IMP in HOLCF

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1 Denotational Semantics of Commands in HOLCF

theory *Denotational* **imports** *HOLCF Natural* **begin**

1.1 Definition

constdefs
```
  dlift :: "(('a::type) discr -> 'b::pcpo) => ('a lift -> 'b)"
  "dlift f == (LAM x. case x of UU => UU | Def y => f·(Discr y))"
```

consts *D :: "com => state discr -> state lift"*

primrec
```
  "D(skip) = (LAM s. Def(undiscr s))"
  "D(X :== a) = (LAM s. Def((undiscr s)[X ↦ a(undiscr s)]))"
  "D(c0 ; c1) = (dlift(D c1) oo (D c0))"
  "D(if b then c1 else c2) =
        (LAM s. if b (undiscr s) then (D c1)·s else (D c2)·s)"
  "D(while b do c) =
        fix·(LAM w s. if b (undiscr s) then (dlift w)·((D c)·s)
                      else Def(undiscr s))"
```

1.2 Equivalence of Denotational Semantics in HOLCF and Evaluation Semantics in HOL

lemma *dlift_Def [simp]: "dlift f·(Def x) = f·(Discr x)"*
  ⟨*proof*⟩

lemma *cont_dlift [iff]: "cont (%f. dlift f)"*
⟨*proof*⟩

lemma *dlift_is_Def [simp]:*
"(dlift f·l = Def y) = (∃x. l = Def x ∧ f·(Discr x) = Def y)"
⟨*proof*⟩

lemma *eval_implies_D: "⟨c,s⟩ $\longrightarrow_c$ t ==> D c·(Discr s) = (Def t)"*
⟨*proof*⟩

lemma *D_implies_eval: "!s t. D c·(Discr s) = (Def t) --> ⟨c,s⟩ $\longrightarrow_c$ t"*
⟨*proof*⟩

theorem *D_is_eval: "(D c·(Discr s) = (Def t)) = (⟨c,s⟩ $\longrightarrow_c$ t)"*
⟨*proof*⟩

end

2 Correctness of Hoare by Fixpoint Reasoning

theory *HoareEx* **imports** *Denotational* **begin**

An example from the HOLCF paper by Müller, Nipkow, Oheimb, Slotosch [1]. It demonstrates fixpoint reasoning by showing the correctness of the Hoare rule for while-loops.

types *assn = "state => bool"*

constdefs
hoare_valid :: "[assn, com, assn] => bool"   ("|= {(1_)}/ (_)/ {(1_)}" 50)
"|= {A} c {B} == ∀s t. A s ∧ D c $(Discr s) = Def t --> B t"

lemma *WHILE_rule_sound:*
"|= {A} c {A} ==> |= {A} while b do c {λs. A s ∧ ¬ b s}"
⟨*proof*⟩

end

References

[1] O. Müller, T. Nipkow, D. v. Oheimb, and O. Slotosch. HOLCF = HOL + LCF. *J. Functional Programming*, 9:191–223, 1999.