Euclidean Vector Space (1A)

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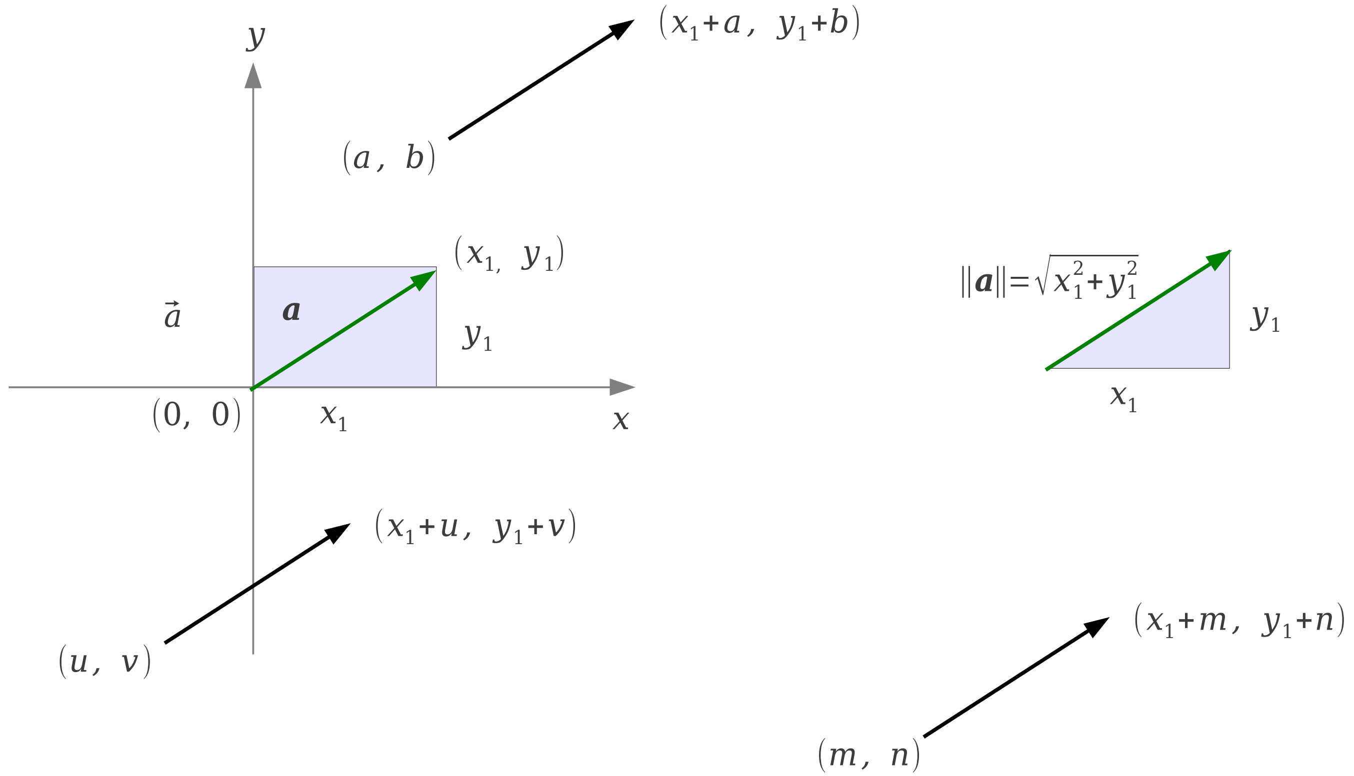
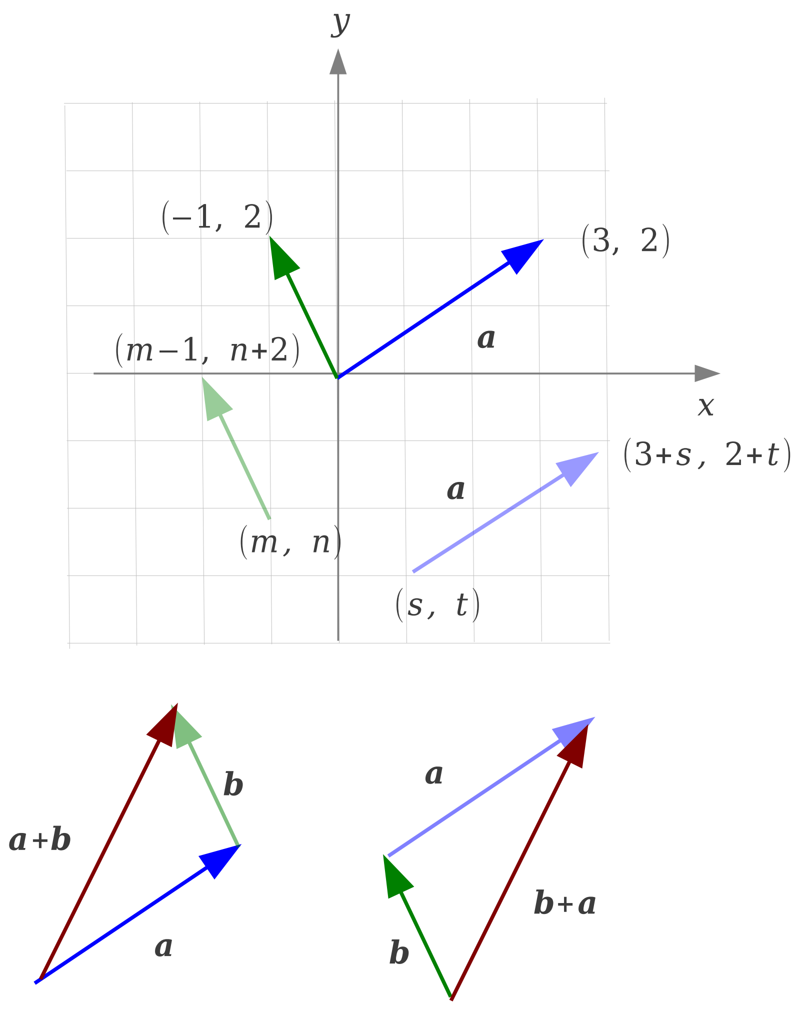
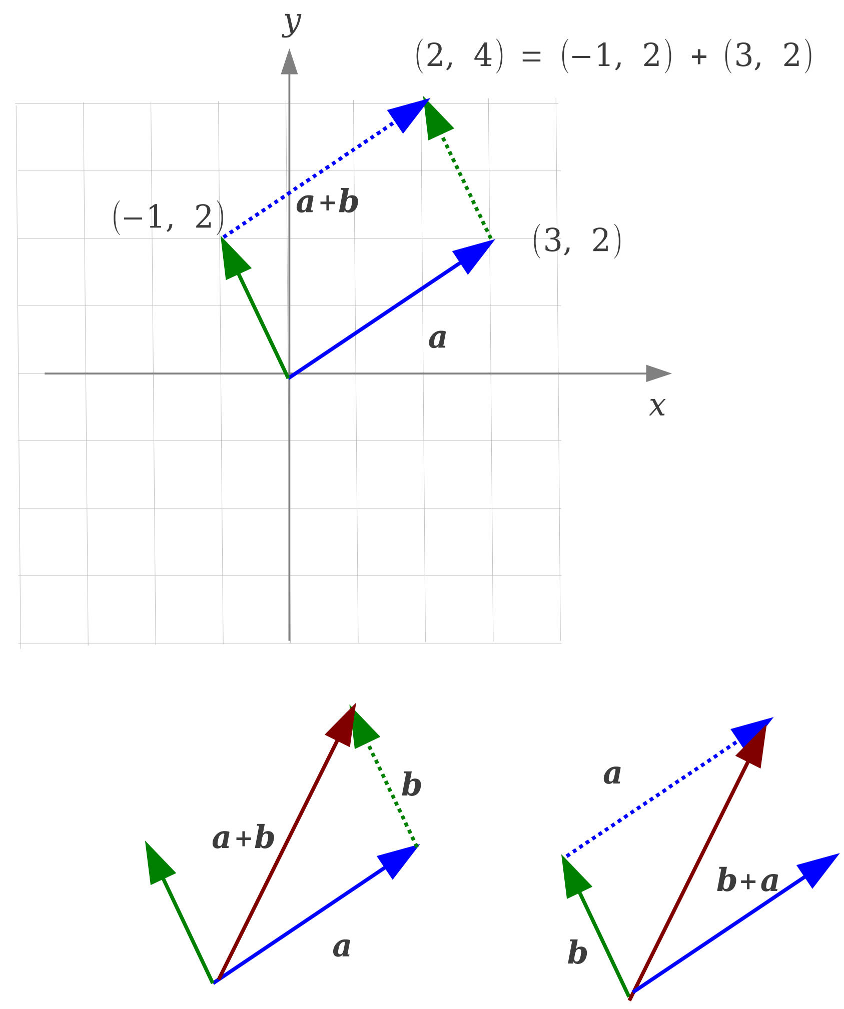
Vectors



