General Vector Space (2A)

Young Won Lim

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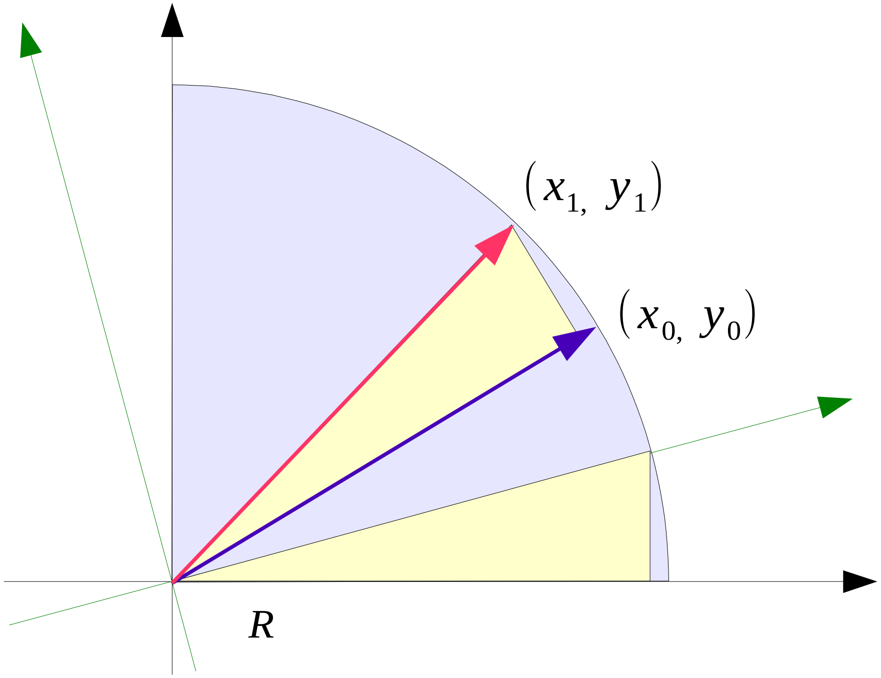
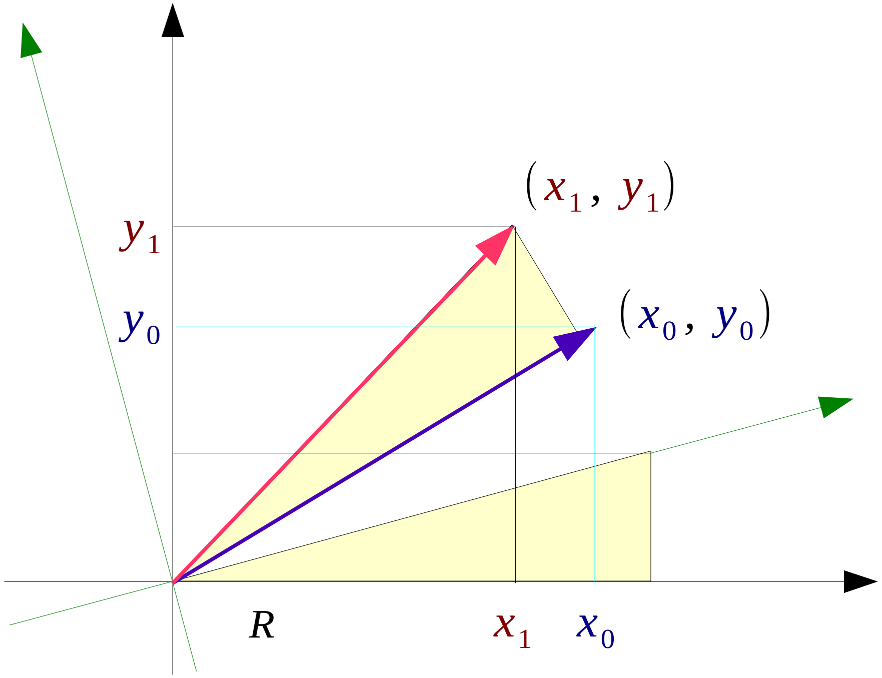
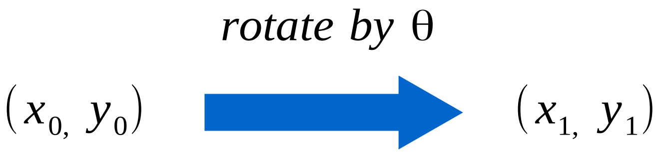
Vector Rotation (1)



