CTFS (1B)

- Continuous Time Fourier Series

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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Fourier Series

$$f(x) = a_0 + \sum_{k=1}^{\infty} (a_k \cos kx + b_k \sin kx)$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{+\pi} f(x)\, dx$$

$$a_k = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \cos kx\, dx$$

$$b_k = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \sin kx\, dx$$

$$k = 1,\ 2,\ 3,\ ...$$

one-sided spectrum
only positive frequencies

Trigonometric Identities

$$\cos\theta\,\cos\phi = \frac{1}{2}\left(\cos(\theta-\phi) + \cos(\theta+\phi)\right)$$

$$\sin\theta\,\sin\phi = \frac{1}{2}\left(\cos(\theta-\phi) - \cos(\theta+\phi)\right)$$

$$\sin\theta\,\cos\phi = \frac{1}{2}\left(\sin(\theta+\phi) + \sin(\theta-\phi)\right)$$

$$\cos\theta\,\sin\phi = \frac{1}{2}\left(\sin(\theta+\phi) - \sin(\theta-\phi)\right)$$

$$\frac{1}{2}\left(1 + \cos(\theta+\phi)\right) \quad when \quad \theta=\phi$$

$$\frac{1}{2}\left(1 - \cos(\theta+\phi)\right) \quad when \quad \theta=\phi$$

$$\frac{1}{2}\left(\sin(\theta+\phi)\right) \quad when \quad \theta=\phi$$

$$\frac{1}{2}\left(\sin(\theta+\phi)\right) \quad when \quad \theta=\phi$$

$$\int_{-\pi}^{+\pi} \cos nx\,\cos mx\,dx = 0 \quad (n \neq m)$$

$$\int_{-\pi}^{+\pi} \sin nx\,\sin mx\,dx = 0 \quad (n \neq m)$$

$$\int_{-\pi}^{+\pi} \sin nx\,\cos mx\,dx = 0$$

$$\int_{-\pi}^{+\pi} \cos nx\,\sin mx\,dx = 0$$

$$\int_{-\pi}^{+\pi} \cos nx\,\cos mx\,dx = \pi \quad (n = m)$$

$$\int_{-\pi}^{+\pi} \sin nx\,\sin mx\,dx = \pi \quad (n = m)$$

Trigonometric Orthogonality

$$f(x) = a_0 + \sum_{k=1}^{\infty} (a_k \cos kx + b_k \sin kx)$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{+\pi} f(x)\, dx$$

$$a_k = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \cos kx\, dx$$

$$b_k = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \sin kx\, dx$$

$$k = 1, 2, 3, \ldots$$

$$\int_{-\pi}^{+\pi} \cos n x \cos m x\, dx = 0 \quad (n \neq m)$$

$$\int_{-\pi}^{+\pi} \sin n x \sin m x\, dx = 0 \quad (n \neq m)$$

$$\int_{-\pi}^{+\pi} \sin n x \cos m x\, dx = 0$$

$$\int_{-\pi}^{+\pi} \cos n x \sin m x\, dx = 0$$

$$\int_{-\pi}^{+\pi} \cos n x \cos m x\, dx = \pi \quad (n = m)$$

$$\int_{-\pi}^{+\pi} \sin n x \sin m x\, dx = \pi \quad (n = m)$$

$$a_k \leftarrow \quad f(x) \cdot \cos k x = a_0 \cdot \cos k x + \sum_{m=1}^{\infty} (a_m \cos m x \cdot \cos k x + b_m \sin m x \cdot \cos k x)$$

$$b_k \leftarrow \quad f(x) \cdot \sin k x = a_0 \cdot \sin k x + \sum_{m=1}^{\infty} (a_m \cos m x \cdot \sin n x + b_m \sin m x \cdot \sin k x)$$