(x_1+a, y_1+b)
y
(a, b)
(x_1, y_1)
$\vec{a}$
$\boldsymbol{a}$
y_1
$(0, 0)$
x_1
x
(x_1+u, y_1+v)
(u, v)
$\|\boldsymbol{a}\| = \sqrt{x_1^2 + y_1^2}$
y_1
x_1
(x_1+m, y_1+n)
(m, n)

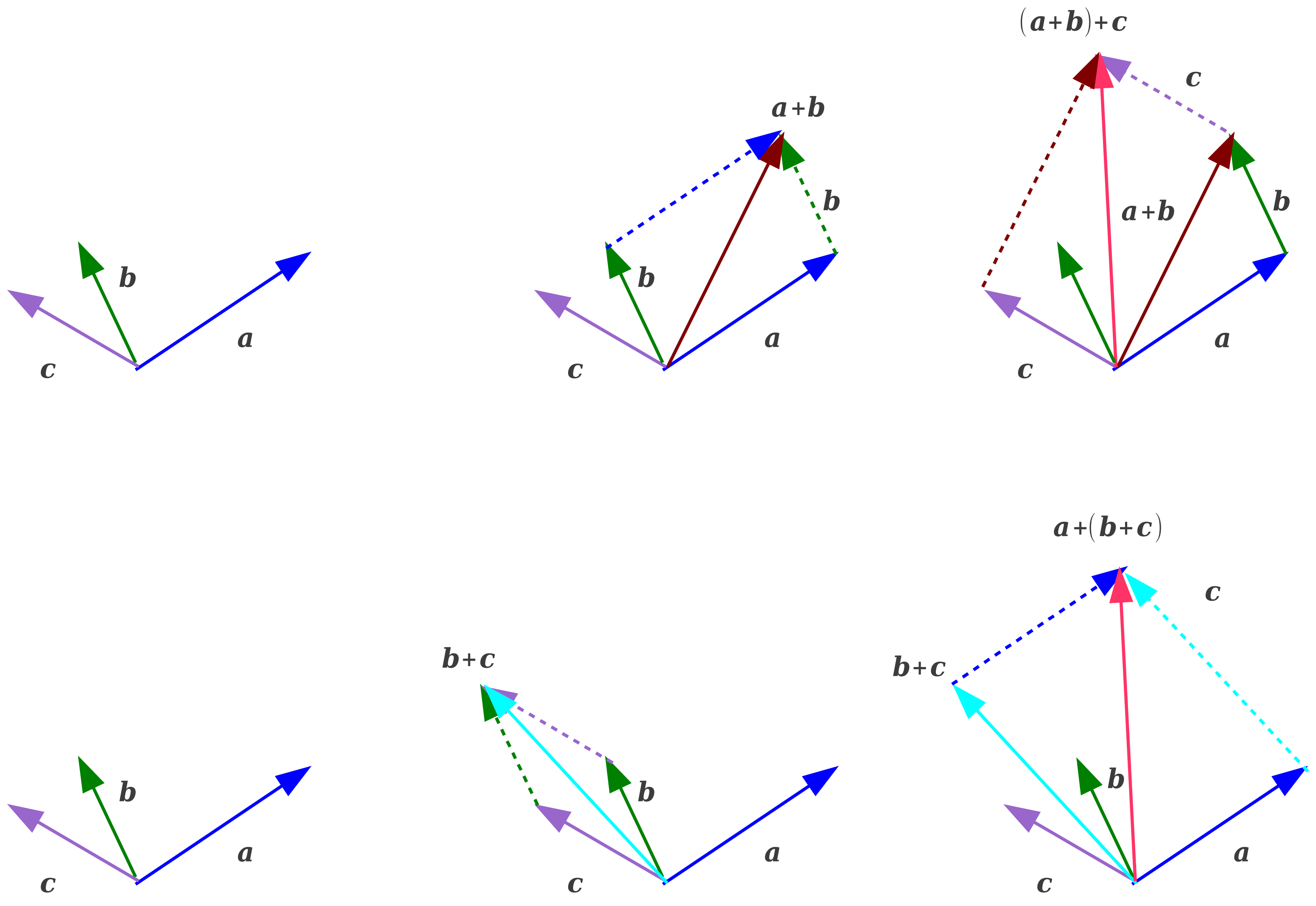
Vector Addition





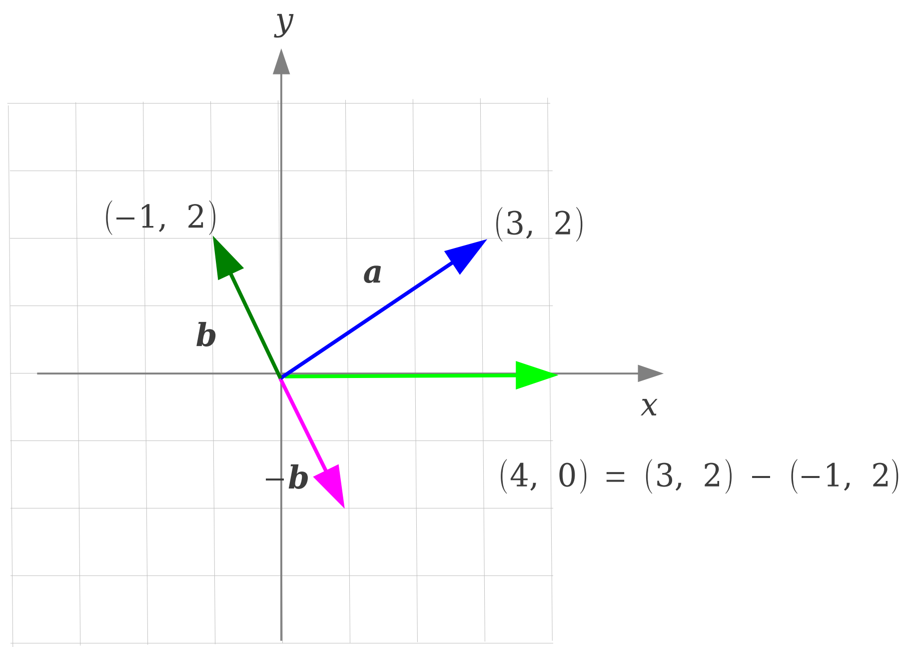
$$\boldsymbol{a}+\boldsymbol{b} = \boldsymbol{b}+\boldsymbol{a}$$

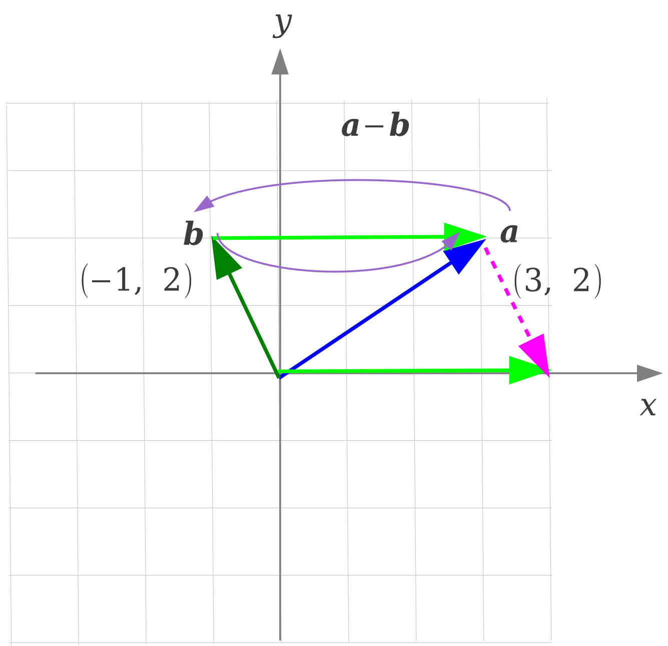
Vector Addition

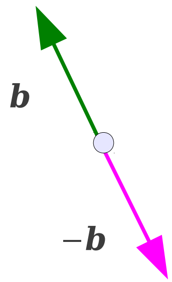


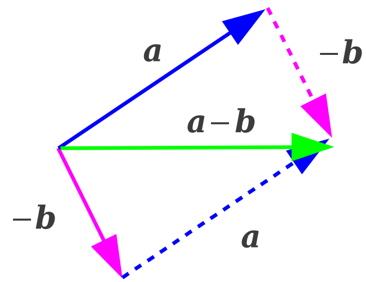
$$(\boldsymbol{a}+\boldsymbol{b})+\boldsymbol{c} \;=\; \boldsymbol{a}+(\boldsymbol{b}+\boldsymbol{c})$$

