `B`.

```
theorem A_fairness_NRO:
     "[|con_K ∈ used evs;
        NRO ∈ parts (spies evs);
        con_K = Crypt (priK TTP)
                      {|Number f_con, Agent A, Agent B, Nonce L, Key K|};
        NRO = Crypt (priK A) {|Number f_nro, Agent B, Nonce L, Crypt K M|};
        NRR = Crypt (priK B) {|Number f_nrr, Agent A, Nonce L, Crypt K M|};
        A ∉ bad;  evs ∈ zg |]
    ==> Gets A {|Number f_nrr, Agent A, Nonce L, NRR|} ∈ set evs"
apply clarify
apply (erule rev_mp)
apply (erule rev_mp)
apply (erule zg.induct)
apply (frule_tac [5] ZG2_msg_in_parts_spies, simp_all)
```

Fake

```
   apply (simp add: parts_insert_knows_A)
   apply (blast dest: Fake_parts_sing_imp_Un)
```

ZG1

```
  apply (blast dest: Crypt_used_imp_L_used)
```

ZG2

```
 apply (blast dest: parts_cut)
```

ZG4

```
apply (blast intro: sub_K_implies_NRR [OF refl]
             dest: Gets_imp_knows_Spy [THEN parts.Inj])
done
```

Fairness for `B`: NRR exists at all, then `B` holds NRO. `B` must be uncompromised, but there is no assumption about `A`.

```
theorem B_fairness_NRR:
     "[|NRR ∈ used evs;
        NRR = Crypt (priK B) {|Number f_nrr, Agent A, Nonce L, C|};
        NRO = Crypt (priK A) {|Number f_nro, Agent B, Nonce L, C|};
        B ∉ bad; evs ∈ zg |]
    ==> Gets B {|Number f_nro, Agent B, Nonce L, C, NRO|} ∈ set evs"
apply clarify
apply (erule rev_mp)
apply (erule zg.induct)
apply (frule_tac [5] ZG2_msg_in_parts_spies, simp_all)
```

Fake

```
apply (blast dest!: Fake_parts_sing_imp_Un)
```

ZG2

```
apply (blast dest: parts_cut)
done
```

If *con_K* exists at all, then *B* can get it, by *con_K_validity*. Cannot conclude that also NRO is available to *B*, because if *A* were unfair, *A* could build message 3 without building message 1, which contains NRO.

end

14 Verifying the Needham-Schroeder Public-Key Protocol

```
theory NS_Public_Bad imports Public begin

consts  ns_public  :: "event list set"

inductive ns_public
  intros

   Nil:  "[] ∈ ns_public"


   Fake: "⟦evsf ∈ ns_public;  X ∈ synth (analz (spies evsf))⟧
          ⟹ Says Spy B X  # evsf ∈ ns_public"


   NS1:  "⟦evs1 ∈ ns_public;  Nonce NA ∉ used evs1⟧
          ⟹ Says A B (Crypt (pubEK B) {|Nonce NA, Agent A|})
                # evs1  ∈  ns_public"


   NS2:  "⟦evs2 ∈ ns_public;  Nonce NB ∉ used evs2;
           Says A' B (Crypt (pubEK B) {|Nonce NA, Agent A|}) ∈ set evs2⟧
          ⟹ Says B A (Crypt (pubEK A) {|Nonce NA, Nonce NB|})
                # evs2  ∈  ns_public"
```

```
  NS3:  "⟦evs3 ∈ ns_public;
          Says A  B (Crypt (pubEK B) {|Nonce NA, Agent A|}) ∈ set evs3;
          Says B' A (Crypt (pubEK A) {|Nonce NA, Nonce NB|}) ∈ set evs3⟧
         ⟹ Says A B (Crypt (pubEK B) (Nonce NB)) # evs3 ∈ ns_public"

declare knows_Spy_partsEs [elim]
declare analz_into_parts [dest]
declare Fake_parts_insert_in_Un  [dest]
declare image_eq_UN [simp]


lemma "∃NB. ∃evs ∈ ns_public. Says A B (Crypt (pubEK B) (Nonce NB)) ∈ set
evs"
apply (intro exI bexI)
apply (rule_tac [2] ns_public.Nil [THEN ns_public.NS1, THEN ns_public.NS2,

                                   THEN ns_public.NS3])
by possibility




lemma Spy_see_priEK [simp]:
      "evs ∈ ns_public ⟹ (Key (priEK A) ∈ parts (spies evs)) = (A ∈ bad)"
by (erule ns_public.induct, auto)

lemma Spy_analz_priEK [simp]:
      "evs ∈ ns_public ⟹ (Key (priEK A) ∈ analz (spies evs)) = (A ∈ bad)"
by auto




lemma no_nonce_NS1_NS2 [rule_format]:
      "evs ∈ ns_public
       ⟹ Crypt (pubEK C) {|NA', Nonce NA|} ∈ parts (spies evs) ⟶
           Crypt (pubEK B) {|Nonce NA, Agent A|} ∈ parts (spies evs) ⟶
           Nonce NA ∈ analz (spies evs)"
apply (erule ns_public.induct, simp_all)
apply (blast intro: analz_insertI)+
done



lemma unique_NA:
     "⟦Crypt(pubEK B)  {|Nonce NA, Agent A |} ∈ parts(spies evs);
       Crypt(pubEK B') {|Nonce NA, Agent A'|} ∈ parts(spies evs);
       Nonce NA ∉ analz (spies evs); evs ∈ ns_public⟧
      ⟹ A=A' ∧ B=B'"
apply (erule rev_mp, erule rev_mp, erule rev_mp)
```

```
apply (erule ns_public.induct, simp_all)

apply (blast intro!: analz_insertI)+
done



theorem Spy_not_see_NA:
      "⟦Says A B (Crypt(pubEK B) {|Nonce NA, Agent A|}) ∈ set evs;
        A ∉ bad;  B ∉ bad;  evs ∈ ns_public⟧
       ⟹ Nonce NA ∉ analz (spies evs)"
apply (erule rev_mp)
apply (erule ns_public.induct, simp_all, spy_analz)
apply (blast dest: unique_NA intro: no_nonce_NS1_NS2)+
done



lemma A_trusts_NS2_lemma [rule_format]:
   "⟦A ∉ bad;  B ∉ bad;  evs ∈ ns_public⟧
    ⟹ Crypt (pubEK A) {|Nonce NA, Nonce NB|} ∈ parts (spies evs) ⟶
        Says A B (Crypt(pubEK B) {|Nonce NA, Agent A|}) ∈ set evs ⟶
        Says B A (Crypt(pubEK A) {|Nonce NA, Nonce NB|}) ∈ set evs"
apply (erule ns_public.induct)
apply (auto dest: Spy_not_see_NA unique_NA)
done

theorem A_trusts_NS2:
     "⟦Says A  B (Crypt(pubEK B) {|Nonce NA, Agent A|}) ∈ set evs;
       Says B' A (Crypt(pubEK A) {|Nonce NA, Nonce NB|}) ∈ set evs;
       A ∉ bad;  B ∉ bad;  evs ∈ ns_public⟧
      ⟹ Says B A (Crypt(pubEK A) {|Nonce NA, Nonce NB|}) ∈ set evs"
by (blast intro: A_trusts_NS2_lemma)



lemma B_trusts_NS1 [rule_format]:
     "evs ∈ ns_public
      ⟹ Crypt (pubEK B) {|Nonce NA, Agent A|} ∈ parts (spies evs) ⟶
          Nonce NA ∉ analz (spies evs) ⟶
          Says A B (Crypt (pubEK B) {|Nonce NA, Agent A|}) ∈ set evs"
apply (erule ns_public.induct, simp_all)

apply (blast intro!: analz_insertI)
done




lemma unique_NB [dest]:
     "⟦Crypt(pubEK A)  {|Nonce NA, Nonce NB|} ∈ parts(spies evs);
       Crypt(pubEK A') {|Nonce NA', Nonce NB|} ∈ parts(spies evs);
```

```
       Nonce NB ∉ analz (spies evs); evs ∈ ns_public⟧
     ⟹ A=A' ∧ NA=NA'"
apply (erule rev_mp, erule rev_mp, erule rev_mp)
apply (erule ns_public.induct, simp_all)

apply (blast intro!: analz_insertI)+
done



theorem Spy_not_see_NB [dest]:
     "⟦Says B A (Crypt (pubEK A) {|Nonce NA, Nonce NB|}) ∈ set evs;
       ∀C. Says A C (Crypt (pubEK C) (Nonce NB)) ∉ set evs;
       A ∉ bad;  B ∉ bad;  evs ∈ ns_public⟧
     ⟹ Nonce NB ∉ analz (spies evs)"
apply (erule rev_mp, erule rev_mp)
apply (erule ns_public.induct, simp_all, spy_analz)
apply (simp_all add: all_conj_distrib)
apply (blast intro: no_nonce_NS1_NS2)+
done



lemma B_trusts_NS3_lemma [rule_format]:
     "⟦A ∉ bad;  B ∉ bad;  evs ∈ ns_public⟧
      ⟹ Crypt (pubEK B) (Nonce NB) ∈ parts (spies evs) ⟶
          Says B A  (Crypt (pubEK A) {|Nonce NA, Nonce NB|}) ∈ set evs ⟶
          (∃C. Says A C (Crypt (pubEK C) (Nonce NB)) ∈ set evs)"
apply (erule ns_public.induct, auto)
by (blast intro: no_nonce_NS1_NS2)+

theorem B_trusts_NS3:
     "⟦Says B A  (Crypt (pubEK A) {|Nonce NA, Nonce NB|}) ∈ set evs;
       Says A' B (Crypt (pubEK B) (Nonce NB)) ∈ set evs;
       A ∉ bad;  B ∉ bad;  evs ∈ ns_public⟧
      ⟹ ∃C. Says A C (Crypt (pubEK C) (Nonce NB)) ∈ set evs"
by (blast intro: B_trusts_NS3_lemma)



lemma "⟦A ∉ bad;  B ∉ bad;  evs ∈ ns_public⟧
       ⟹ Says B A (Crypt (pubEK A) {|Nonce NA, Nonce NB|}) ∈ set evs
           ⟶ Nonce NB ∉ analz (spies evs)"
apply (erule ns_public.induct, simp_all, spy_analz)

apply blast

apply (blast intro: no_nonce_NS1_NS2)

apply clarify
apply (frule_tac A' = A in
       Says_imp_knows_Spy [THEN parts.Inj, THEN unique_NB], auto)
apply (rename_tac C B' evs3)
```

This is the attack!

```
1. ⋀C B' evs3.
     ⟦A ∉ bad; B ∉ bad; evs3 ∈ ns_public;
      Says A C (Crypt (pubK C) {|Nonce NA, Agent A|}) ∈ set evs3;
      Says B' A (Crypt (pubK A) {|Nonce NA, Nonce NB|})
      ∈ set evs3;
      C ∈ bad;
      Says B A (Crypt (pubK A) {|Nonce NA, Nonce NB|})
      ∈ set evs3;
      Nonce NB ∉ analz (knows Spy evs3)⟧
     ⟹ False
```

oops

end

15 Verifying the Needham-Schroeder-Lowe Public-Key Protocol

theory *NS_Public* **imports** *Public* **begin**

consts *ns_public* :: "event list set"

inductive *ns_public*
intros

```
Nil:  "[] ∈ ns_public"

Fake: "⟦evsf ∈ ns_public;  X ∈ synth (analz (spies evsf))⟧
       ⟹ Says Spy B X  # evsf ∈ ns_public"

NS1:  "⟦evs1 ∈ ns_public;  Nonce NA ∉ used evs1⟧
       ⟹ Says A B (Crypt (pubEK B) {|Nonce NA, Agent A|})
            # evs1  ∈  ns_public"

NS2:  "⟦evs2 ∈ ns_public;  Nonce NB ∉ used evs2;
        Says A' B (Crypt (pubEK B) {|Nonce NA, Agent A|}) ∈ set evs2⟧
       ⟹ Says B A (Crypt (pubEK A) {|Nonce NA, Nonce NB, Agent B|})
            # evs2  ∈  ns_public"

NS3:  "⟦evs3 ∈ ns_public;
        Says A  B (Crypt (pubEK B) {|Nonce NA, Agent A|}) ∈ set evs3;
        Says B' A (Crypt (pubEK A) {|Nonce NA, Nonce NB, Agent B|})
           ∈ set evs3⟧
       ⟹ Says A B (Crypt (pubEK B) (Nonce NB)) # evs3 ∈ ns_public"
```

declare *knows_Spy_partsEs [elim]*
declare *knows_Spy_partsEs [elim]*
declare *analz_into_parts [dest]*
declare *Fake_parts_insert_in_Un [dest]*
declare *image_eq_UN [simp]*

lemma *"∃NB. ∃evs ∈ ns_public. Says A B (Crypt (pubEK B) (Nonce NB)) ∈ set evs"*
apply *(intro exI bexI)*
apply *(rule_tac [2] ns_public.Nil [THEN ns_public.NS1, THEN ns_public.NS2, THEN ns_public.NS3], possibility)*
done

lemma *Spy_see_priEK [simp]:*
"evs ∈ ns_public ⟹ (Key (priEK A) ∈ parts (spies evs)) = (A ∈ bad)"
by *(erule ns_public.induct, auto)*

lemma *Spy_analz_priEK [simp]:*
"evs ∈ ns_public ⟹ (Key (priEK A) ∈ analz (spies evs)) = (A ∈ bad)"
by *auto*

15.1 Authenticity properties obtained from NS2

lemma *no_nonce_NS1_NS2 [rule_format]:*
"evs ∈ ns_public
⟹ Crypt (pubEK C) {|NA', Nonce NA, Agent D|} ∈ parts (spies evs) ⟶
Crypt (pubEK B) {|Nonce NA, Agent A|} ∈ parts (spies evs) ⟶
Nonce NA ∈ analz (spies evs)"
apply *(erule ns_public.induct, simp_all)*
apply *(blast intro: analz_insertI)+*
done

lemma *unique_NA:*
"⟦Crypt(pubEK B) {|Nonce NA, Agent A |} ∈ parts(spies evs);
Crypt(pubEK B') {|Nonce NA, Agent A'|} ∈ parts(spies evs);
Nonce NA ∉ analz (spies evs); evs ∈ ns_public⟧
⟹ A=A' ∧ B=B'"
apply *(erule rev_mp, erule rev_mp, erule rev_mp)*
apply *(erule ns_public.induct, simp_all)*

apply *(blast intro: analz_insertI)+*
done

theorem *Spy_not_see_NA:*
"⟦Says A B (Crypt(pubEK B) {|Nonce NA, Agent A|}) ∈ set evs;

```
        A ∉ bad;  B ∉ bad;  evs ∈ ns_public⟧
       ⟹ Nonce NA ∉ analz (spies evs)"
apply (erule rev_mp)
apply (erule ns_public.induct, simp_all, spy_analz)
apply (blast dest: unique_NA intro: no_nonce_NS1_NS2)+
done



lemma A_trusts_NS2_lemma [rule_format]:
   "⟦A ∉ bad;  B ∉ bad;  evs ∈ ns_public⟧
    ⟹ Crypt (pubEK A) {|Nonce NA, Nonce NB, Agent B|} ∈ parts (spies evs)
⟶
        Says A B (Crypt(pubEK B) {|Nonce NA, Agent A|}) ∈ set evs ⟶
        Says B A (Crypt(pubEK A) {|Nonce NA, Nonce NB, Agent B|}) ∈ set evs"
apply (erule ns_public.induct, simp_all)

apply (blast dest: Spy_not_see_NA)+
done

theorem A_trusts_NS2:
     "⟦Says A  B (Crypt(pubEK B) {|Nonce NA, Agent A|}) ∈ set evs;
       Says B' A (Crypt(pubEK A) {|Nonce NA, Nonce NB, Agent B|}) ∈ set evs;
       A ∉ bad;  B ∉ bad;  evs ∈ ns_public⟧
      ⟹ Says B A (Crypt(pubEK A) {|Nonce NA, Nonce NB, Agent B|}) ∈ set evs"
by (blast intro: A_trusts_NS2_lemma)



lemma B_trusts_NS1 [rule_format]:
     "evs ∈ ns_public
      ⟹ Crypt (pubEK B) {|Nonce NA, Agent A|} ∈ parts (spies evs) ⟶
         Nonce NA ∉ analz (spies evs) ⟶
         Says A B (Crypt (pubEK B) {|Nonce NA, Agent A|}) ∈ set evs"
apply (erule ns_public.induct, simp_all)

apply (blast intro!: analz_insertI)
done
```

15.2 Authenticity properties obtained from NS2

```
lemma unique_NB [dest]:
     "⟦Crypt(pubEK A)  {|Nonce NA, Nonce NB, Agent B|} ∈ parts(spies evs);
       Crypt(pubEK A') {|Nonce NA', Nonce NB, Agent B'|} ∈ parts(spies evs);
       Nonce NB ∉ analz (spies evs); evs ∈ ns_public⟧
      ⟹ A=A' ∧ NA=NA' ∧ B=B'"
apply (erule rev_mp, erule rev_mp, erule rev_mp)
apply (erule ns_public.induct, simp_all)

apply (blast intro: analz_insertI)+
done
```

```
theorem Spy_not_see_NB [dest]:
     "[[Says B A (Crypt (pubEK A) {|Nonce NA, Nonce NB, Agent B|}) ∈ set evs;
       A ∉ bad;  B ∉ bad;  evs ∈ ns_public]]
      ⟹ Nonce NB ∉ analz (spies evs)"
apply (erule rev_mp)
apply (erule ns_public.induct, simp_all, spy_analz)
apply (blast intro: no_nonce_NS1_NS2)+
done


lemma B_trusts_NS3_lemma [rule_format]:
     "[[A ∉ bad;  B ∉ bad;  evs ∈ ns_public]] ⟹
      Crypt (pubEK B) (Nonce NB) ∈ parts (spies evs) ⟶
      Says B A (Crypt (pubEK A) {|Nonce NA, Nonce NB, Agent B|}) ∈ set evs
⟶
      Says A B (Crypt (pubEK B) (Nonce NB)) ∈ set evs"
by (erule ns_public.induct, auto)

theorem B_trusts_NS3:
     "[[Says B A  (Crypt (pubEK A) {|Nonce NA, Nonce NB, Agent B|}) ∈ set evs;
       Says A' B (Crypt (pubEK B) (Nonce NB)) ∈ set evs;
       A ∉ bad;  B ∉ bad;  evs ∈ ns_public]]
      ⟹ Says A B (Crypt (pubEK B) (Nonce NB)) ∈ set evs"
by (blast intro: B_trusts_NS3_lemma)
```

15.3 Overall guarantee for B

```
theorem B_trusts_protocol:
     "[[A ∉ bad;  B ∉ bad;  evs ∈ ns_public]] ⟹
      Crypt (pubEK B) (Nonce NB) ∈ parts (spies evs) ⟶
      Says B A  (Crypt (pubEK A) {|Nonce NA, Nonce NB, Agent B|}) ∈ set evs
⟶
      Says A B (Crypt (pubEK B) {|Nonce NA, Agent A|}) ∈ set evs"
by (erule ns_public.induct, auto)

end
```

16 The TLS Protocol: Transport Layer Security

```
theory TLS imports Public NatPair begin

constdefs
  certificate      :: "[agent,key] => msg"
    "certificate A KA == Crypt (priSK Server) {|Agent A, Key KA|}"
```

TLS apparently does not require separate keypairs for encryption and signature. Therefore, we formalize signature as encryption using the private encryption key.

```
datatype role = ClientRole | ServerRole

consts
```

```
  PRF  :: "nat*nat*nat => nat"


  sessionK :: "(nat*nat*nat) * role => key"

syntax
    clientK :: "nat*nat*nat => key"
    serverK :: "nat*nat*nat => key"

translations
  "clientK X" == "sessionK(X, ClientRole)"
  "serverK X" == "sessionK(X, ServerRole)"

specification (PRF)
  inj_PRF: "inj PRF"
  — the pseudo-random function is collision-free
   apply (rule exI [of _ "%(x,y,z). nat2_to_nat(x, nat2_to_nat(y,z))"])
   apply (simp add: inj_on_def)
   apply (blast dest!: nat2_to_nat_inj [THEN injD])
   done

specification (sessionK)
  inj_sessionK: "inj sessionK"
  — sessionK is collision-free; also, no clientK clashes with any serverK.
   apply (rule exI [of _
         "%((x,y,z), r). nat2_to_nat(role_case 0 1 r,
                          nat2_to_nat(x, nat2_to_nat(y,z)))"])
   apply (simp add: inj_on_def split: role.split)
   apply (blast dest!: nat2_to_nat_inj [THEN injD])
   done

axioms
  — sessionK makes symmetric keys
  isSym_sessionK: "sessionK nonces ∈ symKeys"

  — sessionK never clashes with a long-term symmetric key (they don't exist in TLS
anyway)
  sessionK_neq_shrK [iff]: "sessionK nonces ≠ shrK A"


consts     tls :: "event list set"
inductive tls
  intros
   Nil:  — The initial, empty trace
         "[] ∈ tls"

   Fake: — The Spy may say anything he can say. The sender field is correct, but
agents don't use that information.
         "[| evsf ∈ tls;  X ∈ synth (analz (spies evsf)) |]
          ==> Says Spy B X # evsf ∈ tls"

   SpyKeys: — The spy may apply PRF and sessionK to available nonces
         "[| evsSK ∈ tls;
             {Nonce NA, Nonce NB, Nonce M} <= analz (spies evsSK) |]
```

```
 ==> Notes Spy {| Nonce (PRF(M,NA,NB)),
                  Key (sessionK((NA,NB,M),role)) |} # evsSK ∈ tls"
```

ClientHello:

— (7.4.1.2) PA represents *CLIENT_VERSION*, *CIPHER_SUITES* and *COMPRESSION_METHODS*. It is uninterpreted but will be confirmed in the FINISHED messages. NA is CLIENT RANDOM, while SID is *SESSION_ID*. UNIX TIME is omitted because the protocol doesn't use it. May assume *NA ∉ range PRF* because CLIENT RANDOM is 28 bytes while MASTER SECRET is 48 bytes

```
"[| evsCH ∈ tls;  Nonce NA ∉ used evsCH;  NA ∉ range PRF |]
 ==> Says A B {|Agent A, Nonce NA, Number SID, Number PA|}
       # evsCH  ∈  tls"
```

ServerHello:

— 7.4.1.3 of the TLS Internet-Draft PB represents *CLIENT_VERSION*, *CIPHER_SUITE* and *COMPRESSION_METHOD*. SERVER CERTIFICATE (7.4.2) is always present. *CERTIFICATE_REQUEST* (7.4.4) is implied.

```
"[| evsSH ∈ tls;  Nonce NB ∉ used evsSH;  NB ∉ range PRF;
    Says A' B {|Agent A, Nonce NA, Number SID, Number PA|}
      ∈ set evsSH |]
 ==> Says B A {|Nonce NB, Number SID, Number PB|} # evsSH  ∈  tls"
```

Certificate:

— SERVER (7.4.2) or CLIENT (7.4.6) CERTIFICATE.

```
"evsC ∈ tls ==> Says B A (certificate B (pubK B)) # evsC  ∈  tls"
```

ClientKeyExch:

— CLIENT KEY EXCHANGE (7.4.7). The client, A, chooses PMS, the PREMASTER SECRET. She encrypts PMS using the supplied KB, which ought to be pubK B. We assume *PMS ∉ range PRF* because a clash betweem the PMS and another MASTER SECRET is highly unlikely (even though both items have the same length, 48 bytes). The Note event records in the trace that she knows PMS (see REMARK at top).

```
"[| evsCX ∈ tls;  Nonce PMS ∉ used evsCX;  PMS ∉ range PRF;
    Says B' A (certificate B KB) ∈ set evsCX |]
 ==> Says A B (Crypt KB (Nonce PMS))
      # Notes A {|Agent B, Nonce PMS|}
      # evsCX  ∈  tls"
```

CertVerify:

— The optional Certificate Verify (7.4.8) message contains the specific components listed in the security analysis, F.1.1.2. It adds the pre-master-secret, which is also essential! Checking the signature, which is the only use of A's certificate, assures B of A's presence

```
"[| evsCV ∈ tls;
    Says B' A {|Nonce NB, Number SID, Number PB|} ∈ set evsCV;
    Notes A {|Agent B, Nonce PMS|} ∈ set evsCV |]
 ==> Says A B (Crypt (priK A) (Hash{|Nonce NB, Agent B, Nonce PMS|}))
      # evsCV  ∈  tls"
```

— Finally come the FINISHED messages (7.4.8), confirming PA and PB among other things. The master-secret is PRF(PMS,NA,NB). Either party may send its message first.

ClientFinished:

— The occurrence of Notes A —Agent B, Nonce PMS— stops the rule's applying when the Spy has satisfied the "Says A B" by repaying messages sent by the true client; in that case, the Spy does not know PMS and could not send ClientFinished. One could simply put *A ≠ Spy* into the rule, but one should not expect the spy to be well-behaved.

```
"[| evsCF ∈ tls;
    Says A  B {|Agent A, Nonce NA, Number SID, Number PA|}
      ∈ set evsCF;
    Says B' A {|Nonce NB, Number SID, Number PB|} ∈ set evsCF;
    Notes A {|Agent B, Nonce PMS|} ∈ set evsCF;
    M = PRF(PMS,NA,NB) |]
 ==> Says A B (Crypt (clientK(NA,NB,M))
                (Hash{|Number SID, Nonce M,
                       Nonce NA, Number PA, Agent A,
                       Nonce NB, Number PB, Agent B|}))
     # evsCF  ∈  tls"
```

ServerFinished:

— Keeping A' and A" distinct means B cannot even check that the two messages originate from the same source.

```
"[| evsSF ∈ tls;
    Says A' B  {|Agent A, Nonce NA, Number SID, Number PA|}
      ∈ set evsSF;
    Says B  A  {|Nonce NB, Number SID, Number PB|} ∈ set evsSF;
    Says A'' B (Crypt (pubK B) (Nonce PMS)) ∈ set evsSF;
    M = PRF(PMS,NA,NB) |]
 ==> Says B A (Crypt (serverK(NA,NB,M))
                (Hash{|Number SID, Nonce M,
                       Nonce NA, Number PA, Agent A,
                       Nonce NB, Number PB, Agent B|}))
     # evsSF  ∈  tls"
```

ClientAccepts:

— Having transmitted ClientFinished and received an identical message encrypted with serverK, the client stores the parameters needed to resume this session. The "Notes A ..." premise is used to prove *Notes_master_imp_Crypt_PMS*.

```
"[| evsCA ∈ tls;
    Notes A {|Agent B, Nonce PMS|} ∈ set evsCA;
    M = PRF(PMS,NA,NB);
    X = Hash{|Number SID, Nonce M,
              Nonce NA, Number PA, Agent A,
              Nonce NB, Number PB, Agent B|};
    Says A  B (Crypt (clientK(NA,NB,M)) X) ∈ set evsCA;
    Says B' A (Crypt (serverK(NA,NB,M)) X) ∈ set evsCA |]
 ==>
    Notes A {|Number SID, Agent A, Agent B, Nonce M|} # evsCA  ∈
tls"
```

ServerAccepts:

— Having transmitted ServerFinished and received an identical message encrypted with clientK, the server stores the parameters needed to resume this session. The "Says A" B ..." premise is used to prove *Notes_master_imp_Crypt_PMS*.

```
"[| evsSA ∈ tls;
```

```
            A ≠ B;
            Says A'' B (Crypt (pubK B) (Nonce PMS)) ∈ set evsSA;
            M = PRF(PMS,NA,NB);
            X = Hash{|Number SID, Nonce M,
                      Nonce NA, Number PA, Agent A,
                      Nonce NB, Number PB, Agent B|};
            Says B  A (Crypt (serverK(NA,NB,M)) X) ∈ set evsSA;
            Says A' B (Crypt (clientK(NA,NB,M)) X) ∈ set evsSA |]
         ==>
            Notes B {|Number SID, Agent A, Agent B, Nonce M|} # evsSA  ∈
tls"
```

ClientResume:
— If A recalls the *SESSION_ID*, then she sends a FINISHED message using the new nonces and stored MASTER SECRET.

```
        "[| evsCR ∈ tls;
            Says A  B {|Agent A, Nonce NA, Number SID, Number PA|}: set evsCR;
            Says B' A {|Nonce NB, Number SID, Number PB|} ∈ set evsCR;
            Notes A {|Number SID, Agent A, Agent B, Nonce M|} ∈ set evsCR
|]
         ==> Says A B (Crypt (clientK(NA,NB,M))
                      (Hash{|Number SID, Nonce M,
                             Nonce NA, Number PA, Agent A,
                             Nonce NB, Number PB, Agent B|}))
             # evsCR  ∈  tls"
```

ServerResume:
— Resumption (7.3): If B finds the *SESSION_ID* then he can send a FINISHED message using the recovered MASTER SECRET

```
        "[| evsSR ∈ tls;
            Says A' B {|Agent A, Nonce NA, Number SID, Number PA|}: set evsSR;
            Says B  A {|Nonce NB, Number SID, Number PB|} ∈ set evsSR;
            Notes B {|Number SID, Agent A, Agent B, Nonce M|} ∈ set evsSR
|]
         ==> Says B A (Crypt (serverK(NA,NB,M))
                      (Hash{|Number SID, Nonce M,
                             Nonce NA, Number PA, Agent A,
                             Nonce NB, Number PB, Agent B|})) # evsSR
               ∈  tls"
```

Oops:
— The most plausible compromise is of an old session key. Losing the MASTER SECRET or PREMASTER SECRET is more serious but rather unlikely. The assumption *A ≠ Spy* is essential: otherwise the Spy could learn session keys merely by replaying messages!

```
        "[| evso ∈ tls;  A ≠ Spy;
            Says A B (Crypt (sessionK((NA,NB,M),role)) X) ∈ set evso |]
         ==> Says A Spy (Key (sessionK((NA,NB,M),role))) # evso  ∈  tls"
```

declare *Says_imp_knows_Spy [THEN analz.Inj, dest]*
declare *parts.Body [dest]*
declare *analz_into_parts [dest]*

declare *Fake_parts_insert_in_Un [dest]*

Automatically unfold the definition of "certificate"

declare *certificate_def [simp]*

Injectiveness of key-generating functions

declare *inj_PRF [THEN inj_eq, iff]*
declare *inj_sessionK [THEN inj_eq, iff]*
declare *isSym_sessionK [simp]*

lemma *pubK_neq_sessionK [iff]: "publicKey b A ≠ sessionK arg"*
by *(simp add: symKeys_neq_imp_neq)*

declare *pubK_neq_sessionK [THEN not_sym, iff]*

lemma *priK_neq_sessionK [iff]: "invKey (publicKey b A) ≠ sessionK arg"*
by *(simp add: symKeys_neq_imp_neq)*

declare *priK_neq_sessionK [THEN not_sym, iff]*

lemmas *keys_distinct = pubK_neq_sessionK priK_neq_sessionK*

16.1 Protocol Proofs

Possibility properties state that some traces run the protocol to the end. Four paths and 12 rules are considered.