Any Period *p = 2L*

$$g(v) = a_0 + \sum_{k=1}^{\infty}(a_k \cos kv + b_k \sin kv)$$

$$f(x) = a_0 + \sum_{k=1}^{\infty}\left(a_k \cos\frac{k\pi}{L}x + b_k \sin\frac{k\pi}{L}x\right)$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{+\pi}g(v)\,dv$$

$$a_k = \frac{1}{\pi}\int_{-\pi}^{+\pi}g(v)\cos kv\,dv$$

$$b_k = \frac{1}{\pi}\int_{-\pi}^{+\pi}g(v)\sin kv\,dv$$

$$k = 1,\ 2,\ ...$$
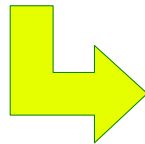
$$a_0 = \frac{1}{2L}\int_{-L}^{+L}f(x)\,dx$$

$$a_k = \frac{1}{L}\int_{-L}^{+L}f(x)\cos\frac{k\pi x}{L}\,dx$$

$$b_k = \frac{1}{L}\int_{-L}^{+L}f(x)\sin\frac{k\pi x}{L}\,dx$$

$$k = 1,\ 2,\ 3,\ ...$$

$$v:\ [-\pi,\ +\pi]$$

$$x:\ [-L,\ +L]$$

$$v = \frac{\pi}{L}x$$

$$dv = \frac{\pi}{L}dx$$
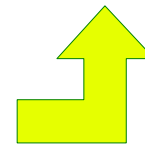
Time and Frequency

$$f(x) = a_0 + \sum_{k=1}^{\infty} \left(a_k \cos \frac{k\pi}{L} x + b_k \sin \frac{k\pi}{L} x \right)$$

$$x(t) = a_0 + \sum_{k=1}^{\infty} \left(a_k \cos \frac{2\pi k}{T} t + b_k \sin \frac{2\pi k}{T} t \right)$$

$$a_0 = \frac{1}{2L} \int_{-L}^{+L} f(x)\, dx$$

$$a_0 = \frac{1}{T} \int_{0}^{T} x(t)\, dt$$

$$a_k = \frac{1}{L} \int_{-L}^{+L} f(x) \cos \frac{k\pi x}{L}\, dx$$

$$a_k = \frac{2}{T} \int_{0}^{T} x(t) \cos \frac{2\pi k t}{T}\, dt$$

$$b_k = \frac{1}{L} \int_{-L}^{+L} f(x) \sin \frac{k\pi x}{L}\, dx$$

$$b_k = \frac{2}{T} \int_{0}^{T} x(t) \sin \frac{2\pi k t}{T}\, dt$$

$$k = 1,\ 2,\ 3,\ ...$$

$$k = 1,\ 2,\ ...$$

$$x:\ [-L,\ +L]$$

$$t:\ [0,\ T]$$

$$2L = T$$

Continuous Time Periodic Signal $x(t)$

Harmonic Frquency

$$x(t) = a_0 + \sum_{k=1}^{\infty} \left(a_k \cos\frac{2\pi k}{T} t + b_k \sin\frac{2\pi k}{T} t \right)$$

$$x(t) = a_0 + \sum_{k=1}^{\infty} \left(a_k \cos(2\pi k f_0 t) + b_k \sin(2\pi k f_0 t) \right)$$
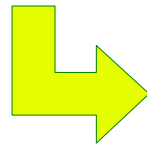
$$a_0 = \frac{1}{T} \int_0^T x(t)\, dt$$

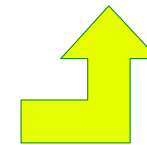$$a_k = \frac{2}{T} \int_0^T x(t) \cos\frac{2\pi k t}{T}\, dt$$

$$b_k = \frac{2}{T} \int_0^T x(t) \sin\frac{2\pi k t}{T}\, dt$$

$$k = 1, 2, \ldots$$

$$a_0 = \frac{1}{T} \int_0^T x(t)\, dt$$

$$a_k = \frac{2}{T} \int_0^T x(t) \cos(2\pi k f_0 t)\, dt \quad k = 1, 2, \ldots$$

$$b_k = \frac{2}{T} \int_0^T x(t) \sin(2\pi k f_0 t)\, dt \quad k = 1, 2, \ldots$$

$$t : [0, \ T]$$

$$t : [0, \ T]$$

resolution frequency $\qquad f_0 = \dfrac{1}{T}$

n-th harmonic frequency $\qquad f_n = n f_0 = n \dfrac{1}{T}$

Radial Frequency

$$x(t) = a_0 + \sum_{n=1}^{\infty}\left(a_k \cos(k\,2\pi f_0 t) + b_k \sin(k\,2\pi f_0 t)\right)$$

$$x(t) = a_0 + \sum_{n=1}^{\infty}\left(a_k \cos(\boldsymbol{k}\,\omega_0 t) + b_k \sin(\boldsymbol{k}\,\omega_0 t)\right)$$