Vector Subtraction





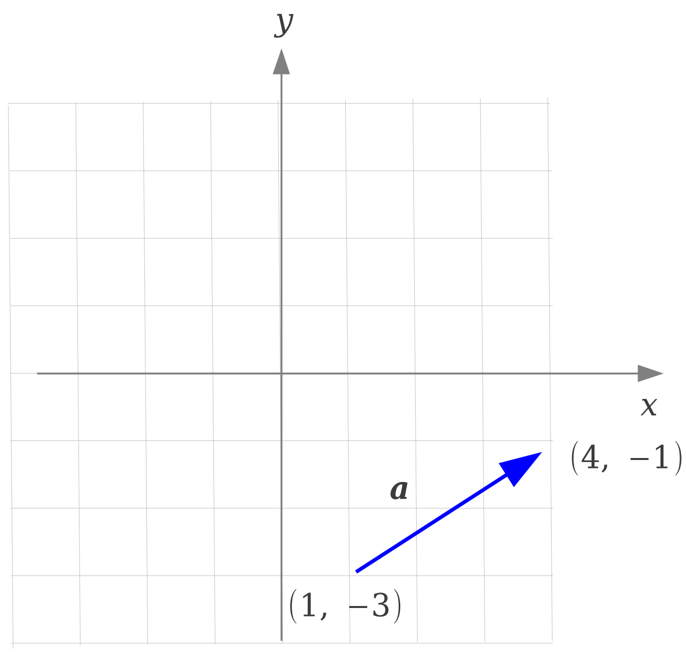


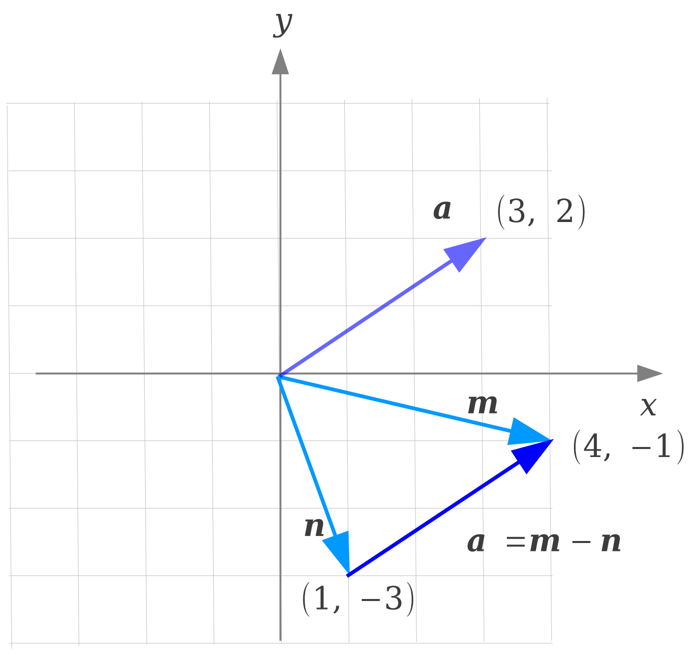


a − b

subtract **a** from **b**

arrow from **b** to **a**





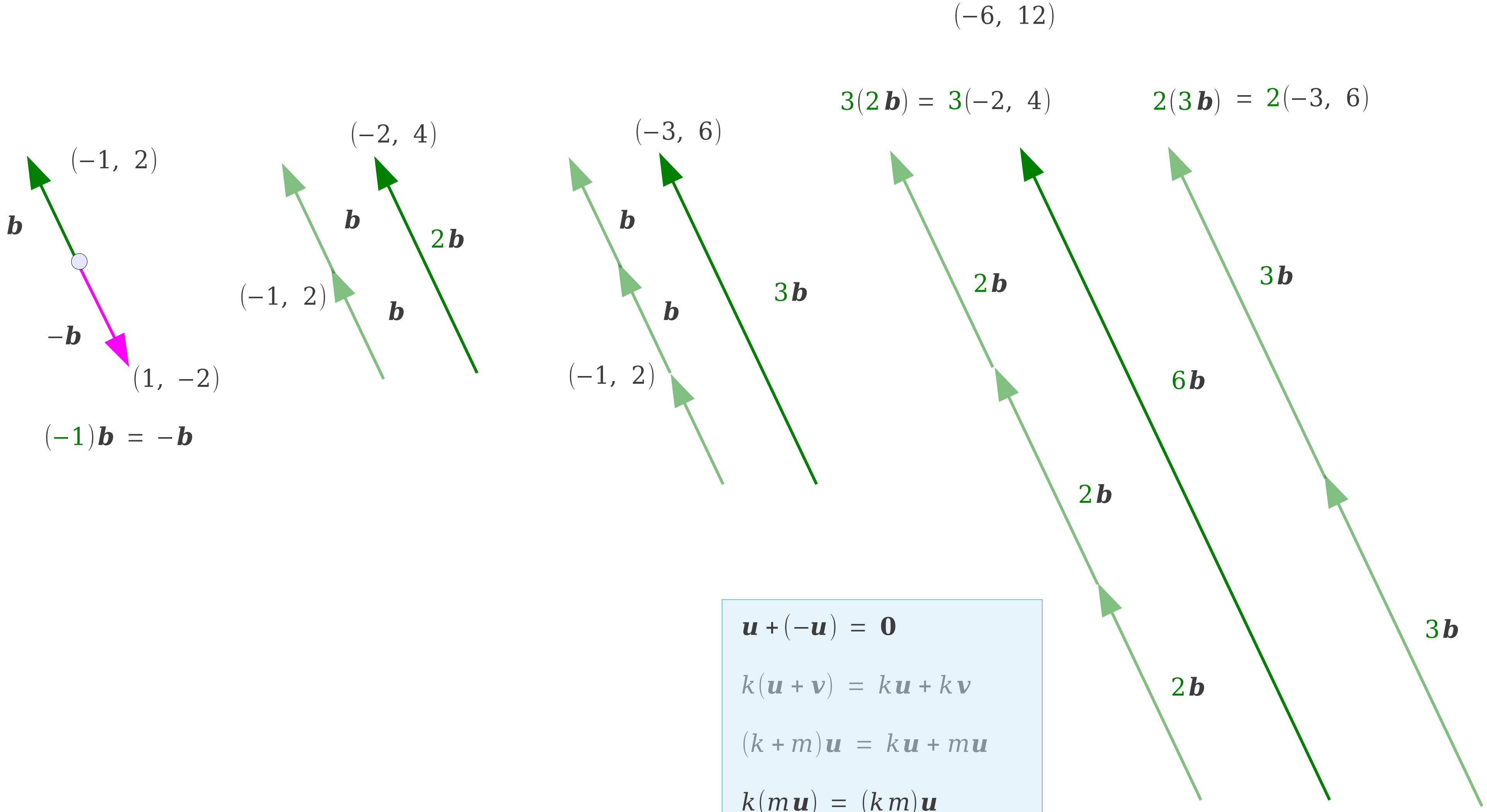
$$\boldsymbol{m} = (4,\ -1)$$

$$\boldsymbol{n} = (1,\ -3)$$