Possibility property ending with ClientAccepts.

```
lemma "[| ∀evs. (@ N. Nonce N ∉ used evs) ∉ range PRF;  A ≠ B |]
      ==> ∃SID M. ∃evs ∈ tls.
            Notes A {|Number SID, Agent A, Agent B, Nonce M|} ∈ set evs"
apply (intro exI bexI)
apply (rule_tac [2] tls.Nil
                    [THEN tls.ClientHello, THEN tls.ServerHello,
                     THEN tls.Certificate, THEN tls.ClientKeyExch,
                     THEN tls.ClientFinished, THEN tls.ServerFinished,
                     THEN tls.ClientAccepts], possibility, blast+)
done
```

And one for ServerAccepts. Either FINISHED message may come first.

```
lemma "[| ∀evs. (@ N. Nonce N ∉ used evs) ∉ range PRF; A ≠ B |]
      ==> ∃SID NA PA NB PB M. ∃evs ∈ tls.
           Notes B {|Number SID, Agent A, Agent B, Nonce M|} ∈ set evs"
apply (intro exI bexI)
apply (rule_tac [2] tls.Nil
                    [THEN tls.ClientHello, THEN tls.ServerHello,
                     THEN tls.Certificate, THEN tls.ClientKeyExch,
                     THEN tls.ServerFinished, THEN tls.ClientFinished,
                     THEN tls.ServerAccepts], possibility, blast+)
done
```

Another one, for CertVerify (which is optional)

```
lemma "[| ∀evs. (@ N. Nonce N ∉ used evs) ∉ range PRF;  A ≠ B |]
       ==> ∃NB PMS. ∃evs ∈ tls.
              Says A B (Crypt (priK A) (Hash{|Nonce NB, Agent B, Nonce PMS|}))

                ∈ set evs"
apply (intro exI bexI)
apply (rule_tac [2] tls.Nil
                    [THEN tls.ClientHello, THEN tls.ServerHello,
                     THEN tls.Certificate, THEN tls.ClientKeyExch,
                     THEN tls.CertVerify], possibility, blast+)
done
```

Another one, for session resumption (both ServerResume and ClientResume). NO tls.Nil here: we refer to a previous session, not the empty trace.

```
lemma "[| evs0 ∈ tls;
          Notes A {|Number SID, Agent A, Agent B, Nonce M|} ∈ set evs0;
          Notes B {|Number SID, Agent A, Agent B, Nonce M|} ∈ set evs0;
          ∀evs. (@ N. Nonce N ∉ used evs) ∉ range PRF;
          A ≠ B |]
      ==> ∃NA PA NB PB X. ∃evs ∈ tls.
                X = Hash{|Number SID, Nonce M,
                          Nonce NA, Number PA, Agent A,
                          Nonce NB, Number PB, Agent B|}  &
                Says A B (Crypt (clientK(NA,NB,M)) X) ∈ set evs  &
                Says B A (Crypt (serverK(NA,NB,M)) X) ∈ set evs"
apply (intro exI bexI)
apply (rule_tac [2] tls.ClientHello
                    [THEN tls.ServerHello,
                     THEN tls.ServerResume, THEN tls.ClientResume], possibility,
blast+)
done
```

16.2 Inductive proofs about tls

Spy never sees a good agent's private key!

```
lemma Spy_see_priK [simp]:
     "evs ∈ tls ==> (Key (privateKey b A) ∈ parts (spies evs)) = (A ∈ bad)"
by (erule tls.induct, force, simp_all, blast)

lemma Spy_analz_priK [simp]:
     "evs ∈ tls ==> (Key (privateKey b A) ∈ analz (spies evs)) = (A ∈ bad)"
by auto

lemma Spy_see_priK_D [dest!]:
    "[|Key (privateKey b A) ∈ parts (knows Spy evs);  evs ∈ tls|] ==> A ∈
bad"
by (blast dest: Spy_see_priK)
```

This lemma says that no false certificates exist. One might extend the model to include bogus certificates for the agents, but there seems little point in doing so: the loss of their private keys is a worse breach of security.

lemma *certificate_valid:*
```
    "[| certificate B KB ∈ parts (spies evs);  evs ∈ tls |] ==> KB = pubK
B"
```
apply *(erule rev_mp)*
apply *(erule tls.induct, force, simp_all, blast)*
done

lemmas *CX_KB_is_pubKB = Says_imp_spies [THEN parts.Inj, THEN certificate_valid]*

16.2.1 Properties of items found in Notes

lemma *Notes_Crypt_parts_spies:*
```
     "[| Notes A {|Agent B, X|} ∈ set evs;  evs ∈ tls |]
      ==> Crypt (pubK B) X ∈ parts (spies evs)"
```
apply *(erule rev_mp)*
apply *(erule tls.induct,*
frule_tac [7] CX_KB_is_pubKB, force, simp_all)
apply *(blast intro: parts_insertI)*
done

C may be either A or B

lemma *Notes_master_imp_Crypt_PMS:*
```
     "[| Notes C {|s, Agent A, Agent B, Nonce(PRF(PMS,NA,NB))|} ∈ set evs;
         evs ∈ tls |]
      ==> Crypt (pubK B) (Nonce PMS) ∈ parts (spies evs)"
```
apply *(erule rev_mp)*
apply *(erule tls.induct, force, simp_all)*

Fake

apply *(blast intro: parts_insertI)*

Client, Server Accept

apply *(blast dest!: Notes_Crypt_parts_spies)+*
done

Compared with the theorem above, both premise and conclusion are stronger

lemma *Notes_master_imp_Notes_PMS:*
```
     "[| Notes A {|s, Agent A, Agent B, Nonce(PRF(PMS,NA,NB))|} ∈ set evs;
         evs ∈ tls |]
      ==> Notes A {|Agent B, Nonce PMS|} ∈ set evs"
```
apply *(erule rev_mp)*
apply *(erule tls.induct, force, simp_all)*

ServerAccepts

apply *blast*
done

16.2.2 Protocol goal: if B receives CertVerify, then A sent it

B can check A's signature if he has received A's certificate.

lemma *TrustCertVerify_lemma:*
```
     "[| X ∈ parts (spies evs);
```

```
        X = Crypt (priK A) (Hash{|nb, Agent B, pms|});
        evs ∈ tls;  A ∉ bad |]
     ==> Says A B X ∈ set evs"
apply (erule rev_mp, erule ssubst)
apply (erule tls.induct, force, simp_all, blast)
done
```

Final version: B checks X using the distributed KA instead of priK A

```
lemma TrustCertVerify:
     "[| X ∈ parts (spies evs);
         X = Crypt (invKey KA) (Hash{|nb, Agent B, pms|});
         certificate A KA ∈ parts (spies evs);
         evs ∈ tls;  A ∉ bad |]
      ==> Says A B X ∈ set evs"
by (blast dest!: certificate_valid intro!: TrustCertVerify_lemma)
```

If CertVerify is present then A has chosen PMS.

```
lemma UseCertVerify_lemma:
     "[| Crypt (priK A) (Hash{|nb, Agent B, Nonce PMS|}) ∈ parts (spies evs);
         evs ∈ tls;  A ∉ bad |]
      ==> Notes A {|Agent B, Nonce PMS|} ∈ set evs"
apply (erule rev_mp)
apply (erule tls.induct, force, simp_all, blast)
done
```

Final version using the distributed KA instead of priK A

```
lemma UseCertVerify:
     "[| Crypt (invKey KA) (Hash{|nb, Agent B, Nonce PMS|})
           ∈ parts (spies evs);
         certificate A KA ∈ parts (spies evs);
         evs ∈ tls;  A ∉ bad |]
      ==> Notes A {|Agent B, Nonce PMS|} ∈ set evs"
by (blast dest!: certificate_valid intro!: UseCertVerify_lemma)


lemma no_Notes_A_PRF [simp]:
     "evs ∈ tls ==> Notes A {|Agent B, Nonce (PRF x)|} ∉ set evs"
apply (erule tls.induct, force, simp_all)
```

ClientKeyExch: PMS is assumed to differ from any PRF.

```
apply blast
done


lemma MS_imp_PMS [dest!]:
     "[| Nonce (PRF (PMS,NA,NB)) ∈ parts (spies evs);  evs ∈ tls |]
      ==> Nonce PMS ∈ parts (spies evs)"
apply (erule rev_mp)
apply (erule tls.induct, force, simp_all)
```

Fake

```
apply (blast intro: parts_insertI)
```

Easy, e.g. by freshness

```
apply (blast dest: Notes_Crypt_parts_spies)+
done
```

16.2.3 Unicity results for PMS, the pre-master-secret

PMS determines B.

```
lemma Crypt_unique_PMS:
     "[| Crypt(pubK B)  (Nonce PMS) ∈ parts (spies evs);
         Crypt(pubK B') (Nonce PMS) ∈ parts (spies evs);
         Nonce PMS ∉ analz (spies evs);
         evs ∈ tls |]
      ==> B=B'"
apply (erule rev_mp, erule rev_mp, erule rev_mp)
apply (erule tls.induct, analz_mono_contra, force, simp_all (no_asm_simp))
```

Fake, ClientKeyExch

```
apply blast+
done
```

In A's internal Note, PMS determines A and B.

```
lemma Notes_unique_PMS:
     "[| Notes A  {|Agent B,  Nonce PMS|} ∈ set evs;
         Notes A' {|Agent B', Nonce PMS|} ∈ set evs;
         evs ∈ tls |]
      ==> A=A' & B=B'"
apply (erule rev_mp, erule rev_mp)
apply (erule tls.induct, force, simp_all)
```

ClientKeyExch

```
apply (blast dest!: Notes_Crypt_parts_spies)
done
```

16.3 Secrecy Theorems

Key compromise lemma needed to prove *analz_image_keys*. No collection of keys can help the spy get new private keys.

```
lemma analz_image_priK [rule_format]:
     "evs ∈ tls
      ==> ∀KK. (Key(priK B) ∈ analz (Key‘KK Un (spies evs))) =
          (priK B ∈ KK | B ∈ bad)"
apply (erule tls.induct)
apply (simp_all (no_asm_simp)
                del: image_insert
                add: image_Un [THEN sym]
                     insert_Key_image Un_assoc [THEN sym])
```

Fake

```
apply spy_analz
done
```

slightly speeds up the big simplification below

```
lemma range_sessionkeys_not_priK:
     "KK <= range sessionK ==> priK B ∉ KK"
by blast
```

Lemma for the trivial direction of the if-and-only-if

```
lemma analz_image_keys_lemma:
     "(X ∈ analz (G Un H)) --> (X ∈ analz H)  ==>
      (X ∈ analz (G Un H))  =  (X ∈ analz H)"
by (blast intro: analz_mono [THEN subsetD])



lemma analz_image_keys [rule_format]:
     "evs ∈ tls ==>
      ∀KK. KK <= range sessionK -->
               (Nonce N ∈ analz (Key‘KK Un (spies evs))) =
               (Nonce N ∈ analz (spies evs))"
apply (erule tls.induct, frule_tac [7] CX_KB_is_pubKB)
apply (safe del: iffI)
apply (safe del: impI iffI intro!: analz_image_keys_lemma)
apply (simp_all (no_asm_simp)
                del: image_insert imp_disjL
                add: image_Un [THEN sym]  Un_assoc [THEN sym]
                     insert_Key_singleton
                     range_sessionkeys_not_priK analz_image_priK)
apply (simp_all add: insert_absorb)
```

Fake

```
apply spy_analz
done
```

Knowing some session keys is no help in getting new nonces

```
lemma analz_insert_key [simp]:
     "evs ∈ tls ==>
      (Nonce N ∈ analz (insert (Key (sessionK z)) (spies evs))) =
      (Nonce N ∈ analz (spies evs))"
by (simp del: image_insert
         add: insert_Key_singleton analz_image_keys)
```

16.3.1 Protocol goal: serverK(Na,Nb,M) and clientK(Na,Nb,M) remain secure

Lemma: session keys are never used if PMS is fresh. Nonces don't have to agree, allowing session resumption. Converse doesn't hold; revealing PMS doesn't force the keys to be sent. THEY ARE NOT SUITABLE AS SAFE ELIM RULES.

```
lemma PMS_lemma:
     "[| Nonce PMS ∉ parts (spies evs);
         K = sessionK((Na, Nb, PRF(PMS,NA,NB)), role);
         evs ∈ tls |]
   ==> Key K ∉ parts (spies evs) & (∀Y. Crypt K Y ∉ parts (spies evs))"
apply (erule rev_mp, erule ssubst)
apply (erule tls.induct, frule_tac [7] CX_KB_is_pubKB)
```

apply *(force, simp_all (no_asm_simp))*

Fake

apply *(blast intro: parts_insertI)*

SpyKeys

apply *blast*

Many others

apply *(force dest!: Notes_Crypt_parts_spies Notes_master_imp_Crypt_PMS)+*
done

lemma *PMS_sessionK_not_spied:*
```
     "[| Key (sessionK((Na, Nb, PRF(PMS,NA,NB)), role)) ∈ parts (spies evs);
         evs ∈ tls |]
      ==> Nonce PMS ∈ parts (spies evs)"
```
by *(blast dest: PMS_lemma)*

lemma *PMS_Crypt_sessionK_not_spied:*
```
     "[| Crypt (sessionK((Na, Nb, PRF(PMS,NA,NB)), role)) Y
           ∈ parts (spies evs);  evs ∈ tls |]
      ==> Nonce PMS ∈ parts (spies evs)"
```
by *(blast dest: PMS_lemma)*

Write keys are never sent if M (MASTER SECRET) is secure. Converse fails; betraying M doesn't force the keys to be sent! The strong Oops condition can be weakened later by unicity reasoning, with some effort. NO LONGER USED: see *clientK_not_spied* and *serverK_not_spied*

lemma *sessionK_not_spied:*
```
     "[| ∀A. Says A Spy (Key (sessionK((NA,NB,M),role))) ∉ set evs;
         Nonce M ∉ analz (spies evs);  evs ∈ tls |]
      ==> Key (sessionK((NA,NB,M),role)) ∉ parts (spies evs)"
```
apply *(erule rev_mp, erule rev_mp)*
apply *(erule tls.induct, analz_mono_contra)*
apply *(force, simp_all (no_asm_simp))*

Fake, SpyKeys

apply *blast+*
done

If A sends ClientKeyExch to an honest B, then the PMS will stay secret.

lemma *Spy_not_see_PMS:*
```
     "[| Notes A {|Agent B, Nonce PMS|} ∈ set evs;
         evs ∈ tls;  A ∉ bad;  B ∉ bad |]
      ==> Nonce PMS ∉ analz (spies evs)"
```
apply *(erule rev_mp, erule tls.induct, frule_tac [7] CX_KB_is_pubKB)*
apply *(force, simp_all (no_asm_simp))*

Fake

apply *spy_analz*

SpyKeys

```
apply force
apply (simp_all add: insert_absorb)
```

ClientHello, ServerHello, ClientKeyExch: mostly freshness reasoning

```
apply (blast dest: Notes_Crypt_parts_spies)
apply (blast dest: Notes_Crypt_parts_spies)
apply (blast dest: Notes_Crypt_parts_spies)
```

ClientAccepts and ServerAccepts: because *PMS* $\notin$ *range PRF*

```
apply force+
done
```

If A sends ClientKeyExch to an honest B, then the MASTER SECRET will stay secret.

```
lemma Spy_not_see_MS:
     "[| Notes A {|Agent B, Nonce PMS|} ∈ set evs;
         evs ∈ tls;  A ∉ bad;  B ∉ bad |]
      ==> Nonce (PRF(PMS,NA,NB)) ∉ analz (spies evs)"
apply (erule rev_mp, erule tls.induct, frule_tac [7] CX_KB_is_pubKB)
apply (force, simp_all (no_asm_simp))
```

Fake

```
apply spy_analz
```

SpyKeys: by secrecy of the PMS, Spy cannot make the MS

```
apply (blast dest!: Spy_not_see_PMS)
apply (simp_all add: insert_absorb)
```

ClientAccepts and ServerAccepts: because PMS was already visible; others, freshness etc.

```
apply (blast dest: Notes_Crypt_parts_spies Spy_not_see_PMS
                   Notes_imp_knows_Spy [THEN analz.Inj])+
done
```

16.3.2 Weakening the Oops conditions for leakage of clientK

If A created PMS then nobody else (except the Spy in replays) would send a message using a clientK generated from that PMS.

```
lemma Says_clientK_unique:
     "[| Says A' B' (Crypt (clientK(Na,Nb,PRF(PMS,NA,NB))) Y) ∈ set evs;
         Notes A {|Agent B, Nonce PMS|} ∈ set evs;
         evs ∈ tls;  A' ≠ Spy |]
      ==> A = A'"
apply (erule rev_mp, erule rev_mp)
apply (erule tls.induct, frule_tac [7] CX_KB_is_pubKB)
apply (force, simp_all)
```

ClientKeyExch

```
apply (blast dest!: PMS_Crypt_sessionK_not_spied)
```

ClientFinished, ClientResume: by unicity of PMS

```
apply (blast dest!: Notes_master_imp_Notes_PMS
```

```
            intro: Notes_unique_PMS [THEN conjunct1])+
done
```

If A created PMS and has not leaked her clientK to the Spy, then it is completely secure: not even in parts!

```
lemma clientK_not_spied:
     "[| Notes A {|Agent B, Nonce PMS|} ∈ set evs;
         Says A Spy (Key (clientK(Na,Nb,PRF(PMS,NA,NB)))) ∉ set evs;
         A ∉ bad;  B ∉ bad;
         evs ∈ tls |]
      ==> Key (clientK(Na,Nb,PRF(PMS,NA,NB))) ∉ parts (spies evs)"
apply (erule rev_mp, erule rev_mp)
apply (erule tls.induct, frule_tac [7] CX_KB_is_pubKB)
apply (force, simp_all (no_asm_simp))
```

ClientKeyExch

```
apply blast
```

SpyKeys

```
apply (blast dest!: Spy_not_see_MS)
```

ClientKeyExch

```
apply (blast dest!: PMS_sessionK_not_spied)
```

Oops

```
apply (blast intro: Says_clientK_unique)
done
```

16.3.3 Weakening the Oops conditions for leakage of serverK

If A created PMS for B, then nobody other than B or the Spy would send a message using a serverK generated from that PMS.

```
lemma Says_serverK_unique:
     "[| Says B' A' (Crypt (serverK(Na,Nb,PRF(PMS,NA,NB))) Y) ∈ set evs;
         Notes A {|Agent B, Nonce PMS|} ∈ set evs;
         evs ∈ tls;  A ∉ bad;  B ∉ bad;  B' ≠ Spy |]
      ==> B = B'"
apply (erule rev_mp, erule rev_mp)
apply (erule tls.induct, frule_tac [7] CX_KB_is_pubKB)
apply (force, simp_all)
```

ClientKeyExch

```
apply (blast dest!: PMS_Crypt_sessionK_not_spied)
```

ServerResume, ServerFinished: by unicity of PMS

```
apply (blast dest!: Notes_master_imp_Crypt_PMS
            dest: Spy_not_see_PMS Notes_Crypt_parts_spies Crypt_unique_PMS)+
done
```

If A created PMS for B, and B has not leaked his serverK to the Spy, then it is completely secure: not even in parts!

```
lemma serverK_not_spied:
     "[| Notes A {|Agent B, Nonce PMS|} ∈ set evs;
         Says B Spy (Key(serverK(Na,Nb,PRF(PMS,NA,NB)))) ∉ set evs;
         A ∉ bad;  B ∉ bad;  evs ∈ tls |]
      ==> Key (serverK(Na,Nb,PRF(PMS,NA,NB))) ∉ parts (spies evs)"
apply (erule rev_mp, erule rev_mp)
apply (erule tls.induct, frule_tac [7] CX_KB_is_pubKB)
apply (force, simp_all (no_asm_simp))
```

Fake

```
apply blast
```

SpyKeys

```
apply (blast dest!: Spy_not_see_MS)
```

ClientKeyExch

```
apply (blast dest!: PMS_sessionK_not_spied)
```

Oops

```
apply (blast intro: Says_serverK_unique)
done
```

16.3.4 Protocol goals: if A receives ServerFinished, then B is present and has used the quoted values PA, PB, etc. Note that it is up to A to compare PA with what she originally sent.

The mention of her name (A) in X assures A that B knows who she is.

```
lemma TrustServerFinished [rule_format]:
     "[| X = Crypt (serverK(Na,Nb,M))
               (Hash{|Number SID, Nonce M,
                      Nonce Na, Number PA, Agent A,
                      Nonce Nb, Number PB, Agent B|});
         M = PRF(PMS,NA,NB);
         evs ∈ tls;  A ∉ bad;  B ∉ bad |]
      ==> Says B Spy (Key(serverK(Na,Nb,M))) ∉ set evs -->
          Notes A {|Agent B, Nonce PMS|} ∈ set evs -->
          X ∈ parts (spies evs) --> Says B A X ∈ set evs"
apply (erule ssubst)+
apply (erule tls.induct, frule_tac [7] CX_KB_is_pubKB)
apply (force, simp_all (no_asm_simp))
```

Fake: the Spy doesn't have the critical session key!

```
apply (blast dest: serverK_not_spied)
```

ClientKeyExch

```
apply (blast dest!: PMS_Crypt_sessionK_not_spied)
done
```

This version refers not to ServerFinished but to any message from B. We don't assume B has received CertVerify, and an intruder could have changed A's identity in all other messages, so we can't be sure that B sends his message to

A. If CLIENT KEY EXCHANGE were augmented to bind A's identity with PMS, then we could replace A' by A below.

```
lemma TrustServerMsg [rule_format]:
     "[| M = PRF(PMS,NA,NB);  evs ∈ tls;  A ∉ bad;  B ∉ bad |]
      ==> Says B Spy (Key(serverK(Na,Nb,M))) ∉ set evs -->
          Notes A {|Agent B, Nonce PMS|} ∈ set evs -->
          Crypt (serverK(Na,Nb,M)) Y ∈ parts (spies evs)  -->
          (∃A'. Says B A' (Crypt (serverK(Na,Nb,M)) Y) ∈ set evs)"
apply (erule ssubst)
apply (erule tls.induct, frule_tac [7] CX_KB_is_pubKB)
apply (force, simp_all (no_asm_simp) add: ex_disj_distrib)
```

Fake: the Spy doesn't have the critical session key!

```
apply (blast dest: serverK_not_spied)
```

ClientKeyExch

```
apply (clarify, blast dest!: PMS_Crypt_sessionK_not_spied)
```

ServerResume, ServerFinished: by unicity of PMS

```
apply (blast dest!: Notes_master_imp_Crypt_PMS
            dest: Spy_not_see_PMS Notes_Crypt_parts_spies Crypt_unique_PMS)+
done
```

16.3.5 Protocol goal: if B receives any message encrypted with clientK then A has sent it

ASSUMING that A chose PMS. Authentication is assumed here; B cannot verify it. But if the message is ClientFinished, then B can then check the quoted values PA, PB, etc.

```
lemma TrustClientMsg [rule_format]:
     "[| M = PRF(PMS,NA,NB);  evs ∈ tls;  A ∉ bad;  B ∉ bad |]
      ==> Says A Spy (Key(clientK(Na,Nb,M))) ∉ set evs -->
          Notes A {|Agent B, Nonce PMS|} ∈ set evs -->
          Crypt (clientK(Na,Nb,M)) Y ∈ parts (spies evs) -->
          Says A B (Crypt (clientK(Na,Nb,M)) Y) ∈ set evs"
apply (erule ssubst)
apply (erule tls.induct, frule_tac [7] CX_KB_is_pubKB)
apply (force, simp_all (no_asm_simp))
```

Fake: the Spy doesn't have the critical session key!

```
apply (blast dest: clientK_not_spied)
```

ClientKeyExch

```
apply (blast dest!: PMS_Crypt_sessionK_not_spied)
```

ClientFinished, ClientResume: by unicity of PMS

```
apply (blast dest!: Notes_master_imp_Notes_PMS dest: Notes_unique_PMS)+
done
```

16.3.6 Protocol goal: if B receives ClientFinished, and if B is able to check a CertVerify from A, then A has used the quoted values PA, PB, etc. Even this one requires A to be uncompromised.

lemma *AuthClientFinished:*
```
"[| M = PRF(PMS,NA,NB);
    Says A Spy (Key(clientK(Na,Nb,M))) ∉ set evs;
    Says A' B (Crypt (clientK(Na,Nb,M)) Y) ∈ set evs;
    certificate A KA ∈ parts (spies evs);
    Says A'' B (Crypt (invKey KA) (Hash{|nb, Agent B, Nonce PMS|}))
      ∈ set evs;
    evs ∈ tls;  A ∉ bad;  B ∉ bad |]
 ==> Says A B (Crypt (clientK(Na,Nb,M)) Y) ∈ set evs"
```
by *(blast intro!: TrustClientMsg UseCertVerify)*

end

17 The Certified Electronic Mail Protocol by Abadi et al.

theory *CertifiedEmail* **imports** *Public* **begin**

syntax
```
  TTP         :: agent
  RPwd        :: "agent => key"
```

translations
```
  "TTP"   == "Server "
  "RPwd"  == "shrK "
```

consts
```
  NoAuth   :: nat
  TTPAuth  :: nat
  SAuth    :: nat
  BothAuth :: nat
```

We formalize a fixed way of computing responses. Could be better.

constdefs

```
  "response"    :: "agent => agent => nat => msg"
   "response S R q == Hash {|Agent S, Key (shrK R), Nonce q|}"


consts  certified_mail   :: "event list set"
inductive "certified_mail"
  intros

Nil: — The empty trace
     "[] ∈ certified_mail"

Fake: — The Spy may say anything he can say. The sender field is correct, but agents
don't use that information.
      "[| evsf ∈ certified_mail; X ∈ synth(analz(spies evsf))|]
       ==> Says Spy B X # evsf ∈ certified_mail"

FakeSSL: — The Spy may open SSL sessions with TTP, who is the only agent
equipped with the necessary credentials to serve as an SSL server.
         "[| evsfssl ∈ certified_mail; X ∈ synth(analz(spies evsfssl))|]
          ==> Notes TTP {|Agent Spy, Agent TTP, X|} # evsfssl ∈ certified_mail"

CM1: — The sender approaches the recipient. The message is a number.
 "[|evs1 ∈ certified_mail;
    Key K ∉ used evs1;
    K ∈ symKeys;
    Nonce q ∉ used evs1;
    hs = Hash{|Number cleartext, Nonce q, response S R q, Crypt K (Number m)|};
    S2TTP = Crypt(pubEK TTP) {|Agent S, Number BothAuth, Key K, Agent R, hs|}|]
  ==> Says S R {|Agent S, Agent TTP, Crypt K (Number m), Number BothAuth,
                 Number cleartext, Nonce q, S2TTP|} # evs1
       ∈ certified_mail"

CM2: — The recipient records S2TTP while transmitting it and her password to TTP
over an SSL channel.
 "[|evs2 ∈ certified_mail;
    Gets R {|Agent S, Agent TTP, em, Number BothAuth, Number cleartext,
             Nonce q, S2TTP|} ∈ set evs2;
    TTP ≠ R;
    hr = Hash {|Number cleartext, Nonce q, response S R q, em|} |]
  ==>
   Notes TTP {|Agent R, Agent TTP, S2TTP, Key(RPwd R), hr|} # evs2
     ∈ certified_mail"

CM3: — TTP simultaneously reveals the key to the recipient and gives a receipt to the
sender. The SSL channel does not authenticate the client (R), but TTP accepts the
message only if the given password is that of the claimed sender, R. He replies over
the established SSL channel.
 "[|evs3 ∈ certified_mail;
    Notes TTP {|Agent R, Agent TTP, S2TTP, Key(RPwd R), hr|} ∈ set evs3;
    S2TTP = Crypt (pubEK TTP)
                    {|Agent S, Number BothAuth, Key k, Agent R, hs|};
    TTP ≠ R;  hs = hr;  k ∈ symKeys|]
  ==>
   Notes R {|Agent TTP, Agent R, Key k, hr|} #
```

```
   Gets S (Crypt (priSK TTP) S2TTP) #
   Says TTP S (Crypt (priSK TTP) S2TTP) # evs3 ∈ certified_mail"

Reception:
 "[|evsr ∈ certified_mail; Says A B X ∈ set evsr|]
  ==> Gets B X#evsr ∈ certified_mail"


declare Says_imp_knows_Spy [THEN analz.Inj, dest]
declare analz_into_parts [dest]


lemma "[| Key K ∉ used []; K ∈ symKeys |] ==>
       ∃S2TTP. ∃evs ∈ certified_mail.
           Says TTP S (Crypt (priSK TTP) S2TTP) ∈ set evs"
apply (intro exI bexI)
apply (rule_tac [2] certified_mail.Nil
                    [THEN certified_mail.CM1, THEN certified_mail.Reception,
                     THEN certified_mail.CM2,
                     THEN certified_mail.CM3])
apply (possibility, auto)
done


lemma Gets_imp_Says:
 "[| Gets B X ∈ set evs; evs ∈ certified_mail |] ==> ∃A. Says A B X ∈ set
evs"
apply (erule rev_mp)
apply (erule certified_mail.induct, auto)
done


lemma Gets_imp_parts_knows_Spy:
     "[|Gets A X ∈ set evs; evs ∈ certified_mail|] ==> X ∈ parts(spies evs)"
apply (drule Gets_imp_Says, simp)
apply (blast dest: Says_imp_knows_Spy parts.Inj)
done

lemma CM2_S2TTP_analz_knows_Spy:
 "[|Gets R {|Agent A, Agent B, em, Number AO, Number cleartext,
              Nonce q, S2TTP|} ∈ set evs;
    evs ∈ certified_mail|]
  ==> S2TTP ∈ analz(spies evs)"
apply (drule Gets_imp_Says, simp)
apply (blast dest: Says_imp_knows_Spy analz.Inj)
done

lemmas CM2_S2TTP_parts_knows_Spy =
    CM2_S2TTP_analz_knows_Spy [THEN analz_subset_parts [THEN subsetD]]

lemma hr_form_lemma [rule_format]:
 "evs ∈ certified_mail
  ==> hr ∉ synth (analz (spies evs)) -->
      (∀S2TTP. Notes TTP {|Agent R, Agent TTP, S2TTP, pwd, hr|}
```

```
          ∈ set evs -->
      (∃clt q S em. hr = Hash {|Number clt, Nonce q, response S R q, em|}))"
apply (erule certified_mail.induct)
apply (synth_analz_mono_contra, simp_all, blast+)
done
```

Cannot strengthen the first disjunct to $R \neq Spy$ because the fakessl rule allows Spy to spoof the sender's name. Maybe can strengthen the second disjunct with $R \neq Spy$.

```
lemma hr_form:
 "[|Notes TTP {|Agent R, Agent TTP, S2TTP, pwd, hr|} ∈ set evs;
    evs ∈ certified_mail|]
  ==> hr ∈ synth (analz (spies evs)) |
      (∃clt q S em. hr = Hash {|Number clt, Nonce q, response S R q, em|})"
by (blast intro: hr_form_lemma)

lemma Spy_dont_know_private_keys [dest!]:
    "[|Key (privateKey b A) ∈ parts (spies evs); evs ∈ certified_mail|]
     ==> A ∈ bad"
apply (erule rev_mp)
apply (erule certified_mail.induct, simp_all)
```

Fake

```
apply (blast dest: Fake_parts_insert_in_Un)
```

Message 1

```
apply blast
```

Message 3

```
apply (frule_tac hr_form, assumption)
apply (elim disjE exE)
apply (simp_all add: parts_insert2)
 apply (force dest!: parts_insert_subset_Un [THEN [2] rev_subsetD]
                     analz_subset_parts [THEN subsetD], blast)
done

lemma Spy_know_private_keys_iff [simp]:
    "evs ∈ certified_mail
     ==> (Key (privateKey b A) ∈ parts (spies evs)) = (A ∈ bad)"
by blast

lemma Spy_dont_know_TTPKey_parts [simp]:
     "evs ∈ certified_mail ==> Key (privateKey b TTP) ∉ parts(spies evs)"

by simp

lemma Spy_dont_know_TTPKey_analz [simp]:
     "evs ∈ certified_mail ==> Key (privateKey b TTP) ∉ analz(spies evs)"
by auto
```

Thus, prove any goal that assumes that Spy knows a private key belonging to TTP

```
declare Spy_dont_know_TTPKey_parts [THEN [2] rev_notE, elim!]
```

```
lemma CM3_k_parts_knows_Spy:
 "[| evs ∈ certified_mail;
     Notes TTP {|Agent A, Agent TTP,
                 Crypt (pubEK TTP) {|Agent S, Number AO, Key K,
                 Agent R, hs|}, Key (RPwd R), hs|} ∈ set evs|]
  ==> Key K ∈ parts(spies evs)"
apply (rotate_tac 1)
apply (erule rev_mp)
apply (erule certified_mail.induct, simp_all)
   apply (blast  intro:parts_insertI)
```

Fake SSL

```
apply (blast dest: parts.Body)
```

Message 2

```
apply (blast dest!: Gets_imp_Says elim!: knows_Spy_partsEs)
```

Message 3

```
apply (frule_tac hr_form, assumption)
apply (elim disjE exE)
apply (simp_all add: parts_insert2)
apply (blast intro: subsetD [OF parts_mono [OF Set.subset_insertI]])
done

lemma Spy_dont_know_RPwd [rule_format]:
    "evs ∈ certified_mail ==> Key (RPwd A) ∈ parts(spies evs) --> A ∈ bad"
apply (erule certified_mail.induct, simp_all)
```

Fake

```
apply (blast dest: Fake_parts_insert_in_Un)
```

Message 1

```
apply blast
```

Message 3

```
apply (frule CM3_k_parts_knows_Spy, assumption)
apply (frule_tac hr_form, assumption)
apply (elim disjE exE)
apply (simp_all add: parts_insert2)
apply (force dest!: parts_insert_subset_Un [THEN [2] rev_subsetD]
                    analz_subset_parts [THEN subsetD])
done


lemma Spy_know_RPwd_iff [simp]:
    "evs ∈ certified_mail ==> (Key (RPwd A) ∈ parts(spies evs)) = (A∈bad)"
by (auto simp add: Spy_dont_know_RPwd)

lemma Spy_analz_RPwd_iff [simp]:
    "evs ∈ certified_mail ==> (Key (RPwd A) ∈ analz(spies evs)) = (A∈bad)"
by (auto simp add: Spy_dont_know_RPwd [OF _ analz_subset_parts [THEN subsetD]])
```

Unused, but a guarantee of sorts

```
theorem CertAutenticity:
     "[|Crypt (priSK TTP) X ∈ parts (spies evs); evs ∈ certified_mail|]
      ==> ∃A. Says TTP A (Crypt (priSK TTP) X) ∈ set evs"
apply (erule rev_mp)
apply (erule certified_mail.induct, simp_all)
```

Fake

```
apply (blast dest: Spy_dont_know_private_keys Fake_parts_insert_in_Un)
```

Message 1

```
apply blast
```

Message 3

```
apply (frule_tac hr_form, assumption)
apply (elim disjE exE)
apply (simp_all add: parts_insert2 parts_insert_knows_A)
 apply (blast dest!: Fake_parts_sing_imp_Un, blast)
done
```

17.1 Proving Confidentiality Results

```
lemma analz_image_freshK [rule_format]:
 "evs ∈ certified_mail ==>
   ∀K KK. invKey (pubEK TTP) ∉ KK -->
          (Key K ∈ analz (Key‘KK Un (spies evs))) =
          (K ∈ KK | Key K ∈ analz (spies evs))"
apply (erule certified_mail.induct)
apply (drule_tac [6] A=TTP in symKey_neq_priEK)
apply (erule_tac [6] disjE [OF hr_form])
apply (drule_tac [5] CM2_S2TTP_analz_knows_Spy)
prefer 9
apply (elim exE)
apply (simp_all add: synth_analz_insert_eq
                     subset_trans [OF _ subset_insertI]
                     subset_trans [OF _ Un_upper2]
                del: image_insert image_Un add: analz_image_freshK_simps)
done


lemma analz_insert_freshK:
  "[| evs ∈ certified_mail;  KAB ≠ invKey (pubEK TTP) |] ==>
      (Key K ∈ analz (insert (Key KAB) (spies evs))) =
      (K = KAB | Key K ∈ analz (spies evs))"
by (simp only: analz_image_freshK analz_image_freshK_simps)
```

S2TTP must have originated from a valid sender provided *K* is secure. Proof is surprisingly hard.

```
lemma Notes_SSL_imp_used:
     "[|Notes B {|Agent A, Agent B, X|} ∈ set evs|] ==> X ∈ used evs"
by (blast dest!: Notes_imp_used)
```

```
lemma S2TTP_sender_lemma [rule_format]:
 "evs ∈ certified_mail ==>
    Key K ∉ analz (spies evs) -->
    (∀AO. Crypt (pubEK TTP)
           {|Agent S, Number AO, Key K, Agent R, hs|} ∈ used evs -->
    (∃m ctxt q.
        hs = Hash{|Number ctxt, Nonce q, response S R q, Crypt K (Number m)|}
&
        Says S R
           {|Agent S, Agent TTP, Crypt K (Number m), Number AO,
             Number ctxt, Nonce q,
             Crypt (pubEK TTP)
              {|Agent S, Number AO, Key K, Agent R, hs |}|} ∈ set evs))"
apply (erule certified_mail.induct, analz_mono_contra)
apply (drule_tac [5] CM2_S2TTP_parts_knows_Spy, simp)
apply (simp add: used_Nil Crypt_notin_initState, simp_all)
```

Fake

```
apply (blast dest: Fake_parts_sing [THEN subsetD]
            dest!: analz_subset_parts [THEN subsetD])
```

Fake SSL

```
apply (blast dest: Fake_parts_sing [THEN subsetD]
            dest: analz_subset_parts [THEN subsetD])
```

Message 1

```
apply (clarsimp, blast)
```

Message 2

```
apply (simp add: parts_insert2, clarify)
apply (drule parts_cut, assumption, simp)
apply (blast intro: usedI)
```

Message 3

```
apply (blast dest: Notes_SSL_imp_used used_parts_subset_parts)
done

lemma S2TTP_sender:
 "[|Crypt (pubEK TTP) {|Agent S, Number AO, Key K, Agent R, hs|} ∈ used evs;
    Key K ∉ analz (spies evs);
    evs ∈ certified_mail|]
  ==> ∃m ctxt q.
        hs = Hash{|Number ctxt, Nonce q, response S R q, Crypt K (Number m)|}
&
        Says S R
           {|Agent S, Agent TTP, Crypt K (Number m), Number AO,
             Number ctxt, Nonce q,
             Crypt (pubEK TTP)
              {|Agent S, Number AO, Key K, Agent R, hs |}|} ∈ set evs"
by (blast intro: S2TTP_sender_lemma)
```

Nobody can have used non-existent keys!

```
lemma new_keys_not_used [simp]:
    "[|Key K ∉ used evs; K ∈ symKeys; evs ∈ certified_mail|]
     ==> K ∉ keysFor (parts (spies evs))"
apply (erule rev_mp)
apply (erule certified_mail.induct, simp_all)
```

Fake

```
apply (force dest!: keysFor_parts_insert)
```

Message 1

```
apply blast
```

Message 3

```
apply (frule CM3_k_parts_knows_Spy, assumption)
apply (frule_tac hr_form, assumption)
apply (force dest!: keysFor_parts_insert)
done
```