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt$$

$$a_k = \frac{2}{T}\int_0^T x(t)\cos(k\,2\pi f_0 t)\,dt \quad k = 1,\,2,\,...$$

$$b_k = \frac{2}{T}\int_0^T x(t)\sin(k\,2\pi f_0 t)\,dt \quad k = 1,\,2,\,...$$

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt$$

$$a_k = \frac{2}{T}\int_0^T x(t)\cos(\boldsymbol{k}\,\omega_0 t)\,dt$$

$$b_k = \frac{2}{T}\int_0^T x(t)\sin(\boldsymbol{k}\,\omega_0 t)\,dt$$

$$k = 1,\,2,\,...$$

$$t:\ [0,\ T]$$

$$t:\ [0,\ T]$$

linear frequency $\qquad f$

angular (radial) frequency $\qquad \omega = 2\pi f$

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Complex Fourier Series Coefficients

$$x(t) = a_0 + \sum_{k=1}^{\infty} \left(a_k \cos(k\,\omega_0 t) + b_k \sin(k\,\omega_0 t) \right)$$

$$x(t) = A_0 + \sum_{k=1}^{\infty} \left(A_k\, e^{jk\omega_0 t} + B_k\, e^{-jk\omega_0 t} \right)$$

$$a_0 = \frac{1}{T} \int_0^T x(t)\, dt$$

$$a_k = \frac{2}{T} \int_0^T x(t) \cos(k\,\omega_0 t)\, dt$$

$$b_k = \frac{2}{T} \int_0^T x(t) \sin(k\,\omega_0 t)\, dt$$

$$k = 1,\ 2,\ ...$$

$$A_0 = \frac{1}{T} \int_0^T x(t)\, dt$$

$$A_k = \frac{1}{T} \int_0^T x(t)\, e^{-jk\omega_0 t}\, dt$$

$$B_k = \frac{1}{T} \int_0^T x(t)\, e^{+jk\omega_0 t}\, dt$$

$$t:\ [0,\ T]$$

$$t:\ [0,\ T]$$

Real coefficients
$$a_0,\ a_k,\ b_k,\quad k = 1,\ 2,\ ...$$

➡ **one-sided spectrum**
only positive frequencies

Complex coefficients
$$A_0,\ A_k,\ B_k,\quad k = 1,\ 2,\ ...$$

➡ **two-sided spectrum**
Both pos and neg frequencies

Euler Equation (1)

$$x(t) = a_0 + \sum_{k=1}^{\infty} \left(a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t) \right)$$

$$a_0 = \frac{1}{T} \int_0^T x(t)\, dt$$

$$a_k = \frac{2}{T} \int_0^T x(t) \cos(k\omega_0 t)\, dt$$

$$b_k = \frac{2}{T} \int_0^T x(t) \sin(k\omega_0 t)\, dt$$

$$k = 1, 2, \ldots$$

$$e^{+j\omega t} = \cos\omega t + j\sin\omega t$$

$$e^{-j\omega t} = \cos\omega t - j\sin\omega t$$

$$\cos\omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$\sin\omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)$$

$$= a_k \frac{1}{2} \left(e^{jk\omega_0 t} + e^{-jk\omega_0 t} \right) + b_k \frac{1}{2j} \left(e^{jk\omega_0 t} - e^{-jk\omega_0 t} \right)$$

$$= \frac{(a_k - jb_k)}{2} e^{jk\omega_0 t} + \frac{(a_k + jb_k)}{2} e^{-jk\omega_0 t}$$

$$= A_k e^{jk\omega_0 t} + B_k e^{-jk\omega_0 t}$$

$$x(t) = A_0 + \sum_{k=1}^{\infty} \left(A_k e^{jk\omega_0 t} + B_k e^{-jk\omega_0 t} \right)$$

zero freq ➡ $A_0 = a_0$

pos freq ➡ $A_k = \frac{1}{2} \left(a_k - jb_k \right)$

neg freq ➡ $B_k = \frac{1}{2} \left(a_k + jb_k \right)$

only positive frequencies

Euler Equation (2)