$$\boldsymbol{a} = \boldsymbol{m} - \boldsymbol{n} = (4,\ -1) - (1,\ -3) = (3,\ 2)$$

Finding the **Component Form** of a vector

Scalar Multiplication (1)



$\boldsymbol{u} + (-\boldsymbol{u}) = \boldsymbol{0}$

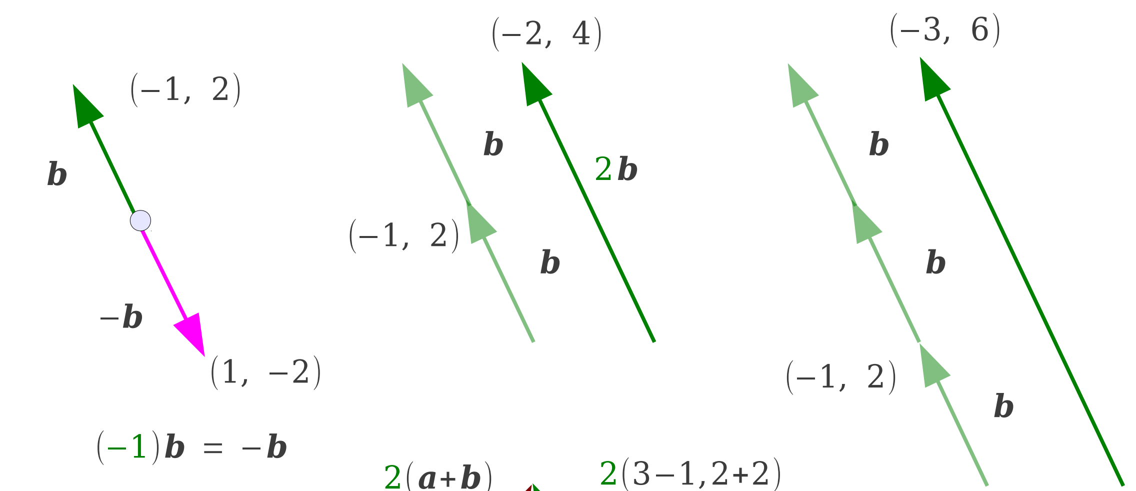
$k(\boldsymbol{u} + \boldsymbol{v}) = k\boldsymbol{u} + k\boldsymbol{v}$