Less easy to prove `m' = m`. Maybe needs a separate unicity theorem for ciphertexts of the form `Crypt K (Number m)`, where `K` is secure.

```
lemma Key_unique_lemma [rule_format]:
     "evs ∈ certified_mail ==>
       Key K ∉ analz (spies evs) -->
       (∀m cleartext q hs.
        Says S R
            {|Agent S, Agent TTP, Crypt K (Number m), Number AO,
              Number cleartext, Nonce q,
              Crypt (pubEK TTP) {|Agent S, Number AO, Key K, Agent R, hs|}|}
          ∈ set evs -->
       (∀m' cleartext' q' hs'.
       Says S' R'
           {|Agent S', Agent TTP, Crypt K (Number m'), Number AO',
             Number cleartext', Nonce q',
             Crypt (pubEK TTP) {|Agent S', Number AO', Key K, Agent R', hs'|}|}
          ∈ set evs --> R' = R & S' = S & AO' = AO & hs' = hs))"
apply (erule certified_mail.induct, analz_mono_contra, simp_all)
 prefer 2
```

Message 1

```
 apply (blast dest!: Says_imp_knows_Spy [THEN parts.Inj] new_keys_not_used
Crypt_imp_keysFor)
```

Fake

```
apply (auto dest!: usedI S2TTP_sender analz_subset_parts [THEN subsetD])
done
```

The key determines the sender, recipient and protocol options.

```
lemma Key_unique:
      "[|Says S R
           {|Agent S, Agent TTP, Crypt K (Number m), Number AO,
             Number cleartext, Nonce q,
             Crypt (pubEK TTP) {|Agent S, Number AO, Key K, Agent R, hs|}|}
```

```
          ∈ set evs;
         Says S' R'
           {|Agent S', Agent TTP, Crypt K (Number m'), Number AO',
             Number cleartext', Nonce q',
             Crypt (pubEK TTP) {|Agent S', Number AO', Key K, Agent R', hs'|}|}
          ∈ set evs;
         Key K ∉ analz (spies evs);
         evs ∈ certified_mail|]
       ==> R' = R & S' = S & AO' = AO & hs' = hs"
by (rule Key_unique_lemma, assumption+)
```

17.2 The Guarantees for Sender and Recipient

A Sender's guarantee: If Spy gets the key then `R` is bad and `S` moreover gets his return receipt (and therefore has no grounds for complaint).

```
theorem S_fairness_bad_R:
      "[|Says S R {|Agent S, Agent TTP, Crypt K (Number m), Number AO,
                    Number cleartext, Nonce q, S2TTP|} ∈ set evs;
         S2TTP = Crypt (pubEK TTP) {|Agent S, Number AO, Key K, Agent R, hs|};
         Key K ∈ analz (spies evs);
         evs ∈ certified_mail;
         S≠Spy|]
       ==> R ∈ bad & Gets S (Crypt (priSK TTP) S2TTP) ∈ set evs"
apply (erule rev_mp)
apply (erule ssubst)
apply (erule rev_mp)
apply (erule certified_mail.induct, simp_all)
```

Fake

```
apply spy_analz
```

Fake SSL

```
apply spy_analz
```

Message 3

```
apply (frule_tac hr_form, assumption)
apply (elim disjE exE)
apply (simp_all add: synth_analz_insert_eq
                     subset_trans [OF _ subset_insertI]
                     subset_trans [OF _ Un_upper2]
                del: image_insert image_Un add: analz_image_freshK_simps)
apply (simp_all add: symKey_neq_priEK analz_insert_freshK)
apply (blast dest: Notes_SSL_imp_used S2TTP_sender Key_unique)+
done
```

Confidentially for the symmetric key

```
theorem Spy_not_see_encrypted_key:
      "[|Says S R {|Agent S, Agent TTP, Crypt K (Number m), Number AO,
                    Number cleartext, Nonce q, S2TTP|} ∈ set evs;
         S2TTP = Crypt (pubEK TTP) {|Agent S, Number AO, Key K, Agent R, hs|};
         evs ∈ certified_mail;
         S≠Spy; R ∉ bad|]
```

```
     ==> Key K ∉ analz(spies evs)"
by (blast dest: S_fairness_bad_R)
```

Agent `R`, who may be the Spy, doesn't receive the key until `S` has access to the return receipt.

```
theorem S_guarantee:
     "[|Says S R {|Agent S, Agent TTP, Crypt K (Number m), Number AO,
                   Number cleartext, Nonce q, S2TTP|} ∈ set evs;
        S2TTP = Crypt (pubEK TTP) {|Agent S, Number AO, Key K, Agent R, hs|};
        Notes R {|Agent TTP, Agent R, Key K, hs|} ∈ set evs;
        S≠Spy;  evs ∈ certified_mail|]
     ==> Gets S (Crypt (priSK TTP) S2TTP) ∈ set evs"
apply (erule rev_mp)
apply (erule ssubst)
apply (erule rev_mp)
apply (erule certified_mail.induct, simp_all)
```

Message 1

```
apply (blast dest: Notes_imp_used)
```

Message 3

```
apply (blast dest: Notes_SSL_imp_used S2TTP_sender Key_unique S_fairness_bad_R)
```

```
done
```

If `R` sends message 2, and a delivery certificate exists, then `R` receives the necessary key. This result is also important to `S`, as it confirms the validity of the return receipt.

```
theorem RR_validity:
  "[|Crypt (priSK TTP) S2TTP ∈ used evs;
     S2TTP = Crypt (pubEK TTP)
               {|Agent S, Number AO, Key K, Agent R,
                 Hash {|Number cleartext, Nonce q, r, em|}|};
     hr = Hash {|Number cleartext, Nonce q, r, em|};
     R≠Spy;  evs ∈ certified_mail|]
  ==> Notes R {|Agent TTP, Agent R, Key K, hr|} ∈ set evs"
apply (erule rev_mp)
apply (erule ssubst)
apply (erule ssubst)
apply (erule certified_mail.induct, simp_all)
```

Fake

```
apply (blast dest: Fake_parts_sing [THEN subsetD]
             dest!: analz_subset_parts [THEN subsetD])
```

Fake SSL

```
apply (blast dest: Fake_parts_sing [THEN subsetD]
            dest!: analz_subset_parts [THEN subsetD])
```

Message 2

```
apply (drule CM2_S2TTP_parts_knows_Spy, assumption)
apply (force dest: parts_cut)
```

Message 3

apply *(frule_tac hr_form, assumption)*
apply *(elim disjE exE, simp_all)*
apply *(blast dest: Fake_parts_sing [THEN subsetD]*
dest!: analz_subset_parts [THEN subsetD])
done

end

18 Extensions to Standard Theories

theory *Extensions* **imports** *Event* **begin**

18.1 Extensions to Theory *Set*

lemma *eq: "[| !!x. x:A ==> x:B; !!x. x:B ==> x:A |] ==> A=B"*
by *auto*

lemma *insert_Un: "P ({x} Un A) ==> P (insert x A)"*
by *simp*

lemma *in_sub: "x:A ==> {x}<=A"*
by *auto*

18.2 Extensions to Theory *List*

18.2.1 "minus l x" erase the first element of "l" equal to "x"

consts *minus :: "'a list => 'a => 'a list"*

primrec
"minus [] y = []"
"minus (x#xs) y = (if x=y then xs else x # minus xs y)"

lemma *set_minus: "set (minus l x) <= set l"*
by *(induct l, auto)*

18.3 Extensions to Theory *Message*

18.3.1 declarations for tactics

declare *analz_subset_parts [THEN subsetD, dest]*
declare *image_eq_UN [simp]*
declare *parts_insert2 [simp]*
declare *analz_cut [dest]*
declare *split_if_asm [split]*
declare *analz_insertI [intro]*
declare *Un_Diff [simp]*

18.3.2 extract the agent number of an Agent message

consts *agt_nb :: "msg => agent"*

recdef *agt_nb "measure size"*
"agt_nb (Agent A) = A"

18.3.3 messages that are pairs

constdefs *is_MPair :: "msg => bool"*
"is_MPair X == EX Y Z. X = {|Y,Z|}"

declare *is_MPair_def [simp]*

lemma *MPair_is_MPair [iff]: "is_MPair {|X,Y|}"*
by *simp*

lemma *Agent_isnt_MPair [iff]: "~ is_MPair (Agent A)"*
by *simp*

lemma *Number_isnt_MPair [iff]: "~ is_MPair (Number n)"*
by *simp*

lemma *Key_isnt_MPair [iff]: "~ is_MPair (Key K)"*
by *simp*

lemma *Nonce_isnt_MPair [iff]: "~ is_MPair (Nonce n)"*
by *simp*

lemma *Hash_isnt_MPair [iff]: "~ is_MPair (Hash X)"*
by *simp*

lemma *Crypt_isnt_MPair [iff]: "~ is_MPair (Crypt K X)"*
by *simp*

syntax *not_MPair :: "msg => bool"*

translations *"not_MPair X" == "~ is_MPair X"*

lemma *is_MPairE: "[| is_MPair X ==> P; not_MPair X ==> P |] ==> P"*
by *auto*

declare *is_MPair_def [simp del]*

constdefs *has_no_pair :: "msg set => bool"*
"has_no_pair H == ALL X Y. {|X,Y|} ~:H"

declare *has_no_pair_def [simp]*

18.3.4 well-foundedness of messages

lemma *wf_Crypt1 [iff]: "Crypt K X ~= X"*
by *(induct X, auto)*

lemma *wf_Crypt2 [iff]: "X ~= Crypt K X"*
by *(induct X, auto)*

lemma *parts_size: "X:parts {Y} ==> X=Y | size X < size Y"*
by *(erule parts.induct, auto)*

lemma *wf_Crypt_parts [iff]: "Crypt K X ~:parts {X}"*
by *(auto dest: parts_size)*

18.3.5 lemmas on keysFor

constdefs *usekeys :: "msg set => key set"*
"usekeys G == {K. EX Y. Crypt K Y:G}"

lemma *finite_keysFor [intro]: "finite G ==> finite (keysFor G)"*
apply *(simp add: keysFor_def)*
apply *(rule finite_UN_I, auto)*
apply *(erule finite_induct, auto)*
apply *(case_tac "EX K X. x = Crypt K X", clarsimp)*
apply *(subgoal_tac "{Ka. EX Xa. (Ka=K & Xa=X) | Crypt Ka Xa:F}*
= insert K (usekeys F)", auto simp: usekeys_def)
by *(subgoal_tac "{K. EX X. Crypt K X = x | Crypt K X:F} = usekeys F",*
auto simp: usekeys_def)

18.3.6 lemmas on parts

lemma *parts_sub: "[| X:parts G; G<=H |] ==> X:parts H"*
by *(auto dest: parts_mono)*

lemma *parts_Diff [dest]: "X:parts (G - H) ==> X:parts G"*
by *(erule parts_sub, auto)*

lemma *parts_Diff_notin: "[| Y ~:H; Nonce n ~:parts (H - {Y}) |]*
==> Nonce n ~:parts H"
by *simp*

lemmas *parts_insert_substI = parts_insert [THEN ssubst]*
lemmas *parts_insert_substD = parts_insert [THEN sym, THEN ssubst]*

lemma *finite_parts_msg [iff]: "finite (parts {X})"*
by *(induct X, auto)*

lemma *finite_parts [intro]: "finite H ==> finite (parts H)"*
apply *(erule finite_induct, simp)*
by *(rule parts_insert_substI, simp)*

lemma *parts_parts: "[| X:parts {Y}; Y:parts G |] ==> X:parts G"*
by *(frule parts_cut, auto)*

lemma *parts_parts_parts: "[| X:parts {Y}; Y:parts {Z}; Z:parts G |] ==> X:parts*
G"
by *(auto dest: parts_parts)*

lemma *parts_parts_Crypt: "[| Crypt K X:parts G; Nonce n:parts {X} |]*
==> Nonce n:parts G"
by *(blast intro: parts.Body dest: parts_parts)*

18.3.7 lemmas on synth

lemma *synth_sub: "[| X:synth G; G<=H |] ==> X:synth H"*
by *(auto dest: synth_mono)*

lemma *Crypt_synth [rule_format]: "[| X:synth G; Key K ~:G |] ==>*
Crypt K Y:parts {X} --> Crypt K Y:parts G"
by *(erule synth.induct, auto dest: parts_sub)*

18.3.8 lemmas on analz

lemma *analz_UnI1 [intro]: "X:analz G ==> X:analz (G Un H)"*
by *(subgoal_tac "G <= G Un H", auto dest: analz_mono)*

lemma *analz_sub: "[| X:analz G; G <= H |] ==> X:analz H"*
by *(auto dest: analz_mono)*

lemma *analz_Diff [dest]: "X:analz (G - H) ==> X:analz G"*
by *(erule analz.induct, auto)*

lemmas *in_analz_subset_cong = analz_subset_cong [THEN subsetD]*

lemma *analz_eq: "A=A' ==> analz A = analz A'"*
by *auto*

lemmas *insert_commute_substI = insert_commute [THEN ssubst]*

lemma *analz_insertD:*
"[| Crypt K Y:H; Key (invKey K):H |] ==> analz (insert Y H) = analz H"
by *(blast intro: analz.Decrypt analz_insert_eq)*

lemma *must_decrypt [rule_format,dest]: "[| X:analz H; has_no_pair H |] ==>*
X ~:H --> (EX K Y. Crypt K Y:H & Key (invKey K):H)"
by *(erule analz.induct, auto)*

lemma *analz_needs_only_finite: "X:analz H ==> EX G. G <= H & finite G"*
by *(erule analz.induct, auto)*

lemma *notin_analz_insert: "X ~:analz (insert Y G) ==> X ~:analz G"*
by *auto*

18.3.9 lemmas on parts, synth and analz

lemma *parts_invKey [rule_format,dest]:"X:parts {Y} ==>*
X:analz (insert (Crypt K Y) H) --> X ~:analz H --> Key (invKey K):analz H"
by *(erule parts.induct, auto dest: parts.Fst parts.Snd parts.Body)*

lemma *in_analz: "Y:analz H ==> EX X. X:H & Y:parts {X}"*
by *(erule analz.induct, auto intro: parts.Fst parts.Snd parts.Body)*

lemmas *in_analz_subset_parts = analz_subset_parts [THEN subsetD]*

lemma *Crypt_synth_insert: "[| Crypt K X:parts (insert Y H);*
Y:synth (analz H); Key K ~:analz H |] ==> Crypt K X:parts H"
apply *(drule parts_insert_substD, clarify)*

apply *(frule in_sub)*
apply *(frule parts_mono)*
by *auto*

18.3.10 greatest nonce used in a message

consts *greatest_msg :: "msg => nat"*

recdef *greatest_msg "measure size"*
"greatest_msg (Nonce n) = n"
"greatest_msg {|X,Y|} = max (greatest_msg X) (greatest_msg Y)"
"greatest_msg (Crypt K X) = greatest_msg X"
"greatest_msg other = 0"

lemma *greatest_msg_is_greatest: "Nonce n:parts {X} ==> n <= greatest_msg X"*
by *(induct X, auto, arith+)*

18.3.11 sets of keys

constdefs *keyset :: "msg set => bool"*
"keyset G == ALL X. X:G --> (EX K. X = Key K)"

lemma *keyset_in [dest]: "[| keyset G; X:G |] ==> EX K. X = Key K"*
by *(auto simp: keyset_def)*

lemma *MPair_notin_keyset [simp]: "keyset G ==> {|X,Y|} ~:G"*
by *auto*

lemma *Crypt_notin_keyset [simp]: "keyset G ==> Crypt K X ~:G"*
by *auto*

lemma *Nonce_notin_keyset [simp]: "keyset G ==> Nonce n ~:G"*
by *auto*

lemma *parts_keyset [simp]: "keyset G ==> parts G = G"*
by *(auto, erule parts.induct, auto)*

18.3.12 keys a priori necessary for decrypting the messages of G

constdefs *keysfor :: "msg set => msg set"*
"keysfor G == Key ‘ keysFor (parts G)"

lemma *keyset_keysfor [iff]: "keyset (keysfor G)"*
by *(simp add: keyset_def keysfor_def, blast)*

lemma *keyset_Diff_keysfor [simp]: "keyset H ==> keyset (H - keysfor G)"*
by *(auto simp: keyset_def)*

lemma *keysfor_Crypt: "Crypt K X:parts G ==> Key (invKey K):keysfor G"*
by *(auto simp: keysfor_def Crypt_imp_invKey_keysFor)*

lemma *no_key_no_Crypt: "Key K ~:keysfor G ==> Crypt (invKey K) X ~:parts G"*
by *(auto dest: keysfor_Crypt)*

lemma *finite_keysfor [intro]: "finite G ==> finite (keysfor G)"*
by *(auto simp: keysfor_def intro: finite_UN_I)*

18.3.13 only the keys necessary for G are useful in analz

lemma *analz_keyset: "keyset H ==>*
analz (G Un H) = H - keysfor G Un (analz (G Un (H Int keysfor G)))"
apply *(rule eq)*
apply *(erule analz.induct, blast)*
apply *(simp, blast dest: Un_upper1)*
apply *(simp, blast dest: Un_upper2)*
apply *(case_tac "Key (invKey K):H - keysfor G", clarsimp)*
apply *(drule_tac X=X* **in** *no_key_no_Crypt)*
by *(auto intro: analz_sub)*

lemmas *analz_keyset_substD = analz_keyset [THEN sym, THEN ssubst]*

18.4 Extensions to Theory *Event*

18.4.1 general protocol properties

constdefs *is_Says :: "event => bool"*
"is_Says ev == (EX A B X. ev = Says A B X)"

lemma *is_Says_Says [iff]: "is_Says (Says A B X)"*
by *(simp add: is_Says_def)*

constdefs *Gets_correct :: "event list set => bool"*
"Gets_correct p == ALL evs B X. evs:p --> Gets B X:set evs
--> (EX A. Says A B X:set evs)"

lemma *Gets_correct_Says: "[| Gets_correct p; Gets B X # evs:p |]*
==> EX A. Says A B X:set evs"
apply *(simp add: Gets_correct_def)*
by *(drule_tac x="Gets B X # evs"* **in** *spec, auto)*

constdefs *one_step :: "event list set => bool"*
"one_step p == ALL evs ev. ev#evs:p --> evs:p"

lemma *one_step_Cons [dest]: "[| one_step p; ev#evs:p |] ==> evs:p"*
by *(unfold one_step_def, blast)*

lemma *one_step_app: "[| evs@evs':p; one_step p; []:p |] ==> evs':p"*
by *(induct evs, auto)*

lemma *trunc: "[| evs @ evs':p; one_step p |] ==> evs':p"*
by *(induct evs, auto)*

constdefs *has_only_Says :: "event list set => bool"*
"has_only_Says p == ALL evs ev. evs:p --> ev:set evs
--> (EX A B X. ev = Says A B X)"

lemma *has_only_SaysD: "[| ev:set evs; evs:p; has_only_Says p |]*

```
==> EX A B X. ev = Says A B X"
by (unfold has_only_Says_def, blast)

lemma in_has_only_Says [dest]: "[| has_only_Says p; evs:p; ev:set evs |]
==> EX A B X. ev = Says A B X"
by (auto simp: has_only_Says_def)

lemma has_only_Says_imp_Gets_correct [simp]: "has_only_Says p
==> Gets_correct p"
by (auto simp: has_only_Says_def Gets_correct_def)
```

18.4.2 lemma on knows

```
lemma Says_imp_spies2: "Says A B {|X,Y|}:set evs ==> Y:parts (spies evs)"
by (drule Says_imp_spies, drule parts.Inj, drule parts.Snd, simp)

lemma Says_not_parts: "[| Says A B X:set evs; Y ~:parts (spies evs) |]
==> Y ~:parts {X}"
by (auto dest: Says_imp_spies parts_parts)
```

18.4.3 knows without initState

```
consts knows' :: "agent => event list => msg set"

primrec
knows'_Nil:
 "knows' A [] = {}"

knows'_Cons0:
 "knows' A (ev # evs) = (
   if A = Spy then (
     case ev of
       Says A' B X => insert X (knows' A evs)
     | Gets A' X => knows' A evs
     | Notes A' X => if A':bad then insert X (knows' A evs) else knows' A evs
   ) else (
     case ev of
       Says A' B X => if A=A' then insert X (knows' A evs) else knows' A evs
     | Gets A' X => if A=A' then insert X (knows' A evs) else knows' A evs
     | Notes A' X => if A=A' then insert X (knows' A evs) else knows' A evs
   ))"

translations "spies" == "knows Spy"

syntax spies' :: "event list => msg set"

translations "spies'" == "knows' Spy"
```

18.4.4 decomposition of knows into knows' and initState

```
lemma knows_decomp: "knows A evs = knows' A evs Un (initState A)"
by (induct evs, auto split: event.split simp: knows.simps)

lemmas knows_decomp_substI = knows_decomp [THEN ssubst]
lemmas knows_decomp_substD = knows_decomp [THEN sym, THEN ssubst]
```

lemma *knows'_sub_knows: "knows' A evs <= knows A evs"*
by *(auto simp: knows_decomp)*

lemma *knows'_Cons: "knows' A (ev#evs) = knows' A [ev] Un knows' A evs"*
by *(induct ev, auto)*

lemmas *knows'_Cons_substI = knows'_Cons [THEN ssubst]*
lemmas *knows'_Cons_substD = knows'_Cons [THEN sym, THEN ssubst]*

lemma *knows_Cons: "knows A (ev#evs) = initState A Un knows' A [ev]*
Un knows A evs"
apply *(simp only: knows_decomp)*
apply *(rule_tac s="(knows' A [ev] Un knows' A evs) Un initState A"* **in** *trans)*
apply *(simp only: knows'_Cons [of A ev evs] Un_ac)*
apply *blast*
done

lemmas *knows_Cons_substI = knows_Cons [THEN ssubst]*
lemmas *knows_Cons_substD = knows_Cons [THEN sym, THEN ssubst]*

lemma *knows'_sub_spies': "[| evs:p; has_only_Says p; one_step p |]*
==> knows' A evs <= spies' evs"
by *(induct evs, auto split: event.splits)*

18.4.5 knows' is finite

lemma *finite_knows' [iff]: "finite (knows' A evs)"*
by *(induct evs, auto split: event.split simp: knows.simps)*

18.4.6 monotonicity of knows

lemma *knows_sub_Cons: "knows A evs <= knows A (ev#evs)"*
by*(cases A, induct evs, auto simp: knows.simps split:event.split)*

lemma *knows_ConsI: "X:knows A evs ==> X:knows A (ev#evs)"*
by *(auto dest: knows_sub_Cons [THEN subsetD])*

lemma *knows_sub_app: "knows A evs <= knows A (evs @ evs')"*
apply *(induct evs, auto)*
apply *(simp add: knows_decomp)*
by *(case_tac a, auto simp: knows.simps)*

18.4.7 maximum knowledge an agent can have includes messages sent to the agent

consts *knows_max' :: "agent => event list => msg set"*

primrec
knows_max'_def_Nil: "knows_max' A [] = {}"
knows_max'_def_Cons: "knows_max' A (ev # evs) = (
if A=Spy then (
case ev of
Says A' B X => insert X (knows_max' A evs)

```
    | Gets A' X => knows_max' A evs
    | Notes A' X =>
      if A':bad then insert X (knows_max' A evs) else knows_max' A evs
  ) else (
    case ev of
      Says A' B X =>
      if A=A' | A=B then insert X (knows_max' A evs) else knows_max' A evs
    | Gets A' X =>
      if A=A' then insert X (knows_max' A evs) else knows_max' A evs
    | Notes A' X =>
      if A=A' then insert X (knows_max' A evs) else knows_max' A evs
  ))"

constdefs knows_max :: "agent => event list => msg set"
"knows_max A evs == knows_max' A evs Un initState A"

consts spies_max :: "event list => msg set"

translations "spies_max evs" == "knows_max Spy evs"
```

18.4.8 basic facts about *knows_max*

```
lemma spies_max_spies [iff]: "spies_max evs = spies evs"
by (induct evs, auto simp: knows_max_def split: event.splits)

lemma knows_max'_Cons: "knows_max' A (ev#evs)
= knows_max' A [ev] Un knows_max' A evs"
by (auto split: event.splits)

lemmas knows_max'_Cons_substI = knows_max'_Cons [THEN ssubst]
lemmas knows_max'_Cons_substD = knows_max'_Cons [THEN sym, THEN ssubst]

lemma knows_max_Cons: "knows_max A (ev#evs)
= knows_max' A [ev] Un knows_max A evs"
apply (simp add: knows_max_def del: knows_max'_def_Cons)
apply (rule_tac evs1=evs in knows_max'_Cons_substI)
by blast

lemmas knows_max_Cons_substI = knows_max_Cons [THEN ssubst]
lemmas knows_max_Cons_substD = knows_max_Cons [THEN sym, THEN ssubst]

lemma finite_knows_max' [iff]: "finite (knows_max' A evs)"
by (induct evs, auto split: event.split)

lemma knows_max'_sub_spies': "[| evs:p; has_only_Says p; one_step p |]
==> knows_max' A evs <= spies' evs"
by (induct evs, auto split: event.splits)

lemma knows_max'_in_spies' [dest]: "[| evs:p; X:knows_max' A evs;
has_only_Says p; one_step p |] ==> X:spies' evs"
by (rule knows_max'_sub_spies' [THEN subsetD], auto)

lemma knows_max'_app: "knows_max' A (evs @ evs')
= knows_max' A evs Un knows_max' A evs'"
```

by *(induct evs, auto split: event.splits)*

lemma *Says_to_knows_max': "Says A B X:set evs ==> X:knows_max' B evs"*
by *(simp add: in_set_conv_decomp, clarify, simp add: knows_max'_app)*

lemma *Says_from_knows_max': "Says A B X:set evs ==> X:knows_max' A evs"*
by *(simp add: in_set_conv_decomp, clarify, simp add: knows_max'_app)*

18.4.9 used without initState

consts *used' :: "event list => msg set"*

primrec
```
"used' [] = {}"
"used' (ev # evs) = (
  case ev of
    Says A B X => parts {X} Un used' evs
    | Gets A X => used' evs
    | Notes A X => parts {X} Un used' evs
  )"
```

constdefs *init :: "msg set"*
"init == used []"

lemma *used_decomp: "used evs = init Un used' evs"*
by *(induct evs, auto simp: init_def split: event.split)*

lemma *used'_sub_app: "used' evs <= used' (evs@evs')"*
by *(induct evs, auto split: event.split)*

lemma *used'_parts [rule_format]: "X:used' evs ==> Y:parts {X} --> Y:used' evs"*
apply *(induct evs, simp)*
apply *(case_tac a, simp_all)*
apply *(blast dest: parts_trans)+*
done

18.4.10 monotonicity of used

lemma *used_sub_Cons: "used evs <= used (ev#evs)"*
by *(induct evs, (induct ev, auto)+)*

lemma *used_ConsI: "X:used evs ==> X:used (ev#evs)"*
by *(auto dest: used_sub_Cons [THEN subsetD])*

lemma *notin_used_ConsD: "X ~:used (ev#evs) ==> X ~:used evs"*
by *(auto dest: used_sub_Cons [THEN subsetD])*

lemma *used_appD [dest]: "X:used (evs @ evs') ==> X:used evs | X:used evs'"*
by *(induct evs, auto, case_tac a, auto)*

lemma *used_ConsD: "X:used (ev#evs) ==> X:used [ev] | X:used evs"*
by *(case_tac ev, auto)*

lemma *used_sub_app: "used evs <= used (evs@evs')"*

by *(auto simp: used_decomp dest: used'_sub_app [THEN subsetD])*

lemma *used_appIL: "X:used evs ==> X:used (evs' @ evs)"*
by *(induct evs', auto intro: used_ConsI)*

lemma *used_appIR: "X:used evs ==> X:used (evs @ evs')"*
by *(erule used_sub_app [THEN subsetD])*

lemma *used_parts: "[| X:parts {Y}; Y:used evs |] ==> X:used evs"*
apply *(auto simp: used_decomp dest: used'_parts)*
by *(auto simp: init_def used_Nil dest: parts_trans)*

lemma *parts_Says_used: "[| Says A B X:set evs; Y:parts {X} |] ==> Y:used evs"*
by *(induct evs, simp_all, safe, auto intro: used_ConsI)*

lemma *parts_used_app: "X:parts {Y} ==> X:used (evs @ Says A B Y # evs')"*
apply *(drule_tac evs="[Says A B Y]"* **in** *used_parts, simp, blast)*
apply *(drule_tac evs'=evs'* **in** *used_appIR)*
apply *(drule_tac evs'=evs* **in** *used_appIL)*
by *simp*

18.4.11 lemmas on used and knows

lemma *initState_used: "X:parts (initState A) ==> X:used evs"*
by *(induct evs, auto simp: used.simps split: event.split)*

lemma *Says_imp_used: "Says A B X:set evs ==> parts {X} <= used evs"*
by *(induct evs, auto intro: used_ConsI)*

lemma *not_used_not_spied: "X ~:used evs ==> X ~:parts (spies evs)"*
by *(induct evs, auto simp: used_Nil)*

lemma *not_used_not_parts: "[| Y ~:used evs; Says A B X:set evs |]*
==> Y ~:parts {X}"
by *(induct evs, auto intro: used_ConsI)*

lemma *not_used_parts_false: "[| X ~:used evs; Y:parts (spies evs) |]*
==> X ~:parts {Y}"
by *(auto dest: parts_parts)*

lemma *known_used [rule_format]: "[| evs:p; Gets_correct p; one_step p |]*
==> X:parts (knows A evs) --> X:used evs"
apply *(case_tac "A=Spy", blast dest: parts_knows_Spy_subset_used)*
apply *(induct evs)*
apply *(simp add: used.simps, blast)*
apply *(frule_tac ev=a* **and** *evs=evs* **in** *one_step_Cons, simp, clarify)*
apply *(drule_tac P="%G. X:parts G"* **in** *knows_Cons_substD, safe)*
apply *(erule initState_used)*
apply *(case_tac a, auto)*
apply *(drule_tac B=A* **and** *X=msg* **and** *evs=evs* **in** *Gets_correct_Says)*
by *(auto dest: Says_imp_used intro: used_ConsI)*

lemma *known_max_used [rule_format]: "[| evs:p; Gets_correct p; one_step p*

```
|]
==> X:parts (knows_max A evs) --> X:used evs"
apply (case_tac "A=Spy")
apply (simp, blast dest: parts_knows_Spy_subset_used)
apply (induct evs)
apply (simp add: knows_max_def used.simps, blast)
apply (frule_tac ev=a and evs=evs in one_step_Cons, simp, clarify)
apply (drule_tac P="%G. X:parts G" in knows_max_Cons_substD, safe)
apply (case_tac a, auto)
apply (drule_tac B=A and X=msg and evs=evs in Gets_correct_Says)
by (auto simp: knows_max'_Cons dest: Says_imp_used intro: used_ConsI)

lemma not_used_not_known: "[| evs:p; X ~:used evs;
Gets_correct p; one_step p |] ==> X ~:parts (knows A evs)"
by (case_tac "A=Spy", auto dest: not_used_not_spied known_used)

lemma not_used_not_known_max: "[| evs:p; X ~:used evs;
Gets_correct p; one_step p |] ==> X ~:parts (knows_max A evs)"
by (case_tac "A=Spy", auto dest: not_used_not_spied known_max_used)
```

18.4.12 a nonce or key in a message cannot equal a fresh nonce or key

```
lemma Nonce_neq [dest]: "[| Nonce n' ~:used evs;
Says A B X:set evs; Nonce n:parts {X} |] ==> n ~= n'"
by (drule not_used_not_spied, auto dest: Says_imp_knows_Spy parts_sub)

lemma Key_neq [dest]: "[| Key n' ~:used evs;
Says A B X:set evs; Key n:parts {X} |] ==> n ~= n'"
by (drule not_used_not_spied, auto dest: Says_imp_knows_Spy parts_sub)
```

18.4.13 message of an event

```
consts msg :: "event => msg"

recdef msg "measure size"
"msg (Says A B X) = X"
"msg (Gets A X) = X"
"msg (Notes A X) = X"

lemma used_sub_parts_used: "X:used (ev # evs) ==> X:parts {msg ev} Un used
evs"
by (induct ev, auto)



end
```

19 Decomposition of Analz into two parts

theory *Analz* **imports** *Extensions* **begin**

decomposition of `analz` into two parts: `pparts` (for pairs) and analz of `kparts`

19.1 messages that do not contribute to analz

consts *pparts :: "msg set => msg set"*

inductive *"pparts H"*
intros
Inj [intro]: "[| X:H; is_MPair X |] ==> X:pparts H"
Fst [dest]: "[| {|X,Y|}:pparts H; is_MPair X |] ==> X:pparts H"
Snd [dest]: "[| {|X,Y|}:pparts H; is_MPair Y |] ==> Y:pparts H"

19.2 basic facts about *pparts*

lemma *pparts_is_MPair [dest]: "X:pparts H ==> is_MPair X"*
by *(erule pparts.induct, auto)*

lemma *Crypt_notin_pparts [iff]: "Crypt K X ~:pparts H"*
by *auto*

lemma *Key_notin_pparts [iff]: "Key K ~:pparts H"*
by *auto*

lemma *Nonce_notin_pparts [iff]: "Nonce n ~:pparts H"*
by *auto*

lemma *Number_notin_pparts [iff]: "Number n ~:pparts H"*
by *auto*

lemma *Agent_notin_pparts [iff]: "Agent A ~:pparts H"*
by *auto*

lemma *pparts_empty [iff]: "pparts {} = {}"*
by *(auto, erule pparts.induct, auto)*

lemma *pparts_insertI [intro]: "X:pparts H ==> X:pparts (insert Y H)"*
by *(erule pparts.induct, auto)*

lemma *pparts_sub: "[| X:pparts G; G<=H |] ==> X:pparts H"*
by *(erule pparts.induct, auto)*

lemma *pparts_insert2 [iff]: "pparts (insert X (insert Y H))*
= pparts {X} Un pparts {Y} Un pparts H"
by *(rule eq, (erule pparts.induct, auto)+)*

lemma *pparts_insert_MPair [iff]: "pparts (insert {|X,Y|} H)*
= insert {|X,Y|} (pparts ({X,Y} Un H))"
apply *(rule eq, (erule pparts.induct, auto)+)*
apply *(rule_tac Y=Y* **in** *pparts.Fst, auto)*
apply *(erule pparts.induct, auto)*
by *(rule_tac X=X* **in** *pparts.Snd, auto)*

lemma *pparts_insert_Nonce [iff]: "pparts (insert (Nonce n) H) = pparts H"*
by *(rule eq, erule pparts.induct, auto)*

lemma *pparts_insert_Crypt [iff]: "pparts (insert (Crypt K X) H) = pparts*
H"

by *(rule eq, erule pparts.induct, auto)*

lemma *pparts_insert_Key [iff]: "pparts (insert (Key K) H) = pparts H"*
by *(rule eq, erule pparts.induct, auto)*

lemma *pparts_insert_Agent [iff]: "pparts (insert (Agent A) H) = pparts H"*
by *(rule eq, erule pparts.induct, auto)*

lemma *pparts_insert_Number [iff]: "pparts (insert (Number n) H) = pparts H"*
by *(rule eq, erule pparts.induct, auto)*

lemma *pparts_insert_Hash [iff]: "pparts (insert (Hash X) H) = pparts H"*
by *(rule eq, erule pparts.induct, auto)*

lemma *pparts_insert: "X:pparts (insert Y H) ==> X:pparts {Y} Un pparts H"*
by *(erule pparts.induct, blast+)*

lemma *insert_pparts: "X:pparts {Y} Un pparts H ==> X:pparts (insert Y H)"*
by *(safe, erule pparts.induct, auto)*

lemma *pparts_Un [iff]: "pparts (G Un H) = pparts G Un pparts H"*
by *(rule eq, erule pparts.induct, auto dest: pparts_sub)*

lemma *pparts_pparts [iff]: "pparts (pparts H) = pparts H"*
by *(rule eq, erule pparts.induct, auto)*

lemma *pparts_insert_eq: "pparts (insert X H) = pparts {X} Un pparts H"*
by *(rule_tac A=H* **in** *insert_Un, rule pparts_Un)*

lemmas *pparts_insert_substI = pparts_insert_eq [THEN ssubst]*

lemma *in_pparts: "Y:pparts H ==> EX X. X:H & Y:pparts {X}"*
by *(erule pparts.induct, auto)*