$$x(t) = a_0 + \sum_{k=1}^{\infty} \left(a_k \cos(k\,\omega_0 t) + b_k \sin(k\,\omega_0 t) \right)$$

$$a_0 = \frac{1}{T} \int_0^T x(t)\, dt$$

$$a_k = \frac{2}{T} \int_0^T x(t) \cos(k\,\omega_0 t)\, dt$$

$$b_k = \frac{2}{T} \int_0^T x(t) \sin(k\,\omega_0 t)\, dt$$

$$k = 1, 2, \dots$$

$$x(t) = A_0 + \sum_{k=1}^{\infty} \left(A_k\, e^{jk\omega_0 t} + B_k\, e^{-jk\omega_0 t} \right)$$

zero freq ➡ $A_0 = a_0$

pos freq ➡ $A_k = \frac{1}{2} \left(a_k - jb_k \right)$

neg freq ➡ $B_k = \frac{1}{2} \left(a_k + jb_k \right)$

only positive frequencies

$$A_k = \frac{1}{T} \int_0^T x(t) \left(\cos(k\,\omega_0 t) - j\sin(k\,\omega_0 t) \right) dt$$

$$B_k = \frac{1}{T} \int_0^T x(t) \left(\cos(k\,\omega_0 t) + j\sin(k\,\omega_0 t) \right) dt$$

$$A_k = \frac{1}{T} \int_0^T x(t)\, e^{-jk\omega_0 t}\, dt$$

$$B_k = \frac{1}{T} \int_0^T x(t)\, e^{+jk\omega_0 t}\, dt$$

$$x(t) = A_0 + \sum_{k=1}^{\infty} \left(A_k\, e^{+jk\omega_0 t} + B_k\, e^{-jk\omega_0 t} \right)$$

$$x(t) = \sum_{k=0}^{\infty} \left(A_k\, e^{+jk\omega_0 t} + B_k\, e^{-jk\omega_0 t} \right)$$

Complex Fourier Series

$$x(t) = a_0 + \sum_{k=1}^{\infty} \left(a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t) \right)$$

$$x(t) = \sum_{k=0}^{\infty} \left(A_k\, e^{+jk\omega_0 t} + B_k\, e^{-jk\omega_0 t} \right)$$

$$a_0 = \frac{1}{T} \int_0^T x(t)\, dt$$

$$a_k = \frac{2}{T} \int_0^T x(t) \cos(k\omega_0 t)\, dt$$

$$b_k = \frac{2}{T} \int_0^T x(t) \sin(k\omega_0 t)\, dt$$

$$k = 1, 2, \ldots$$

$$A_k = \frac{1}{T} \int_0^T x(t)\, e^{-jk\omega_0 t}\, dt$$

$$k = 0, 1, 2, \ldots$$

$$B_k = \frac{1}{T} \int_0^T x(t)\, e^{+jk\omega_0 t}\, dt$$

$$k = 1, 2, \ldots$$

$$x(t) = A_0 + \sum_{k=1}^{\infty} \left(A_k\, e^{jk\omega_0 t} + B_k\, e^{-jk\omega_0 t} \right)$$

$$x(t) = \sum_{k=-\infty}^{+\infty} C_k\, e^{+jk\omega_0 t}$$

$$A_0 = a_0$$

$$A_k = \frac{1}{2} \left(a_k - j b_k \right)$$

$$B_k = \frac{1}{2} \left(a_k + j b_k \right)$$

$$k = 1, 2, \ldots$$

$$C_k = \frac{1}{T} \int_0^T x(t)\, e^{-jk\omega_0 t}\, dt$$

$$k = -2, -1, 0, +1, +2, \ldots$$

$$C_k = \begin{cases} A_0 & (k = 0) \\ A_k & (k > 0) \\ B_k & (k < 0) \end{cases}$$