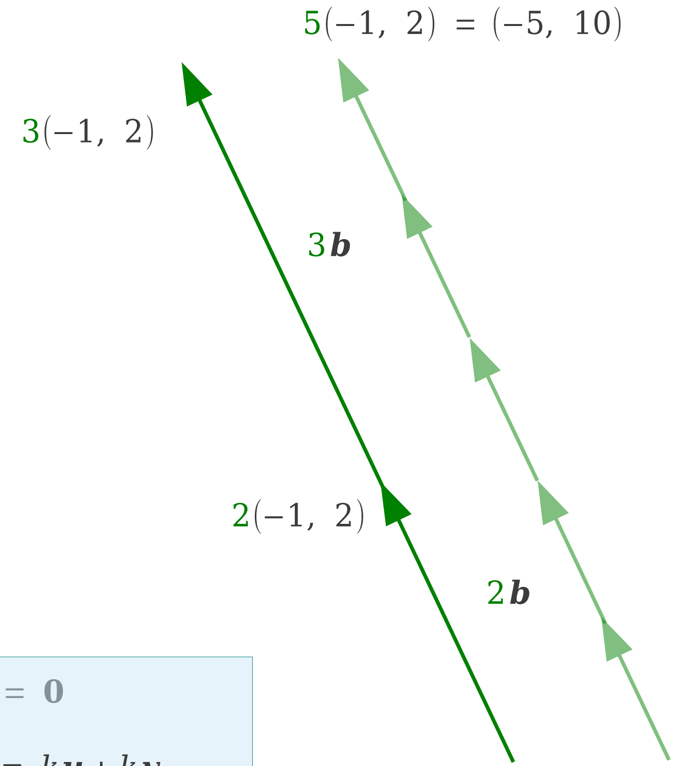
$(k + m)\boldsymbol{u} = k\boldsymbol{u} + m\boldsymbol{u}$

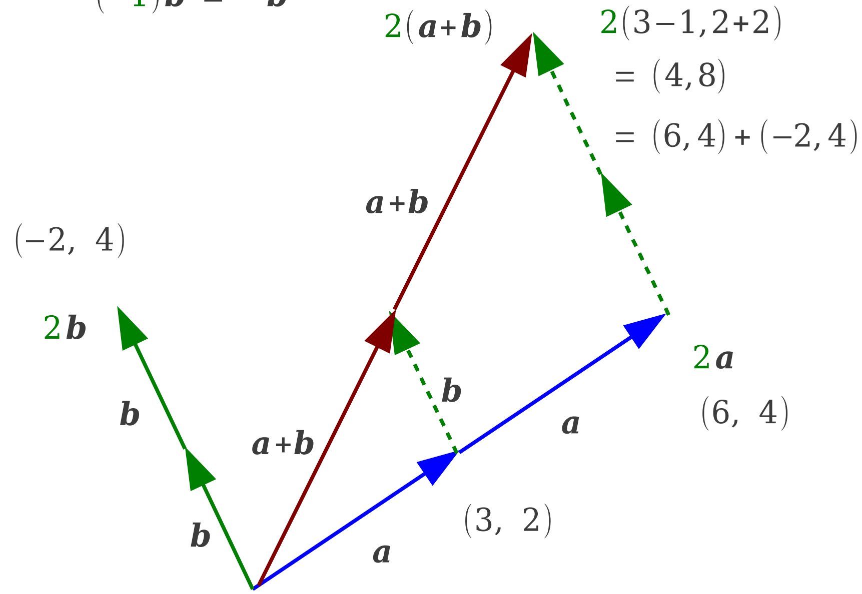
$k(m\boldsymbol{u}) = (km)\boldsymbol{u}$

$1\boldsymbol{u} = \boldsymbol{u}$

Scalar Multiplication (2)







$\boldsymbol{u} + (-\boldsymbol{u}) = \boldsymbol{0}$

$k(\boldsymbol{u} + \boldsymbol{v}) = k\boldsymbol{u} + k\boldsymbol{v}$

$(k + m)\boldsymbol{u} = k\boldsymbol{u} + m\boldsymbol{u}$

$k(m\boldsymbol{u}) = (km)\boldsymbol{u}$

$1\boldsymbol{u} = \boldsymbol{u}$

n-Space

Ordered 2-tuples $(v_1, \ v_2)$ $v_1, \ v_2 \in R$

R^2 ⬄ 2-space ⬄ $\{ \mathit{all\ ordered\ 2-tuples} \ (v_1, \ v_2) \}$

Ordered 3-tuples $(v_1, \ v_2, \ v_3)$ $v_1, \ v_2, \ v_3 \in R$

R^3 ⬄ 3-space ⬄ $\{ \mathit{all\ ordered\ 3-tuples} \ (v_1, \ v_2, \ v_3) \}$

Ordered n-tuples $(v_1, \ v_2, \ \cdots \ , \ v_n)$ $v_1, \ v_2, \ \cdots \ , \ v_n \in R$

R^n ⬄ n-space ⬄ $\{ \mathit{all\ ordered\ n-tuples} \ (v_1, \ v_2, \ \cdots \ , \ v_n) \}$

set

Properties of Real Vector Spaces

$\boldsymbol{u}, \ \boldsymbol{v}, \ \boldsymbol{w}$ vectors in R^n

$k, \ m$ scalars

$$\boldsymbol{u} + \boldsymbol{v} = \boldsymbol{v} + \boldsymbol{u}$$

$$(\boldsymbol{u} + \boldsymbol{v}) + \boldsymbol{w} = \boldsymbol{u} + (\boldsymbol{v} + \boldsymbol{w})$$

$$\boldsymbol{u} + \boldsymbol{0} = \boldsymbol{0} + \boldsymbol{u} = \boldsymbol{u}$$

$$\boldsymbol{u} + (-\boldsymbol{u}) = \boldsymbol{0}$$

$$k(\boldsymbol{u} + \boldsymbol{v}) = k\boldsymbol{u} + k\boldsymbol{v}$$

$$(k + m)\boldsymbol{u} = k\boldsymbol{u} + m\boldsymbol{u}$$

$$k(m\boldsymbol{u}) = (km)\boldsymbol{u}$$

$$1\boldsymbol{u} = \boldsymbol{u}$$

Linear Combination

$\boldsymbol{w}$, $\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_r$ vectors in R^n

$k_1, k_2, \cdots, k_r$ scalars

$\boldsymbol{w}$ is a linear combination of vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_r$

$$\boldsymbol{w} = k_1 \boldsymbol{v}_1 + k_2 \boldsymbol{v}_2 + \cdots + k_r \boldsymbol{v}_r$$

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = k_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= m_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + m_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= n_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + n_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + n_3 \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

Vector Magnitude

- Norm
- Length
- Magnitude

$\|\boldsymbol{v}\|$

R^2 $\quad \boldsymbol{v} = (v_1, \ v_2)$ $\quad \|\boldsymbol{v}\| = \sqrt{v_1^2 + v_2^2} \geq 0$

R^3 $\quad \boldsymbol{v} = (v_1, \ v_2, \ v_3)$ $\quad \|\boldsymbol{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2} \geq 0$

