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Xiu 2010 p.15, Prop.2.11, point 2

$U \sim U(0,1)$ = random variable U having the uniform distribution on (0,1)

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$\sim$ = tilde notation; latex code \sim is used for similarity for asymptotic, so it is better to use a different symbol to prevent confusion. Note that some authors (not everyone) in probability use the "tilde" notation. A possible alternative notation would be \doteqdot, i.e., $\doteqdot$, which is not used often. So we can write:

$U \doteqdot U(0,1)$ = random variable U having the uniform distribution on (0,1)

$X \doteqdot N(\mu,\sigma)$ = random variable X having the normal distribution with mean μ and standard deviation σ (or variance σ^2)

Point 3:

Point 2:

$$U \doteqdot U(0,1) \Rightarrow X := F_X^{-1}(U) \text{ has } F_X(\cdot) \text{ as cdf}$$

U \doteqdot U(0,1) \Rightarrow X := F_X^{-1} (U) \text{ has } F_X(\cdot) \text{ as cdf}

Point 3:

$$X := F_X^{-1}(U) \Rightarrow F_X(X) = F_X(F_X^{-1}(U)) = U$$

X := F_X^{-1} (U) \Rightarrow F_X (X) = F_X (F_X^{-1} (U)) = U

So Point 2 implies Point 3.

To prove Point 2, there are two ways.

Method 1:

Follow the logics of Fig.4.2 to build up a proof.

Method 2:

$$[A \Rightarrow B] \Leftrightarrow [\cancel{B} \Rightarrow \cancel{A}]$$

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Show that if the random variable $F_X^{-1}(U)$ does not have $F_X(\cdot)$ as cdf, i.e., has a different function as cdf, then the random variable U cannot have the uniform distribution $U(0,1)$.