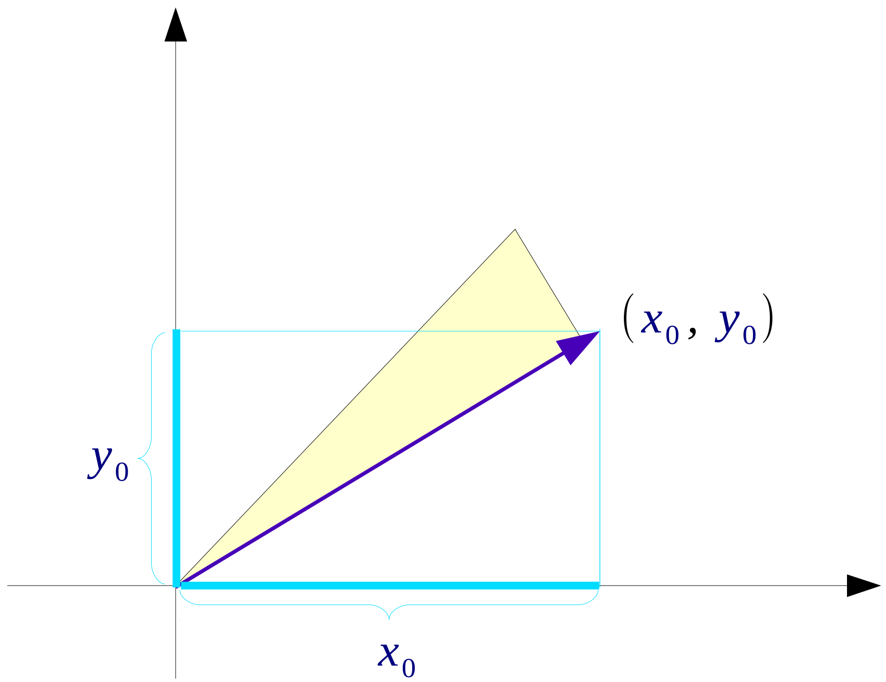


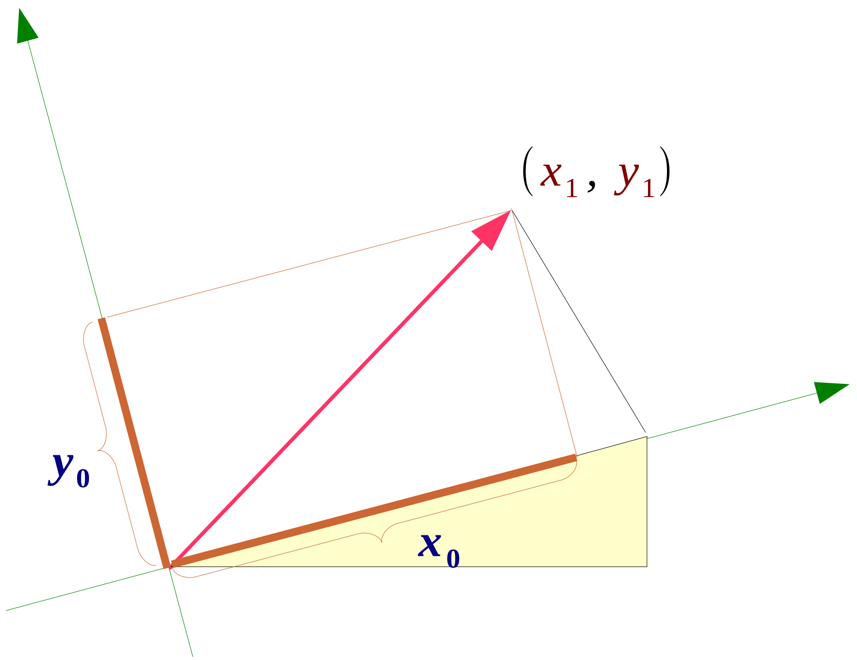


$$x_1 = x_0 \cos\theta - y_0 \sin\theta$$

$$y_1 = x_0 \sin\theta + y_0 \cos\theta$$

Vector Rotation (2)





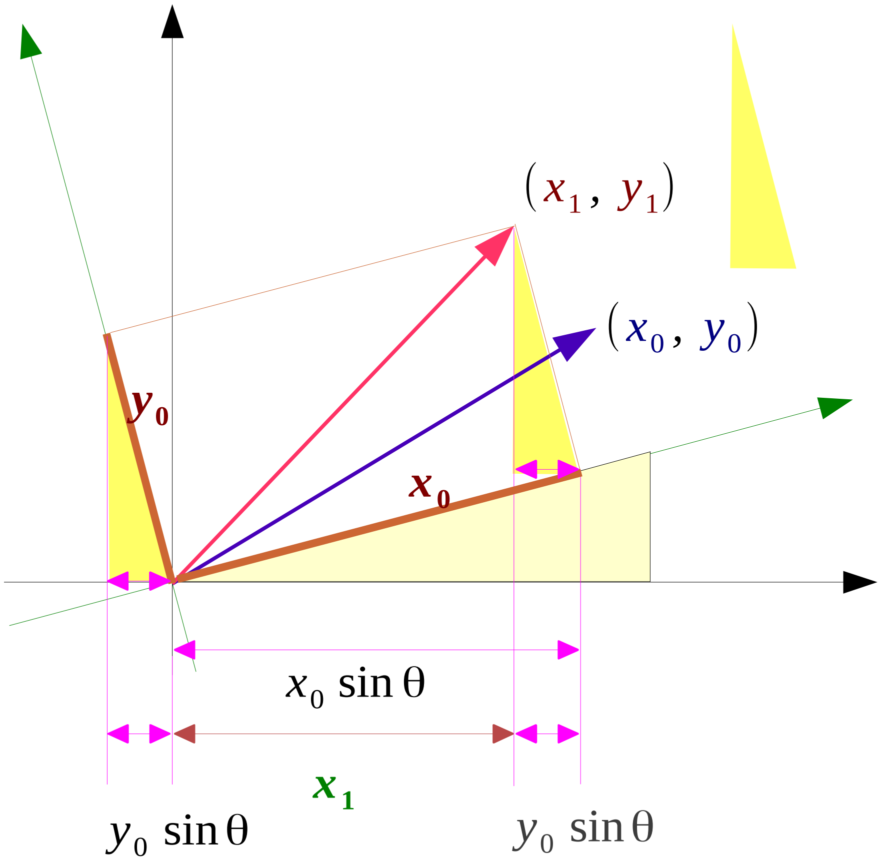
In the rotated coordinate

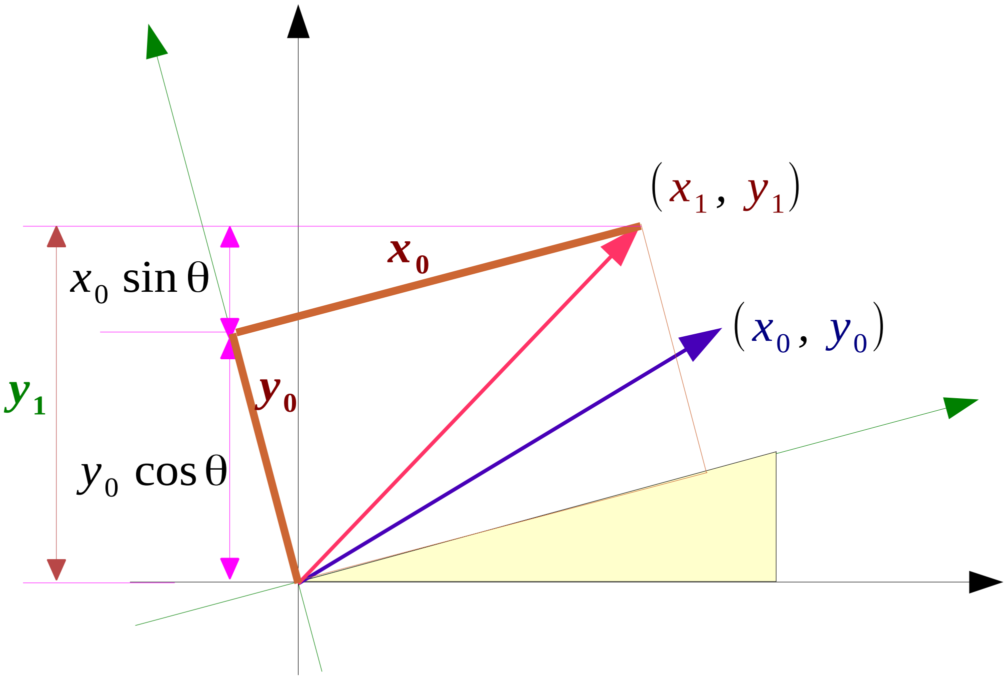
invariant length x_0, y_0

Vector Rotation (3)