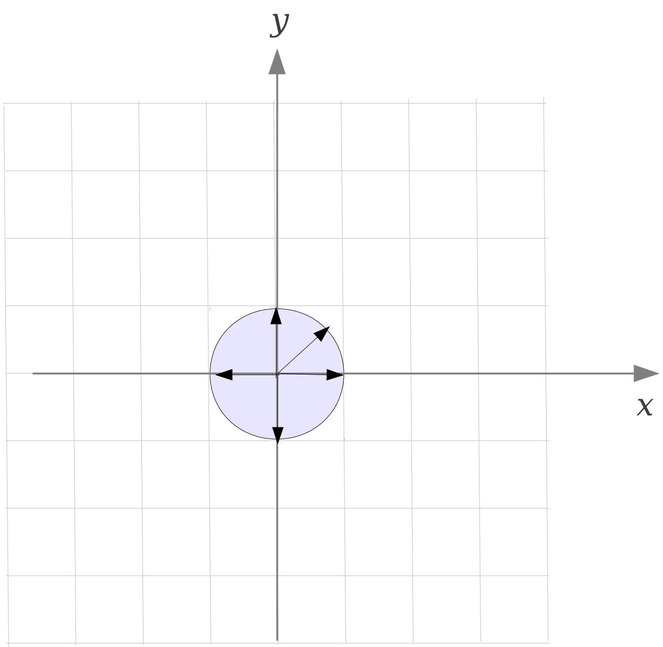
R^n $\quad \boldsymbol{v} = (v_1, \ v_2, \ \cdots \ , v_n)$ $\quad \|\boldsymbol{v}\| = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2} \geq 0$

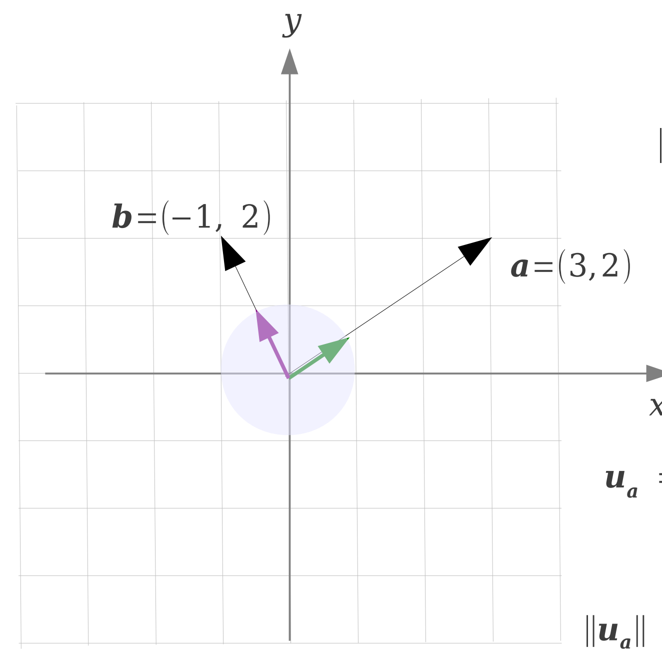
$$\|\boldsymbol{v}\| \geq 0$$

$$\|\boldsymbol{v}\| = 0 \qquad \boldsymbol{v} = \boldsymbol{0}$$

$$\|k\,\boldsymbol{v}\| = |k| \, \|\boldsymbol{v}\| \geq 0$$

Unit Vector





$$|\boldsymbol{a}| = \sqrt{3^2 + 2^2} = \sqrt{13}$$

$$\boldsymbol{u}_a = \left(\frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}}\right)$$

$$\|\boldsymbol{u}_a\| = \sqrt{\frac{3^2}{13} + \frac{2^2}{13}} = \sqrt{\frac{13}{13}} = 1$$

$$\boldsymbol{u} = \frac{\boldsymbol{v}}{\|\boldsymbol{v}\|}$$

$$\|\boldsymbol{u}\| = \left\|\frac{\boldsymbol{v}}{\|\boldsymbol{v}\|}\right\|$$

$$= \frac{\|\boldsymbol{v}\|}{\|\boldsymbol{v}\|} = 1$$

$$\|\boldsymbol{v}\| \geq 0$$

Standard Unit Vectors

R^2 ⇔ 2-space ⇔ $\{ \textit{all ordered } 2\textit{-tuples } (v_1, v_2) \}$

$\boldsymbol{i} = (1, 0)$

$\boldsymbol{j} = (0, 1)$

$\boldsymbol{v} = (v_1, v_2) = v_1(1, 0) + v_2(0, 1)$

R^3 ⇔ 3-space ⇔ $\{ \textit{all ordered } 3\textit{-tuples } (v_1, v_2, v_3) \}$

$\boldsymbol{i} = (1, 0, 0)$

$\boldsymbol{j} = (0, 1, 0)$

$\boldsymbol{k} = (0, 0, 1)$

$\boldsymbol{v} = (v_1, v_2, v_3) = v_1(1, 0, 0) + v_2(0, 1, 0) + v_3(0, 0, 1)$

R^n ⇔ n-space ⇔ $\{ \textit{all ordered } n\textit{-tuples } (v_1, v_2, \cdots, v_n) \}$

$\boldsymbol{e_1} = (1, 0, \cdots, 0)$

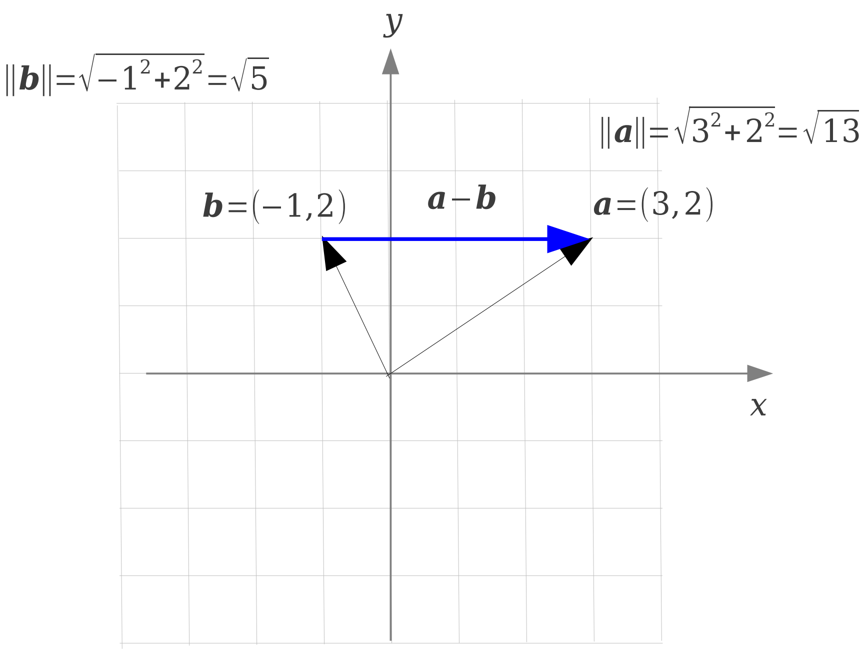
$\boldsymbol{e_2} = (0, 1, \cdots, 0)$

$\cdots$

$\boldsymbol{e_n} = (0, 0, \cdots, 1)$

$\boldsymbol{v} = (v_1, v_2, \cdots, v_n) = v_1(1, 0, \cdots, 0) + v_2(0, 1, \cdots, 0) + \cdots + v_n(0, 0, \cdots, 1)$