19.3 facts about *pparts* and *parts*

lemma *pparts_no_Nonce [dest]: "[| X:pparts {Y}; Nonce n ~:parts {Y} |] ==> Nonce n ~:parts {X}"*
by *(erule pparts.induct, simp_all)*

19.4 facts about *pparts* and *analz*

lemma *pparts_analz: "X:pparts H ==> X:analz H"*
by *(erule pparts.induct, auto)*

lemma *pparts_analz_sub: "[| X:pparts G; G<=H |] ==> X:analz H"*
by *(auto dest: pparts_sub pparts_analz)*

19.5 messages that contribute to analz

consts *kparts :: "msg set => msg set"*

inductive *"kparts H"*

intros
```
Inj [intro]: "[| X:H; not_MPair X |] ==> X:kparts H"
Fst [intro]: "[| {|X,Y|}:pparts H; not_MPair X |] ==> X:kparts H"
Snd [intro]: "[| {|X,Y|}:pparts H; not_MPair Y |] ==> Y:kparts H"
```

19.6 basic facts about *kparts*

lemma *kparts_not_MPair [dest]: "X:kparts H ==> not_MPair X"*
by *(erule kparts.induct, auto)*

lemma *kparts_empty [iff]: "kparts {} = {}"*
by *(rule eq, erule kparts.induct, auto)*

lemma *kparts_insertI [intro]: "X:kparts H ==> X:kparts (insert Y H)"*
by *(erule kparts.induct, auto dest: pparts_insertI)*

lemma *kparts_insert2 [iff]: "kparts (insert X (insert Y H))*
= kparts {X} Un kparts {Y} Un kparts H"
by *(rule eq, (erule kparts.induct, auto)+)*

lemma *kparts_insert_MPair [iff]: "kparts (insert {|X,Y|} H)*
= kparts ({X,Y} Un H)"
by *(rule eq, (erule kparts.induct, auto)+)*

lemma *kparts_insert_Nonce [iff]: "kparts (insert (Nonce n) H)*
= insert (Nonce n) (kparts H)"
by *(rule eq, erule kparts.induct, auto)*

lemma *kparts_insert_Crypt [iff]: "kparts (insert (Crypt K X) H)*
= insert (Crypt K X) (kparts H)"
by *(rule eq, erule kparts.induct, auto)*

lemma *kparts_insert_Key [iff]: "kparts (insert (Key K) H)*
= insert (Key K) (kparts H)"
by *(rule eq, erule kparts.induct, auto)*

lemma *kparts_insert_Agent [iff]: "kparts (insert (Agent A) H)*
= insert (Agent A) (kparts H)"
by *(rule eq, erule kparts.induct, auto)*

lemma *kparts_insert_Number [iff]: "kparts (insert (Number n) H)*
= insert (Number n) (kparts H)"
by *(rule eq, erule kparts.induct, auto)*

lemma *kparts_insert_Hash [iff]: "kparts (insert (Hash X) H)*
= insert (Hash X) (kparts H)"
by *(rule eq, erule kparts.induct, auto)*

lemma *kparts_insert: "X:kparts (insert X H) ==> X:kparts {X} Un kparts H"*
by *(erule kparts.induct, (blast dest: pparts_insert)+)*

lemma *kparts_insert_fst [rule_format,dest]: "X:kparts (insert Z H) ==>*
X ~:kparts H --> X:kparts {Z}"
by *(erule kparts.induct, (blast dest: pparts_insert)+)*

lemma *kparts_sub: "[| X:kparts G; G<=H |] ==> X:kparts H"*
by *(erule kparts.induct, auto dest: pparts_sub)*

lemma *kparts_Un [iff]: "kparts (G Un H) = kparts G Un kparts H"*
by *(rule eq, erule kparts.induct, auto dest: kparts_sub)*

lemma *pparts_kparts [iff]: "pparts (kparts H) = {}"*
by *(rule eq, erule pparts.induct, auto)*

lemma *kparts_kparts [iff]: "kparts (kparts H) = kparts H"*
by *(rule eq, erule kparts.induct, auto)*

lemma *kparts_insert_eq: "kparts (insert X H) = kparts {X} Un kparts H"*
by *(rule_tac A=H* **in** *insert_Un, rule kparts_Un)*

lemmas *kparts_insert_substI = kparts_insert_eq [THEN ssubst]*

lemma *in_kparts: "Y:kparts H ==> EX X. X:H & Y:kparts {X}"*
by *(erule kparts.induct, auto dest: in_pparts)*

lemma *kparts_has_no_pair [iff]: "has_no_pair (kparts H)"*
by *auto*

19.7 facts about *kparts* and *parts*

lemma *kparts_no_Nonce [dest]: "[| X:kparts {Y}; Nonce n ~:parts {Y} |]*
==> Nonce n ~:parts {X}"
by *(erule kparts.induct, auto)*

lemma *kparts_parts: "X:kparts H ==> X:parts H"*
by *(erule kparts.induct, auto dest: pparts_analz)*

lemma *parts_kparts: "X:parts (kparts H) ==> X:parts H"*
by *(erule parts.induct, auto dest: kparts_parts*
intro: parts.Fst parts.Snd parts.Body)

lemma *Crypt_kparts_Nonce_parts [dest]: "[| Crypt K Y:kparts {Z};*
Nonce n:parts {Y} |] ==> Nonce n:parts {Z}"
by *auto*

19.8 facts about *kparts* and *analz*

lemma *kparts_analz: "X:kparts H ==> X:analz H"*
by *(erule kparts.induct, auto dest: pparts_analz)*

lemma *kparts_analz_sub: "[| X:kparts G; G<=H |] ==> X:analz H"*
by *(erule kparts.induct, auto dest: pparts_analz_sub)*

lemma *analz_kparts [rule_format,dest]: "X:analz H ==>*
Y:kparts {X} --> Y:analz H"
by *(erule analz.induct, auto dest: kparts_analz_sub)*

lemma *analz_kparts_analz: "X:analz (kparts H) ==> X:analz H"*

by *(erule analz.induct, auto dest: kparts_analz)*

lemma *analz_kparts_insert: "X:analz (kparts (insert Z H)) ==>*
X:analz (kparts {Z} Un kparts H)"
by *(rule analz_sub, auto)*

lemma *Nonce_kparts_synth [rule_format]: "Y:synth (analz G)*
==> Nonce n:kparts {Y} --> Nonce n:analz G"
by *(erule synth.induct, auto)*

lemma *kparts_insert_synth: "[| Y:parts (insert X G); X:synth (analz G);*
Nonce n:kparts {Y}; Nonce n ~:analz G |] ==> Y:parts G"
apply *(drule parts_insert_substD, clarify)*
apply *(drule in_sub, drule_tac X=Y* **in** *parts_sub, simp)*
by *(auto dest: Nonce_kparts_synth)*

lemma *Crypt_insert_synth: "[| Crypt K Y:parts (insert X G); X:synth (analz*
G);
Nonce n:kparts {Y}; Nonce n ~:analz G |] ==> Crypt K Y:parts G"
apply *(drule parts_insert_substD, clarify)*
apply *(drule in_sub, drule_tac X="Crypt K Y"* **in** *parts_sub, simp, clarsimp)*
apply *(ind_cases "Crypt K Y:synth (analz G)")*
by *(auto dest: Nonce_kparts_synth)*

19.9 analz is pparts + analz of kparts

lemma *analz_pparts_kparts: "X:analz H ==> X:pparts H | X:analz (kparts H)"*
apply *(erule analz.induct)*
apply *(rule_tac X=X* **in** *is_MPairE, blast, blast)*
apply *(erule disjE, rule_tac X=X* **in** *is_MPairE, blast, blast, blast)*
by *(erule disjE, rule_tac X=Y* **in** *is_MPairE, blast+)*

lemma *analz_pparts_kparts_eq: "analz H = pparts H Un analz (kparts H)"*
by *(rule eq, auto dest: analz_pparts_kparts pparts_analz analz_kparts_analz)*

lemmas *analz_pparts_kparts_substI = analz_pparts_kparts_eq [THEN ssubst]*
lemmas *analz_pparts_kparts_substD*
= analz_pparts_kparts_eq [THEN sym, THEN ssubst]

end

20 Protocol-Independent Confidentiality Theorem on Nonces

theory *Guard* **imports** *Analz Extensions* **begin**

consts *guard :: "nat => key set => msg set"*

inductive *"guard n Ks"*
intros

```
No_Nonce [intro]: "Nonce n ~:parts {X} ==> X:guard n Ks"
Guard_Nonce [intro]: "invKey K:Ks ==> Crypt K X:guard n Ks"
Crypt [intro]: "X:guard n Ks ==> Crypt K X:guard n Ks"
Pair [intro]: "[| X:guard n Ks; Y:guard n Ks |] ==> {|X,Y|}:guard n Ks"
```

20.1 basic facts about *guard*

lemma *Key_is_guard [iff]: "Key K:guard n Ks"*
by *auto*

lemma *Agent_is_guard [iff]: "Agent A:guard n Ks"*
by *auto*

lemma *Number_is_guard [iff]: "Number r:guard n Ks"*
by *auto*

lemma *Nonce_notin_guard: "X:guard n Ks ==> X ~= Nonce n"*
by *(erule guard.induct, auto)*

lemma *Nonce_notin_guard_iff [iff]: "Nonce n ~:guard n Ks"*
by *(auto dest: Nonce_notin_guard)*

lemma *guard_has_Crypt [rule_format]: "X:guard n Ks ==> Nonce n:parts {X}*
--> (EX K Y. Crypt K Y:kparts {X} & Nonce n:parts {Y})"
by *(erule guard.induct, auto)*

lemma *Nonce_notin_kparts_msg: "X:guard n Ks ==> Nonce n ~:kparts {X}"*
by *(erule guard.induct, auto)*

lemma *Nonce_in_kparts_imp_no_guard: "Nonce n:kparts H*
==> EX X. X:H & X ~:guard n Ks"
apply *(drule in_kparts, clarify)*
apply *(rule_tac x=X* **in** *exI, clarify)*
by *(auto dest: Nonce_notin_kparts_msg)*

lemma *guard_kparts [rule_format]: "X:guard n Ks ==>*
Y:kparts {X} --> Y:guard n Ks"
by *(erule guard.induct, auto)*

lemma *guard_Crypt: "[| Crypt K Y:guard n Ks; K ~:invKey`Ks |] ==> Y:guard*
n Ks"
by *(ind_cases "Crypt K Y:guard n Ks", auto)*

lemma *guard_MPair [iff]: "({|X,Y|}:guard n Ks) = (X:guard n Ks & Y:guard*
n Ks)"
by *(auto, (ind_cases "{|X,Y|}:guard n Ks", auto)+)*

lemma *guard_not_guard [rule_format]: "X:guard n Ks ==>*
Crypt K Y:kparts {X} --> Nonce n:kparts {Y} --> Y ~:guard n Ks"
by *(erule guard.induct, auto dest: guard_kparts)*

lemma *guard_extand: "[| X:guard n Ks; Ks <= Ks' |] ==> X:guard n Ks'"*
by *(erule guard.induct, auto)*

20.2 guarded sets

constdefs *Guard :: "nat => key set => msg set => bool"*
"Guard n Ks H == ALL X. X:H --> X:guard n Ks"

20.3 basic facts about *Guard*

lemma *Guard_empty [iff]: "Guard n Ks {}"*
by *(simp add: Guard_def)*

lemma *notin_parts_Guard [intro]: "Nonce n ~:parts G ==> Guard n Ks G"*
apply *(unfold Guard_def, clarify)*
apply *(subgoal_tac "Nonce n ~:parts {X}")*
by *(auto dest: parts_sub)*

lemma *Nonce_notin_kparts [simplified]: "Guard n Ks H ==> Nonce n ~:kparts H"*
by *(auto simp: Guard_def dest: in_kparts Nonce_notin_kparts_msg)*

lemma *Guard_must_decrypt: "[| Guard n Ks H; Nonce n:analz H |] ==>*
EX K Y. Crypt K Y:kparts H & Key (invKey K):kparts H"
apply *(drule_tac P="%G. Nonce n:G"* **in** *analz_pparts_kparts_substD, simp)*
by *(drule must_decrypt, auto dest: Nonce_notin_kparts)*

lemma *Guard_kparts [intro]: "Guard n Ks H ==> Guard n Ks (kparts H)"*
by *(auto simp: Guard_def dest: in_kparts guard_kparts)*

lemma *Guard_mono: "[| Guard n Ks H; G <= H |] ==> Guard n Ks G"*
by *(auto simp: Guard_def)*

lemma *Guard_insert [iff]: "Guard n Ks (insert X H)*
= (Guard n Ks H & X:guard n Ks)"
by *(auto simp: Guard_def)*

lemma *Guard_Un [iff]: "Guard n Ks (G Un H) = (Guard n Ks G & Guard n Ks H)"*
by *(auto simp: Guard_def)*

lemma *Guard_synth [intro]: "Guard n Ks G ==> Guard n Ks (synth G)"*
by *(auto simp: Guard_def, erule synth.induct, auto)*

lemma *Guard_analz [intro]: "[| Guard n Ks G; ALL K. K:Ks --> Key K ~:analz G |]*
==> Guard n Ks (analz G)"
apply *(auto simp: Guard_def)*
apply *(erule analz.induct, auto)*
by *(ind_cases "Crypt K Xa:guard n Ks", auto)*

lemma *in_Guard [dest]: "[| X:G; Guard n Ks G |] ==> X:guard n Ks"*
by *(auto simp: Guard_def)*

lemma *in_synth_Guard: "[| X:synth G; Guard n Ks G |] ==> X:guard n Ks"*
by *(drule Guard_synth, auto)*

lemma *in_analz_Guard: "[| X:analz G; Guard n Ks G;*
ALL K. K:Ks --> Key K ~:analz G |] ==> X:guard n Ks"

by *(drule Guard_analz, auto)*

lemma *Guard_keyset [simp]: "keyset G ==> Guard n Ks G"*
by *(auto simp: Guard_def)*

lemma *Guard_Un_keyset: "[| Guard n Ks G; keyset H |] ==> Guard n Ks (G Un H)"*
by *auto*

lemma *in_Guard_kparts: "[| X:G; Guard n Ks G; Y:kparts {X} |] ==> Y:guard n Ks"*
by *blast*

lemma *in_Guard_kparts_neq: "[| X:G; Guard n Ks G; Nonce n':kparts {X} |] ==> n ~= n'"*
by *(blast dest: in_Guard_kparts)*

lemma *in_Guard_kparts_Crypt: "[| X:G; Guard n Ks G; is_MPair X; Crypt K Y:kparts {X}; Nonce n:kparts {Y} |] ==> invKey K:Ks"*
apply *(drule in_Guard, simp)*
apply *(frule guard_not_guard, simp+)*
apply *(drule guard_kparts, simp)*
by *(ind_cases "Crypt K Y:guard n Ks", auto)*

lemma *Guard_extand: "[| Guard n Ks G; Ks <= Ks' |] ==> Guard n Ks' G"*
by *(auto simp: Guard_def dest: guard_extand)*

lemma *guard_invKey [rule_format]: "[| X:guard n Ks; Nonce n:kparts {Y} |] ==>*
Crypt K Y:kparts {X} --> invKey K:Ks"
by *(erule guard.induct, auto)*

lemma *Crypt_guard_invKey [rule_format]: "[| Crypt K Y:guard n Ks; Nonce n:kparts {Y} |] ==> invKey K:Ks"*
by *(auto dest: guard_invKey)*

20.4 set obtained by decrypting a message

syntax *decrypt :: "msg set => key => msg => msg set"*

translations *"decrypt H K Y" => "insert Y (H - {Crypt K Y})"*

lemma *analz_decrypt: "[| Crypt K Y:H; Key (invKey K):H; Nonce n:analz H |] ==> Nonce n:analz (decrypt H K Y)"*
apply *(drule_tac P="%H. Nonce n:analz H"* **in** *ssubst [OF insert_Diff])*
apply *assumption*
apply *(simp only: analz_Crypt_if, simp)*
done

lemma *parts_decrypt: "[| Crypt K Y:H; X:parts (decrypt H K Y) |] ==> X:parts H"*
by *(erule parts.induct, auto intro: parts.Fst parts.Snd parts.Body)*

20.5 number of Crypt's in a message

consts *crypt_nb :: "msg => nat"*

recdef *crypt_nb "measure size"*
"crypt_nb (Crypt K X) = Suc (crypt_nb X)"
"crypt_nb {|X,Y|} = crypt_nb X + crypt_nb Y"
"crypt_nb X = 0"

20.6 basic facts about *crypt_nb*

lemma *non_empty_crypt_msg: "Crypt K Y:parts {X} ==> 0 < crypt_nb X"*
by *(induct X, simp_all, safe, simp_all)*

20.7 number of Crypt's in a message list

consts *cnb :: "msg list => nat"*

recdef *cnb "measure size"*
"cnb [] = 0"
"cnb (X#l) = crypt_nb X + cnb l"

20.8 basic facts about *cnb*

lemma *cnb_app [simp]: "cnb (l @ l') = cnb l + cnb l'"*
by *(induct l, auto)*

lemma *mem_cnb_minus: "x mem l ==> cnb l = crypt_nb x + (cnb l - crypt_nb x)"*
by *(induct l, auto)*

lemmas *mem_cnb_minus_substI = mem_cnb_minus [THEN ssubst]*

lemma *cnb_minus [simp]: "x mem l ==> cnb (minus l x) = cnb l - crypt_nb x"*
apply *(induct l, auto)*
by *(erule_tac l1=l* **and** *x1=x* **in** *mem_cnb_minus_substI, simp)*

lemma *parts_cnb: "Z:parts (set l) ==>*
cnb l = (cnb l - crypt_nb Z) + crypt_nb Z"
by *(erule parts.induct, auto simp: in_set_conv_decomp)*

lemma *non_empty_crypt: "Crypt K Y:parts (set l) ==> 0 < cnb l"*
by *(induct l, auto dest: non_empty_crypt_msg parts_insert_substD)*

20.9 list of kparts

lemma *kparts_msg_set: "EX l. kparts {X} = set l & cnb l = crypt_nb X"*
apply *(induct X, simp_all)*
apply *(rule_tac x="[Agent agent]"* **in** *exI, simp)*
apply *(rule_tac x="[Number nat]"* **in** *exI, simp)*
apply *(rule_tac x="[Nonce nat]"* **in** *exI, simp)*
apply *(rule_tac x="[Key nat]"* **in** *exI, simp)*
apply *(rule_tac x="[Hash X]"* **in** *exI, simp)*
apply *(clarify, rule_tac x="l@la"* **in** *exI, simp)*
by *(clarify, rule_tac x="[Crypt nat X]"* **in** *exI, simp)*

lemma *kparts_set: "EX l'. kparts (set l) = set l' & cnb l' = cnb l"*
apply *(induct l)*
apply *(rule_tac x="[]"* **in** *exI, simp, clarsimp)*
apply *(subgoal_tac "EX l''. kparts {a} = set l'' & cnb l'' = crypt_nb a", clarify)*
apply *(rule_tac x="l''@l'"* **in** *exI, simp)*
apply *(rule kparts_insert_substI, simp)*
by *(rule kparts_msg_set)*

20.10 list corresponding to "decrypt"

constdefs *decrypt' :: "msg list => key => msg => msg list"*
"decrypt' l K Y == Y # minus l (Crypt K Y)"

declare *decrypt'_def [simp]*

20.11 basic facts about *decrypt'*

lemma *decrypt_minus: "decrypt (set l) K Y <= set (decrypt' l K Y)"*
by *(induct l, auto)*

20.12 if the analyse of a finite guarded set gives n then it must also gives one of the keys of Ks

lemma *Guard_invKey_by_list [rule_format]: "ALL l. cnb l = p*
--> Guard n Ks (set l) --> Nonce n:analz (set l)
--> (EX K. K:Ks & Key K:analz (set l))"
apply *(induct p)*

apply *(clarify, drule Guard_must_decrypt, simp, clarify)*
apply *(drule kparts_parts, drule non_empty_crypt, simp)*

apply *(clarify, frule Guard_must_decrypt, simp, clarify)*
apply *(drule_tac P="%G. Nonce n:G"* **in** *analz_pparts_kparts_substD, simp)*
apply *(frule analz_decrypt, simp_all)*
apply *(subgoal_tac "EX l'. kparts (set l) = set l' & cnb l' = cnb l", clarsimp)*
apply *(drule_tac G="insert Y (set l' - {Crypt K Y})"*
and *H="set (decrypt' l' K Y)"* **in** *analz_sub, rule decrypt_minus)*
apply *(rule_tac analz_pparts_kparts_substI, simp)*
apply *(case_tac "K:invKey`Ks")*

apply *(clarsimp, blast)*

apply *(subgoal_tac "Guard n Ks (set (decrypt' l' K Y))")*
apply *(drule_tac x="decrypt' l' K Y"* **in** *spec, simp add: mem_iff)*
apply *(subgoal_tac "Crypt K Y:parts (set l)")*
apply *(drule parts_cnb, rotate_tac -1, simp)*
apply *(clarify, drule_tac X="Key Ka"* **and** *H="insert Y (set l')"* **in** *analz_sub)*
apply *(rule insert_mono, rule set_minus)*
apply *(simp add: analz_insertD, blast)*

apply *(blast dest: kparts_parts)*

apply *(rule_tac H="insert Y (set l')"* **in** *Guard_mono)*
apply *(subgoal_tac "Guard n Ks (set l')", simp)*
apply *(rule_tac K=K* **in** *guard_Crypt, simp add: Guard_def, simp)*
apply *(drule_tac t="set l'"* **in** *sym, simp)*
apply *(rule Guard_kparts, simp, simp)*
apply *(rule_tac B="set l'"* **in** *subset_trans, rule set_minus, blast)*
by *(rule kparts_set)*

lemma *Guard_invKey_finite: "[| Nonce n:analz G; Guard n Ks G; finite G |]*
==> EX K. K:Ks & Key K:analz G"
apply *(drule finite_list, clarify)*
by *(rule Guard_invKey_by_list, auto)*

lemma *Guard_invKey: "[| Nonce n:analz G; Guard n Ks G |]*
==> EX K. K:Ks & Key K:analz G"
by *(auto dest: analz_needs_only_finite Guard_invKey_finite)*

20.13 if the analyse of a finite guarded set and a (possibly infinite) set of keys gives n then it must also gives Ks

lemma *Guard_invKey_keyset: "[| Nonce n:analz (G Un H); Guard n Ks G; finite G;*
keyset H |] ==> EX K. K:Ks & Key K:analz (G Un H)"
apply *(frule_tac P="%G. Nonce n:G"* **and** *G2=G* **in** *analz_keyset_substD, simp_all)*
apply *(drule_tac G="G Un (H Int keysfor G)"* **in** *Guard_invKey_finite)*
by *(auto simp: Guard_def intro: analz_sub)*

end

theory *Guard_Public* **imports** *Guard Public Extensions* **begin**

20.14 Extensions to Theory *Public*

declare *initState.simps [simp del]*

20.14.1 signature

constdefs *sign :: "agent => msg => msg"*
"sign A X == {|Agent A, X, Crypt (priK A) (Hash X)|}"

lemma *sign_inj [iff]: "(sign A X = sign A' X') = (A=A' & X=X')"*
by *(auto simp: sign_def)*

20.14.2 agent associated to a key

constdefs *agt :: "key => agent"*
"agt K == @A. K = priK A | K = pubK A"

lemma *agt_priK [simp]: "agt (priK A) = A"*
by *(simp add: agt_def)*

lemma *agt_pubK [simp]: "agt (pubK A) = A"*

by *(simp add: agt_def)*

20.14.3 basic facts about *initState*

lemma *no_Crypt_in_parts_init [simp]: "Crypt K X ~:parts (initState A)"*
by *(cases A, auto simp: initState.simps)*

lemma *no_Crypt_in_analz_init [simp]: "Crypt K X ~:analz (initState A)"*
by *auto*

lemma *no_priK_in_analz_init [simp]: "A ~:bad*
==> Key (priK A) ~:analz (initState Spy)"
by *(auto simp: initState.simps)*

lemma *priK_notin_initState_Friend [simp]: "A ~= Friend C*
==> Key (priK A) ~: parts (initState (Friend C))"
by *(auto simp: initState.simps)*

lemma *keyset_init [iff]: "keyset (initState A)"*
by *(cases A, auto simp: keyset_def initState.simps)*

20.14.4 sets of private keys

constdefs *priK_set :: "key set => bool"*
"priK_set Ks == ALL K. K:Ks --> (EX A. K = priK A)"

lemma *in_priK_set: "[| priK_set Ks; K:Ks |] ==> EX A. K = priK A"*
by *(simp add: priK_set_def)*

lemma *priK_set1 [iff]: "priK_set {priK A}"*
by *(simp add: priK_set_def)*

lemma *priK_set2 [iff]: "priK_set {priK A, priK B}"*
by *(simp add: priK_set_def)*

20.14.5 sets of good keys

constdefs *good :: "key set => bool"*
"good Ks == ALL K. K:Ks --> agt K ~:bad"

lemma *in_good: "[| good Ks; K:Ks |] ==> agt K ~:bad"*
by *(simp add: good_def)*

lemma *good1 [simp]: "A ~:bad ==> good {priK A}"*
by *(simp add: good_def)*

lemma *good2 [simp]: "[| A ~:bad; B ~:bad |] ==> good {priK A, priK B}"*
by *(simp add: good_def)*

20.14.6 greatest nonce used in a trace, 0 if there is no nonce

consts *greatest :: "event list => nat"*

recdef *greatest "measure size"*
"greatest [] = 0"

```
"greatest (ev # evs) = max (greatest_msg (msg ev)) (greatest evs)"

lemma greatest_is_greatest: "Nonce n:used evs ==> n <= greatest evs"
apply (induct evs, auto simp: initState.simps)
apply (drule used_sub_parts_used, safe)
apply (drule greatest_msg_is_greatest, arith)
by (simp, arith)
```

20.14.7 function giving a new nonce

```
constdefs new :: "event list => nat"
"new evs == Suc (greatest evs)"

lemma new_isnt_used [iff]: "Nonce (new evs) ~:used evs"
by (clarify, drule greatest_is_greatest, auto simp: new_def)
```

20.15 Proofs About Guarded Messages

20.15.1 small hack necessary because priK is defined as the inverse of pubK

```
lemma pubK_is_invKey_priK: "pubK A = invKey (priK A)"
by simp

lemmas pubK_is_invKey_priK_substI = pubK_is_invKey_priK [THEN ssubst]

lemmas invKey_invKey_substI = invKey [THEN ssubst]

lemma "Nonce n:parts {X} ==> Crypt (pubK A) X:guard n {priK A}"
apply (rule pubK_is_invKey_priK_substI, rule invKey_invKey_substI)
by (rule Guard_Nonce, simp+)
```

20.15.2 guardedness results

```
lemma sign_guard [intro]: "X:guard n Ks ==> sign A X:guard n Ks"
by (auto simp: sign_def)

lemma Guard_init [iff]: "Guard n Ks (initState B)"
by (induct B, auto simp: Guard_def initState.simps)

lemma Guard_knows_max': "Guard n Ks (knows_max' C evs)
==> Guard n Ks (knows_max C evs)"
by (simp add: knows_max_def)

lemma Nonce_not_used_Guard_spies [dest]: "Nonce n ~:used evs
==> Guard n Ks (spies evs)"
by (auto simp: Guard_def dest: not_used_not_known parts_sub)

lemma Nonce_not_used_Guard [dest]: "[| evs:p; Nonce n ~:used evs;
Gets_correct p; one_step p |] ==> Guard n Ks (knows (Friend C) evs)"
by (auto simp: Guard_def dest: known_used parts_trans)

lemma Nonce_not_used_Guard_max [dest]: "[| evs:p; Nonce n ~:used evs;
Gets_correct p; one_step p |] ==> Guard n Ks (knows_max (Friend C) evs)"
by (auto simp: Guard_def dest: known_max_used parts_trans)
```

lemma *Nonce_not_used_Guard_max' [dest]: "[| evs:p; Nonce n ~:used evs; Gets_correct p; one_step p |] ==> Guard n Ks (knows_max' (Friend C) evs)"*
apply *(rule_tac H="knows_max (Friend C) evs"* **in** *Guard_mono)*
by *(auto simp: knows_max_def)*

20.15.3 regular protocols

constdefs *regular :: "event list set => bool"*
"regular p == ALL evs A. evs:p --> (Key (priK A):parts (spies evs)) = (A:bad)"

lemma *priK_parts_iff_bad [simp]: "[| evs:p; regular p |] ==>*
(Key (priK A):parts (spies evs)) = (A:bad)"
by *(auto simp: regular_def)*

lemma *priK_analz_iff_bad [simp]: "[| evs:p; regular p |] ==>*
(Key (priK A):analz (spies evs)) = (A:bad)"
by *auto*

lemma *Guard_Nonce_analz: "[| Guard n Ks (spies evs); evs:p;*
priK_set Ks; good Ks; regular p |] ==> Nonce n ~:analz (spies evs)"
apply *(clarify, simp only: knows_decomp)*
apply *(drule Guard_invKey_keyset, simp+, safe)*
apply *(drule in_good, simp)*
apply *(drule in_priK_set, simp+, clarify)*
apply *(frule_tac A=A* **in** *priK_analz_iff_bad)*
by *(simp add: knows_decomp)+*

end

21 Lists of Messages and Lists of Agents

theory *List_Msg* **imports** *Extensions* **begin**

21.1 Implementation of Lists by Messages

21.1.1 nil is represented by any message which is not a pair

syntax *cons :: "msg => msg => msg"*

translations *"cons x l" => "{|x,l|}"*

21.1.2 induction principle

lemma *lmsg_induct: "[| !!x. not_MPair x ==> P x; !!x l. P l ==> P (cons x l) |]*
==> P l"
by *(induct l, auto)*

21.1.3 head

consts *head :: "msg => msg"*

recdef *head "measure size"*
"head (cons x l) = x"

21.1.4 tail

consts *tail :: "msg => msg"*

recdef *tail "measure size"*
"tail (cons x l) = l"

21.1.5 length

consts *len :: "msg => nat"*

recdef *len "measure size"*
"len (cons x l) = Suc (len l)"
"len other = 0"

lemma *len_not_empty: "n < len l ==> EX x l'. l = cons x l'"*
by *(cases l, auto)*

21.1.6 membership

consts *isin :: "msg * msg => bool"*

recdef *isin "measure (%(x,l). size l)"*
"isin (x, cons y l) = (x=y | isin (x,l))"
"isin (x, other) = False"

21.1.7 delete an element

consts *del :: "msg * msg => msg"*

recdef *del "measure (%(x,l). size l)"*
"del (x, cons y l) = (if x=y then l else cons y (del (x,l)))"
"del (x, other) = other"

lemma *notin_del [simp]: "~ isin (x,l) ==> del (x,l) = l"*
by *(induct l, auto)*

lemma *isin_del [rule_format]: "isin (y, del (x,l)) --> isin (y,l)"*
by *(induct l, auto)*

21.1.8 concatenation

consts *app :: "msg * msg => msg"*

recdef *app "measure (%(l,l'). size l)"*
"app (cons x l, l') = cons x (app (l,l'))"
"app (other, l') = l'"

lemma *isin_app [iff]: "isin (x, app(l,l')) = (isin (x,l) | isin (x,l'))"*
by *(induct l, auto)*

21.1.9 replacement

consts *repl :: "msg * nat * msg => msg"*

recdef *repl "measure (%(l,i,x'). i)"*

```
"repl (cons x l, Suc i, x') = cons x (repl (l,i,x'))"
"repl (cons x l, 0, x') = cons x' l"
"repl (other, i, M') = other"
```

21.1.10 ith element

consts `ith :: "msg * nat => msg"`

recdef `ith "measure (%(l,i). i)"`
`"ith (cons x l, Suc i) = ith (l,i)"`
`"ith (cons x l, 0) = x"`
`"ith (other, i) = other"`

lemma `ith_head: "0 < len l ==> ith (l,0) = head l"`
by `(cases l, auto)`

21.1.11 insertion

consts `ins :: "msg * nat * msg => msg"`

recdef `ins "measure (%(l,i,x). i)"`
`"ins (cons x l, Suc i, y) = cons x (ins (l,i,y))"`
`"ins (l, 0, y) = cons y l"`

lemma `ins_head [simp]: "ins (l,0,y) = cons y l"`
by `(cases l, auto)`

21.1.12 truncation

consts `trunc :: "msg * nat => msg"`

recdef `trunc "measure (%(l,i). i)"`
`"trunc (l,0) = l"`
`"trunc (cons x l, Suc i) = trunc (l,i)"`

lemma `trunc_zero [simp]: "trunc (l,0) = l"`
by `(cases l, auto)`

21.2 Agent Lists

21.2.1 set of well-formed agent-list messages

syntax `nil :: msg`

translations `"nil" == "Number 0"`

consts `agl :: "msg set"`

inductive `agl`
intros
`Nil[intro]: "nil:agl"`
`Cons[intro]: "[| A:agent; I:agl |] ==> cons (Agent A) I :agl"`