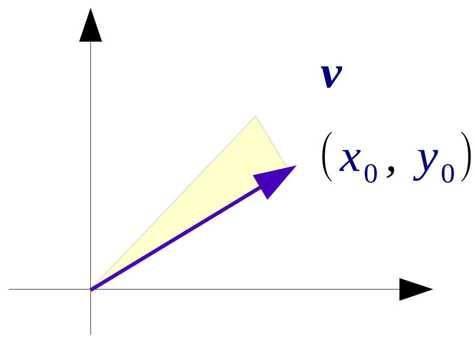
$$x_1 = x_0 \cos\theta - y_0 \sin\theta$$

$$y_1 = x_0 \sin\theta + y_0 \cos\theta$$





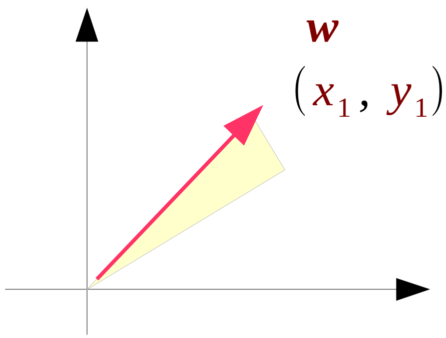
Transformation



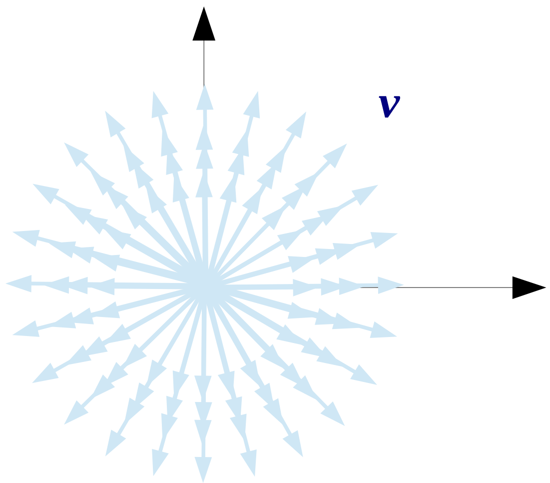
$f : V \rightarrow W$

transformation map

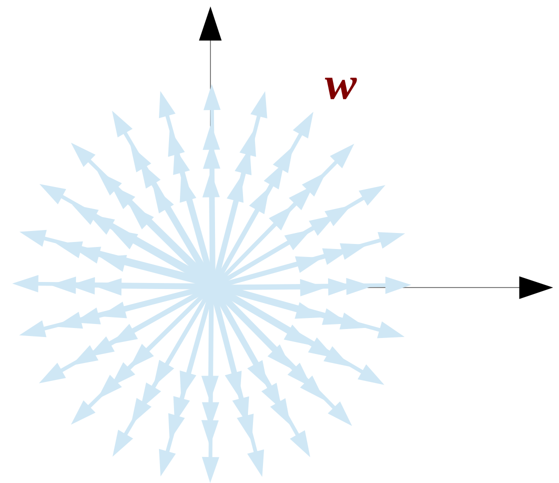
cf) operator



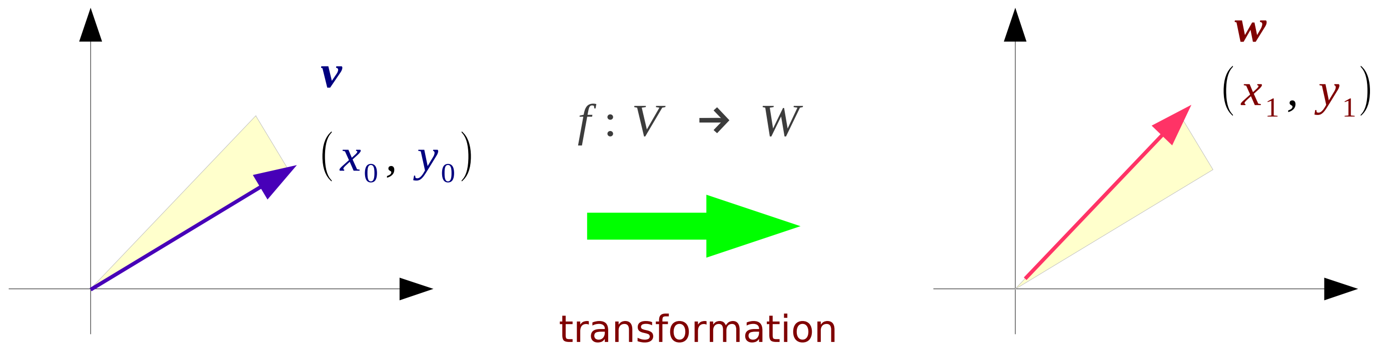
V W



$f : V \rightarrow W$



Matrix Transformation



$$\begin{aligned} x_1 &= x_0 \cos\theta - y_0 \sin\theta \\ y_1 &= x_0 \sin\theta + y_0 \cos\theta \end{aligned}$$