Single-Sided Spectrum

$$x(t) = a_0 + \sum_{k=1}^{\infty} \left(a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t) \right)$$

$$x(t) = g_0 + \sum_{k=1}^{\infty} g_k \cos(k\omega_0 t + \phi_k)$$

$$a_0 = \frac{1}{T} \int_0^T x(t)\, dt$$

$$a_k = \frac{2}{T} \int_0^T x(t) \cos(k\omega_0 t)\, dt$$

$$b_k = \frac{2}{T} \int_0^T x(t) \sin(k\omega_0 t)\, dt$$

$$k = +1, +2, \ldots$$

$$g_0 = a_0$$

$$g_k = \sqrt{a_k^2 + b_k^2}$$

$$\phi_k = \tan^{-1}\left(-\frac{b_k}{a_k} \right)$$

$$k = +1, +2, \ldots$$

$$\cos(\alpha+\beta) = \underline{\cos(\alpha)\cos(\beta)} - \underline{\sin(\alpha)\sin(\beta)}$$

$$g_k \cos(k\omega_0 t + \phi_k) = \underline{g_k \cos(\phi_k)} \cos(k\omega_0 t) - \underline{g_k \sin(\phi_k)} \sin(k\omega_0 t)$$

$$\underline{a_k} \cos(k\omega_0 t) + \underline{b_k} \sin(k\omega_0 t)$$

$$a_k = g_k \cos(\phi_k)$$
$$-b_k = g_k \sin(\phi_k)$$

$$a_k^2 + b_k^2 = g_k^2$$

$$-\frac{b_k}{a_k} = \tan(\phi_k)$$

Phasor Representation (1)

$$x(t) = a_0 + \sum_{k=1}^{\infty} \left(a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t) \right)$$

$$x(t) = g_0 + \sum_{k=1}^{\infty} g_k \cos(k\omega_0 t + \phi_k)$$

$$a_0 = \frac{1}{T} \int_0^T x(t)\, dt$$

$$a_k = \frac{2}{T} \int_0^T x(t) \cos(k\omega_0 t)\, dt$$

$$b_k = \frac{2}{T} \int_0^T x(t) \sin(k\omega_0 t)\, dt$$

$$k = 1,\ 2,\ ...$$

$$g_0 = a_0$$

$$g_k = \sqrt{a_k^2 + b_k^2}$$

$$\phi_k = \tan^{-1}\left(-\frac{b_k}{a_k} \right)$$

$$k = 1,\ 2,\ ...$$

$$x(t) = g_0 + \sum_{k=1}^{\infty} g_k \cos(k\omega_0 t + \phi_k)$$

$$x(t) = g_0 + \sum_{k=1}^{\infty} g_k \,\Re\left\{ e^{+j(k\omega_0 t + \phi_k)} \right\}$$

$$x(t) = g_0 + \sum_{k=1}^{\infty} \Re\left\{ g_k \cdot e^{+j\phi_k} \cdot e^{+jk\omega_0 t} \right\}$$

$$x(t) = X_0 + \sum_{k=1}^{\infty} \Re\left\{ X_k e^{+jk\omega_0 t} \right\}$$

$$X_0 = g_0$$

$$X_k = g_k \cdot e^{+j\phi_k}$$

$$k = 1,\ 2,\ ...$$

Phasor Representation (2)

$$x(t) = g_0 + \sum_{k=1}^{\infty} \frac{g_k}{2} \cdot \left(e^{+j(k\omega_0 t + \phi_k)} + e^{-j(k\omega_0 t + \phi_k)} \right)$$

$$x(t) = g_0 + \sum_{k=1}^{\infty} g_k \cos(k\omega_0 t + \phi_k)$$

$$x(t) = g_0 + \sum_{k=1}^{\infty} \left(\frac{g_k}{2} e^{+j\phi_k} e^{+jk\omega_0 t} + \frac{g_k}{2} e^{-j\phi_k} e^{-jk\omega_0 t} \right)$$

$$g_0 = a_0$$

$$g_k = \sqrt{a_k^2 + b_k^2}$$

$$x(t) = g_0 + \sum_{k=1}^{\infty} \left(\frac{g_k e^{+j\phi_k}}{2} e^{+jk\omega_0 t} + \frac{g_k e^{-j\phi_k}}{2} e^{-jk\omega_0 t} \right)$$