21.2.2 basic facts about agent lists

lemma `del_in_agl [intro]: "I:agl ==> del (a,I):agl"`

by *(erule agl.induct, auto)*

lemma *app_in_agl [intro]: "[| I:agl; J:agl |] ==> app (I,J):agl"*
by *(erule agl.induct, auto)*

lemma *no_Key_in_agl: "I:agl ==> Key K ~:parts {I}"*
by *(erule agl.induct, auto)*

lemma *no_Nonce_in_agl: "I:agl ==> Nonce n ~:parts {I}"*
by *(erule agl.induct, auto)*

lemma *no_Key_in_appdel: "[| I:agl; J:agl |] ==>*
Key K ~:parts {app (J, del (Agent B, I))}"
by *(rule no_Key_in_agl, auto)*

lemma *no_Nonce_in_appdel: "[| I:agl; J:agl |] ==>*
Nonce n ~:parts {app (J, del (Agent B, I))}"
by *(rule no_Nonce_in_agl, auto)*

lemma *no_Crypt_in_agl: "I:agl ==> Crypt K X ~:parts {I}"*
by *(erule agl.induct, auto)*

lemma *no_Crypt_in_appdel: "[| I:agl; J:agl |] ==>*
Crypt K X ~:parts {app (J, del (Agent B,I))}"
by *(rule no_Crypt_in_agl, auto)*

end

22 Protocol P1

theory *P1* **imports** *Guard_Public List_Msg* **begin**

22.1 Protocol Definition

22.1.1 offer chaining: B chains his offer for A with the head offer of L for sending it to C

constdefs *chain :: "agent => nat => agent => msg => agent => msg"*
"chain B ofr A L C ==
let m1= Crypt (pubK A) (Nonce ofr) in
let m2= Hash {|head L, Agent C|} in
sign B {|m1,m2|}"

declare *Let_def [simp]*

lemma *chain_inj [iff]: "(chain B ofr A L C = chain B' ofr' A' L' C')*
= (B=B' & ofr=ofr' & A=A' & head L = head L' & C=C')"
by *(auto simp: chain_def Let_def)*

lemma *Nonce_in_chain [iff]: "Nonce ofr:parts {chain B ofr A L C}"*
by *(auto simp: chain_def sign_def)*

22.1.2 agent whose key is used to sign an offer

consts *shop :: "msg => msg"*

recdef *shop "measure size"*
"shop {|B,X,Crypt K H|} = Agent (agt K)"

lemma *shop_chain [simp]: "shop (chain B ofr A L C) = Agent B"*
by *(simp add: chain_def sign_def)*

22.1.3 nonce used in an offer

consts *nonce :: "msg => msg"*

recdef *nonce "measure size"*
"nonce {|B,{|Crypt K ofr,m2|},CryptH|} = ofr"

lemma *nonce_chain [simp]: "nonce (chain B ofr A L C) = Nonce ofr"*
by *(simp add: chain_def sign_def)*

22.1.4 next shop

consts *next_shop :: "msg => agent"*

recdef *next_shop "measure size"*
"next_shop {|B,{|m1,Hash{|headL,Agent C|}|},CryptH|} = C"

lemma *next_shop_chain [iff]: "next_shop (chain B ofr A L C) = C"*
by *(simp add: chain_def sign_def)*

22.1.5 anchor of the offer list

constdefs *anchor :: "agent => nat => agent => msg"*
"anchor A n B == chain A n A (cons nil nil) B"

lemma *anchor_inj [iff]: "(anchor A n B = anchor A' n' B')*
= (A=A' & n=n' & B=B')"
by *(auto simp: anchor_def)*

lemma *Nonce_in_anchor [iff]: "Nonce n:parts {anchor A n B}"*
by *(auto simp: anchor_def)*

lemma *shop_anchor [simp]: "shop (anchor A n B) = Agent A"*
by *(simp add: anchor_def)*

lemma *nonce_anchor [simp]: "nonce (anchor A n B) = Nonce n"*
by *(simp add: anchor_def)*

lemma *next_shop_anchor [iff]: "next_shop (anchor A n B) = B"*
by *(simp add: anchor_def)*

22.1.6 request event

constdefs *reqm :: "agent => nat => nat => msg => agent => msg"*
"reqm A r n I B == {|Agent A, Number r, cons (Agent A) (cons (Agent B) I),
cons (anchor A n B) nil|}"

lemma *reqm_inj [iff]: "(reqm A r n I B = reqm A' r' n' I' B')*
= (A=A' & r=r' & n=n' & I=I' & B=B')"
by *(auto simp: reqm_def)*

lemma *Nonce_in_reqm [iff]: "Nonce n:parts {reqm A r n I B}"*
by *(auto simp: reqm_def)*

constdefs *req :: "agent => nat => nat => msg => agent => event"*
"req A r n I B == Says A B (reqm A r n I B)"

lemma *req_inj [iff]: "(req A r n I B = req A' r' n' I' B')*
= (A=A' & r=r' & n=n' & I=I' & B=B')"
by *(auto simp: req_def)*

22.1.7 propose event

constdefs *prom :: "agent => nat => agent => nat => msg => msg =>*
msg => agent => msg"
"prom B ofr A r I L J C == {|Agent A, Number r,
app (J, del (Agent B, I)), cons (chain B ofr A L C) L|}"

lemma *prom_inj [dest]: "prom B ofr A r I L J C*
= prom B' ofr' A' r' I' L' J' C'
==> B=B' & ofr=ofr' & A=A' & r=r' & L=L' & C=C'"
by *(auto simp: prom_def)*

lemma *Nonce_in_prom [iff]: "Nonce ofr:parts {prom B ofr A r I L J C}"*
by *(auto simp: prom_def)*

constdefs *pro :: "agent => nat => agent => nat => msg => msg =>*
msg => agent => event"
"pro B ofr A r I L J C == Says B C (prom B ofr A r I L J C)"

lemma *pro_inj [dest]: "pro B ofr A r I L J C = pro B' ofr' A' r' I' L' J'*
C'
==> B=B' & ofr=ofr' & A=A' & r=r' & L=L' & C=C'"
by *(auto simp: pro_def dest: prom_inj)*

22.1.8 protocol

consts *p1 :: "event list set"*

inductive *p1*
intros

Nil: "[]:p1"

Fake: "[| evsf:p1; X:synth (analz (spies evsf)) |] ==> Says Spy B X # evsf
: p1"

Request: "[| evsr:p1; Nonce n ~:used evsr; I:agl |] ==> req A r n I B # evsr
: p1"

Propose: "[| evsp:p1; Says A' B {|Agent A,Number r,I,cons M L|}:set evsp;

```
I:agl; J:agl; isin (Agent C, app (J, del (Agent B, I)));
Nonce ofr ~:used evsp |] ==> pro B ofr A r I (cons M L) J C # evsp : p1"
```

22.1.9 Composition of Traces

```
lemma "evs':p1 ==>
       evs:p1 & (ALL n. Nonce n:used evs' --> Nonce n ~:used evs) -->
       evs'@evs : p1"
apply (erule p1.induct, safe)
apply (simp_all add: used_ConsI)
apply (erule p1.Fake, erule synth_sub, rule analz_mono, rule knows_sub_app)
apply (erule p1.Request, safe, simp_all add: req_def, force)
apply (erule_tac A'=A' in p1.Propose, simp_all)
apply (drule_tac x=ofr in spec, simp add: pro_def, blast)
apply (erule_tac A'=A' in p1.Propose, auto simp: pro_def)
done
```

22.1.10 Valid Offer Lists

```
consts valid :: "agent => nat => agent => msg set"

inductive "valid A n B"
intros
Request [intro]: "cons (anchor A n B) nil:valid A n B"

Propose [intro]: "L:valid A n B
==> cons (chain (next_shop (head L)) ofr A L C) L:valid A n B"
```

22.1.11 basic properties of valid

```
lemma valid_not_empty: "L:valid A n B ==> EX M L'. L = cons M L'"
by (erule valid.cases, auto)

lemma valid_pos_len: "L:valid A n B ==> 0 < len L"
by (erule valid.induct, auto)
```

22.1.12 offers of an offer list

```
constdefs offer_nonces :: "msg => msg set"
"offer_nonces L == {X. X:parts {L} & (EX n. X = Nonce n)}"
```

22.1.13 the originator can get the offers

```
lemma "L:valid A n B ==> offer_nonces L <= analz (insert L (initState A))"
by (erule valid.induct, auto simp: anchor_def chain_def sign_def
offer_nonces_def initState.simps)
```

22.1.14 list of offers

```
consts offers :: "msg => msg"

recdef offers "measure size"
"offers (cons M L) = cons {|shop M, nonce M|} (offers L)"
"offers other = nil"
```

22.1.15 list of agents whose keys are used to sign a list of offers

consts *shops :: "msg => msg"*

recdef *shops "measure size"*
"shops (cons M L) = cons (shop M) (shops L)"
"shops other = other"

lemma *shops_in_agl: "L:valid A n B ==> shops L:agl"*
by *(erule valid.induct, auto simp: anchor_def chain_def sign_def)*

22.1.16 builds a trace from an itinerary

consts *offer_list :: "agent * nat * agent * msg * nat => msg"*

recdef *offer_list "measure (%(A,n,B,I,ofr). size I)"*
"offer_list (A,n,B,nil,ofr) = cons (anchor A n B) nil"
"offer_list (A,n,B,cons (Agent C) I,ofr) = (
let L = offer_list (A,n,B,I,Suc ofr) in
cons (chain (next_shop (head L)) ofr A L C) L)"

lemma *"I:agl ==> ALL ofr. offer_list (A,n,B,I,ofr):valid A n B"*
by *(erule agl.induct, auto)*

consts *trace :: "agent * nat * agent * nat * msg * msg * msg*
=> event list"

recdef *trace "measure (%(B,ofr,A,r,I,L,K). size K)"*
"trace (B,ofr,A,r,I,L,nil) = []"
"trace (B,ofr,A,r,I,L,cons (Agent D) K) = (
let C = (if K=nil then B else agt_nb (head K)) in
let I' = (if K=nil then cons (Agent A) (cons (Agent B) I)
else cons (Agent A) (app (I, cons (head K) nil))) in
let I'' = app (I, cons (head K) nil) in
pro C (Suc ofr) A r I' L nil D
 # trace (B,Suc ofr,A,r,I'',tail L,K))"

constdefs *trace' :: "agent => nat => nat => msg => agent => nat => event list"*
"trace' A r n I B ofr == (
let AI = cons (Agent A) I in
let L = offer_list (A,n,B,AI,ofr) in
trace (B,ofr,A,r,nil,L,AI))"

declare *trace'_def [simp]*

22.1.17 there is a trace in which the originator receives a valid answer

lemma *p1_not_empty: "evs:p1 ==> req A r n I B:set evs -->*
(EX evs'. evs'@evs:p1 & pro B' ofr A r I' L J A:set evs' & L:valid A n B)"
oops

22.2 properties of protocol P1

publicly verifiable forward integrity: anyone can verify the validity of an offer list

22.2.1 strong forward integrity: except the last one, no offer can be modified

lemma *strong_forward_integrity: "ALL L. Suc i < len L*
--> L:valid A n B & repl (L,Suc i,M):valid A n B --> M = ith (L,Suc i)"
apply *(induct i)*

apply *clarify*
apply *(frule len_not_empty, clarsimp)*
apply *(frule len_not_empty, clarsimp)*
apply *(ind_cases "{|x,xa,l'a|}:valid A n B")*
apply *(ind_cases "{|x,M,l'a|}:valid A n B")*
apply *(simp add: chain_def)*

apply *clarify*
apply *(frule len_not_empty, clarsimp)*
apply *(ind_cases "{|x,repl(l',Suc na,M)|}:valid A n B")*
apply *(frule len_not_empty, clarsimp)*
apply *(ind_cases "{|x,l'|}:valid A n B")*
by *(drule_tac x=l'* **in** *spec, simp, blast)*

22.2.2 insertion resilience: except at the beginning, no offer can be inserted

lemma *chain_isnt_head [simp]: "L:valid A n B ==>*
head L ~= chain (next_shop (head L)) ofr A L C"
by *(erule valid.induct, auto simp: chain_def sign_def anchor_def)*

lemma *insertion_resilience: "ALL L. L:valid A n B --> Suc i < len L*
--> ins (L,Suc i,M) ~:valid A n B"
apply *(induct i)*

apply *clarify*
apply *(frule len_not_empty, clarsimp)*
apply *(ind_cases "{|x,l'|}:valid A n B", simp)*
apply *(ind_cases "{|x,M,l'|}:valid A n B", clarsimp)*
apply *(ind_cases "{|head l',l'|}:valid A n B", simp, simp)*

apply *clarify*
apply *(frule len_not_empty, clarsimp)*
apply *(ind_cases "{|x,l'|}:valid A n B")*
apply *(frule len_not_empty, clarsimp)*
apply *(ind_cases "{|x,ins(l',Suc na,M)|}:valid A n B")*
apply *(frule len_not_empty, clarsimp)*
by *(drule_tac x=l'* **in** *spec, clarsimp)*

22.2.3 truncation resilience: only shop i can truncate at offer i

lemma *truncation_resilience: "ALL L. L:valid A n B --> Suc i < len L*

```
--> cons M (trunc (L,Suc i)):valid A n B --> shop M = shop (ith (L,i))"
apply (induct i)

apply clarify
apply (frule len_not_empty, clarsimp)
apply (ind_cases "{|x,l'|}:valid A n B")
apply (frule len_not_empty, clarsimp)
apply (ind_cases "{|M,l'|}:valid A n B")
apply (frule len_not_empty, clarsimp, simp)

apply clarify
apply (frule len_not_empty, clarsimp)
apply (ind_cases "{|x,l'|}:valid A n B")
apply (frule len_not_empty, clarsimp)
by (drule_tac x=l' in spec, clarsimp)
```

22.2.4 declarations for tactics

```
declare knows_Spy_partsEs [elim]
declare Fake_parts_insert [THEN subsetD, dest]
declare initState.simps [simp del]
```

22.2.5 get components of a message

```
lemma get_ML [dest]: "Says A' B {|A,r,I,M,L|}:set evs ==>
M:parts (spies evs) & L:parts (spies evs)"
by blast
```

22.2.6 general properties of p1

```
lemma reqm_neq_prom [iff]:
"reqm A r n I B ~= prom B' ofr A' r' I' (cons M L) J C"
by (auto simp: reqm_def prom_def)

lemma prom_neq_reqm [iff]:
"prom B' ofr A' r' I' (cons M L) J C ~= reqm A r n I B"
by (auto simp: reqm_def prom_def)

lemma req_neq_pro [iff]: "req A r n I B ~= pro B' ofr A' r' I' (cons M L)
J C"
by (auto simp: req_def pro_def)

lemma pro_neq_req [iff]: "pro B' ofr A' r' I' (cons M L) J C ~= req A r n
I B"
by (auto simp: req_def pro_def)

lemma p1_has_no_Gets: "evs:p1 ==> ALL A X. Gets A X ~:set evs"
by (erule p1.induct, auto simp: req_def pro_def)

lemma p1_is_Gets_correct [iff]: "Gets_correct p1"
by (auto simp: Gets_correct_def dest: p1_has_no_Gets)

lemma p1_is_one_step [iff]: "one_step p1"
by (unfold one_step_def, clarify, ind_cases "ev#evs:p1", auto)
```

```
lemma p1_has_only_Says' [rule_format]: "evs:p1 ==>
ev:set evs --> (EX A B X. ev=Says A B X)"
by (erule p1.induct, auto simp: req_def pro_def)

lemma p1_has_only_Says [iff]: "has_only_Says p1"
by (auto simp: has_only_Says_def dest: p1_has_only_Says')

lemma p1_is_regular [iff]: "regular p1"
apply (simp only: regular_def, clarify)
apply (erule_tac p1.induct)
apply (simp_all add: initState.simps knows.simps pro_def prom_def
                     req_def reqm_def anchor_def chain_def sign_def)
by (auto dest: no_Key_in_agl no_Key_in_appdel parts_trans)
```

22.2.7 private keys are safe

```
lemma priK_parts_Friend_imp_bad [rule_format,dest]:
     "[| evs:p1; Friend B ~= A |]
      ==> (Key (priK A):parts (knows (Friend B) evs)) --> (A:bad)"
apply (erule p1.induct)
apply (simp_all add: initState.simps knows.simps pro_def prom_def
                req_def reqm_def anchor_def chain_def sign_def, blast)
apply (blast dest: no_Key_in_agl)
apply (auto del: parts_invKey disjE  dest: parts_trans
            simp add: no_Key_in_appdel)
done

lemma priK_analz_Friend_imp_bad [rule_format,dest]:
     "[| evs:p1; Friend B ~= A |]
==> (Key (priK A):analz (knows (Friend B) evs)) --> (A:bad)"
by auto

lemma priK_notin_knows_max_Friend: "[| evs:p1; A ~:bad; A ~= Friend C |]
==> Key (priK A) ~:analz (knows_max (Friend C) evs)"
apply (rule not_parts_not_analz, simp add: knows_max_def, safe)
apply (drule_tac H="spies' evs" in parts_sub)
apply (rule_tac p=p1 in knows_max'_sub_spies', simp+)
apply (drule_tac H="spies evs" in parts_sub)
by (auto dest: knows'_sub_knows [THEN subsetD] priK_notin_initState_Friend)
```

22.2.8 general guardedness properties

```
lemma agl_guard [intro]: "I:agl ==> I:guard n Ks"
by (erule agl.induct, auto)

lemma Says_to_knows_max'_guard: "[| Says A' C {|A'',r,I,L|}:set evs;
Guard n Ks (knows_max' C evs) |] ==> L:guard n Ks"
by (auto dest: Says_to_knows_max')

lemma Says_from_knows_max'_guard: "[| Says C A' {|A'',r,I,L|}:set evs;
Guard n Ks (knows_max' C evs) |] ==> L:guard n Ks"
by (auto dest: Says_from_knows_max')

lemma Says_Nonce_not_used_guard: "[| Says A' B {|A'',r,I,L|}:set evs;
Nonce n ~:used evs |] ==> L:guard n Ks"
```

by *(drule not_used_not_parts, auto)*

22.2.9 guardedness of messages

lemma *chain_guard [iff]: "chain B ofr A L C:guard n {priK A}"*
by *(case_tac "ofr=n", auto simp: chain_def sign_def)*

lemma *chain_guard_Nonce_neq [intro]: "n ~= ofr*
==> chain B ofr A' L C:guard n {priK A}"
by *(auto simp: chain_def sign_def)*

lemma *anchor_guard [iff]: "anchor A n' B:guard n {priK A}"*
by *(case_tac "n'=n", auto simp: anchor_def)*

lemma *anchor_guard_Nonce_neq [intro]: "n ~= n'*
==> anchor A' n' B:guard n {priK A}"
by *(auto simp: anchor_def)*

lemma *reqm_guard [intro]: "I:agl ==> reqm A r n' I B:guard n {priK A}"*
by *(case_tac "n'=n", auto simp: reqm_def)*

lemma *reqm_guard_Nonce_neq [intro]: "[| n ~= n'; I:agl |]*
==> reqm A' r n' I B:guard n {priK A}"
by *(auto simp: reqm_def)*

lemma *prom_guard [intro]: "[| I:agl; J:agl; L:guard n {priK A} |]*
==> prom B ofr A r I L J C:guard n {priK A}"
by *(auto simp: prom_def)*

lemma *prom_guard_Nonce_neq [intro]: "[| n ~= ofr; I:agl; J:agl;*
L:guard n {priK A} |] ==> prom B ofr A' r I L J C:guard n {priK A}"
by *(auto simp: prom_def)*

22.2.10 Nonce uniqueness

lemma *uniq_Nonce_in_chain [dest]: "Nonce k:parts {chain B ofr A L C} ==>*
k=ofr"
by *(auto simp: chain_def sign_def)*

lemma *uniq_Nonce_in_anchor [dest]: "Nonce k:parts {anchor A n B} ==> k=n"*
by *(auto simp: anchor_def chain_def sign_def)*

lemma *uniq_Nonce_in_reqm [dest]: "[| Nonce k:parts {reqm A r n I B};*
I:agl |] ==> k=n"
by *(auto simp: reqm_def dest: no_Nonce_in_agl)*

lemma *uniq_Nonce_in_prom [dest]: "[| Nonce k:parts {prom B ofr A r I L J*
C};
I:agl; J:agl; Nonce k ~:parts {L} |] ==> k=ofr"
by *(auto simp: prom_def dest: no_Nonce_in_agl no_Nonce_in_appdel)*

22.2.11 requests are guarded

lemma *req_imp_Guard [rule_format]: "[| evs:p1; A ~:bad |] ==>*
req A r n I B:set evs --> Guard n {priK A} (spies evs)"

```
apply (erule p1.induct, simp)
apply (simp add: req_def knows.simps, safe)
apply (erule in_synth_Guard, erule Guard_analz, simp)
by (auto simp: req_def pro_def dest: Says_imp_knows_Spy)

lemma req_imp_Guard_Friend: "[| evs:p1; A ~:bad; req A r n I B:set evs |]
==> Guard n {priK A} (knows_max (Friend C) evs)"
apply (rule Guard_knows_max')
apply (rule_tac H="spies evs" in Guard_mono)
apply (rule req_imp_Guard, simp+)
apply (rule_tac B="spies' evs" in subset_trans)
apply (rule_tac p=p1 in knows_max'_sub_spies', simp+)
by (rule knows'_sub_knows)
```

22.2.12 propositions are guarded

```
lemma pro_imp_Guard [rule_format]: "[| evs:p1; B ~:bad; A ~:bad |] ==>
pro B ofr A r I (cons M L) J C:set evs --> Guard ofr {priK A} (spies evs)"
apply (erule p1.induct)

apply simp

apply (simp add: pro_def, safe)

apply (erule in_synth_Guard, drule Guard_analz, simp, simp)

apply simp

apply (simp, simp add: req_def pro_def, blast)

apply (simp add: pro_def)
apply (blast dest: prom_inj Says_Nonce_not_used_guard Nonce_not_used_Guard)

apply simp
apply safe
apply (simp add: pro_def)
apply (blast dest: prom_inj Says_Nonce_not_used_guard)

apply (simp add: pro_def)
apply (blast dest: Says_imp_knows_Spy)

apply (simp add: pro_def)
apply (blast dest: prom_inj Says_Nonce_not_used_guard Nonce_not_used_Guard)

apply simp
apply safe

apply (simp add: pro_def)
apply (blast dest: prom_inj Says_Nonce_not_used_guard)

apply (simp add: pro_def)
by (blast dest: Says_imp_knows_Spy)

lemma pro_imp_Guard_Friend: "[| evs:p1; B ~:bad; A ~:bad;
```

```
pro B ofr A r I (cons M L) J C:set evs |]
==> Guard ofr {priK A} (knows_max (Friend D) evs)"
apply (rule Guard_knows_max')
apply (rule_tac H="spies evs" in Guard_mono)
apply (rule pro_imp_Guard, simp+)
apply (rule_tac B="spies' evs" in subset_trans)
apply (rule_tac p=p1 in knows_max'_sub_spies', simp+)
by (rule knows'_sub_knows)
```

22.2.13 data confidentiality: no one other than the originator can decrypt the offers

```
lemma Nonce_req_notin_spies: "[| evs:p1; req A r n I B:set evs; A ~:bad |]
==> Nonce n ~:analz (spies evs)"
by (frule req_imp_Guard, simp+, erule Guard_Nonce_analz, simp+)

lemma Nonce_req_notin_knows_max_Friend: "[| evs:p1; req A r n I B:set evs;
A ~:bad; A ~= Friend C |] ==> Nonce n ~:analz (knows_max (Friend C) evs)"
apply (clarify, frule_tac C=C in req_imp_Guard_Friend, simp+)
apply (simp add: knows_max_def, drule Guard_invKey_keyset, simp+)
by (drule priK_notin_knows_max_Friend, auto simp: knows_max_def)

lemma Nonce_pro_notin_spies: "[| evs:p1; B ~:bad; A ~:bad;
pro B ofr A r I (cons M L) J C:set evs |] ==> Nonce ofr ~:analz (spies evs)"
by (frule pro_imp_Guard, simp+, erule Guard_Nonce_analz, simp+)

lemma Nonce_pro_notin_knows_max_Friend: "[| evs:p1; B ~:bad; A ~:bad;
A ~= Friend D; pro B ofr A r I (cons M L) J C:set evs |]
==> Nonce ofr ~:analz (knows_max (Friend D) evs)"
apply (clarify, frule_tac A=A in pro_imp_Guard_Friend, simp+)
apply (simp add: knows_max_def, drule Guard_invKey_keyset, simp+)
by (drule priK_notin_knows_max_Friend, auto simp: knows_max_def)
```

22.2.14 non repudiability: an offer signed by B has been sent by B

```
lemma Crypt_reqm: "[| Crypt (priK A) X:parts {reqm A' r n I B}; I:agl |]
==> A=A'"
by (auto simp: reqm_def anchor_def chain_def sign_def dest: no_Crypt_in_agl)

lemma Crypt_prom: "[| Crypt (priK A) X:parts {prom B ofr A' r I L J C};
I:agl; J:agl |] ==> A=B | Crypt (priK A) X:parts {L}"
apply (simp add: prom_def anchor_def chain_def sign_def)
by (blast dest: no_Crypt_in_agl no_Crypt_in_appdel)

lemma Crypt_safeness: "[| evs:p1; A ~:bad |] ==> Crypt (priK A) X:parts (spies
evs)
--> (EX B Y. Says A B Y:set evs & Crypt (priK A) X:parts {Y})"
apply (erule p1.induct)

apply simp

apply clarsimp
apply (drule_tac P="%G. Crypt (priK A) X:G" in parts_insert_substD, simp)
apply (erule disjE)
```

```
apply (drule_tac K="priK A" in Crypt_synth, simp+, blast, blast)

apply (simp add: req_def, clarify)
apply (drule_tac P="%G. Crypt (priK A) X:G" in parts_insert_substD, simp)
apply (erule disjE)
apply (frule Crypt_reqm, simp, clarify)
apply (rule_tac x=B in exI, rule_tac x="reqm A r n I B" in exI, simp, blast)

apply (simp add: pro_def, clarify)
apply (drule_tac P="%G. Crypt (priK A) X:G" in parts_insert_substD, simp)
apply (rotate_tac -1, erule disjE)
apply (frule Crypt_prom, simp, simp)
apply (rotate_tac -1, erule disjE)
apply (rule_tac x=C in exI)
apply (rule_tac x="prom B ofr Aa r I (cons M L) J C" in exI, blast)
apply (subgoal_tac "cons M L:parts (spies evsp)")
apply (drule_tac G="{cons M L}" and H="spies evsp" in parts_trans, blast,
blast)
apply (drule Says_imp_spies, rotate_tac -1, drule parts.Inj)
apply (drule parts.Snd, drule parts.Snd, drule parts.Snd)
by auto

lemma Crypt_Hash_imp_sign: "[| evs:p1; A ~:bad |] ==>
Crypt (priK A) (Hash X):parts (spies evs)
--> (EX B Y. Says A B Y:set evs & sign A X:parts {Y})"
apply (erule p1.induct)

apply simp

apply clarsimp
apply (drule_tac P="%G. Crypt (priK A) (Hash X):G" in parts_insert_substD)
apply simp
apply (erule disjE)
apply (drule_tac K="priK A" in Crypt_synth, simp+, blast, blast)

apply (simp add: req_def, clarify)
apply (drule_tac P="%G. Crypt (priK A) (Hash X):G" in parts_insert_substD)
apply simp
apply (erule disjE)
apply (frule Crypt_reqm, simp+)
apply (rule_tac x=B in exI, rule_tac x="reqm Aa r n I B" in exI)
apply (simp add: reqm_def sign_def anchor_def no_Crypt_in_agl)
apply (simp add: chain_def sign_def, blast)

apply (simp add: pro_def, clarify)
apply (drule_tac P="%G. Crypt (priK A) (Hash X):G" in parts_insert_substD)
apply simp
apply (rotate_tac -1, erule disjE)
apply (simp add: prom_def sign_def no_Crypt_in_agl no_Crypt_in_appdel)
apply (simp add: chain_def sign_def)
apply (rotate_tac -1, erule disjE)
apply (rule_tac x=C in exI)
apply (rule_tac x="prom B ofr Aa r I (cons M L) J C" in exI)
apply (simp add: prom_def chain_def sign_def)
```

```
apply (erule impE)
apply (blast dest: get_ML parts_sub)
apply (blast del: MPair_parts)+
done

lemma sign_safeness: "[| evs:p1; A ~:bad |] ==> sign A X:parts (spies evs)
--> (EX B Y. Says A B Y:set evs & sign A X:parts {Y})"
apply (clarify, simp add: sign_def, frule parts.Snd)
apply (blast dest: Crypt_Hash_imp_sign [unfolded sign_def])
done

end
```

23 Protocol P2

```
theory P2 imports Guard_Public List_Msg begin
```

23.1 Protocol Definition

Like P1 except the definitions of *chain*, *shop*, *next_shop* and *nonce*

23.1.1 offer chaining: B chains his offer for A with the head offer of L for sending it to C

```
constdefs chain :: "agent => nat => agent => msg => agent => msg"
"chain B ofr A L C ==
let m1= sign B (Nonce ofr) in
let m2= Hash {|head L, Agent C|} in
{|Crypt (pubK A) m1, m2|}"

declare Let_def [simp]

lemma chain_inj [iff]: "(chain B ofr A L C = chain B' ofr' A' L' C')
= (B=B' & ofr=ofr' & A=A' & head L = head L' & C=C')"
by (auto simp: chain_def Let_def)

lemma Nonce_in_chain [iff]: "Nonce ofr:parts {chain B ofr A L C}"
by (auto simp: chain_def sign_def)
```

23.1.2 agent whose key is used to sign an offer

```
consts shop :: "msg => msg"

recdef shop "measure size"
"shop {|Crypt K {|B,ofr,Crypt K' H|},m2|} = Agent (agt K')"

lemma shop_chain [simp]: "shop (chain B ofr A L C) = Agent B"
by (simp add: chain_def sign_def)
```

23.1.3 nonce used in an offer

```
consts nonce :: "msg => msg"
```

recdef *nonce "measure size"*
"nonce {|Crypt K {|B,ofr,CryptH|},m2|} = ofr"

lemma *nonce_chain [simp]: "nonce (chain B ofr A L C) = Nonce ofr"*
by *(simp add: chain_def sign_def)*

23.1.4 next shop

consts *next_shop :: "msg => agent"*

recdef *next_shop "measure size"*
"next_shop {|m1,Hash {|headL,Agent C|}|} = C"

lemma *"next_shop (chain B ofr A L C) = C"*
by *(simp add: chain_def sign_def)*

23.1.5 anchor of the offer list

constdefs *anchor :: "agent => nat => agent => msg"*
"anchor A n B == chain A n A (cons nil nil) B"

lemma *anchor_inj [iff]:*
"(anchor A n B = anchor A' n' B') = (A=A' & n=n' & B=B')"
by *(auto simp: anchor_def)*

lemma *Nonce_in_anchor [iff]: "Nonce n:parts {anchor A n B}"*
by *(auto simp: anchor_def)*

lemma *shop_anchor [simp]: "shop (anchor A n B) = Agent A"*
by *(simp add: anchor_def)*

23.1.6 request event

constdefs *reqm :: "agent => nat => nat => msg => agent => msg"*
"reqm A r n I B == {|Agent A, Number r, cons (Agent A) (cons (Agent B) I),
cons (anchor A n B) nil|}"

lemma *reqm_inj [iff]: "(reqm A r n I B = reqm A' r' n' I' B')*
= (A=A' & r=r' & n=n' & I=I' & B=B')"
by *(auto simp: reqm_def)*

lemma *Nonce_in_reqm [iff]: "Nonce n:parts {reqm A r n I B}"*
by *(auto simp: reqm_def)*

constdefs *req :: "agent => nat => nat => msg => agent => event"*
"req A r n I B == Says A B (reqm A r n I B)"

lemma *req_inj [iff]: "(req A r n I B = req A' r' n' I' B')*
= (A=A' & r=r' & n=n' & I=I' & B=B')"
by *(auto simp: req_def)*

23.1.7 propose event

constdefs *prom :: "agent => nat => agent => nat => msg => msg =>*
msg => agent => msg"

```
"prom B ofr A r I L J C == {|Agent A, Number r,
app (J, del (Agent B, I)), cons (chain B ofr A L C) L|}"
```

lemma *prom_inj [dest]: "prom B ofr A r I L J C = prom B' ofr' A' r' I' L' J' C'*
==> B=B' & ofr=ofr' & A=A' & r=r' & L=L' & C=C'"
by *(auto simp: prom_def)*

lemma *Nonce_in_prom [iff]: "Nonce ofr:parts {prom B ofr A r I L J C}"*
by *(auto simp: prom_def)*

constdefs *pro :: "agent => nat => agent => nat => msg => msg =>*
msg => agent => event"
"pro B ofr A r I L J C == Says B C (prom B ofr A r I L J C)"

lemma *pro_inj [dest]: "pro B ofr A r I L J C = pro B' ofr' A' r' I' L' J' C'*
==> B=B' & ofr=ofr' & A=A' & r=r' & L=L' & C=C'"
by *(auto simp: pro_def dest: prom_inj)*

23.1.8 protocol

consts *p2 :: "event list set"*

inductive *p2*
intros

Nil: "[]:p2"

Fake: "[| evsf:p2; X:synth (analz (spies evsf)) |] ==> Says Spy B X # evsf : p2"

Request: "[| evsr:p2; Nonce n ~:used evsr; I:agl |] ==> req A r n I B # evsr : p2"

Propose: "[| evsp:p2; Says A' B {|Agent A,Number r,I,cons M L|}:set evsp; I:agl; J:agl; isin (Agent C, app (J, del (Agent B, I)));
Nonce ofr ~:used evsp |] ==> pro B ofr A r I (cons M L) J C # evsp : p2"

23.1.9 valid offer lists

consts *valid :: "agent => nat => agent => msg set"*

inductive *"valid A n B"*
intros
Request [intro]: "cons (anchor A n B) nil:valid A n B"

Propose [intro]: "L:valid A n B
==> cons (chain (next_shop (head L)) ofr A L C) L:valid A n B"

23.1.10 basic properties of valid

lemma *valid_not_empty: "L:valid A n B ==> EX M L'. L = cons M L'"*
by *(erule valid.cases, auto)*

lemma *valid_pos_len: "L:valid A n B ==> 0 < len L"*
by *(erule valid.induct, auto)*

23.1.11 list of offers

consts *offers :: "msg => msg"*

recdef *offers "measure size"*
"offers (cons M L) = cons {|shop M, nonce M|} (offers L)"
"offers other = nil"

23.2 Properties of Protocol P2

same as *P1_Prop* except that publicly verifiable forward integrity is replaced by forward privacy

23.3 strong forward integrity: except the last one, no offer can be modified

lemma *strong_forward_integrity: "ALL L. Suc i < len L*
--> L:valid A n B --> repl (L,Suc i,M):valid A n B --> M = ith (L,Suc i)"
apply *(induct i)*

apply *clarify*
apply *(frule len_not_empty, clarsimp)*
apply *(frule len_not_empty, clarsimp)*
apply *(ind_cases "{|x,xa,l'a|}:valid A n B")*
apply *(ind_cases "{|x,M,l'a|}:valid A n B")*
apply *(simp add: chain_def)*

apply *clarify*
apply *(frule len_not_empty, clarsimp)*
apply *(ind_cases "{|x,repl(l',Suc na,M)|}:valid A n B")*
apply *(frule len_not_empty, clarsimp)*
apply *(ind_cases "{|x,l'|}:valid A n B")*
by *(drule_tac x=l'* **in** *spec, simp, blast)*

