

Mtg 14: Tue, 26 Jan 10

[14-1

Thm: Simple Simpson's rule

$$E_2 = -\frac{(b-a)^5}{2880} f^{(4)}(\xi), \quad \xi \in [a, b]$$

$$= -\frac{h^5}{90} f^{(4)}(\xi), \quad h = \frac{b-a}{2}$$

Ref: 1) A. p. 257, pf. based on divided diff. ///

2) Suli & Mayers = S&M, p. 205 ///

use Lagrange interp. error

Rem: If  $f \in \mathcal{P}_3 \Rightarrow f^{(4)} = 0$

$$\Rightarrow E_2 = 0$$

Simple Simpson exact for poly. deg  $\leq 3$ . ///

Pf: (Similar tech. of pf for h. interp. error)

Shift origin of x axis to pt.  $x_1$ :

Transf. of var.  $x(t) = x_1 + ht, t \in [-1, 1]$   
 $h = (b-a)/2$

$$(1) \begin{cases} t=0, & x(0) = x_1 \\ t=+1, & x(+1) = x_1 + h = x_2 \\ t=-1, & x(-1) = x_0 \end{cases}$$

$$(2) E_2 = I - I_2 = \frac{b-a}{2} e(1) = h e(1)$$

$\underbrace{\hspace{2cm}}_2$                        $\uparrow$  p. 7-2

$$(3) e(t) := \int_{-t}^{+t} \underbrace{f(x(t))}_{F(t)} dt - \frac{t}{3} [F(-t) + 4F(0) + F(t)]$$

"error"  $\uparrow$

= error of int. on  $[-t, +t]$

$$e(1) = \text{" " " } [-1, 1]$$

$\uparrow$   $t=1$

Def:  $G(t) := e(t) - t^5 e(1)$  (4)

Goal:  $G^{(3)}(\xi_3) = 0$

$$(5) = - \frac{2(\xi_3)^2}{3} [F^{(4)}(\xi_4) + 90e(1)]$$

$$\rightarrow e(1) \stackrel{(6)}{=} - \frac{1}{80} F^{(4)}(\xi_4) = - \frac{(b-a)^4}{1440} f^{(4)}(\xi)$$

$$(1) E_2 = h e(t) = - \frac{(b-a)^5}{2880} f^{(4)}(\xi) \quad (14-3)$$

Rem: Similarity w/ pf for (end goal)  
 L. interp. error

$$G(t) := e(t) - t^5 \overset{e}{f}(t) \quad (4) \text{ p.14-2}$$

$$G(x) := E(x) - \frac{f_{n+1}(x)}{f_{n+1}(t)} E(t) \quad (1) \text{ p.10-3}$$

Error: Simpson: (1) p.14-3

$$(b-a)^5 \leftrightarrow t^5$$

L. interp. error: (3) p.10-1

$$f_{n+1}(x) \leftrightarrow f_{n+1}(x)$$

(3) p.10-1

(1) p.10-3

Montesquieu: Darwin (end Rem)  
 NOVA: "What Darwin never knew."