$$\phi_k = \tan^{-1}\left(-\frac{b_k}{a_k} \right)$$

$$k = 1, 2, \dots$$

$$x(t) = \sum_{k=-\infty}^{+\infty} C_k e^{+jk\omega_0 t}$$

$$x(t) = X_0 + \sum_{k=1}^{\infty} \Re\{ X_k e^{+jk\omega_0 t} \}$$

$$C_k = \frac{g_k e^{+j\phi_k}}{2} \quad (k > 0)$$

$$X_0 = g_0$$

$$C_{-k} = \frac{g_k e^{-j\phi_k}}{2} \quad (k < 0)$$

$$X_k = g_k \cdot e^{+j\phi_k}$$

$$k = -2, -1, 0, +1, +2, \dots$$

$$k = 1, 2, \dots$$

Two-Sided Spectrum

$$x(t) = \sum_{k=-\infty}^{+\infty} C_k \, e^{+jk\omega_0 t}$$

$$x(t) = \sum_{k=-\infty}^{+\infty} C_k \, e^{+jk\omega_0 t}$$

$$C_k = \frac{1}{T} \int_0^T x(t) \, e^{-jk\omega_0 t} \, dt$$

$$k = \dots, -2, -1, 0, +1, +2, \dots$$

$$C_k = \frac{1}{T} \int_0^T x(t) \, e^{-jk\omega_0 t} \, dt$$

$$k = \dots, -2, -1, 0, +1, +2, \dots$$

$$C_k = \begin{cases} a_0 & (k = 0) \\ \frac{1}{2}(a_k - jb_k) & (k > 0) \\ \frac{1}{2}(a_k + jb_k) & (k < 0) \end{cases}$$

$$C_k = \begin{cases} a_0 & (k = 0) \\ \frac{1}{2} g_k \, e^{+j\phi_k} & (k > 0) \\ \frac{1}{2} g_k \, e^{-j\phi_k} & (k < 0) \end{cases}$$

$$|C_k| = \begin{cases} a_0 & (k = 0) \\ \frac{1}{2}\sqrt{a_k^2 + b_k^2} & (k \neq 0) \end{cases}$$

$$Arg(C_k) = \begin{cases} \tan^{-1}(-b_k/a_k) & (k > 0) \\ \tan^{-1}(+b_k/a_k) & (k < 0) \end{cases}$$

$$|C_k| = \begin{cases} a_0 & (k = 0) \\ \frac{1}{2}|g_k| & (k \neq 0) \end{cases}$$

$$Arg(C_k) = \begin{cases} +\phi_k & (k > 0) \\ -\phi_k & (k < 0) \end{cases}$$

Power Spectrum   *Two-Sided*

$$|C_k|^2 + |C_{-k}|^2 = \frac{1}{2}|g_k|^2 = \frac{1}{2}(a_k^2 + b_k^2)$$

Periodogram   *One-Sided*

$$2 \cdot |C_k| = |g_k| = \sqrt{a_k^2 + b_k^2}$$

CTFS of Impulse Train (1)

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$
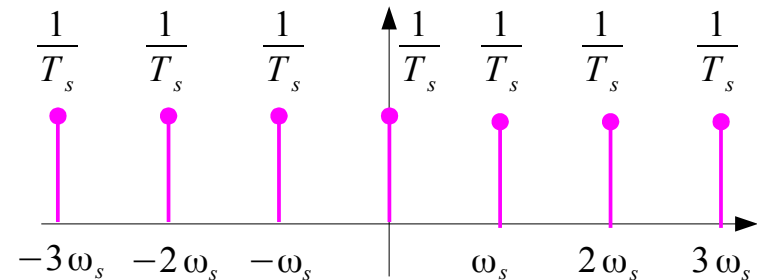
Fourier Series Expansion of Impulse Train

$$p(t) = \sum_{k=-\infty}^{+\infty} a_k \, e^{+jk\omega_s t}$$

Fourier Series Coefficients

$$a_k = \frac{1}{T_s} \int_{-T_s/2}^{+T_s/2} \delta(t) \, e^{-jk\omega_s t} \, dt$$