Distance between Vectors



R^2

$$d(\boldsymbol{a} - \boldsymbol{b}) = \|\boldsymbol{a} - \boldsymbol{b}\|$$

$$= \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2} \geq 0$$

R^3

$$d(\boldsymbol{a} - \boldsymbol{b}) = \|\boldsymbol{a} - \boldsymbol{b}\|$$

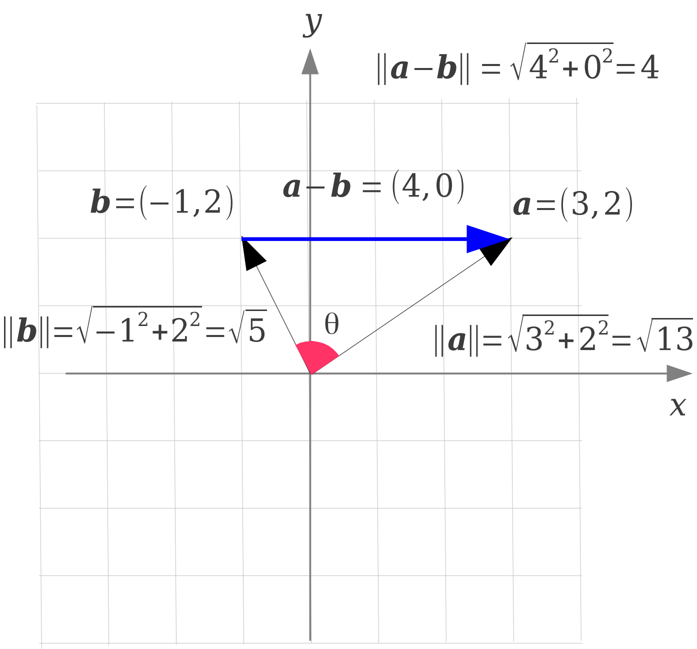
$$= \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2} \geq 0$$

R^n

$$d(\boldsymbol{a} - \boldsymbol{b}) = \|\boldsymbol{a} - \boldsymbol{b}\|$$

$$= \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + \cdots + (a_n - b_n)^2} \geq 0$$

Law of Cosine



$4^2=\sqrt{13}^2+\sqrt{5}^2-2\sqrt{13}\sqrt{5}\cos\theta$

$2=2\sqrt{13}\sqrt{5}\cos\theta$ $\qquad \cos\theta = \dfrac{1}{\sqrt{65}}$

$\boldsymbol{a}\cdot\boldsymbol{b}=3\cdot-1+2\cdot2=-3+4=1$

$$R^2 \quad d(\boldsymbol{a}-\boldsymbol{b}) = \|\boldsymbol{a}-\boldsymbol{b}\| = \sqrt{(a_1-b_1)^2 + (a_2-b_2)^2} \geq 0$$

Law of Cosine

$$\|\boldsymbol{a}-\boldsymbol{b}\|^2 = \|\boldsymbol{a}\|^2 + \|\boldsymbol{b}\|^2 - 2\|\boldsymbol{a}\|\,\|\boldsymbol{b}\|\cos\theta$$

$\boldsymbol{a}-\boldsymbol{b}$

$= (a_1-b_1,\ a_2-b_2)$

$\boldsymbol{a} = (a_1,\ a_2)$

$\boldsymbol{b} = (b_1,\ b_2)$

$(a_1-b_1)^2 + (a_2-b_2)^2$

$a_1^2 + a_2^2$

$b_1^2 + b_2^2$

$-2(a_1 b_1 + a_2 b_2)$

$a_1^2 + a_2^2$ $\qquad -2\sqrt{a_1^2 + a_2^2}$

$b_1^2 + b_2^2$ $\qquad \sqrt{b_1^2 + b_2^2}$

$\cos\theta$

$$\boldsymbol{a}\cdot\boldsymbol{b} = \|\boldsymbol{a}\|\,\|\boldsymbol{b}\|\cos\theta$$

Dot Product

Dot Product
Euclidean Inner Product

$\boldsymbol{a}, \boldsymbol{b}$ Vectors in R^n

θ Angle between $\boldsymbol{a}, \boldsymbol{b}$

$$\boldsymbol{a} \cdot \boldsymbol{b} = \|\boldsymbol{a}\| \, \|\boldsymbol{b}\| \cos\theta$$

⬇

$$\boldsymbol{a} \cdot \boldsymbol{a} = \|\boldsymbol{a}\| \, \|\boldsymbol{a}\| \cos 0^\circ$$

⬇

$$\|\boldsymbol{a}\| = \sqrt{\boldsymbol{a} \cdot \boldsymbol{a}}$$

R^2
$$\boldsymbol{a} \cdot \boldsymbol{b} = \|\boldsymbol{a}\| \, \|\boldsymbol{b}\| \cos\theta$$
$$= a_1 b_1 + a_2 b_2$$

R^3
$$\boldsymbol{a} \cdot \boldsymbol{b} = \|\boldsymbol{a}\| \, \|\boldsymbol{b}\| \cos\theta$$
$$= a_1 b_1 + a_2 b_2 + a_3 b_3$$

R^n
$$\boldsymbol{a} \cdot \boldsymbol{b} = \|\boldsymbol{a}\| \, \|\boldsymbol{b}\| \cos\theta$$
$$= a_1 b_1 + a_2 b_2 + \cdots a_n b_n$$

Properties of Dot Products

$$\boldsymbol{a}\cdot\boldsymbol{b} = \boldsymbol{b}\cdot\boldsymbol{a}$$

$$\boldsymbol{a}\cdot(\boldsymbol{b}+\boldsymbol{c}) = \boldsymbol{a}\cdot\boldsymbol{b} + \boldsymbol{a}\cdot\boldsymbol{c}$$

$$k(\boldsymbol{a}\cdot\boldsymbol{b}) = (k\boldsymbol{a})\cdot\boldsymbol{b}$$

$$\boldsymbol{a}\cdot\boldsymbol{a} \geq 0 \qquad \boldsymbol{a}\cdot\boldsymbol{a} = 0 \Leftrightarrow \boldsymbol{a} = \boldsymbol{0}$$

$$\boldsymbol{0}\cdot\boldsymbol{a} = \boldsymbol{a}\cdot\boldsymbol{0} = 0$$

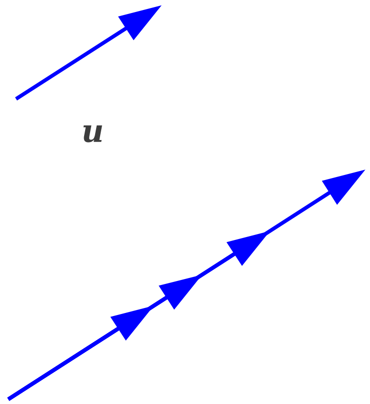
$$(\boldsymbol{a}+\boldsymbol{b})\cdot\boldsymbol{c} = \boldsymbol{a}\cdot\boldsymbol{c} + \boldsymbol{b}\cdot\boldsymbol{c}$$