23.4 insertion resilience: except at the beginning, no offer can be inserted

lemma *chain_isnt_head [simp]: "L:valid A n B ==>*
head L ~= chain (next_shop (head L)) ofr A L C"
by *(erule valid.induct, auto simp: chain_def sign_def anchor_def)*

lemma *insertion_resilience: "ALL L. L:valid A n B --> Suc i < len L*
--> ins (L,Suc i,M) ~:valid A n B"
apply *(induct i)*

apply *clarify*
apply *(frule len_not_empty, clarsimp)*
apply *(ind_cases "{|x,l'|}:valid A n B", simp)*
apply *(ind_cases "{|x,M,l'|}:valid A n B", clarsimp)*
apply *(ind_cases "{|head l',l'|}:valid A n B", simp, simp)*

apply *clarify*
apply *(frule len_not_empty, clarsimp)*
apply *(ind_cases "{|x,l'|}:valid A n B")*
apply *(frule len_not_empty, clarsimp)*
apply *(ind_cases "{|x,ins(l',Suc na,M)|}:valid A n B")*
apply *(frule len_not_empty, clarsimp)*
by *(drule_tac x=l'* **in** *spec, clarsimp)*

23.5 truncation resilience: only shop i can truncate at offer i

lemma *truncation_resilience: "ALL L. L:valid A n B --> Suc i < len L*
--> cons M (trunc (L,Suc i)):valid A n B --> shop M = shop (ith (L,i))"
apply *(induct i)*

apply *clarify*
apply *(frule len_not_empty, clarsimp)*
apply *(ind_cases "{|x,l'|}:valid A n B")*
apply *(frule len_not_empty, clarsimp)*
apply *(ind_cases "{|M,l'|}:valid A n B")*
apply *(frule len_not_empty, clarsimp, simp)*

apply *clarify*
apply *(frule len_not_empty, clarsimp)*
apply *(ind_cases "{|x,l'|}:valid A n B")*
apply *(frule len_not_empty, clarsimp)*
by *(drule_tac x=l'* **in** *spec, clarsimp)*

23.6 declarations for tactics

declare *knows_Spy_partsEs [elim]*
declare *Fake_parts_insert [THEN subsetD, dest]*
declare *initState.simps [simp del]*

23.7 get components of a message

lemma *get_ML [dest]: "Says A' B {|A,R,I,M,L|}:set evs ==>*
M:parts (spies evs) & L:parts (spies evs)"
by *blast*

23.8 general properties of p2

lemma *reqm_neq_prom [iff]:*
"reqm A r n I B ~= prom B' ofr A' r' I' (cons M L) J C"
by *(auto simp: reqm_def prom_def)*

lemma *prom_neq_reqm [iff]:*
"prom B' ofr A' r' I' (cons M L) J C ~= reqm A r n I B"
by *(auto simp: reqm_def prom_def)*

lemma *req_neq_pro [iff]: "req A r n I B ~= pro B' ofr A' r' I' (cons M L)*
J C"
by *(auto simp: req_def pro_def)*

lemma *pro_neq_req [iff]: "pro B' ofr A' r' I' (cons M L) J C ~= req A r n I B"*
by *(auto simp: req_def pro_def)*

lemma *p2_has_no_Gets: "evs:p2 ==> ALL A X. Gets A X ~:set evs"*
by *(erule p2.induct, auto simp: req_def pro_def)*

lemma *p2_is_Gets_correct [iff]: "Gets_correct p2"*
by *(auto simp: Gets_correct_def dest: p2_has_no_Gets)*

lemma *p2_is_one_step [iff]: "one_step p2"*
by *(unfold one_step_def, clarify, ind_cases "ev#evs:p2", auto)*

lemma *p2_has_only_Says' [rule_format]: "evs:p2 ==>*
ev:set evs --> (EX A B X. ev=Says A B X)"
by *(erule p2.induct, auto simp: req_def pro_def)*

lemma *p2_has_only_Says [iff]: "has_only_Says p2"*
by *(auto simp: has_only_Says_def dest: p2_has_only_Says')*

lemma *p2_is_regular [iff]: "regular p2"*
apply *(simp only: regular_def, clarify)*
apply *(erule_tac p2.induct)*
apply *(simp_all add: initState.simps knows.simps pro_def prom_def req_def reqm_def anchor_def chain_def sign_def)*
by *(auto dest: no_Key_in_agl no_Key_in_appdel parts_trans)*

23.9 private keys are safe

lemma *priK_parts_Friend_imp_bad [rule_format,dest]:*
"[| evs:p2; Friend B ~= A |]
==> (Key (priK A):parts (knows (Friend B) evs)) --> (A:bad)"
apply *(erule p2.induct)*
apply *(simp_all add: initState.simps knows.simps pro_def prom_def*
req_def reqm_def anchor_def chain_def sign_def, blast)
apply *(blast dest: no_Key_in_agl)*
apply *(auto del: parts_invKey disjE dest: parts_trans*
simp add: no_Key_in_appdel)
done

lemma *priK_analz_Friend_imp_bad [rule_format,dest]:*
"[| evs:p2; Friend B ~= A |]
==> (Key (priK A):analz (knows (Friend B) evs)) --> (A:bad)"
by *auto*

lemma *priK_notin_knows_max_Friend:*
"[| evs:p2; A ~:bad; A ~= Friend C |]
==> Key (priK A) ~:analz (knows_max (Friend C) evs)"
apply *(rule not_parts_not_analz, simp add: knows_max_def, safe)*
apply *(drule_tac H="spies' evs"* **in** *parts_sub)*
apply *(rule_tac p=p2* **in** *knows_max'_sub_spies', simp+)*
apply *(drule_tac H="spies evs"* **in** *parts_sub)*
by *(auto dest: knows'_sub_knows [THEN subsetD] priK_notin_initState_Friend)*

23.10 general guardedness properties

lemma *agl_guard [intro]: "I:agl ==> I:guard n Ks"*
by *(erule agl.induct, auto)*

lemma *Says_to_knows_max'_guard: "[| Says A' C {|A'',r,I,L|}:set evs; Guard n Ks (knows_max' C evs) |] ==> L:guard n Ks"*
by *(auto dest: Says_to_knows_max')*

lemma *Says_from_knows_max'_guard: "[| Says C A' {|A'',r,I,L|}:set evs; Guard n Ks (knows_max' C evs) |] ==> L:guard n Ks"*
by *(auto dest: Says_from_knows_max')*

lemma *Says_Nonce_not_used_guard: "[| Says A' B {|A'',r,I,L|}:set evs; Nonce n ~:used evs |] ==> L:guard n Ks"*
by *(drule not_used_not_parts, auto)*

23.11 guardedness of messages

lemma *chain_guard [iff]: "chain B ofr A L C:guard n {priK A}"*
by *(case_tac "ofr=n", auto simp: chain_def sign_def)*

lemma *chain_guard_Nonce_neq [intro]: "n ~= ofr ==> chain B ofr A' L C:guard n {priK A}"*
by *(auto simp: chain_def sign_def)*

lemma *anchor_guard [iff]: "anchor A n' B:guard n {priK A}"*
by *(case_tac "n'=n", auto simp: anchor_def)*

lemma *anchor_guard_Nonce_neq [intro]: "n ~= n' ==> anchor A' n' B:guard n {priK A}"*
by *(auto simp: anchor_def)*

lemma *reqm_guard [intro]: "I:agl ==> reqm A r n' I B:guard n {priK A}"*
by *(case_tac "n'=n", auto simp: reqm_def)*

lemma *reqm_guard_Nonce_neq [intro]: "[| n ~= n'; I:agl |] ==> reqm A' r n' I B:guard n {priK A}"*
by *(auto simp: reqm_def)*

lemma *prom_guard [intro]: "[| I:agl; J:agl; L:guard n {priK A} |] ==> prom B ofr A r I L J C:guard n {priK A}"*
by *(auto simp: prom_def)*

lemma *prom_guard_Nonce_neq [intro]: "[| n ~= ofr; I:agl; J:agl; L:guard n {priK A} |] ==> prom B ofr A' r I L J C:guard n {priK A}"*
by *(auto simp: prom_def)*

23.12 Nonce uniqueness

lemma *uniq_Nonce_in_chain [dest]: "Nonce k:parts {chain B ofr A L C} ==> k=ofr"*
by *(auto simp: chain_def sign_def)*

lemma *uniq_Nonce_in_anchor [dest]: "Nonce k:parts {anchor A n B} ==> k=n"*

by *(auto simp: anchor_def chain_def sign_def)*

lemma *uniq_Nonce_in_reqm [dest]: "[| Nonce k:parts {reqm A r n I B};*
I:agl |] ==> k=n"
by *(auto simp: reqm_def dest: no_Nonce_in_agl)*

lemma *uniq_Nonce_in_prom [dest]: "[| Nonce k:parts {prom B ofr A r I L J*
C};
I:agl; J:agl; Nonce k ~:parts {L} |] ==> k=ofr"
by *(auto simp: prom_def dest: no_Nonce_in_agl no_Nonce_in_appdel)*

23.13 requests are guarded

lemma *req_imp_Guard [rule_format]: "[| evs:p2; A ~:bad |] ==>*
req A r n I B:set evs --> Guard n {priK A} (spies evs)"
apply *(erule p2.induct, simp)*
apply *(simp add: req_def knows.simps, safe)*
apply *(erule in_synth_Guard, erule Guard_analz, simp)*
by *(auto simp: req_def pro_def dest: Says_imp_knows_Spy)*

lemma *req_imp_Guard_Friend: "[| evs:p2; A ~:bad; req A r n I B:set evs |]*
==> Guard n {priK A} (knows_max (Friend C) evs)"
apply *(rule Guard_knows_max')*
apply *(rule_tac H="spies evs"* **in** *Guard_mono)*
apply *(rule req_imp_Guard, simp+)*
apply *(rule_tac B="spies' evs"* **in** *subset_trans)*
apply *(rule_tac p=p2* **in** *knows_max'_sub_spies', simp+)*
by *(rule knows'_sub_knows)*

23.14 propositions are guarded

lemma *pro_imp_Guard [rule_format]: "[| evs:p2; B ~:bad; A ~:bad |] ==>*
pro B ofr A r I (cons M L) J C:set evs --> Guard ofr {priK A} (spies evs)"
apply *(erule p2.induct)*

apply *simp*

apply *(simp add: pro_def, safe)*

apply *(erule in_synth_Guard, drule Guard_analz, simp, simp)*

apply *simp*

apply *(simp, simp add: req_def pro_def, blast)*

apply *(simp add: pro_def)*
apply *(blast dest: prom_inj Says_Nonce_not_used_guard Nonce_not_used_Guard)*

apply *simp*
apply *safe*
apply *(simp add: pro_def)*
apply *(blast dest: prom_inj Says_Nonce_not_used_guard)*

apply *(simp add: pro_def)*

apply *(blast dest: Says_imp_knows_Spy)*

apply *(simp add: pro_def)*
apply *(blast dest: prom_inj Says_Nonce_not_used_guard Nonce_not_used_Guard)*

apply *simp*
apply *safe*

apply *(simp add: pro_def)*
apply *(blast dest: prom_inj Says_Nonce_not_used_guard)*

apply *(simp add: pro_def)*
by *(blast dest: Says_imp_knows_Spy)*

lemma *pro_imp_Guard_Friend: "[| evs:p2; B ~:bad; A ~:bad;*
pro B ofr A r I (cons M L) J C:set evs |]
==> Guard ofr {priK A} (knows_max (Friend D) evs)"
apply *(rule Guard_knows_max')*
apply *(rule_tac H="spies evs"* **in** *Guard_mono)*
apply *(rule pro_imp_Guard, simp+)*
apply *(rule_tac B="spies' evs"* **in** *subset_trans)*
apply *(rule_tac p=p2* **in** *knows_max'_sub_spies', simp+)*
by *(rule knows'_sub_knows)*

23.15 data confidentiality: no one other than the originator can decrypt the offers

lemma *Nonce_req_notin_spies: "[| evs:p2; req A r n I B:set evs; A ~:bad |]*
==> Nonce n ~:analz (spies evs)"
by *(frule req_imp_Guard, simp+, erule Guard_Nonce_analz, simp+)*

lemma *Nonce_req_notin_knows_max_Friend: "[| evs:p2; req A r n I B:set evs;*
A ~:bad; A ~= Friend C |] ==> Nonce n ~:analz (knows_max (Friend C) evs)"
apply *(clarify, frule_tac C=C* **in** *req_imp_Guard_Friend, simp+)*
apply *(simp add: knows_max_def, drule Guard_invKey_keyset, simp+)*
by *(drule priK_notin_knows_max_Friend, auto simp: knows_max_def)*

lemma *Nonce_pro_notin_spies: "[| evs:p2; B ~:bad; A ~:bad;*
pro B ofr A r I (cons M L) J C:set evs |] ==> Nonce ofr ~:analz (spies evs)"
by *(frule pro_imp_Guard, simp+, erule Guard_Nonce_analz, simp+)*

lemma *Nonce_pro_notin_knows_max_Friend: "[| evs:p2; B ~:bad; A ~:bad;*
A ~= Friend D; pro B ofr A r I (cons M L) J C:set evs |]
==> Nonce ofr ~:analz (knows_max (Friend D) evs)"
apply *(clarify, frule_tac A=A* **in** *pro_imp_Guard_Friend, simp+)*
apply *(simp add: knows_max_def, drule Guard_invKey_keyset, simp+)*
by *(drule priK_notin_knows_max_Friend, auto simp: knows_max_def)*

23.16 forward privacy: only the originator can know the identity of the shops

lemma *forward_privacy_Spy: "[| evs:p2; B ~:bad; A ~:bad;*
pro B ofr A r I (cons M L) J C:set evs |]
==> sign B (Nonce ofr) ~:analz (spies evs)"

by *(auto simp:sign_def dest: Nonce_pro_notin_spies)*

lemma *forward_privacy_Friend: "[| evs:p2; B ~:bad; A ~:bad; A ~= Friend D; pro B ofr A r I (cons M L) J C:set evs |] ==> sign B (Nonce ofr) ~:analz (knows_max (Friend D) evs)"*
by *(auto simp:sign_def dest:Nonce_pro_notin_knows_max_Friend)*

23.17 non repudiability: an offer signed by B has been sent by B

lemma *Crypt_reqm: "[| Crypt (priK A) X:parts {reqm A' r n I B}; I:agl |] ==> A=A'"*
by *(auto simp: reqm_def anchor_def chain_def sign_def dest: no_Crypt_in_agl)*

lemma *Crypt_prom: "[| Crypt (priK A) X:parts {prom B ofr A' r I L J C}; I:agl; J:agl |] ==> A=B | Crypt (priK A) X:parts {L}"*
apply *(simp add: prom_def anchor_def chain_def sign_def)*
by *(blast dest: no_Crypt_in_agl no_Crypt_in_appdel)*

lemma *Crypt_safeness: "[| evs:p2; A ~:bad |] ==> Crypt (priK A) X:parts (spies evs) --> (EX B Y. Says A B Y:set evs & Crypt (priK A) X:parts {Y})"*
apply *(erule p2.induct)*

apply *simp*

apply *clarsimp*
apply *(drule_tac P="%G. Crypt (priK A) X:G"* **in** *parts_insert_substD, simp)*
apply *(erule disjE)*
apply *(drule_tac K="priK A"* **in** *Crypt_synth, simp+, blast, blast)*

apply *(simp add: req_def, clarify)*
apply *(drule_tac P="%G. Crypt (priK A) X:G"* **in** *parts_insert_substD, simp)*
apply *(erule disjE)*
apply *(frule Crypt_reqm, simp, clarify)*
apply *(rule_tac x=B* **in** *exI, rule_tac x="reqm A r n I B"* **in** *exI, simp, blast)*

apply *(simp add: pro_def, clarify)*
apply *(drule_tac P="%G. Crypt (priK A) X:G"* **in** *parts_insert_substD, simp)*
apply *(rotate_tac -1, erule disjE)*
apply *(frule Crypt_prom, simp, simp)*
apply *(rotate_tac -1, erule disjE)*
apply *(rule_tac x=C* **in** *exI)*
apply *(rule_tac x="prom B ofr Aa r I (cons M L) J C"* **in** *exI, blast)*
apply *(subgoal_tac "cons M L:parts (spies evsp)")*
apply *(drule_tac G="{cons M L}"* **and** *H="spies evsp"* **in** *parts_trans, blast, blast)*
apply *(drule Says_imp_spies, rotate_tac -1, drule parts.Inj)*
apply *(drule parts.Snd, drule parts.Snd, drule parts.Snd)*
by *auto*

lemma *Crypt_Hash_imp_sign: "[| evs:p2; A ~:bad |] ==> Crypt (priK A) (Hash X):parts (spies evs) --> (EX B Y. Says A B Y:set evs & sign A X:parts {Y})"*

apply *(erule p2.induct)*

apply *simp*

apply *clarsimp*
apply *(drule_tac P="%G. Crypt (priK A) (Hash X):G"* **in** *parts_insert_substD)*
apply *simp*
apply *(erule disjE)*
apply *(drule_tac K="priK A"* **in** *Crypt_synth, simp+, blast, blast)*

apply *(simp add: req_def, clarify)*
apply *(drule_tac P="%G. Crypt (priK A) (Hash X):G"* **in** *parts_insert_substD)*
apply *simp*
apply *(erule disjE)*
apply *(frule Crypt_reqm, simp+)*
apply *(rule_tac x=B* **in** *exI, rule_tac x="reqm Aa r n I B"* **in** *exI)*
apply *(simp add: reqm_def sign_def anchor_def no_Crypt_in_agl)*
apply *(simp add: chain_def sign_def, blast)*

apply *(simp add: pro_def, clarify)*
apply *(drule_tac P="%G. Crypt (priK A) (Hash X):G"* **in** *parts_insert_substD)*
apply *simp*
apply *(rotate_tac -1, erule disjE)*
apply *(simp add: prom_def sign_def no_Crypt_in_agl no_Crypt_in_appdel)*
apply *(simp add: chain_def sign_def)*
apply *(rotate_tac -1, erule disjE)*
apply *(rule_tac x=C* **in** *exI)*
apply *(rule_tac x="prom B ofr Aa r I (cons M L) J C"* **in** *exI)*
apply *(simp add: prom_def chain_def sign_def)*
apply *(erule impE)*
apply *(blast dest: get_ML parts_sub)*
apply *(blast del: MPair_parts)+*
done

lemma *sign_safeness: "[| evs:p2; A ~:bad |] ==> sign A X:parts (spies evs)*
--> (EX B Y. Says A B Y:set evs & sign A X:parts {Y})"
apply *(clarify, simp add: sign_def, frule parts.Snd)*
apply *(blast dest: Crypt_Hash_imp_sign [unfolded sign_def])*
done

end

24 Needham-Schroeder-Lowe Public-Key Protocol

theory *Guard_NS_Public* **imports** *Guard_Public* **begin**

24.1 messages used in the protocol

syntax *ns1 :: "agent => agent => nat => event"*

translations *"ns1 A B NA" => "Says A B (Crypt (pubK B) {|Nonce NA, Agent A|})"*

syntax *ns1' :: "agent => agent => agent => nat => event"*

translations *"ns1' A' A B NA"*
=> "Says A' B (Crypt (pubK B) {|Nonce NA, Agent A|})"

syntax *ns2 :: "agent => agent => nat => nat => event"*

translations *"ns2 B A NA NB"*
=> "Says B A (Crypt (pubK A) {|Nonce NA, Nonce NB, Agent B|})"

syntax *ns2' :: "agent => agent => agent => nat => nat => event"*

translations *"ns2' B' B A NA NB"*
=> "Says B' A (Crypt (pubK A) {|Nonce NA, Nonce NB, Agent B|})"

syntax *ns3 :: "agent => agent => nat => event"*

translations *"ns3 A B NB" => "Says A B (Crypt (pubK B) (Nonce NB))"*

24.2 definition of the protocol

consts *nsp :: "event list set"*

inductive *nsp*
intros

Nil: "[]:nsp"

Fake: "[| evs:nsp; X:synth (analz (spies evs)) |] ==> Says Spy B X # evs : nsp"

NS1: "[| evs1:nsp; Nonce NA ~:used evs1 |] ==> ns1 A B NA # evs1 : nsp"

NS2: "[| evs2:nsp; Nonce NB ~:used evs2; ns1' A' A B NA:set evs2 |] ==> ns2 B A NA NB # evs2:nsp"

NS3: "[| evs3:nsp; ns1 A B NA:set evs3; ns2' B' B A NA NB:set evs3 |] ==> ns3 A B NB # evs3:nsp"

24.3 declarations for tactics

declare *knows_Spy_partsEs [elim]*
declare *Fake_parts_insert [THEN subsetD, dest]*
declare *initState.simps [simp del]*

24.4 general properties of nsp

lemma *nsp_has_no_Gets: "evs:nsp ==> ALL A X. Gets A X ~:set evs"*
by *(erule nsp.induct, auto)*

lemma *nsp_is_Gets_correct [iff]: "Gets_correct nsp"*
by *(auto simp: Gets_correct_def dest: nsp_has_no_Gets)*

lemma *nsp_is_one_step [iff]: "one_step nsp"*

by *(unfold one_step_def, clarify, ind_cases "ev#evs:nsp", auto)*

lemma *nsp_has_only_Says' [rule_format]: "evs:nsp ==>*
ev:set evs --> (EX A B X. ev=Says A B X)"
by *(erule nsp.induct, auto)*

lemma *nsp_has_only_Says [iff]: "has_only_Says nsp"*
by *(auto simp: has_only_Says_def dest: nsp_has_only_Says')*

lemma *nsp_is_regular [iff]: "regular nsp"*
apply *(simp only: regular_def, clarify)*
by *(erule nsp.induct, auto simp: initState.simps knows.simps)*

24.5 nonce are used only once

lemma *NA_is_uniq [rule_format]: "evs:nsp ==>*
Crypt (pubK B) {|Nonce NA, Agent A|}:parts (spies evs)
--> Crypt (pubK B') {|Nonce NA, Agent A'|}:parts (spies evs)
--> Nonce NA ~:analz (spies evs) --> A=A' & B=B'"
apply *(erule nsp.induct, simp_all)*
by *(blast intro: analz_insertI)+*

lemma *no_Nonce_NS1_NS2 [rule_format]: "evs:nsp ==>*
Crypt (pubK B') {|Nonce NA', Nonce NA, Agent A'|}:parts (spies evs)
--> Crypt (pubK B) {|Nonce NA, Agent A|}:parts (spies evs)
--> Nonce NA:analz (spies evs)"
apply *(erule nsp.induct, simp_all)*
by *(blast intro: analz_insertI)+*

lemma *no_Nonce_NS1_NS2' [rule_format]:*
"[| Crypt (pubK B') {|Nonce NA', Nonce NA, Agent A'|}:parts (spies evs);
Crypt (pubK B) {|Nonce NA, Agent A|}:parts (spies evs); evs:nsp |]
==> Nonce NA:analz (spies evs)"
by *(rule no_Nonce_NS1_NS2, auto)*

lemma *NB_is_uniq [rule_format]: "evs:nsp ==>*
Crypt (pubK A) {|Nonce NA, Nonce NB, Agent B|}:parts (spies evs)
--> Crypt (pubK A') {|Nonce NA', Nonce NB, Agent B'|}:parts (spies evs)
--> Nonce NB ~:analz (spies evs) --> A=A' & B=B' & NA=NA'"
apply *(erule nsp.induct, simp_all)*
by *(blast intro: analz_insertI)+*

24.6 guardedness of NA

lemma *ns1_imp_Guard [rule_format]: "[| evs:nsp; A ~:bad; B ~:bad |] ==>*
ns1 A B NA:set evs --> Guard NA {priK A,priK B} (spies evs)"
apply *(erule nsp.induct)*

apply *simp_all*

apply *safe*
apply *(erule in_synth_Guard, erule Guard_analz, simp)*

apply *blast*

```
apply blast
apply blast
apply (drule Nonce_neq, simp+, rule No_Nonce, simp)

apply (frule_tac A=A in Nonce_neq, simp+)
apply (case_tac "NAa=NA")
apply (drule Guard_Nonce_analz, simp+)
apply (drule Says_imp_knows_Spy)+
apply (drule_tac B=B and A'=Aa in NA_is_uniq, auto)

apply (case_tac "NB=NA", clarify)
apply (drule Guard_Nonce_analz, simp+)
apply (drule Says_imp_knows_Spy)+
by (drule no_Nonce_NS1_NS2, auto)
```

24.7 guardedness of NB

```
lemma ns2_imp_Guard [rule_format]: "[| evs:nsp; A ~:bad; B ~:bad |] ==>
ns2 B A NA NB:set evs --> Guard NB {priK A,priK B} (spies evs)"
apply (erule nsp.induct)

apply simp_all

apply safe
apply (erule in_synth_Guard, erule Guard_analz, simp)

apply (frule Nonce_neq, simp+, blast, rule No_Nonce, simp)

apply blast
apply blast
apply blast
apply (frule_tac A=B and n=NB in Nonce_neq, simp+)
apply (case_tac "NAa=NB")
apply (drule Guard_Nonce_analz, simp+)
apply (drule Says_imp_knows_Spy)+
apply (drule no_Nonce_NS1_NS2, auto)

apply (case_tac "NBa=NB", clarify)
apply (drule Guard_Nonce_analz, simp+)
apply (drule Says_imp_knows_Spy)+
by (drule_tac A=Aa and A'=A in NB_is_uniq, auto)
```

24.8 Agents' Authentication

```
lemma B_trusts_NS1: "[| evs:nsp; A ~:bad; B ~:bad |] ==>
Crypt (pubK B) {|Nonce NA, Agent A|}:parts (spies evs)
--> Nonce NA ~:analz (spies evs) --> ns1 A B NA:set evs"
apply (erule nsp.induct, simp_all)
by (blast intro: analz_insertI)+

lemma A_trusts_NS2: "[| evs:nsp; A ~:bad; B ~:bad |] ==> ns1 A B NA:set evs
--> Crypt (pubK A) {|Nonce NA, Nonce NB, Agent B|}:parts (spies evs)
--> ns2 B A NA NB:set evs"
apply (erule nsp.induct, simp_all, safe)
```

apply *(frule_tac B=B* **in** *ns1_imp_Guard, simp+)*
apply *(drule Guard_Nonce_analz, simp+, blast)*
apply *(frule_tac B=B* **in** *ns1_imp_Guard, simp+)*
apply *(drule Guard_Nonce_analz, simp+, blast)*
apply *(frule_tac B=B* **in** *ns1_imp_Guard, simp+)*
by *(drule Guard_Nonce_analz, simp+, blast+)*

lemma *B_trusts_NS3: "[| evs:nsp; A ~:bad; B ~:bad |] ==> ns2 B A NA NB:set evs*
--> Crypt (pubK B) (Nonce NB):parts (spies evs) --> ns3 A B NB:set evs"
apply *(erule nsp.induct, simp_all, safe)*
apply *(frule_tac B=B* **in** *ns2_imp_Guard, simp+)*
apply *(drule Guard_Nonce_analz, simp+, blast)*
apply *(frule_tac B=B* **in** *ns2_imp_Guard, simp+)*
apply *(drule Guard_Nonce_analz, simp+, blast)*
apply *(frule_tac B=B* **in** *ns2_imp_Guard, simp+)*
apply *(drule Guard_Nonce_analz, simp+, blast, blast)*
apply *(frule_tac B=B* **in** *ns2_imp_Guard, simp+)*
by *(drule Guard_Nonce_analz, auto dest: Says_imp_knows_Spy NB_is_uniq)*

end

25 protocol-independent confidentiality theorem on keys

theory *GuardK* **imports** *Analz Extensions* **begin**

consts *guardK :: "nat => key set => msg set"*

inductive *"guardK n Ks"*
intros
No_Key [intro]: "Key n ~:parts {X} ==> X:guardK n Ks"
Guard_Key [intro]: "invKey K:Ks ==> Crypt K X:guardK n Ks"
Crypt [intro]: "X:guardK n Ks ==> Crypt K X:guardK n Ks"
Pair [intro]: "[| X:guardK n Ks; Y:guardK n Ks |] ==> {|X,Y|}:guardK n Ks"

25.1 basic facts about *guardK*

lemma *Nonce_is_guardK [iff]: "Nonce p:guardK n Ks"*
by *auto*

lemma *Agent_is_guardK [iff]: "Agent A:guardK n Ks"*
by *auto*

lemma *Number_is_guardK [iff]: "Number r:guardK n Ks"*
by *auto*

lemma *Key_notin_guardK: "X:guardK n Ks ==> X ~= Key n"*
by *(erule guardK.induct, auto)*

lemma *Key_notin_guardK_iff [iff]: "Key n ~:guardK n Ks"*
by *(auto dest: Key_notin_guardK)*

lemma *guardK_has_Crypt [rule_format]: "X:guardK n Ks ==> Key n:parts {X}*
--> (EX K Y. Crypt K Y:kparts {X} & Key n:parts {Y})"
by *(erule guardK.induct, auto)*

lemma *Key_notin_kparts_msg: "X:guardK n Ks ==> Key n ~:kparts {X}"*
by *(erule guardK.induct, auto dest: kparts_parts)*

lemma *Key_in_kparts_imp_no_guardK: "Key n:kparts H*
==> EX X. X:H & X ~:guardK n Ks"
apply *(drule in_kparts, clarify)*
apply *(rule_tac x=X* **in** *exI, clarify)*
by *(auto dest: Key_notin_kparts_msg)*

lemma *guardK_kparts [rule_format]: "X:guardK n Ks ==>*
Y:kparts {X} --> Y:guardK n Ks"
by *(erule guardK.induct, auto dest: kparts_parts parts_sub)*

lemma *guardK_Crypt: "[| Crypt K Y:guardK n Ks; K ~:invKey‘Ks |] ==> Y:guardK*
n Ks"
by *(ind_cases "Crypt K Y:guardK n Ks", auto)*

lemma *guardK_MPair [iff]: "({|X,Y|}:guardK n Ks)*
= (X:guardK n Ks & Y:guardK n Ks)"
by *(auto, (ind_cases "{|X,Y|}:guardK n Ks", auto)+)*

lemma *guardK_not_guardK [rule_format]: "X:guardK n Ks ==>*
Crypt K Y:kparts {X} --> Key n:kparts {Y} --> Y ~:guardK n Ks"
by *(erule guardK.induct, auto dest: guardK_kparts)*

lemma *guardK_extand: "[| X:guardK n Ks; Ks <= Ks’;*
[| K:Ks’; K ~:Ks |] ==> Key K ~:parts {X} |] ==> X:guardK n Ks’"
by *(erule guardK.induct, auto)*

25.2 guarded sets

constdefs *GuardK :: "nat => key set => msg set => bool"*
"GuardK n Ks H == ALL X. X:H --> X:guardK n Ks"

25.3 basic facts about *GuardK*

lemma *GuardK_empty [iff]: "GuardK n Ks {}"*
by *(simp add: GuardK_def)*

lemma *Key_notin_kparts [simplified]: "GuardK n Ks H ==> Key n ~:kparts H"*
by *(auto simp: GuardK_def dest: in_kparts Key_notin_kparts_msg)*

lemma *GuardK_must_decrypt: "[| GuardK n Ks H; Key n:analz H |] ==>*
EX K Y. Crypt K Y:kparts H & Key (invKey K):kparts H"
apply *(drule_tac P="%G. Key n:G"* **in** *analz_pparts_kparts_substD, simp)*
by *(drule must_decrypt, auto dest: Key_notin_kparts)*

lemma *GuardK_kparts [intro]: "GuardK n Ks H ==> GuardK n Ks (kparts H)"*
by *(auto simp: GuardK_def dest: in_kparts guardK_kparts)*

lemma *GuardK_mono: "[| GuardK n Ks H; G <= H |] ==> GuardK n Ks G"*
by *(auto simp: GuardK_def)*

lemma *GuardK_insert [iff]: "GuardK n Ks (insert X H)*
= (GuardK n Ks H & X:guardK n Ks)"
by *(auto simp: GuardK_def)*

lemma *GuardK_Un [iff]: "GuardK n Ks (G Un H) = (GuardK n Ks G & GuardK n*
Ks H)"
by *(auto simp: GuardK_def)*

lemma *GuardK_synth [intro]: "GuardK n Ks G ==> GuardK n Ks (synth G)"*
by *(auto simp: GuardK_def, erule synth.induct, auto)*

lemma *GuardK_analz [intro]: "[| GuardK n Ks G; ALL K. K:Ks --> Key K ~:analz*
G |]
==> GuardK n Ks (analz G)"
apply *(auto simp: GuardK_def)*
apply *(erule analz.induct, auto)*
by *(ind_cases "Crypt K Xa:guardK n Ks", auto)*

lemma *in_GuardK [dest]: "[| X:G; GuardK n Ks G |] ==> X:guardK n Ks"*
by *(auto simp: GuardK_def)*

lemma *in_synth_GuardK: "[| X:synth G; GuardK n Ks G |] ==> X:guardK n Ks"*
by *(drule GuardK_synth, auto)*

lemma *in_analz_GuardK: "[| X:analz G; GuardK n Ks G;*
ALL K. K:Ks --> Key K ~:analz G |] ==> X:guardK n Ks"
by *(drule GuardK_analz, auto)*

lemma *GuardK_keyset [simp]: "[| keyset G; Key n ~:G |] ==> GuardK n Ks G"*
by *(simp only: GuardK_def, clarify, drule keyset_in, auto)*

lemma *GuardK_Un_keyset: "[| GuardK n Ks G; keyset H; Key n ~:H |]*
==> GuardK n Ks (G Un H)"
by *auto*

lemma *in_GuardK_kparts: "[| X:G; GuardK n Ks G; Y:kparts {X} |] ==> Y:guardK*
n Ks"
by *blast*

lemma *in_GuardK_kparts_neq: "[| X:G; GuardK n Ks G; Key n':kparts {X} |]*
==> n ~= n'"
by *(blast dest: in_GuardK_kparts)*

lemma *in_GuardK_kparts_Crypt: "[| X:G; GuardK n Ks G; is_MPair X;*
Crypt K Y:kparts {X}; Key n:kparts {Y} |] ==> invKey K:Ks"
apply *(drule in_GuardK, simp)*
apply *(frule guardK_not_guardK, simp+)*
apply *(drule guardK_kparts, simp)*

by *(ind_cases "Crypt K Y:guardK n Ks", auto)*

lemma *GuardK_extand: "[| GuardK n Ks G; Ks <= Ks';*
[| K:Ks'; K ~:Ks |] ==> Key K ~:parts G |] ==> GuardK n Ks' G"
by *(auto simp: GuardK_def dest: guardK_extand parts_sub)*

25.4 set obtained by decrypting a message

syntax *decrypt :: "msg set => key => msg => msg set"*

translations *"decrypt H K Y" => "insert Y (H - {Crypt K Y})"*

lemma *analz_decrypt: "[| Crypt K Y:H; Key (invKey K):H; Key n:analz H |]*
==> Key n:analz (decrypt H K Y)"
apply *(drule_tac P="%H. Key n:analz H"* **in** *ssubst [OF insert_Diff])*
apply *assumption*
apply *(simp only: analz_Crypt_if, simp)*
done

lemma *parts_decrypt: "[| Crypt K Y:H; X:parts (decrypt H K Y) |] ==> X:parts*
H"
by *(erule parts.induct, auto intro: parts.Fst parts.Snd parts.Body)*