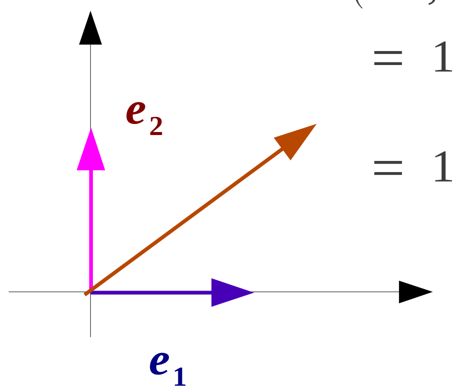
$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_0 \\ y_1 \end{bmatrix}$$

$$\boldsymbol{w} = \boldsymbol{A} \quad \boldsymbol{x}$$

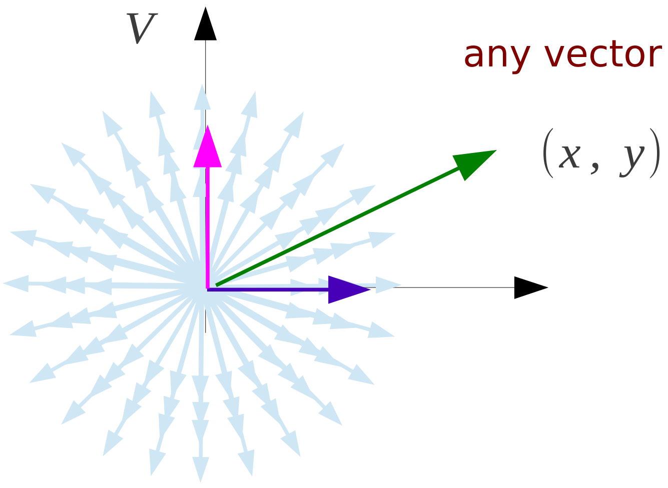
$$\boldsymbol{w} = T_{\boldsymbol{A}}(\boldsymbol{x})$$

$$\boldsymbol{x} \xrightarrow{\quad T_{\boldsymbol{A}} \quad} \boldsymbol{w}$$

Basis and Coordinates

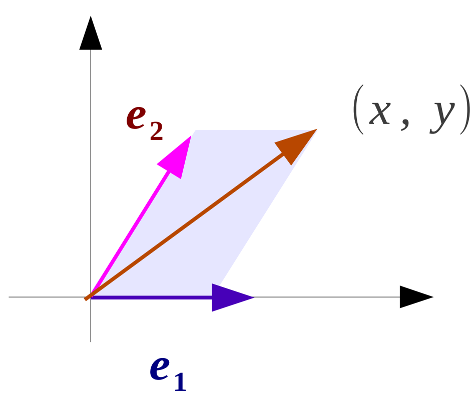


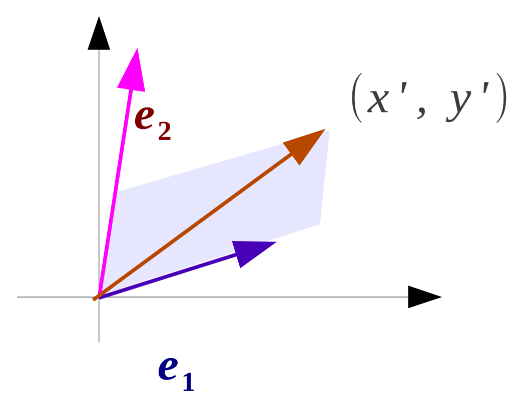
$$(1.5,\ 1.5)$$
$$= 1.5\,\boldsymbol{e}_1 + 1.5\,\boldsymbol{e}_2$$
$$= 1.5\begin{bmatrix}1\\0\end{bmatrix} + 1.5\begin{bmatrix}0\\1\end{bmatrix}$$