$$= \frac{1}{T_s} \int_{-T_s/2}^{+T_s/2} \delta(t) \, e^{-jk\omega_s 0} \, dt$$

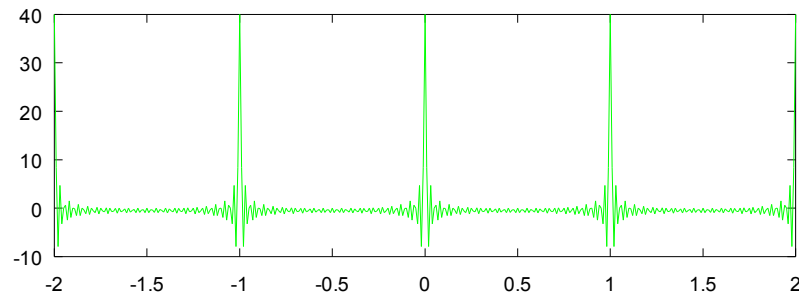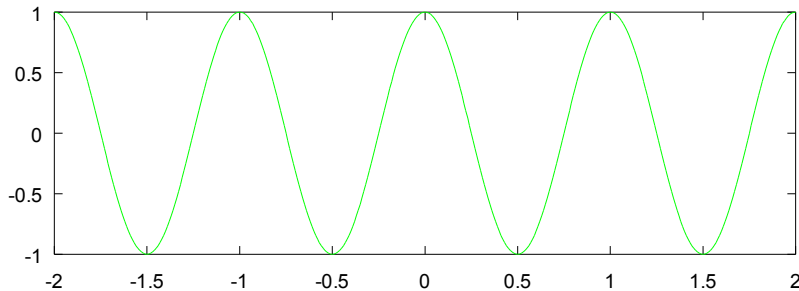$$= \frac{1}{T_s} \int_{-T_s/2}^{+T_s/2} \delta(t) \, dt = \frac{1}{T_s}$$
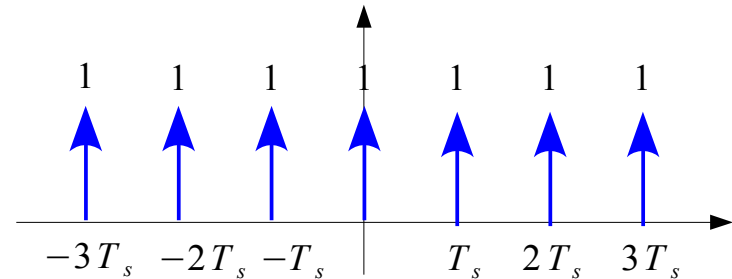
$$\omega_s = \frac{2\pi}{T_s}$$

CTFS of Impulse Train (2)

$$p(t) \;=\; \frac{1}{T_s} \sum_{k=-\infty}^{+\infty} a_k \, e^{+jk\omega_s t} \;=\; \frac{1}{T_s} \sum_{k=-\infty}^{+\infty} \left(\cos k\,\omega_s t - j \sin k\,\omega_s t \right)$$