25.5 number of Crypt's in a message

consts *crypt_nb :: "msg => nat"*

recdef *crypt_nb "measure size"*
"crypt_nb (Crypt K X) = Suc (crypt_nb X)"
"crypt_nb {|X,Y|} = crypt_nb X + crypt_nb Y"
"crypt_nb X = 0"

25.6 basic facts about *crypt_nb*

lemma *non_empty_crypt_msg: "Crypt K Y:parts {X} ==> 0 < crypt_nb X"*
by *(induct X, simp_all, safe, simp_all)*

25.7 number of Crypt's in a message list

consts *cnb :: "msg list => nat"*

recdef *cnb "measure size"*
"cnb [] = 0"
"cnb (X#l) = crypt_nb X + cnb l"

25.8 basic facts about *cnb*

lemma *cnb_app [simp]: "cnb (l @ l') = cnb l + cnb l'"*
by *(induct l, auto)*

lemma *mem_cnb_minus: "x mem l ==> cnb l = crypt_nb x + (cnb l - crypt_nb*
x)"
by *(induct l, auto)*

lemmas *mem_cnb_minus_substI = mem_cnb_minus [THEN ssubst]*

lemma *cnb_minus [simp]: "x mem l ==> cnb (minus l x) = cnb l - crypt_nb x"*
apply *(induct l, auto)*
by *(erule_tac l1=l* **and** *x1=x* **in** *mem_cnb_minus_substI, simp)*

lemma *parts_cnb: "Z:parts (set l) ==>*
cnb l = (cnb l - crypt_nb Z) + crypt_nb Z"
by *(erule parts.induct, auto simp: in_set_conv_decomp)*

lemma *non_empty_crypt: "Crypt K Y:parts (set l) ==> 0 < cnb l"*
by *(induct l, auto dest: non_empty_crypt_msg parts_insert_substD)*

25.9 list of kparts

lemma *kparts_msg_set: "EX l. kparts {X} = set l & cnb l = crypt_nb X"*
apply *(induct X, simp_all)*
apply *(rule_tac x="[Agent agent]"* **in** *exI, simp)*
apply *(rule_tac x="[Number nat]"* **in** *exI, simp)*
apply *(rule_tac x="[Nonce nat]"* **in** *exI, simp)*
apply *(rule_tac x="[Key nat]"* **in** *exI, simp)*
apply *(rule_tac x="[Hash X]"* **in** *exI, simp)*
apply *(clarify, rule_tac x="l@la"* **in** *exI, simp)*
by *(clarify, rule_tac x="[Crypt nat X]"* **in** *exI, simp)*

lemma *kparts_set: "EX l'. kparts (set l) = set l' & cnb l' = cnb l"*
apply *(induct l)*
apply *(rule_tac x="[]"* **in** *exI, simp, clarsimp)*
apply *(subgoal_tac "EX l''. kparts {a} = set l'' & cnb l'' = crypt_nb a",*
clarify)
apply *(rule_tac x="l''@l'"* **in** *exI, simp)*
apply *(rule kparts_insert_substI, simp)*
by *(rule kparts_msg_set)*

25.10 list corresponding to "decrypt"

constdefs *decrypt' :: "msg list => key => msg => msg list"*
"decrypt' l K Y == Y # minus l (Crypt K Y)"

declare *decrypt'_def [simp]*

25.11 basic facts about *decrypt'*

lemma *decrypt_minus: "decrypt (set l) K Y <= set (decrypt' l K Y)"*
by *(induct l, auto)*

if the analysis of a finite guarded set gives n then it must also give one of the keys of Ks

lemma *GuardK_invKey_by_list [rule_format]: "ALL l. cnb l = p*
--> GuardK n Ks (set l) --> Key n:analz (set l)
--> (EX K. K:Ks & Key K:analz (set l))"
apply *(induct p)*

apply *(clarify, drule GuardK_must_decrypt, simp, clarify)*

```
apply (drule kparts_parts, drule non_empty_crypt, simp)

apply (clarify, frule GuardK_must_decrypt, simp, clarify)
apply (drule_tac P="%G. Key n:G" in analz_pparts_kparts_substD, simp)
apply (frule analz_decrypt, simp_all)
apply (subgoal_tac "EX l'. kparts (set l) = set l' & cnb l' = cnb l", clarsimp)
apply (drule_tac G="insert Y (set l' - {Crypt K Y})"
and H="set (decrypt' l' K Y)" in analz_sub, rule decrypt_minus)
apply (rule_tac analz_pparts_kparts_substI, simp)
apply (case_tac "K:invKey`Ks")

apply (clarsimp, blast)

apply (subgoal_tac "GuardK n Ks (set (decrypt' l' K Y))")
apply (drule_tac x="decrypt' l' K Y" in spec, simp add: mem_iff)
apply (subgoal_tac "Crypt K Y:parts (set l)")
apply (drule parts_cnb, rotate_tac -1, simp)
apply (clarify, drule_tac X="Key Ka" and H="insert Y (set l')" in analz_sub)
apply (rule insert_mono, rule set_minus)
apply (simp add: analz_insertD, blast)

apply (blast dest: kparts_parts)

apply (rule_tac H="insert Y (set l')" in GuardK_mono)
apply (subgoal_tac "GuardK n Ks (set l')", simp)
apply (rule_tac K=K in guardK_Crypt, simp add: GuardK_def, simp)
apply (drule_tac t="set l'" in sym, simp)
apply (rule GuardK_kparts, simp, simp)
apply (rule_tac B="set l'" in subset_trans, rule set_minus, blast)
by (rule kparts_set)

lemma GuardK_invKey_finite: "[| Key n:analz G; GuardK n Ks G; finite G |]
==> EX K. K:Ks & Key K:analz G"
apply (drule finite_list, clarify)
by (rule GuardK_invKey_by_list, auto)

lemma GuardK_invKey: "[| Key n:analz G; GuardK n Ks G |]
==> EX K. K:Ks & Key K:analz G"
by (auto dest: analz_needs_only_finite GuardK_invKey_finite)
```

if the analyse of a finite guarded set and a (possibly infinite) set of keys gives n then it must also gives Ks

```
lemma GuardK_invKey_keyset: "[| Key n:analz (G Un H); GuardK n Ks G; finite
G;
keyset H; Key n ~:H |] ==> EX K. K:Ks & Key K:analz (G Un H)"
apply (frule_tac P="%G. Key n:G" and G2=G in analz_keyset_substD, simp_all)
apply (drule_tac G="G Un (H Int keysfor G)" in GuardK_invKey_finite)
apply (auto simp: GuardK_def intro: analz_sub)
by (drule keyset_in, auto)

end


theory Shared imports Event begin
```

```
consts
  shrK    :: "agent => key"

specification (shrK)
  inj_shrK: "inj shrK"
  — No two agents have the same long-term key
   apply (rule exI [of _ "agent_case 0 (λn. n + 2) 1"])
   apply (simp add: inj_on_def split: agent.split)
   done
```

All keys are symmetric

```
defs  all_symmetric_def: "all_symmetric == True"

lemma isSym_keys: "K ∈ symKeys"
by (simp add: symKeys_def all_symmetric_def invKey_symmetric)
```

Server knows all long-term keys; other agents know only their own

```
primrec
  initState_Server:  "initState Server     = Key ‘ range shrK"
  initState_Friend:  "initState (Friend i) = {Key (shrK (Friend i))}"
  initState_Spy:     "initState Spy        = Key‘shrK‘bad"
```

25.12 Basic properties of shrK

```
declare inj_shrK [THEN inj_eq, iff]

lemma invKey_K [simp]: "invKey K = K"
apply (insert isSym_keys)
apply (simp add: symKeys_def)
done


lemma analz_Decrypt' [dest]:
     "[| Crypt K X ∈ analz H;  Key K  ∈ analz H |] ==> X ∈ analz H"
by auto
```

Now cancel the *dest* attribute given to *analz.Decrypt* in its declaration.

```
declare analz.Decrypt [rule del]
```

Rewrites should not refer to *initState (Friend i)* because that expression is not in normal form.

```
lemma keysFor_parts_initState [simp]: "keysFor (parts (initState C)) = {}"
apply (unfold keysFor_def)
apply (induct_tac "C", auto)
done


lemma keysFor_parts_insert:
     "[| K ∈ keysFor (parts (insert X G));  X ∈ synth (analz H) |]
      ==> K ∈ keysFor (parts (G ∪ H)) | Key K ∈ parts H"
by (force dest: Event.keysFor_parts_insert)
```

lemma *Crypt_imp_keysFor: "Crypt K X ∈ H ==> K ∈ keysFor H"*
by *(drule Crypt_imp_invKey_keysFor, simp)*

25.13 Function "knows"

lemma *Spy_knows_Spy_bad [intro!]: "A: bad ==> Key (shrK A) ∈ knows Spy evs"*
apply *(induct_tac "evs")*
apply *(simp_all (no_asm_simp) add: imageI knows_Cons split add: event.split)*
done

lemma *Crypt_Spy_analz_bad: "[| Crypt (shrK A) X ∈ analz (knows Spy evs);*
A: bad |]
==> X ∈ analz (knows Spy evs)"
apply *(force dest!: analz.Decrypt)*
done

lemma *shrK_in_initState [iff]: "Key (shrK A) ∈ initState A"*
by *(induct_tac "A", auto)*

lemma *shrK_in_used [iff]: "Key (shrK A) ∈ used evs"*
by *(rule initState_into_used, blast)*

lemma *Key_not_used [simp]: "Key K ∉ used evs ==> K ∉ range shrK"*
by *blast*

lemma *shrK_neq [simp]: "Key K ∉ used evs ==> shrK B ≠ K"*
by *blast*

declare *shrK_neq [THEN not_sym, simp]*

25.14 Fresh nonces

lemma *Nonce_notin_initState [iff]: "Nonce N ∉ parts (initState B)"*
by *(induct_tac "B", auto)*

lemma *Nonce_notin_used_empty [simp]: "Nonce N ∉ used []"*
apply *(simp (no_asm) add: used_Nil)*
done

25.15 Supply fresh nonces for possibility theorems.

lemma *Nonce_supply_lemma: "∃N. ALL n. N<=n --> Nonce n ∉ used evs"*
apply *(induct_tac "evs")*
apply *(rule_tac x = 0* **in** *exI)*
apply *(simp_all (no_asm_simp) add: used_Cons split add: event.split)*
apply *safe*
apply *(rule msg_Nonce_supply [THEN exE], blast elim!: add_leE)+*
done

```
lemma Nonce_supply1: "∃N. Nonce N ∉ used evs"
by (rule Nonce_supply_lemma [THEN exE], blast)

lemma Nonce_supply2: "∃N N'. Nonce N ∉ used evs & Nonce N' ∉ used evs'
& N ≠ N'"
apply (cut_tac evs = evs in Nonce_supply_lemma)
apply (cut_tac evs = "evs'" in Nonce_supply_lemma, clarify)
apply (rule_tac x = N in exI)
apply (rule_tac x = "Suc (N+Na)" in exI)
apply (simp (no_asm_simp) add: less_not_refl3 le_add1 le_add2 less_Suc_eq_le)
done

lemma Nonce_supply3: "∃N N' N''. Nonce N ∉ used evs & Nonce N' ∉ used evs'
&
                    Nonce N'' ∉ used evs'' & N ≠ N' & N' ≠ N'' & N ≠ N''"
apply (cut_tac evs = evs in Nonce_supply_lemma)
apply (cut_tac evs = "evs'" in Nonce_supply_lemma)
apply (cut_tac evs = "evs''" in Nonce_supply_lemma, clarify)
apply (rule_tac x = N in exI)
apply (rule_tac x = "Suc (N+Na)" in exI)
apply (rule_tac x = "Suc (Suc (N+Na+Nb))" in exI)
apply (simp (no_asm_simp) add: less_not_refl3 le_add1 le_add2 less_Suc_eq_le)
done

lemma Nonce_supply: "Nonce (@ N. Nonce N ∉ used evs) ∉ used evs"
apply (rule Nonce_supply_lemma [THEN exE])
apply (rule someI, blast)
done
```

Unlike the corresponding property of nonces, we cannot prove `finite KK ⟹ ∃K. K ∉ KK ∧ Key K ∉ used evs`. We have infinitely many agents and there is nothing to stop their long-term keys from exhausting all the natural numbers. Instead, possibility theorems must assume the existence of a few keys.

25.16 Tactics for possibility theorems

ML

```
{*
val inj_shrK      = thm "inj_shrK";
val isSym_keys    = thm "isSym_keys";
val Nonce_supply = thm "Nonce_supply";
val invKey_K = thm "invKey_K";
val analz_Decrypt' = thm "analz_Decrypt'";
val keysFor_parts_initState = thm "keysFor_parts_initState";
val keysFor_parts_insert = thm "keysFor_parts_insert";
val Crypt_imp_keysFor = thm "Crypt_imp_keysFor";
val Spy_knows_Spy_bad = thm "Spy_knows_Spy_bad";
val Crypt_Spy_analz_bad = thm "Crypt_Spy_analz_bad";
val shrK_in_initState = thm "shrK_in_initState";
val shrK_in_used = thm "shrK_in_used";
val Key_not_used = thm "Key_not_used";
val shrK_neq = thm "shrK_neq";
val Nonce_notin_initState = thm "Nonce_notin_initState";
```

```
val Nonce_notin_used_empty = thm "Nonce_notin_used_empty";
val Nonce_supply_lemma = thm "Nonce_supply_lemma";
val Nonce_supply1 = thm "Nonce_supply1";
val Nonce_supply2 = thm "Nonce_supply2";
val Nonce_supply3 = thm "Nonce_supply3";
val Nonce_supply = thm "Nonce_supply";
*}
```

ML

```
{*
(*Omitting used_Says makes the tactic much faster: it leaves expressions
    such as  Nonce ?N ∉ used evs that match Nonce_supply*)
fun gen_possibility_tac ss state = state |>
   (REPEAT
    (ALLGOALS (simp_tac (ss delsimps [used_Says, used_Notes, used_Gets]
                         setSolver safe_solver))
     THEN
     REPEAT_FIRST (eq_assume_tac ORELSE'
                   resolve_tac [refl, conjI, Nonce_supply])))

(*Tactic for possibility theorems (ML script version)*)
fun possibility_tac state = gen_possibility_tac (simpset()) state

(*For harder protocols (such as Recur) where we have to set up some
  nonces and keys initially*)
fun basic_possibility_tac st = st |>
    REPEAT
    (ALLGOALS (asm_simp_tac (simpset() setSolver safe_solver))
     THEN
     REPEAT_FIRST (resolve_tac [refl, conjI]))
*}
```

25.17 Specialized Rewriting for Theorems About analz and Image

lemma *subset_Compl_range: "A <= - (range shrK) ==> shrK x ∉ A"*
by *blast*

lemma *insert_Key_singleton: "insert (Key K) H = Key ‘ {K} ∪ H"*
by *blast*

lemma *insert_Key_image: "insert (Key K) (Key‘KK ∪ C) = Key‘(insert K KK)*
∪ C"
by *blast*

lemmas *analz_image_freshK_simps =*
simp_thms mem_simps — these two allow its use with *only:*
disj_comms
image_insert [THEN sym] image_Un [THEN sym] empty_subsetI insert_subset
analz_insert_eq Un_upper2 [THEN analz_mono, THEN [2] rev_subsetD]
insert_Key_singleton subset_Compl_range

```
      Key_not_used insert_Key_image Un_assoc [THEN sym]


lemma analz_image_freshK_lemma:
     "(Key K ∈ analz (Key‘nE ∪ H)) --> (K ∈ nE | Key K ∈ analz H)  ==>
        (Key K ∈ analz (Key‘nE ∪ H)) = (K ∈ nE | Key K ∈ analz H)"
by (blast intro: analz_mono [THEN [2] rev_subsetD])

ML
{*
val analz_image_freshK_lemma = thm "analz_image_freshK_lemma";

val analz_image_freshK_ss =
     simpset() delsimps [image_insert, image_Un]
               delsimps [imp_disjL]    (*reduces blow-up*)
               addsimps thms "analz_image_freshK_simps"
*}



lemma invKey_shrK_iff [iff]:
     "(Key (invKey K) ∈ X) = (Key K ∈ X)"
by auto



method_setup analz_freshK = {*
    Method.no_args
     (Method.METHOD
      (fn facts => EVERY [REPEAT_FIRST (resolve_tac [allI, ballI, impI]),
                          REPEAT_FIRST (rtac analz_image_freshK_lemma),
                          ALLGOALS (asm_simp_tac analz_image_freshK_ss)]))
*}
    "for proving the Session Key Compromise theorem"

method_setup possibility = {*
    Method.ctxt_args (fn ctxt =>
        Method.METHOD (fn facts =>
            gen_possibility_tac (local_simpset_of ctxt))) *}
    "for proving possibility theorems"

lemma knows_subset_knows_Cons: "knows A evs <= knows A (e # evs)"
by (induct e, auto simp: knows_Cons)

end
```

26 lemmas on guarded messages for protocols with symmetric keys

```
theory Guard_Shared imports Guard GuardK Shared begin
```

26.1 Extensions to Theory *Shared*

declare *initState.simps [simp del]*

26.1.1 a little abbreviation

syntax *Ciph :: "agent => msg"*

translations *"Ciph A X" == "Crypt (shrK A) X"*

26.1.2 agent associated to a key

constdefs *agt :: "key => agent"*
"agt K == @A. K = shrK A"

lemma *agt_shrK [simp]: "agt (shrK A) = A"*
by *(simp add: agt_def)*

26.1.3 basic facts about *initState*

lemma *no_Crypt_in_parts_init [simp]: "Crypt K X ~:parts (initState A)"*
by *(cases A, auto simp: initState.simps)*

lemma *no_Crypt_in_analz_init [simp]: "Crypt K X ~:analz (initState A)"*
by *auto*

lemma *no_shrK_in_analz_init [simp]: "A ~:bad*
==> Key (shrK A) ~:analz (initState Spy)"
by *(auto simp: initState.simps)*

lemma *shrK_notin_initState_Friend [simp]: "A ~= Friend C*
==> Key (shrK A) ~: parts (initState (Friend C))"
by *(auto simp: initState.simps)*

lemma *keyset_init [iff]: "keyset (initState A)"*
by *(cases A, auto simp: keyset_def initState.simps)*

26.1.4 sets of symmetric keys

constdefs *shrK_set :: "key set => bool"*
"shrK_set Ks == ALL K. K:Ks --> (EX A. K = shrK A)"

lemma *in_shrK_set: "[| shrK_set Ks; K:Ks |] ==> EX A. K = shrK A"*
by *(simp add: shrK_set_def)*

lemma *shrK_set1 [iff]: "shrK_set {shrK A}"*
by *(simp add: shrK_set_def)*

lemma *shrK_set2 [iff]: "shrK_set {shrK A, shrK B}"*
by *(simp add: shrK_set_def)*

26.1.5 sets of good keys

constdefs *good :: "key set => bool"*
"good Ks == ALL K. K:Ks --> agt K ~:bad"

lemma *in_good: "[| good Ks; K:Ks |] ==> agt K ~:bad"*
by *(simp add: good_def)*

lemma *good1 [simp]: "A ~:bad ==> good {shrK A}"*
by *(simp add: good_def)*

lemma *good2 [simp]: "[| A ~:bad; B ~:bad |] ==> good {shrK A, shrK B}"*
by *(simp add: good_def)*

26.2 Proofs About Guarded Messages

26.2.1 small hack

lemma *shrK_is_invKey_shrK: "shrK A = invKey (shrK A)"*
by *simp*

lemmas *shrK_is_invKey_shrK_substI = shrK_is_invKey_shrK [THEN ssubst]*

lemmas *invKey_invKey_substI = invKey [THEN ssubst]*

lemma *"Nonce n:parts {X} ==> Crypt (shrK A) X:guard n {shrK A}"*
apply *(rule shrK_is_invKey_shrK_substI, rule invKey_invKey_substI)*
by *(rule Guard_Nonce, simp+)*

26.2.2 guardedness results on nonces

lemma *guard_ciph [simp]: "shrK A:Ks ==> Ciph A X:guard n Ks"*
by *(rule Guard_Nonce, simp)*

lemma *guardK_ciph [simp]: "shrK A:Ks ==> Ciph A X:guardK n Ks"*
by *(rule Guard_Key, simp)*

lemma *Guard_init [iff]: "Guard n Ks (initState B)"*
by *(induct B, auto simp: Guard_def initState.simps)*

lemma *Guard_knows_max': "Guard n Ks (knows_max' C evs)*
==> Guard n Ks (knows_max C evs)"
by *(simp add: knows_max_def)*

lemma *Nonce_not_used_Guard_spies [dest]: "Nonce n ~:used evs*
==> Guard n Ks (spies evs)"
by *(auto simp: Guard_def dest: not_used_not_known parts_sub)*

lemma *Nonce_not_used_Guard [dest]: "[| evs:p; Nonce n ~:used evs;*
Gets_correct p; one_step p |] ==> Guard n Ks (knows (Friend C) evs)"
by *(auto simp: Guard_def dest: known_used parts_trans)*

lemma *Nonce_not_used_Guard_max [dest]: "[| evs:p; Nonce n ~:used evs;*
Gets_correct p; one_step p |] ==> Guard n Ks (knows_max (Friend C) evs)"
by *(auto simp: Guard_def dest: known_max_used parts_trans)*

lemma *Nonce_not_used_Guard_max' [dest]: "[| evs:p; Nonce n ~:used evs;*
Gets_correct p; one_step p |] ==> Guard n Ks (knows_max' (Friend C) evs)"
apply *(rule_tac H="knows_max (Friend C) evs"* **in** *Guard_mono)*
by *(auto simp: knows_max_def)*

26.2.3 guardedness results on keys

lemma *GuardK_init [simp]: "n ~:range shrK ==> GuardK n Ks (initState B)"*
by *(induct B, auto simp: GuardK_def initState.simps)*

lemma *GuardK_knows_max': "[| GuardK n A (knows_max' C evs); n ~:range shrK |]*
==> GuardK n A (knows_max C evs)"
by *(simp add: knows_max_def)*

lemma *Key_not_used_GuardK_spies [dest]: "Key n ~:used evs*
==> GuardK n A (spies evs)"
by *(auto simp: GuardK_def dest: not_used_not_known parts_sub)*

lemma *Key_not_used_GuardK [dest]: "[| evs:p; Key n ~:used evs;*
Gets_correct p; one_step p |] ==> GuardK n A (knows (Friend C) evs)"
by *(auto simp: GuardK_def dest: known_used parts_trans)*

lemma *Key_not_used_GuardK_max [dest]: "[| evs:p; Key n ~:used evs;*
Gets_correct p; one_step p |] ==> GuardK n A (knows_max (Friend C) evs)"
by *(auto simp: GuardK_def dest: known_max_used parts_trans)*

lemma *Key_not_used_GuardK_max' [dest]: "[| evs:p; Key n ~:used evs;*
Gets_correct p; one_step p |] ==> GuardK n A (knows_max' (Friend C) evs)"
apply *(rule_tac H="knows_max (Friend C) evs"* **in** *GuardK_mono)*
by *(auto simp: knows_max_def)*

26.2.4 regular protocols

constdefs *regular :: "event list set => bool"*
"regular p == ALL evs A. evs:p --> (Key (shrK A):parts (spies evs)) = (A:bad)"

lemma *shrK_parts_iff_bad [simp]: "[| evs:p; regular p |] ==>*
(Key (shrK A):parts (spies evs)) = (A:bad)"
by *(auto simp: regular_def)*

lemma *shrK_analz_iff_bad [simp]: "[| evs:p; regular p |] ==>*
(Key (shrK A):analz (spies evs)) = (A:bad)"
by *auto*

lemma *Guard_Nonce_analz: "[| Guard n Ks (spies evs); evs:p;*
shrK_set Ks; good Ks; regular p |] ==> Nonce n ~:analz (spies evs)"
apply *(clarify, simp only: knows_decomp)*
apply *(drule Guard_invKey_keyset, simp+, safe)*
apply *(drule in_good, simp)*
apply *(drule in_shrK_set, simp+, clarify)*
apply *(frule_tac A=A* **in** *shrK_analz_iff_bad)*
by *(simp add: knows_decomp)+*

lemma *GuardK_Key_analz: "[| GuardK n Ks (spies evs); evs:p;*
shrK_set Ks; good Ks; regular p; n ~:range shrK |] ==> Key n ~:analz (spies evs)"
apply *(clarify, simp only: knows_decomp)*
apply *(drule GuardK_invKey_keyset, clarify, simp+, simp add: initState.simps)*
apply *clarify*

```
apply (drule in_good, simp)
apply (drule in_shrK_set, simp+, clarify)
apply (frule_tac A=A in shrK_analz_iff_bad)
by (simp add: knows_decomp)+

end
```

27 Otway-Rees Protocol

```
theory Guard_OtwayRees imports Guard_Shared begin
```

27.1 messages used in the protocol

```
syntax nil :: "msg"

translations "nil" == "Number 0"

syntax or1 :: "agent => agent => nat => event"

translations "or1 A B NA"
=> "Says A B {|Nonce NA, Agent A, Agent B,
               Ciph A {|Nonce NA, Agent A, Agent B|}|}"

syntax or1' :: "agent => agent => agent => nat => msg => event"

translations "or1' A' A B NA X"
=> "Says A' B {|Nonce NA, Agent A, Agent B, X|}"

syntax or2 :: "agent => agent => nat => nat => msg => event"

translations "or2 A B NA NB X"
=> "Says B Server {|Nonce NA, Agent A, Agent B, X,
                    Ciph B {|Nonce NA, Nonce NB, Agent A, Agent B|}|}"

syntax or2' :: "agent => agent => agent => nat => nat => event"

translations "or2' B' A B NA NB"
=> "Says B' Server {|Nonce NA, Agent A, Agent B,
                     Ciph A {|Nonce NA, Agent A, Agent B|},
                     Ciph B {|Nonce NA, Nonce NB, Agent A, Agent B|}|}"

syntax or3 :: "agent => agent => nat => nat => key => event"

translations "or3 A B NA NB K"
=> "Says Server B {|Nonce NA, Ciph A {|Nonce NA, Key K|},
                    Ciph B {|Nonce NB, Key K|}|}"

syntax or3':: "agent => msg => agent => agent => nat => nat => key => event"

translations "or3' S Y A B NA NB K"
=> "Says S B {|Nonce NA, Y, Ciph B {|Nonce NB, Key K|}|}"

syntax or4 :: "agent => agent => nat => msg => event"
```

translations *"or4 A B NA X" => "Says B A {|Nonce NA, X, nil|}"*

syntax *or4' :: "agent => agent => nat => msg => event"*

translations *"or4' B' A NA K" =>*
"Says B' A {|Nonce NA, Ciph A {|Nonce NA, Key K|}, nil|}"

27.2 definition of the protocol

consts *or :: "event list set"*

inductive *or*
intros

Nil: "[]:or"

Fake: "[| evs:or; X:synth (analz (spies evs)) |] ==> Says Spy B X # evs:or"

OR1: "[| evs1:or; Nonce NA ~:used evs1 |] ==> or1 A B NA # evs1:or"

OR2: "[| evs2:or; or1' A' A B NA X:set evs2; Nonce NB ~:used evs2 |]
==> or2 A B NA NB X # evs2:or"

OR3: "[| evs3:or; or2' B' A B NA NB:set evs3; Key K ~:used evs3 |]
==> or3 A B NA NB K # evs3:or"

OR4: "[| evs4:or; or2 A B NA NB X:set evs4; or3' S Y A B NA NB K:set evs4 |]
==> or4 A B NA X # evs4:or"

27.3 declarations for tactics

declare *knows_Spy_partsEs [elim]*
declare *Fake_parts_insert [THEN subsetD, dest]*
declare *initState.simps [simp del]*

27.4 general properties of or

lemma *or_has_no_Gets: "evs:or ==> ALL A X. Gets A X ~:set evs"*
by *(erule or.induct, auto)*

lemma *or_is_Gets_correct [iff]: "Gets_correct or"*
by *(auto simp: Gets_correct_def dest: or_has_no_Gets)*

lemma *or_is_one_step [iff]: "one_step or"*
by *(unfold one_step_def, clarify, ind_cases "ev#evs:or", auto)*

lemma *or_has_only_Says' [rule_format]: "evs:or ==>*
ev:set evs --> (EX A B X. ev=Says A B X)"
by *(erule or.induct, auto)*

lemma *or_has_only_Says [iff]: "has_only_Says or"*
by *(auto simp: has_only_Says_def dest: or_has_only_Says')*

27.5 or is regular

lemma *or1'_parts_spies [dest]: "or1' A' A B NA X:set evs*
==> X:parts (spies evs)"
by *blast*

lemma *or2_parts_spies [dest]: "or2 A B NA NB X:set evs*
==> X:parts (spies evs)"
by *blast*

lemma *or3_parts_spies [dest]: "Says S B {|NA, Y, Ciph B {|NB, K|}|}:set evs*
==> K:parts (spies evs)"
by *blast*

lemma *or_is_regular [iff]: "regular or"*
apply *(simp only: regular_def, clarify)*
apply *(erule or.induct, simp_all add: initState.simps knows.simps)*
by *(auto dest: parts_sub)*

27.6 guardedness of KAB

lemma *Guard_KAB [rule_format]: "[| evs:or; A ~:bad; B ~:bad |] ==>*
or3 A B NA NB K:set evs --> GuardK K {shrK A,shrK B} (spies evs)"
apply *(erule or.induct)*

apply *simp_all*

apply *(clarify, erule in_synth_GuardK, erule GuardK_analz, simp)*

apply *blast*

apply *safe*
apply *(blast dest: Says_imp_spies, blast)*

apply *blast*
apply *(drule_tac A=Server* **in** *Key_neq, simp+, rule No_Key, simp)*
apply *(drule_tac A=Server* **in** *Key_neq, simp+, rule No_Key, simp)*

by *(blast dest: Says_imp_spies in_GuardK_kparts)*

27.7 guardedness of NB

lemma *Guard_NB [rule_format]: "[| evs:or; B ~:bad |] ==>*
or2 A B NA NB X:set evs --> Guard NB {shrK B} (spies evs)"
apply *(erule or.induct)*

apply *simp_all*

apply *safe*
apply *(erule in_synth_Guard, erule Guard_analz, simp)*

apply *(drule_tac n=NB* **in** *Nonce_neq, simp+, rule No_Nonce, simp)*
apply *(drule_tac n=NB* **in** *Nonce_neq, simp+, rule No_Nonce, simp)*

apply *blast*

```
apply (drule_tac n=NA in Nonce_neq, simp+, rule No_Nonce, simp)
apply (blast intro!: No_Nonce dest: used_parts)
apply (drule_tac n=NA in Nonce_neq, simp+, rule No_Nonce, simp)
apply (blast intro!: No_Nonce dest: used_parts)
apply (blast dest: Says_imp_spies)
apply (blast dest: Says_imp_spies)
apply (case_tac "Ba=B", clarsimp)
apply (drule_tac n=NB and A=B in Nonce_neq, simp+)
apply (drule Says_imp_spies)
apply (drule_tac n'=NAa in in_Guard_kparts_neq, simp+, rule No_Nonce, simp)

apply (drule Says_imp_spies)
apply (frule_tac n'=NAa in in_Guard_kparts_neq, simp+, rule No_Nonce, simp)
apply (case_tac "Aa=B", clarsimp)
apply (case_tac "NAa=NB", clarsimp)
apply (drule Says_imp_spies)
apply (drule_tac Y="{|Nonce NB, Agent Aa, Agent Ba|}"
                 and K="shrK Aa" in in_Guard_kparts_Crypt, simp+)
apply (simp add: No_Nonce)
apply (case_tac "Ba=B", clarsimp)
apply (case_tac "NBa=NB", clarify)
apply (drule Says_imp_spies)
apply (drule_tac Y="{|Nonce NAa, Nonce NB, Agent Aa, Agent Ba|}"
                 and K="shrK Ba" in in_Guard_kparts_Crypt, simp+)
apply (simp add: No_Nonce)

by (blast dest: Says_imp_spies)+

end
```

28 Yahalom Protocol

```
theory Guard_Yahalom imports Guard_Shared begin
```

28.1 messages used in the protocol

```
syntax ya1 :: "agent => agent => nat => event"

translations "ya1 A B NA" => "Says A B {|Agent A, Nonce NA|}"

syntax ya1' :: "agent => agent => agent => nat => event"

translations "ya1' A' A B NA" => "Says A' B {|Agent A, Nonce NA|}"

syntax ya2 :: "agent => agent => nat => nat => event"

translations "ya2 A B NA NB"
=> "Says B Server {|Agent B, Ciph B {|Agent A, Nonce NA, Nonce NB|}|}"

syntax ya2' :: "agent => agent => agent => nat => nat => event"

translations "ya2' B' A B NA NB"
```

```
=> "Says B' Server {|Agent B, Ciph B {|Agent A, Nonce NA, Nonce NB|}|}"
```

syntax *ya3 :: "agent => agent => nat => nat => key => event"*

translations *"ya3 A B NA NB K"*
=> "Says Server A {|Ciph A {|Agent B, Key K, Nonce NA, Nonce NB|},
Ciph B {|Agent A, Key K|}|}"

syntax *ya3'':: "agent => msg => agent => agent => nat => nat => key => event"*

translations *"ya3' S Y A B NA NB K"*
=> "Says S A {|Ciph A {|Agent B, Key K, Nonce NA, Nonce NB|}, Y|}"

syntax *ya4 :: "agent => agent => nat => nat => msg => event"*

translations *"ya4 A B K NB Y" => "Says A B {|Y, Crypt K (Nonce NB)|}"*

syntax *ya4' :: "agent => agent => nat => nat => msg => event"*

translations *"ya4' A' B K NB Y" => "Says A' B {|Y, Crypt K (Nonce NB)|}"*

28.2 definition of the protocol

consts *ya :: "event list set"*

inductive *ya*
intros

Nil: "[]:ya"

Fake: "[| evs:ya; X:synth (analz (spies evs)) |] ==> Says Spy B X # evs:ya"

YA1: "[| evs1:ya; Nonce NA ~:used evs1 |] ==> ya1 A B NA # evs1:ya"

YA2: "[| evs2:ya; ya1' A' A B NA:set evs2; Nonce NB ~:used evs2 |]
==> ya2 A B NA NB # evs2:ya"

YA3: "[| evs3:ya; ya2' B' A B NA NB:set evs3; Key K ~:used evs3 |]
==> ya3 A B NA NB K # evs3:ya"

YA4: "[| evs4:ya; ya1 A B NA:set evs4; ya3' S Y A B NA NB K:set evs4 |]
==> ya4 A B K NB Y # evs4:ya"

28.3 declarations for tactics

declare *knows_Spy_partsEs [elim]*
declare *Fake_parts_insert [THEN subsetD, dest]*
declare *initState.simps [simp del]*