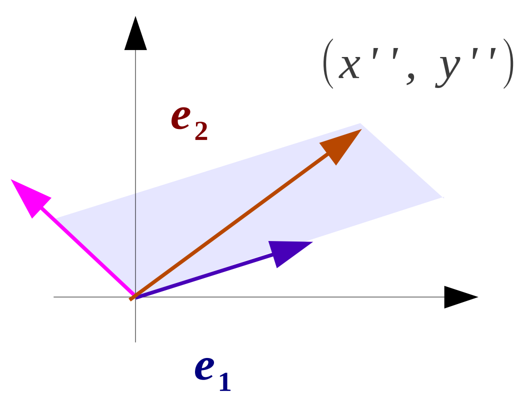


basis $\boldsymbol{e}_1$ $\boldsymbol{e}_2$

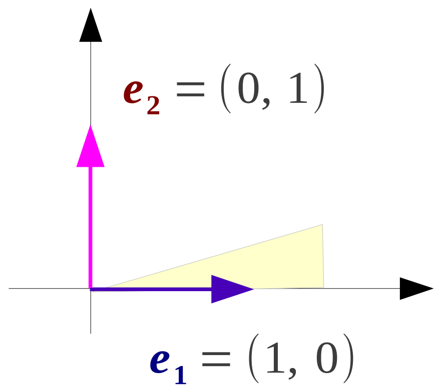
coordinates $(1.5\,,\ 1.5)$





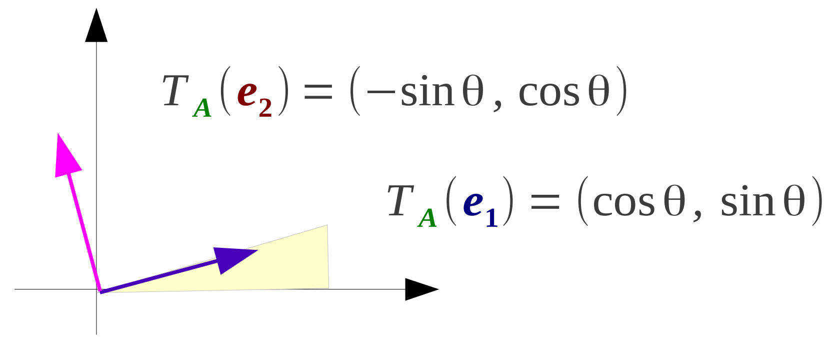


Standard Basis and Matrix



$f : V \rightarrow W$

transformation



standard basis

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_0 \\ y_1 \end{bmatrix}$$

$$\boldsymbol{w} = \boldsymbol{A} \quad \boldsymbol{x}$$

$$\boldsymbol{w} = T_A(\boldsymbol{x})$$

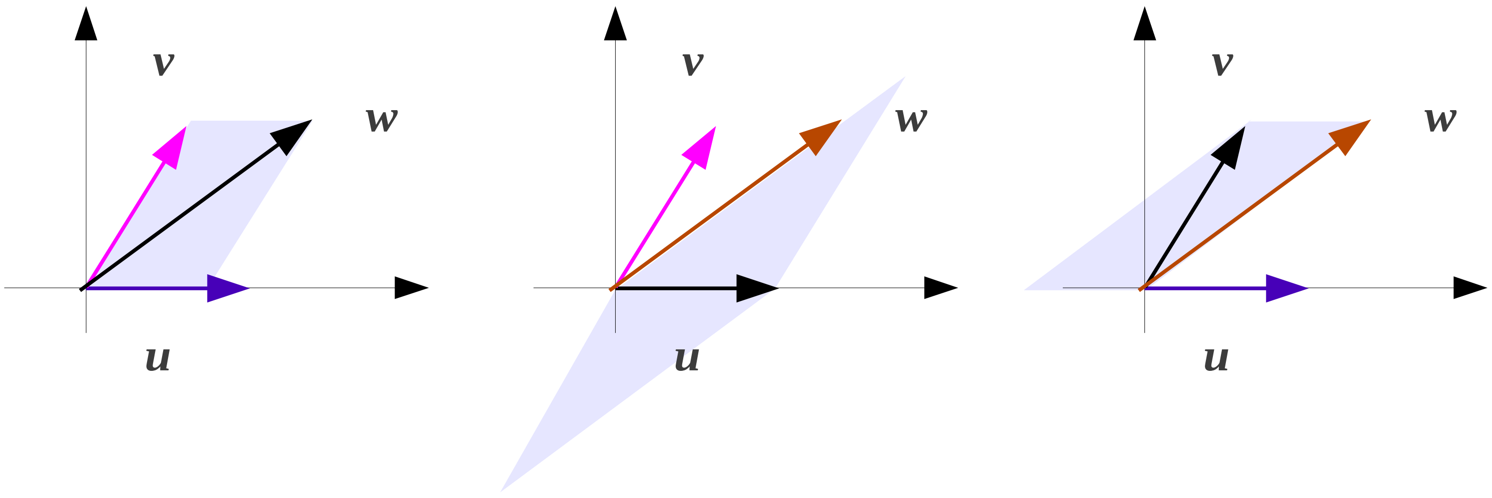
$$\boldsymbol{x} \xrightarrow{T_A} \boldsymbol{w}$$

standard matrix

$$\boldsymbol{A} = \begin{pmatrix} T_A(\boldsymbol{e}_1) & T_A(\boldsymbol{e}_2) & \cdots & T_A(\boldsymbol{e}_n) \end{pmatrix}$$

Dimension

any one vector	line	linearly independent
any two non-collinear vectors	plane	linearly independent
any three or more vectors		linearly dependent



Linear Independent

$S = \{ \boldsymbol{v_1}, \boldsymbol{v_2}, \cdots, \boldsymbol{v_n} \}$ non-empty set of vectors in V

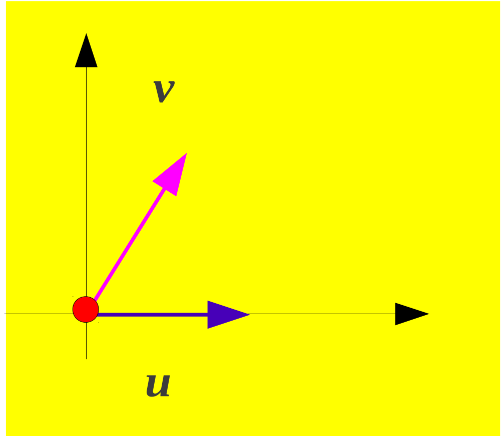
$k_1 \boldsymbol{v_1} + k_2 \boldsymbol{v_2} + \cdots + k_n \boldsymbol{v_n} = \boldsymbol{0}$

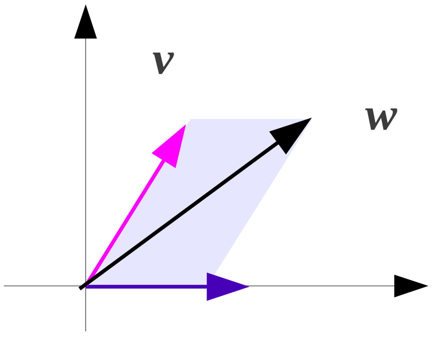
the solution of the above equation

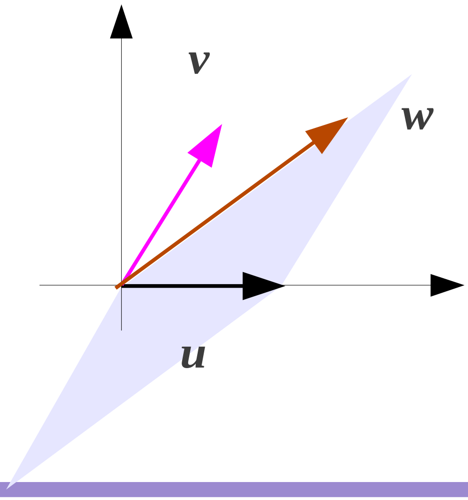
trivial solution: $k_1 = k_2 = \cdots = k_n = 0$

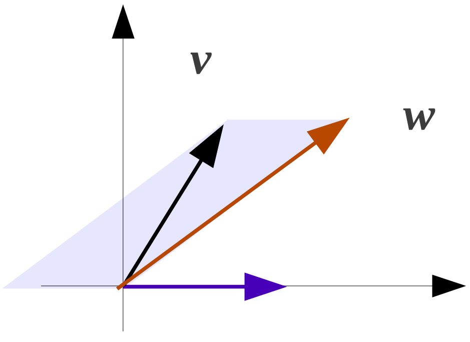
if other solution exists — S linearly dependent

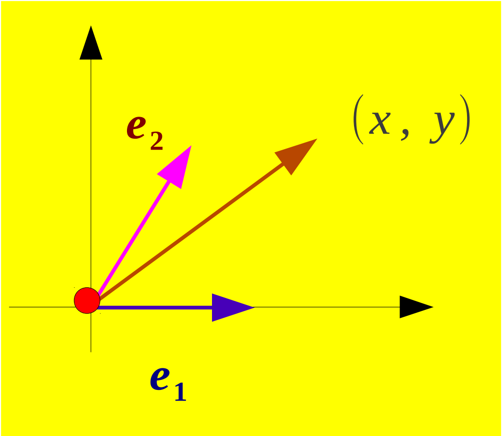
if no other solution exists — S linearly independent