$\cos 2\pi \cdot 1 \cdot t$

$\sin 2\pi \cdot 1 \cdot t$

$$\sum_{k=1}^{40} \cos 2\pi \cdot k \cdot t$$

$$\sum_{k=1}^{40} \sin 2\pi \cdot k \cdot t$$

Young Won Lim
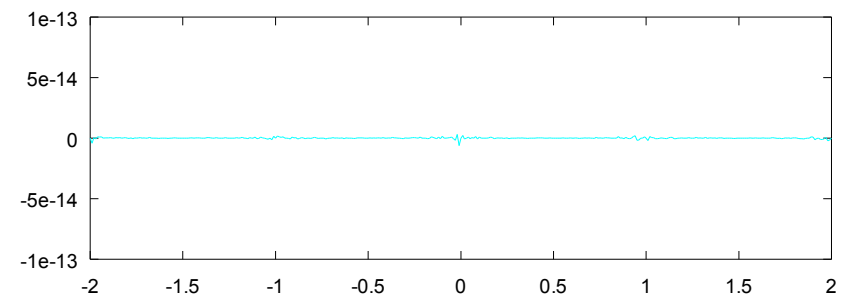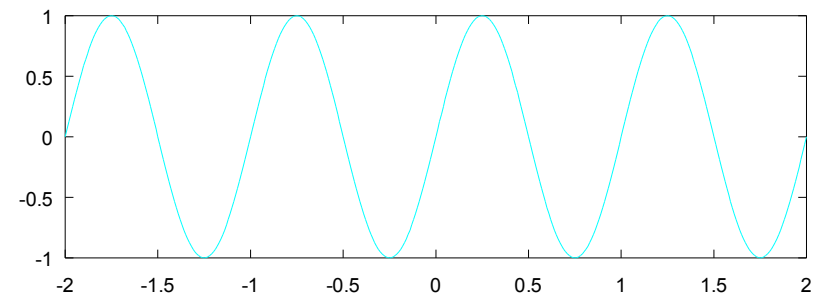8/18/11
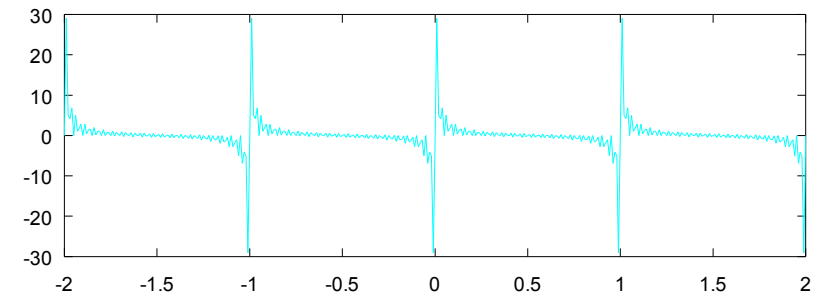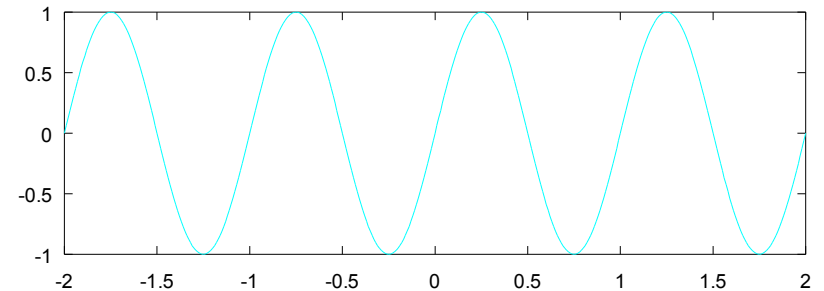
CTFS of Impulse Train (3)

$$p(t) \;=\; \frac{1}{T_s} \sum_{k=-\infty}^{+\infty} a_k \, e^{+jk\omega_s t} \;=\; \frac{1}{T_s} \sum_{k=-\infty}^{+\infty} \left(\cos k\,\omega_s t \,-\, j \sin k\,\omega_s t \right)$$

$\cos 2\pi\cdot 1\cdot t$

$\sin 2\pi\cdot 1\cdot t$
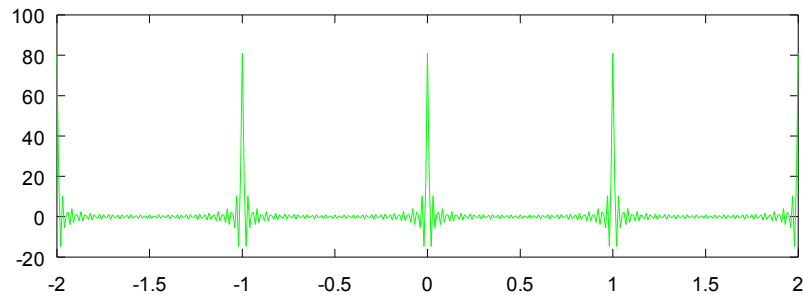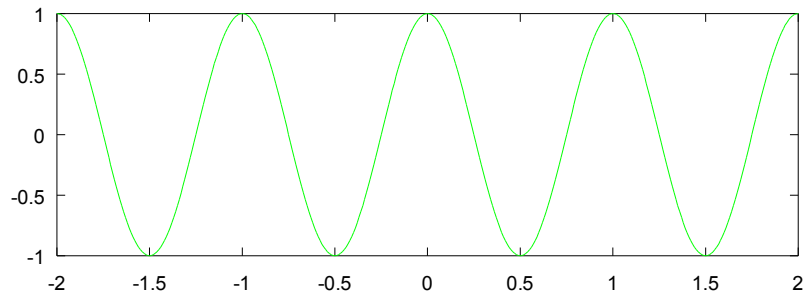
$$\sum_{k=-40}^{40} \cos 2\pi\cdot k\cdot t$$

$$\sum_{k=-40}^{40} \sin 2\pi\cdot k\cdot t$$

References

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