28.4 general properties of ya

lemma *ya_has_no_Gets: "evs:ya ==> ALL A X. Gets A X ~:set evs"*
by *(erule ya.induct, auto)*

lemma *ya_is_Gets_correct [iff]: "Gets_correct ya"*
by *(auto simp: Gets_correct_def dest: ya_has_no_Gets)*

lemma *ya_is_one_step [iff]: "one_step ya"*
by *(unfold one_step_def, clarify, ind_cases "ev#evs:ya", auto)*

lemma *ya_has_only_Says' [rule_format]: "evs:ya ==>*
ev:set evs --> (EX A B X. ev=Says A B X)"
by *(erule ya.induct, auto)*

lemma *ya_has_only_Says [iff]: "has_only_Says ya"*
by *(auto simp: has_only_Says_def dest: ya_has_only_Says')*

lemma *ya_is_regular [iff]: "regular ya"*
apply *(simp only: regular_def, clarify)*
apply *(erule ya.induct, simp_all add: initState.simps knows.simps)*
by *(auto dest: parts_sub)*

28.5 guardedness of KAB

lemma *Guard_KAB [rule_format]: "[| evs:ya; A ~:bad; B ~:bad |] ==>*
ya3 A B NA NB K:set evs --> GuardK K {shrK A,shrK B} (spies evs)"
apply *(erule ya.induct)*

apply *simp_all*

apply *(clarify, erule in_synth_GuardK, erule GuardK_analz, simp)*

apply *safe*
apply *(blast dest: Says_imp_spies)*

apply *blast*
apply *(drule_tac A=Server* **in** *Key_neq, simp+, rule No_Key, simp)*
apply *(drule_tac A=Server* **in** *Key_neq, simp+, rule No_Key, simp)*

apply *(blast dest: Says_imp_spies in_GuardK_kparts)*
by *blast*

28.6 session keys are not symmetric keys

lemma *KAB_isnt_shrK [rule_format]: "evs:ya ==>*
ya3 A B NA NB K:set evs --> K ~:range shrK"
by *(erule ya.induct, auto)*

lemma *ya3_shrK: "evs:ya ==> ya3 A B NA NB (shrK C) ~:set evs"*
by *(blast dest: KAB_isnt_shrK)*

28.7 ya2' implies ya1'

lemma *ya2'_parts_imp_ya1'_parts [rule_format]:*
"[| evs:ya; B ~:bad |] ==>
Ciph B {|Agent A, Nonce NA, Nonce NB|}:parts (spies evs) -->
{|Agent A, Nonce NA|}:spies evs"

by *(erule ya.induct, auto dest: Says_imp_spies intro: parts_parts)*

lemma *ya2'_imp_ya1'_parts: "[| ya2' B' A B NA NB:set evs; evs:ya; B ~:bad |]*
==> {|Agent A, Nonce NA|}:spies evs"
by *(blast dest: Says_imp_spies ya2'_parts_imp_ya1'_parts)*

28.8 uniqueness of NB

lemma *NB_is_uniq_in_ya2'_parts [rule_format]: "[| evs:ya; B ~:bad; B' ~:bad |] ==>*
Ciph B {|Agent A, Nonce NA, Nonce NB|}:parts (spies evs) -->
Ciph B' {|Agent A', Nonce NA', Nonce NB|}:parts (spies evs) -->
A=A' & B=B' & NA=NA'"
apply *(erule ya.induct, simp_all, clarify)*
apply *(drule Crypt_synth_insert, simp+)*
apply *(drule Crypt_synth_insert, simp+, safe)*
apply *(drule not_used_parts_false, simp+)+*
by *(drule Says_not_parts, simp+)+*

lemma *NB_is_uniq_in_ya2': "[| ya2' C A B NA NB:set evs;*
ya2' C' A' B' NA' NB:set evs; evs:ya; B ~:bad; B' ~:bad |]
==> A=A' & B=B' & NA=NA'"
by *(drule NB_is_uniq_in_ya2'_parts, auto dest: Says_imp_spies)*

28.9 ya3' implies ya2'

lemma *ya3'_parts_imp_ya2'_parts [rule_format]: "[| evs:ya; A ~:bad |] ==>*
Ciph A {|Agent B, Key K, Nonce NA, Nonce NB|}:parts (spies evs)
--> Ciph B {|Agent A, Nonce NA, Nonce NB|}:parts (spies evs)"
apply *(erule ya.induct, simp_all)*
apply *(clarify, drule Crypt_synth_insert, simp+)*
apply *(blast intro: parts_sub, blast)*
by *(auto dest: Says_imp_spies parts_parts)*

lemma *ya3'_parts_imp_ya2' [rule_format]: "[| evs:ya; A ~:bad |] ==>*
Ciph A {|Agent B, Key K, Nonce NA, Nonce NB|}:parts (spies evs)
--> (EX B'. ya2' B' A B NA NB:set evs)"
apply *(erule ya.induct, simp_all, safe)*
apply *(drule Crypt_synth_insert, simp+)*
apply *(drule Crypt_synth_insert, simp+, blast)*
apply *blast*
apply *blast*
by *(auto dest: Says_imp_spies2 parts_parts)*

lemma *ya3'_imp_ya2': "[| ya3' S Y A B NA NB K:set evs; evs:ya; A ~:bad |]*
==> (EX B'. ya2' B' A B NA NB:set evs)"
by *(drule ya3'_parts_imp_ya2', auto dest: Says_imp_spies)*

28.10 ya3' implies ya3

lemma *ya3'_parts_imp_ya3 [rule_format]: "[| evs:ya; A ~:bad |] ==>*
Ciph A {|Agent B, Key K, Nonce NA, Nonce NB|}:parts(spies evs)
--> ya3 A B NA NB K:set evs"

```
apply (erule ya.induct, simp_all, safe)
apply (drule Crypt_synth_insert, simp+)
by (blast dest: Says_imp_spies2 parts_parts)

lemma ya3'_imp_ya3: "[| ya3' S Y A B NA NB K:set evs; evs:ya; A ~:bad |]
==> ya3 A B NA NB K:set evs"
by (blast dest: Says_imp_spies ya3'_parts_imp_ya3)
```

28.11 guardedness of NB

```
constdefs ya_keys :: "agent => agent => nat => nat => event list => key set"
"ya_keys A B NA NB evs == {shrK A,shrK B} Un {K. ya3 A B NA NB K:set evs}"

lemma Guard_NB [rule_format]: "[| evs:ya; A ~:bad; B ~:bad |] ==>
ya2 A B NA NB:set evs --> Guard NB (ya_keys A B NA NB evs) (spies evs)"
apply (erule ya.induct)

apply (simp_all add: ya_keys_def)

apply safe
apply (erule in_synth_Guard, erule Guard_analz, simp, clarify)
apply (frule_tac B=B in Guard_KAB, simp+)
apply (drule_tac p=ya in GuardK_Key_analz, simp+)
apply (blast dest: KAB_isnt_shrK, simp)

apply (drule_tac n=NB in Nonce_neq, simp+, rule No_Nonce, simp)

apply blast
apply (drule Says_imp_spies)
apply (drule_tac n=NB in Nonce_neq, simp+)
apply (drule_tac n'=NAa in in_Guard_kparts_neq, simp+)
apply (rule No_Nonce, simp)

apply (rule Guard_extand, simp, blast)
apply (case_tac "NAa=NB", clarify)
apply (frule Says_imp_spies)
apply (frule in_Guard_kparts_Crypt, simp+, blast, simp+)
apply (frule_tac A=A and B=B and NA=NA and NB=NB and C=Ba in ya3_shrK,
simp)
apply (drule ya2'_imp_ya1'_parts, simp, blast, blast)
apply (case_tac "NBa=NB", clarify)
apply (frule Says_imp_spies)
apply (frule in_Guard_kparts_Crypt, simp+, blast, simp+)
apply (frule_tac A=A and B=B and NA=NA and NB=NB and C=Ba in ya3_shrK,
simp)
apply (drule NB_is_uniq_in_ya2', simp+, blast, simp+)
apply (simp add: No_Nonce, blast)

apply (blast dest: Says_imp_spies)
apply (case_tac "NBa=NB", clarify)
apply (frule_tac A=S in Says_imp_spies)
apply (frule in_Guard_kparts_Crypt, simp+)
apply (blast dest: Says_imp_spies)
apply (case_tac "NBa=NB", clarify)
```

```
apply (frule_tac A=S in Says_imp_spies)
apply (frule in_Guard_kparts_Crypt, simp+, blast, simp+)
apply (frule_tac A=A and B=B and NA=NA and NB=NB and C=Aa in ya3_shrK,
simp)
apply (frule ya3'_imp_ya2', simp+, blast, clarify)
apply (frule_tac A=B' in Says_imp_spies)
apply (rotate_tac -1, frule in_Guard_kparts_Crypt, simp+, blast, simp+)
apply (frule_tac A=A and B=B and NA=NA and NB=NB and C=Ba in ya3_shrK,
simp)
apply (drule NB_is_uniq_in_ya2', simp+, blast, clarify)
apply (drule ya3'_imp_ya3, simp+)
apply (simp add: Guard_Nonce)
apply (simp add: No_Nonce)
done

end
```

29 Other Protocol-Independent Results

theory *Proto* **imports** *Guard_Public* **begin**

29.1 protocols

types *rule = "event set * event"*

syntax *msg' :: "rule => msg"*

translations *"msg' R" == "msg (snd R)"*

types *proto = "rule set"*

constdefs *wdef :: "proto => bool"*
"wdef p == ALL R k. R:p --> Number k:parts {msg' R}
--> Number k:parts (msg`(fst R))"

29.2 substitutions

record *subs =*
agent :: "agent => agent"
nonce :: "nat => nat"
nb :: "nat => msg"
key :: "key => key"

consts *apm :: "subs => msg => msg"*

primrec
"apm s (Agent A) = Agent (agent s A)"
"apm s (Nonce n) = Nonce (nonce s n)"
"apm s (Number n) = nb s n"
"apm s (Key K) = Key (key s K)"
"apm s (Hash X) = Hash (apm s X)"
"apm s (Crypt K X) = (
if (EX A. K = pubK A) then Crypt (pubK (agent s (agt K))) (apm s X)

```
else if (EX A. K = priK A) then Crypt (priK (agent s (agt K))) (apm s X)
else Crypt (key s K) (apm s X))"
"apm s {|X,Y|} = {|apm s X, apm s Y|}"

lemma apm_parts: "X:parts {Y} ==> apm s X:parts {apm s Y}"
apply (erule parts.induct, simp_all, blast)
apply (erule parts.Fst)
apply (erule parts.Snd)
by (erule parts.Body)+

lemma Nonce_apm [rule_format]: "Nonce n:parts {apm s X} ==>
(ALL k. Number k:parts {X} --> Nonce n ~:parts {nb s k}) -->
(EX k. Nonce k:parts {X} & nonce s k = n)"
by (induct X, simp_all, blast)

lemma wdef_Nonce: "[| Nonce n:parts {apm s X}; R:p; msg' R = X; wdef p;
Nonce n ~:parts (apm s `(msg `(fst R))) |] ==>
(EX k. Nonce k:parts {X} & nonce s k = n)"
apply (erule Nonce_apm, unfold wdef_def)
apply (drule_tac x=R in spec, drule_tac x=k in spec, clarsimp)
apply (drule_tac x=x in bspec, simp)
apply (drule_tac Y="msg x" and s=s in apm_parts, simp)
by (blast dest: parts_parts)

consts ap :: "subs => event => event"

primrec
"ap s (Says A B X) = Says (agent s A) (agent s B) (apm s X)"
"ap s (Gets A X) = Gets (agent s A) (apm s X)"
"ap s (Notes A X) = Notes (agent s A) (apm s X)"

syntax
ap' :: "rule => msg"
apm' :: "rule => msg"
priK' :: "subs => agent => key"
pubK' :: "subs => agent => key"

translations
"ap' s R" == "ap s (snd R)"
"apm' s R" == "apm s (msg' R)"
"priK' s A" == "priK (agent s A)"
"pubK' s A" == "pubK (agent s A)"
```

29.3 nonces generated by a rule

```
constdefs newn :: "rule => nat set"
"newn R == {n. Nonce n:parts {msg (snd R)} & Nonce n ~:parts (msg`(fst R))}"

lemma newn_parts: "n:newn R ==> Nonce (nonce s n):parts {apm' s R}"
by (auto simp: newn_def dest: apm_parts)
```

29.4 traces generated by a protocol

```
constdefs ok :: "event list => rule => subs => bool"
```

```
"ok evs R s == ((ALL x. x:fst R --> ap s x:set evs)
& (ALL n. n:newn R --> Nonce (nonce s n) ~:used evs))"

consts tr :: "proto => event list set"

inductive "tr p" intros

Nil [intro]: "[]:tr p"

Fake [intro]: "[| evsf:tr p; X:synth (analz (spies evsf)) |]
==> Says Spy B X # evsf:tr p"

Proto [intro]: "[| evs:tr p; R:p; ok evs R s |] ==> ap' s R # evs:tr p"
```

29.5 general properties

```
lemma one_step_tr [iff]: "one_step (tr p)"
apply (unfold one_step_def, clarify)
by (ind_cases "ev # evs:tr p", auto)

constdefs has_only_Says' :: "proto => bool"
"has_only_Says' p == ALL R. R:p --> is_Says (snd R)"

lemma has_only_Says'D: "[| R:p; has_only_Says' p |]
==> (EX A B X. snd R = Says A B X)"
by (unfold has_only_Says'_def is_Says_def, blast)

lemma has_only_Says_tr [simp]: "has_only_Says' p ==> has_only_Says (tr p)"
apply (unfold has_only_Says_def)
apply (rule allI, rule allI, rule impI)
apply (erule tr.induct)
apply (auto simp: has_only_Says'_def ok_def)
by (drule_tac x=a in spec, auto simp: is_Says_def)

lemma has_only_Says'_in_trD: "[| has_only_Says' p; list @ ev # evs1 ∈ tr
p |]
==> (EX A B X. ev = Says A B X)"
by (drule has_only_Says_tr, auto)

lemma ok_not_used: "[| Nonce n ~:used evs; ok evs R s;
ALL x. x:fst R --> is_Says x |] ==> Nonce n ~:parts (apm s `(msg `(fst R)))"
apply (unfold ok_def, clarsimp)
apply (drule_tac x=x in spec, drule_tac x=x in spec)
by (auto simp: is_Says_def dest: Says_imp_spies not_used_not_spied parts_parts)

lemma ok_is_Says: "[| evs' @ ev # evs:tr p; ok evs R s; has_only_Says' p;
R:p; x:fst R |] ==> is_Says x"
apply (unfold ok_def is_Says_def, clarify)
apply (drule_tac x=x in spec, simp)
apply (subgoal_tac "one_step (tr p)")
apply (drule trunc, simp, drule one_step_Cons, simp)
apply (drule has_only_SaysD, simp+)
by (clarify, case_tac x, auto)
```

29.6 types

types *keyfun = "rule => subs => nat => event list => key set"*

types *secfun = "rule => nat => subs => key set => msg"*

29.7 introduction of a fresh guarded nonce

constdefs *fresh :: "proto => rule => subs => nat => key set => event list*
=> bool"
"fresh p R s n Ks evs == (EX evs1 evs2. evs = evs2 @ ap' s R # evs1
& Nonce n ~:used evs1 & R:p & ok evs1 R s & Nonce n:parts {apm' s R}
& apm' s R:guard n Ks)"

lemma *freshD: "fresh p R s n Ks evs ==> (EX evs1 evs2.*
evs = evs2 @ ap' s R # evs1 & Nonce n ~:used evs1 & R:p & ok evs1 R s
& Nonce n:parts {apm' s R} & apm' s R:guard n Ks)"
by *(unfold fresh_def, blast)*

lemma *freshI [intro]: "[| Nonce n ~:used evs1; R:p; Nonce n:parts {apm' s*
R};
ok evs1 R s; apm' s R:guard n Ks |]
==> fresh p R s n Ks (list @ ap' s R # evs1)"
by *(unfold fresh_def, blast)*

lemma *freshI': "[| Nonce n ~:used evs1; (l,r):p;*
Nonce n:parts {apm s (msg r)}; ok evs1 (l,r) s; apm s (msg r):guard n Ks |]
==> fresh p (l,r) s n Ks (evs2 @ ap s r # evs1)"
by *(drule freshI, simp+)*

lemma *fresh_used: "[| fresh p R' s' n Ks evs; has_only_Says' p |]*
==> Nonce n:used evs"
apply *(unfold fresh_def, clarify)*
apply *(drule has_only_Says'D)*
by *(auto intro: parts_used_app)*

lemma *fresh_newn: "[| evs' @ ap' s R # evs:tr p; wdef p; has_only_Says' p;*
Nonce n ~:used evs; R:p; ok evs R s; Nonce n:parts {apm' s R} |]
==> EX k. k:newn R & nonce s k = n"
apply *(drule wdef_Nonce, simp+)*
apply *(frule ok_not_used, simp+)*
apply *(clarify, erule ok_is_Says, simp+)*
apply *(clarify, rule_tac x=k* **in** *exI, simp add: newn_def)*
apply *(clarify, drule_tac Y="msg x"* **and** *s=s* **in** *apm_parts)*
apply *(drule ok_not_used, simp+)*
by *(clarify, erule ok_is_Says, simp+)*

lemma *fresh_rule: "[| evs' @ ev # evs:tr p; wdef p; Nonce n ~:used evs;*
Nonce n:parts {msg ev} |] ==> EX R s. R:p & ap' s R = ev"
apply *(drule trunc, simp, ind_cases "ev # evs:tr p", simp)*
by *(drule_tac x=X* **in** *in_sub, drule parts_sub, simp, simp, blast+)*

lemma *fresh_ruleD: "[| fresh p R' s' n Ks evs; keys R' s' n evs <= Ks; wdef*
p;
has_only_Says' p; evs:tr p; ALL R k s. nonce s k = n --> Nonce n:used evs -->

```
R:p --> k:newn R --> Nonce n:parts {apm' s R} --> apm' s R:guard n Ks -->
apm' s R:parts (spies evs) --> keys R s n evs <= Ks --> P |] ==> P"
apply (frule fresh_used, simp)
apply (unfold fresh_def, clarify)
apply (drule_tac x=R' in spec)
apply (drule fresh_newn, simp+, clarify)
apply (drule_tac x=k in spec)
apply (drule_tac x=s' in spec)
apply (subgoal_tac "apm' s' R':parts (spies (evs2 @ ap' s' R' # evs1))")
apply (case_tac R', drule has_only_Says'D, simp, clarsimp)
apply (case_tac R', drule has_only_Says'D, simp, clarsimp)
apply (rule_tac Y="apm s' X" in parts_parts, blast)
by (rule parts.Inj, rule Says_imp_spies, simp, blast)
```

29.8 safe keys

```
constdefs safe :: "key set => msg set => bool"
"safe Ks G == ALL K. K:Ks --> Key K ~:analz G"

lemma safeD [dest]: "[| safe Ks G; K:Ks |] ==> Key K ~:analz G"
by (unfold safe_def, blast)

lemma safe_insert: "safe Ks (insert X G) ==> safe Ks G"
by (unfold safe_def, blast)

lemma Guard_safe: "[| Guard n Ks G; safe Ks G |] ==> Nonce n ~:analz G"
by (blast dest: Guard_invKey)
```

29.9 guardedness preservation

```
constdefs preserv :: "proto => keyfun => nat => key set => bool"
"preserv p keys n Ks == (ALL evs R' s' R s. evs:tr p -->
Guard n Ks (spies evs) --> safe Ks (spies evs) --> fresh p R' s' n Ks evs -->
keys R' s' n evs <= Ks --> R:p --> ok evs R s --> apm' s R:guard n Ks)"

lemma preservD: "[| preserv p keys n Ks; evs:tr p; Guard n Ks (spies evs);
safe Ks (spies evs); fresh p R' s' n Ks evs; R:p; ok evs R s;
keys R' s' n evs <= Ks |] ==> apm' s R:guard n Ks"
by (unfold preserv_def, blast)

lemma preservD': "[| preserv p keys n Ks; evs:tr p; Guard n Ks (spies evs);
safe Ks (spies evs); fresh p R' s' n Ks evs; (l,Says A B X):p;
ok evs (l,Says A B X) s; keys R' s' n evs <= Ks |] ==> apm s X:guard n Ks"
by (drule preservD, simp+)
```

29.10 monotonic keyfun

```
constdefs monoton :: "proto => keyfun => bool"
"monoton p keys == ALL R' s' n ev evs. ev # evs:tr p -->
keys R' s' n evs <= keys R' s' n (ev # evs)"

lemma monotonD [dest]: "[| keys R' s' n (ev # evs) <= Ks; monoton p keys;
ev # evs:tr p |] ==> keys R' s' n evs <= Ks"
by (unfold monoton_def, blast)
```

29.11 guardedness theorem

lemma *Guard_tr [rule_format]: "[| evs:tr p; has_only_Says' p;*
preserv p keys n Ks; monoton p keys; Guard n Ks (initState Spy) |] ==>
safe Ks (spies evs) --> fresh p R' s' n Ks evs --> keys R' s' n evs <= Ks -->
Guard n Ks (spies evs)"
apply *(erule tr.induct)*

apply *simp*

apply *(clarify, drule freshD, clarsimp)*
apply *(case_tac evs2)*

apply *(frule has_only_Says'D, simp)*
apply *(clarsimp, blast)*

apply *(clarsimp, rule conjI)*
apply *(blast dest: safe_insert)*

apply *(rule in_synth_Guard, simp, rule Guard_analz)*
apply *(blast dest: safe_insert)*
apply *(drule safe_insert, simp add: safe_def)*

apply *(clarify, drule freshD, clarify)*
apply *(case_tac evs2)*

apply *(frule has_only_Says'D, simp)*
apply *(frule_tac R=R'* **in** *has_only_Says'D, simp)*
apply *(case_tac R', clarsimp, blast)*

apply *(frule has_only_Says'D, simp)*
apply *(clarsimp, rule conjI)*
apply *(drule Proto, simp+, blast dest: safe_insert)*

apply *(frule Proto, simp+)*
apply *(erule preservD', simp+)*
apply *(blast dest: safe_insert)*
apply *(blast dest: safe_insert)*
by *(blast, simp, simp, blast)*

29.12 useful properties for guardedness

lemma *newn_neq_used: "[| Nonce n:used evs; ok evs R s; k:newn R |]*
==> n ~= nonce s k"
by *(auto simp: ok_def)*

lemma *ok_Guard: "[| ok evs R s; Guard n Ks (spies evs); x:fst R; is_Says*
x |]
==> apm s (msg x):parts (spies evs) & apm s (msg x):guard n Ks"
apply *(unfold ok_def is_Says_def, clarify)*
apply *(drule_tac x="Says A B X"* **in** *spec, simp)*
by *(drule Says_imp_spies, auto intro: parts_parts)*

lemma *ok_parts_not_new: "[| Y:parts (spies evs); Nonce (nonce s n):parts*
{Y};

```
ok evs R s |] ==> n ~:newn R"
by (auto simp: ok_def dest: not_used_not_spied parts_parts)
```

29.13 unicity

```
constdefs uniq :: "proto => secfun => bool"
"uniq p secret == ALL evs R R' n n' Ks s s'. R:p --> R':p -->
n:newn R --> n':newn R' --> nonce s n = nonce s' n' -->
Nonce (nonce s n):parts {apm' s R} --> Nonce (nonce s n):parts {apm' s' R'}
-->
apm' s R:guard (nonce s n) Ks --> apm' s' R':guard (nonce s n) Ks -->
evs:tr p --> Nonce (nonce s n) ~:analz (spies evs) -->
secret R n s Ks:parts (spies evs) --> secret R' n' s' Ks:parts (spies evs)
-->
secret R n s Ks = secret R' n' s' Ks"

lemma uniqD: "[| uniq p secret; evs: tr p; R:p; R':p; n:newn R; n':newn R';
nonce s n = nonce s' n'; Nonce (nonce s n) ~:analz (spies evs);
Nonce (nonce s n):parts {apm' s R}; Nonce (nonce s n):parts {apm' s' R'};
secret R n s Ks:parts (spies evs); secret R' n' s' Ks:parts (spies evs);
apm' s R:guard (nonce s n) Ks; apm' s' R':guard (nonce s n) Ks |] ==>
secret R n s Ks = secret R' n' s' Ks"
by (unfold uniq_def, blast)

constdefs ord :: "proto => (rule => rule => bool) => bool"
"ord p inf == ALL R R'. R:p --> R':p --> ~ inf R R' --> inf R' R"

lemma ordD: "[| ord p inf; ~ inf R R'; R:p; R':p |] ==> inf R' R"
by (unfold ord_def, blast)

constdefs uniq' :: "proto => (rule => rule => bool) => secfun => bool"
"uniq' p inf secret == ALL evs R R' n n' Ks s s'. R:p --> R':p -->
inf R R' --> n:newn R --> n':newn R' --> nonce s n = nonce s' n' -->
Nonce (nonce s n):parts {apm' s R} --> Nonce (nonce s n):parts {apm' s' R'}
-->
apm' s R:guard (nonce s n) Ks --> apm' s' R':guard (nonce s n) Ks -->
evs:tr p --> Nonce (nonce s n) ~:analz (spies evs) -->
secret R n s Ks:parts (spies evs) --> secret R' n' s' Ks:parts (spies evs)
-->
secret R n s Ks = secret R' n' s' Ks"

lemma uniq'D: "[| uniq' p inf secret; evs: tr p; inf R R'; R:p; R':p; n:newn
R;
n':newn R'; nonce s n = nonce s' n'; Nonce (nonce s n) ~:analz (spies evs);
Nonce (nonce s n):parts {apm' s R}; Nonce (nonce s n):parts {apm' s' R'};
secret R n s Ks:parts (spies evs); secret R' n' s' Ks:parts (spies evs);
apm' s R:guard (nonce s n) Ks; apm' s' R':guard (nonce s n) Ks |] ==>
secret R n s Ks = secret R' n' s' Ks"
by (unfold uniq'_def, blast)

lemma uniq'_imp_uniq: "[| uniq' p inf secret; ord p inf |] ==> uniq p secret"
apply (unfold uniq_def)
apply (rule allI)+
apply (case_tac "inf R R'")
```

apply *(blast dest: uniq'D)*
by *(auto dest: ordD uniq'D intro: sym)*

29.14 Needham-Schroeder-Lowe

constdefs
```
a :: agent "a == Friend 0"
b :: agent "b == Friend 1"
a' :: agent "a' == Friend 2"
b' :: agent "b' == Friend 3"
Na :: nat "Na == 0"
Nb :: nat "Nb == 1"
```

consts
```
ns :: proto
ns1 :: rule
ns2 :: rule
ns3 :: rule
```

translations
```
"ns1" == "({}, Says a b (Crypt (pubK b) {|Nonce Na, Agent a|}))"

"ns2" == "({Says a' b (Crypt (pubK b) {|Nonce Na, Agent a|})},
Says b a (Crypt (pubK a) {|Nonce Na, Nonce Nb, Agent b|}))"

"ns3" == "({Says a b (Crypt (pubK b) {|Nonce Na, Agent a|}),
Says b' a (Crypt (pubK a) {|Nonce Na, Nonce Nb, Agent b|})},
Says a b (Crypt (pubK b) (Nonce Nb)))"
```

inductive *ns* **intros**
```
[iff]: "ns1:ns"
[iff]: "ns2:ns"
[iff]: "ns3:ns"
```

syntax
```
ns3a :: msg
ns3b :: msg
```

translations
```
"ns3a" => "Says a b (Crypt (pubK b) {|Nonce Na, Agent a|})"
"ns3b" => "Says b' a (Crypt (pubK a) {|Nonce Na, Nonce Nb, Agent b|})"
```

constdefs *keys :: "keyfun"*
```
"keys R' s' n evs == {priK' s' a, priK' s' b}"
```

lemma *"monoton ns keys"*
by *(simp add: keys_def monoton_def)*

constdefs *secret :: "secfun"*
```
"secret R n s Ks ==
(if R=ns1 then apm s (Crypt (pubK b) {|Nonce Na, Agent a|})
else if R=ns2 then apm s (Crypt (pubK a) {|Nonce Na, Nonce Nb, Agent b|})
else Number 0)"
```

constdefs *inf :: "rule => rule => bool"*
"inf R R' == (R=ns1 | (R=ns2 & R'~=ns1) | (R=ns3 & R'=ns3))"

lemma *inf_is_ord [iff]: "ord ns inf"*
apply *(unfold ord_def inf_def)*
apply *(rule allI)+*
by *(rule impI, erule ns.cases, simp_all)+*

29.15 general properties

lemma *ns_has_only_Says' [iff]: "has_only_Says' ns"*
apply *(unfold has_only_Says'_def)*
apply *(rule allI, rule impI)*
by *(erule ns.cases, auto)*

lemma *newn_ns1 [iff]: "newn ns1 = {Na}"*
by *(simp add: newn_def)*

lemma *newn_ns2 [iff]: "newn ns2 = {Nb}"*
by *(auto simp: newn_def Na_def Nb_def)*

lemma *newn_ns3 [iff]: "newn ns3 = {}"*
by *(auto simp: newn_def)*

lemma *ns_wdef [iff]: "wdef ns"*
by *(auto simp: wdef_def elim: ns.cases)*

29.16 guardedness for NSL

lemma *"uniq ns secret ==> preserv ns keys n Ks"*
apply *(unfold preserv_def)*
apply *(rule allI)+*
apply *(rule impI, rule impI, rule impI, rule impI, rule impI)*
apply *(erule fresh_ruleD, simp, simp, simp, simp)*
apply *(rule allI)+*
apply *(rule impI, rule impI, rule impI)*
apply *(erule ns.cases)*

apply *(rule impI, rule impI, rule impI, rule impI, rule impI, rule impI)*
apply *(erule ns.cases)*

apply *clarsimp*
apply *(frule newn_neq_used, simp, simp)*
apply *(rule No_Nonce, simp)*

apply *clarsimp*
apply *(frule newn_neq_used, simp, simp)*
apply *(case_tac "nonce sa Na = nonce s Na")*
apply *(frule Guard_safe, simp)*
apply *(frule Crypt_guard_invKey, simp)*
apply *(frule ok_Guard, simp, simp, simp, clarsimp)*
apply *(frule_tac K="pubK' s b"* **in** *Crypt_guard_invKey, simp)*
apply *(frule_tac R=ns1* **and** *R'=ns1* **and** *Ks=Ks* **and** *s=sa* **and** *s'=s* **in** *uniqD, simp+)*

```
apply (simp add: secret_def, simp add: secret_def, force, force)
apply (simp add: secret_def keys_def, blast)
apply (rule No_Nonce, simp)

apply clarsimp
apply (case_tac "nonce sa Na = nonce s Nb")
apply (frule Guard_safe, simp)
apply (frule Crypt_guard_invKey, simp)
apply (frule_tac x=ns3b in ok_Guard, simp, simp, simp, clarsimp)
apply (frule_tac K="pubK' s a" in Crypt_guard_invKey, simp)
apply (frule_tac R=ns1 and R'=ns2 and Ks=Ks and s=sa and s'=s in uniqD,
simp+)
apply (simp add: secret_def, simp add: secret_def, force, force)
apply (simp add: secret_def, rule No_Nonce, simp)

apply (rule impI, rule impI, rule impI, rule impI, rule impI, rule impI)
apply (erule ns.cases)

apply clarsimp
apply (frule newn_neq_used, simp, simp)
apply (rule No_Nonce, simp)

apply clarsimp
apply (frule newn_neq_used, simp, simp)
apply (case_tac "nonce sa Nb = nonce s Na")
apply (frule Guard_safe, simp)
apply (frule Crypt_guard_invKey, simp)
apply (frule ok_Guard, simp, simp, simp, clarsimp)
apply (frule_tac K="pubK' s b" in Crypt_guard_invKey, simp)
apply (frule_tac R=ns2 and R'=ns1 and Ks=Ks and s=sa and s'=s in uniqD,
simp+)
apply (simp add: secret_def, simp add: secret_def, force, force)
apply (simp add: secret_def, rule No_Nonce, simp)

apply clarsimp
apply (case_tac "nonce sa Nb = nonce s Nb")
apply (frule Guard_safe, simp)
apply (frule Crypt_guard_invKey, simp)
apply (frule_tac x=ns3b in ok_Guard, simp, simp, simp, clarsimp)
apply (frule_tac K="pubK' s a" in Crypt_guard_invKey, simp)
apply (frule_tac R=ns2 and R'=ns2 and Ks=Ks and s=sa and s'=s in uniqD,
simp+)
apply (simp add: secret_def, simp add: secret_def, force, force)
apply (simp add: secret_def keys_def, blast)
apply (rule No_Nonce, simp)

by simp
```

29.17 unicity for NSL

```
lemma "uniq' ns inf secret"
apply (unfold uniq'_def)
apply (rule allI)+
apply (rule impI, erule ns.cases)
```

```
apply (rule impI, erule ns.cases)

apply (rule impI, rule impI, rule impI, rule impI)
apply (rule impI, rule impI, rule impI, rule impI)
apply (rule impI, erule tr.induct)

apply (simp add: secret_def)

apply (clarify, simp add: secret_def)
apply (drule notin_analz_insert)
apply (drule Crypt_insert_synth, simp, simp, simp)
apply (drule Crypt_insert_synth, simp, simp, simp, simp)

apply (erule_tac P="ok evsa Ra sa" in rev_mp)
apply (erule ns.cases)

apply (clarify, simp add: secret_def)
apply (erule disjE, erule disjE, clarsimp)
apply (drule ok_parts_not_new, simp, simp, simp)
apply (clarify, drule ok_parts_not_new, simp, simp, simp)

apply (simp add: secret_def)

apply (simp add: secret_def)

apply (rule impI, rule impI, rule impI, rule impI)
apply (rule impI, rule impI, rule impI, rule impI)
apply (rule impI, erule tr.induct)

apply (simp add: secret_def)

apply (clarify, simp add: secret_def)
apply (drule notin_analz_insert)
apply (drule Crypt_insert_synth, simp, simp, simp)
apply (drule_tac n="nonce s' Nb" in Crypt_insert_synth, simp, simp, simp,
simp)

apply (erule_tac P="ok evsa Ra sa" in rev_mp)
apply (erule ns.cases)

apply (clarify, simp add: secret_def)
apply (drule_tac s=sa and n=Na in ok_parts_not_new, simp, simp, simp)

apply (clarify, simp add: secret_def)
apply (drule_tac s=sa and n=Nb in ok_parts_not_new, simp, simp, simp)

apply (simp add: secret_def)

apply simp

apply (rule impI, erule ns.cases)

apply (simp only: inf_def, blast)
```

```
apply (rule impI, rule impI, rule impI, rule impI)
apply (rule impI, rule impI, rule impI, rule impI)
apply (rule impI, erule tr.induct)

apply (simp add: secret_def)

apply (clarify, simp add: secret_def)
apply (drule notin_analz_insert)
apply (drule_tac n="nonce s' Nb" in Crypt_insert_synth, simp, simp, simp)
apply (drule_tac n="nonce s' Nb" in Crypt_insert_synth, simp, simp, simp,
simp)

apply (erule_tac P="ok evsa Ra sa" in rev_mp)
apply (erule ns.cases)

apply (simp add: secret_def)

apply (clarify, simp add: secret_def)
apply (erule disjE, erule disjE, clarsimp, clarsimp)
apply (drule_tac s=sa and n=Nb in ok_parts_not_new, simp, simp, simp)
apply (erule disjE, clarsimp)
apply (drule_tac s=sa and n=Nb in ok_parts_not_new, simp, simp, simp)
by (simp_all add: secret_def)

end
```