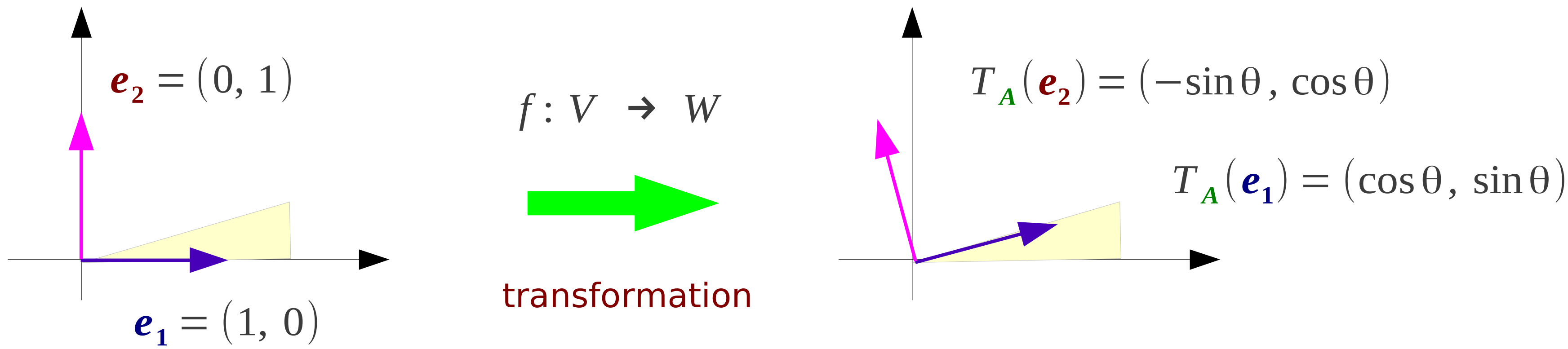








Change of Basis



Old Basis **New Basis**

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_0 \\ y_1 \end{bmatrix}$$

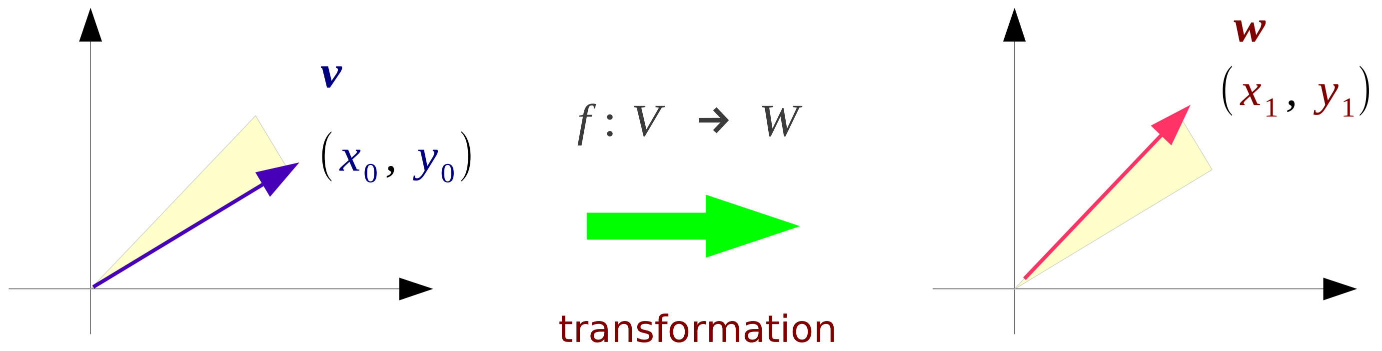
$$\boldsymbol{w} = \boldsymbol{A} \quad \boldsymbol{x}$$

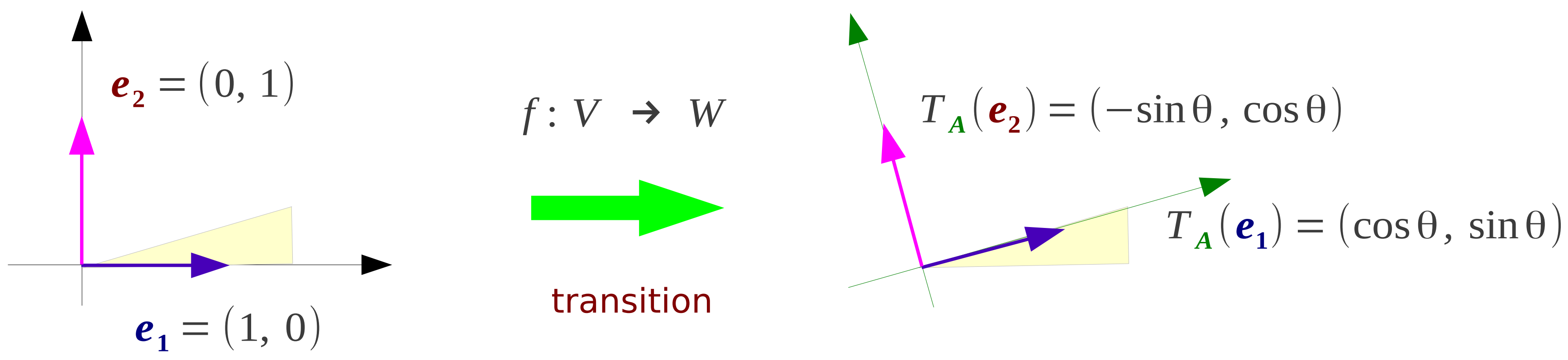
$$\boldsymbol{w} = T_A(\boldsymbol{x})$$

$$\boldsymbol{x} \xrightarrow{T_A} \boldsymbol{w}$$

$$\boldsymbol{A} = \begin{pmatrix} T_A(\boldsymbol{e}_1) & T_A(\boldsymbol{e}_2) & \cdots & T_A(\boldsymbol{e}_n) \end{pmatrix}$$

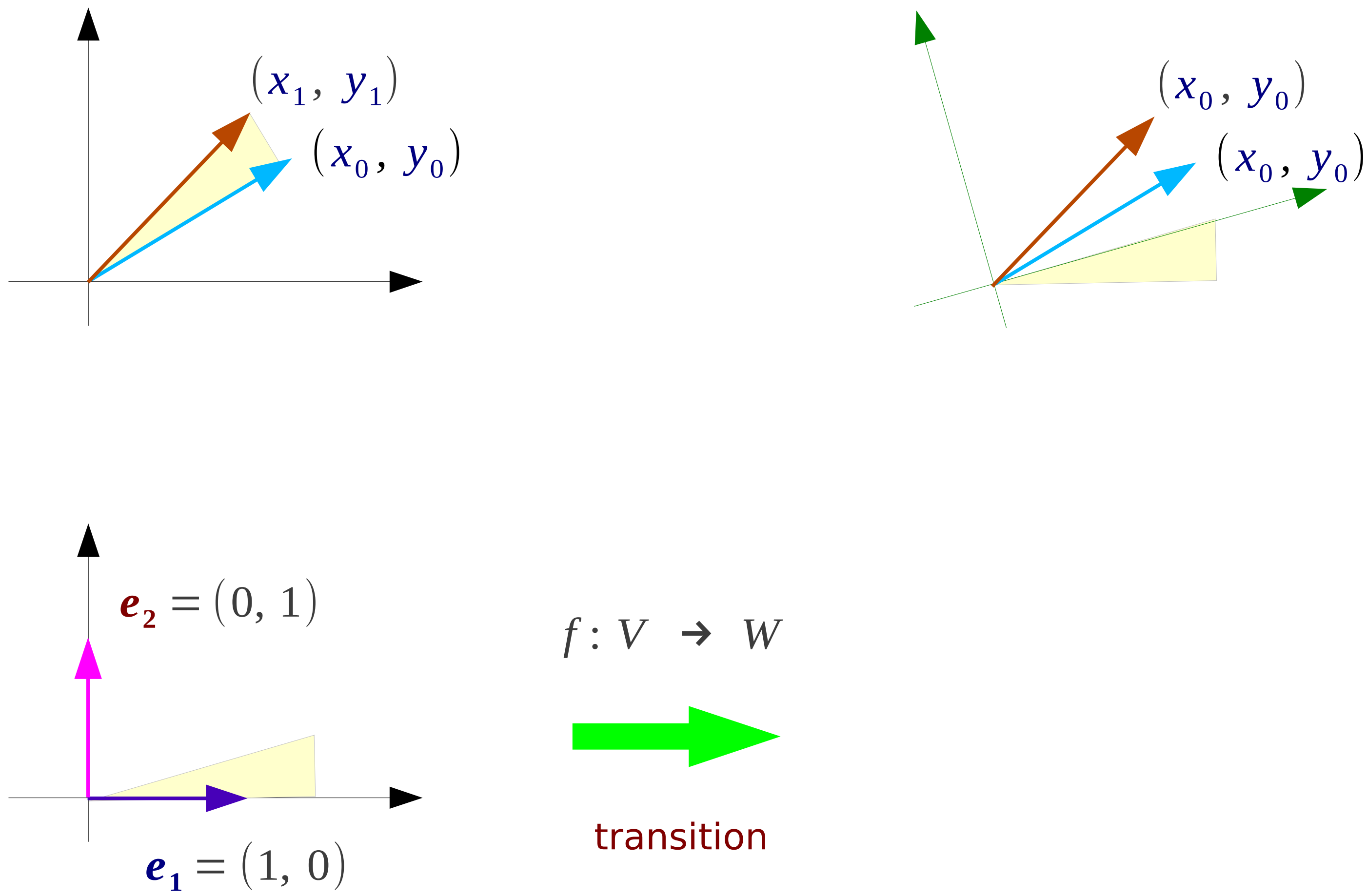
Transformation





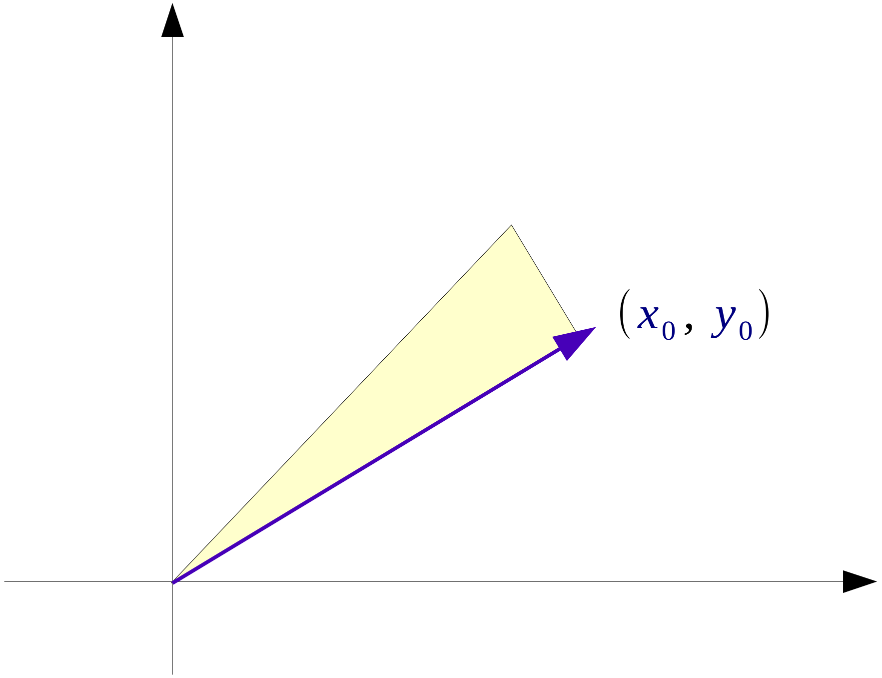
Old Basis **New Basis**

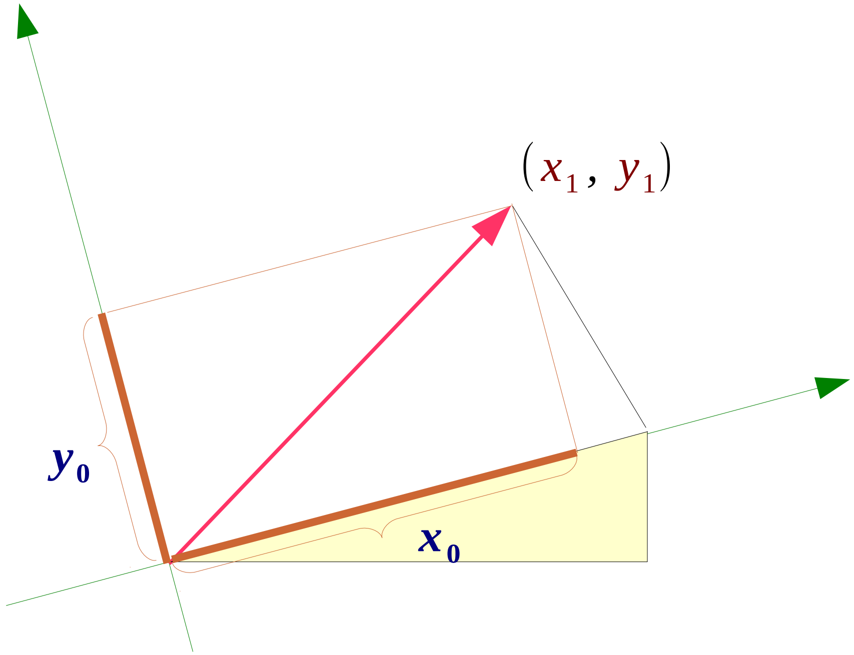
Transformation



Old Basis **New Basis**

Vector Rotation (2)





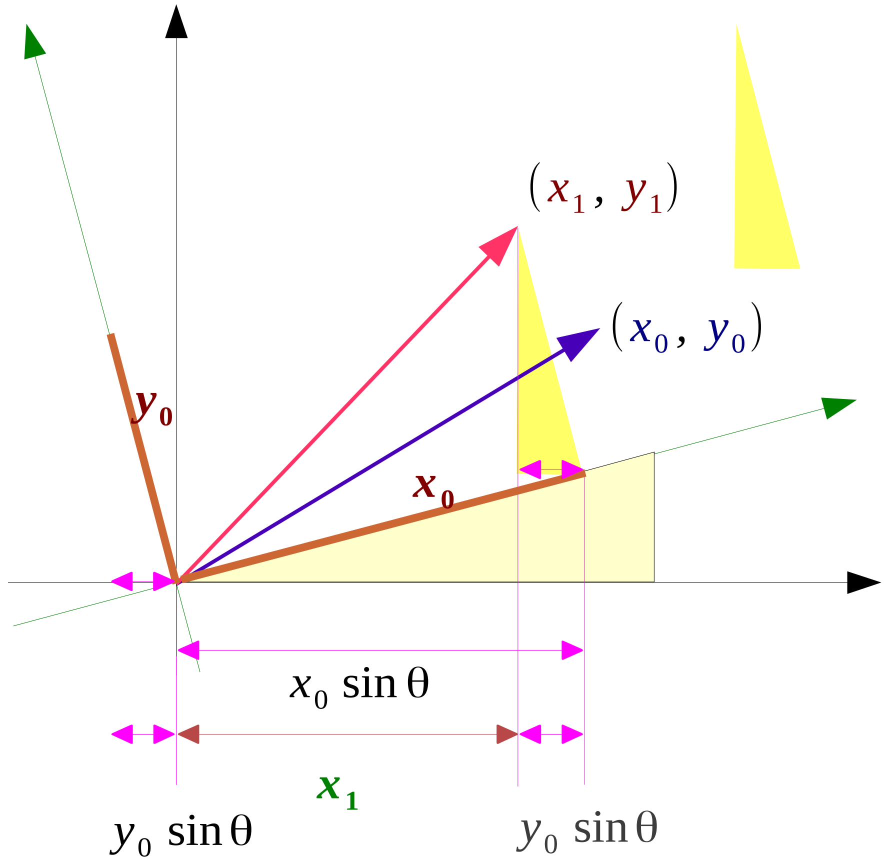
In the rotated coordinate

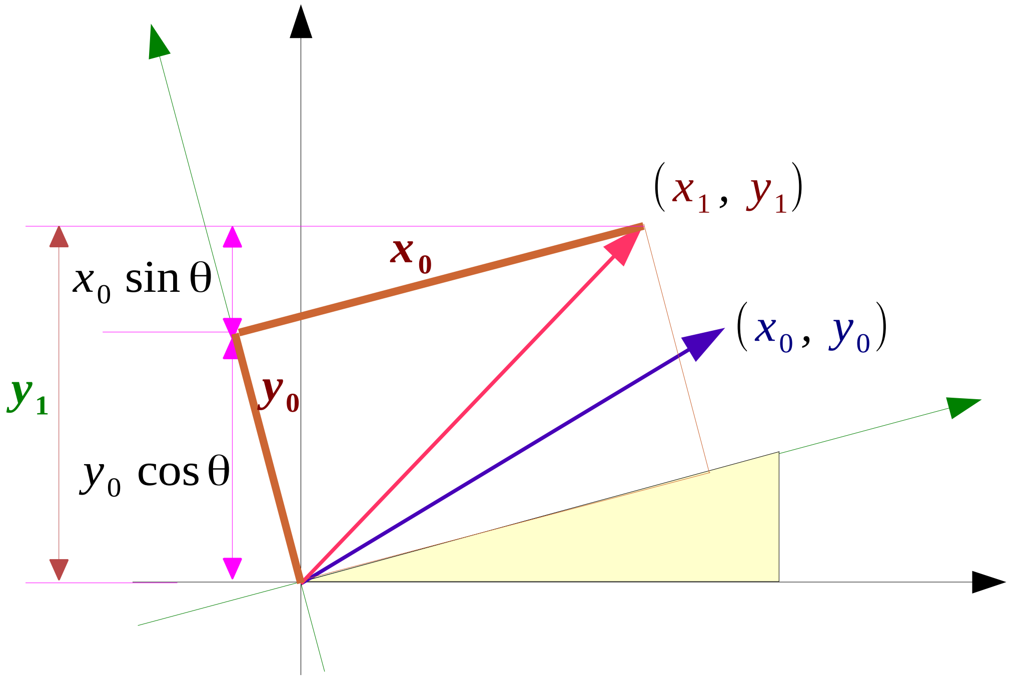
invariant length x_0, y_0

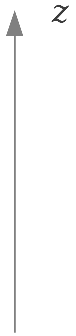
Trasformation

$$x_1 = x_0 \cos\theta - y_0 \sin\theta$$

$$y_1 = x_0 \sin\theta + y_0 \cos\theta$$



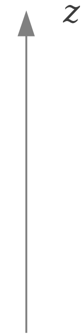




$$\boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \boldsymbol{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \boldsymbol{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \boldsymbol{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

Normal Vector & 3 Points

Non-collinear
3 points



References

[1] http://en.wikipedia.org/
[2] http://planetmath.org/
[3] M.L. Boas, “Mathematical Methods in the Physical Sciences”
[4] D.G. Zill, “Advanced Engineering Mathematics”