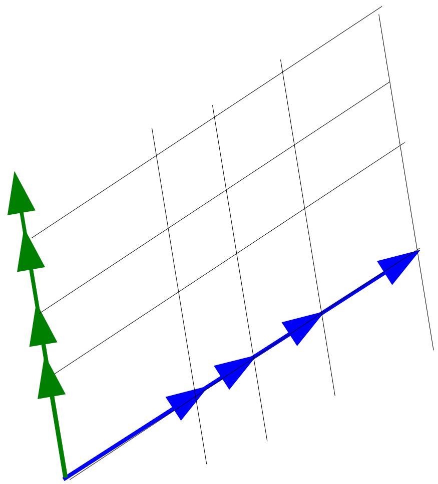
$$\boldsymbol{a}\cdot(\boldsymbol{b}-\boldsymbol{c}) = \boldsymbol{a}\cdot\boldsymbol{b} - \boldsymbol{a}\cdot\boldsymbol{c}$$

$$(\boldsymbol{a}-\boldsymbol{b})\cdot\boldsymbol{c} = \boldsymbol{a}\cdot\boldsymbol{c} - \boldsymbol{b}\cdot\boldsymbol{c}$$

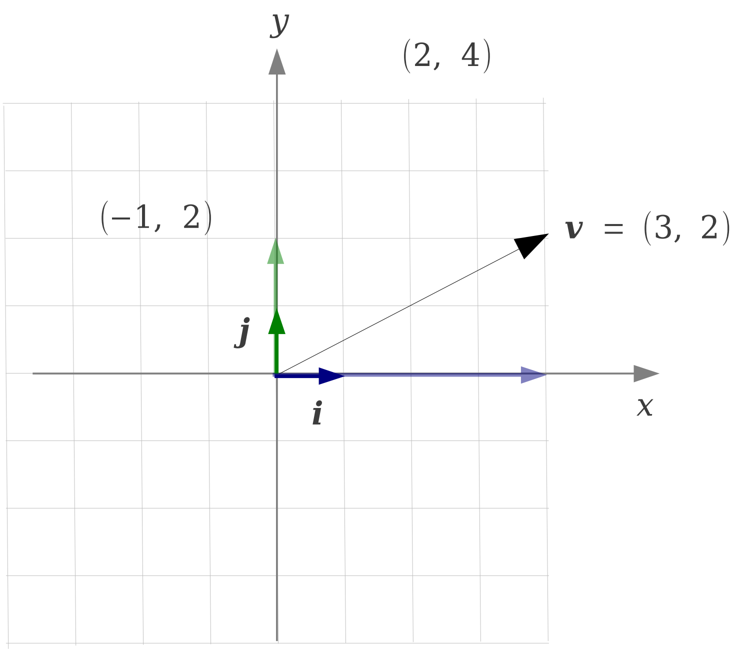
$$k(\boldsymbol{a}\cdot\boldsymbol{b}) = \boldsymbol{a}\cdot(k\boldsymbol{v})$$

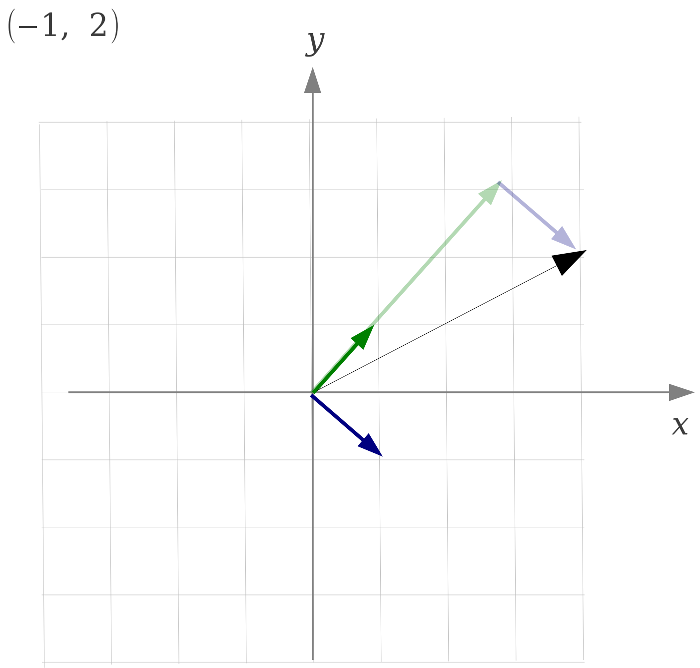
Vector Addition





Basis





$\boldsymbol{v} = 3\boldsymbol{i} + 2\boldsymbol{j}$

$a\begin{bmatrix}1\\0\end{bmatrix} + b\begin{bmatrix}0\\1\end{bmatrix} = 0 \Rightarrow a = b = 0$

$a\begin{bmatrix}1\\1\end{bmatrix} + b\begin{bmatrix}1\\-1\end{bmatrix} = 0 \Rightarrow a = b = 0$

basis

basis

Cross Product (1)

Determinant of order 3

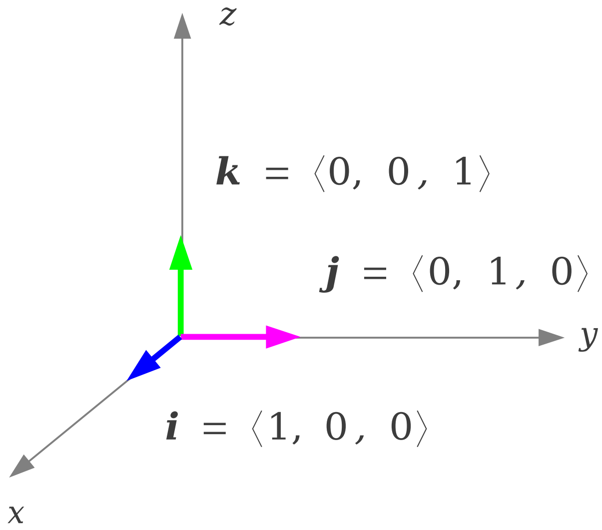
$$\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k} = \langle a_1, a_2, a_3\rangle$$

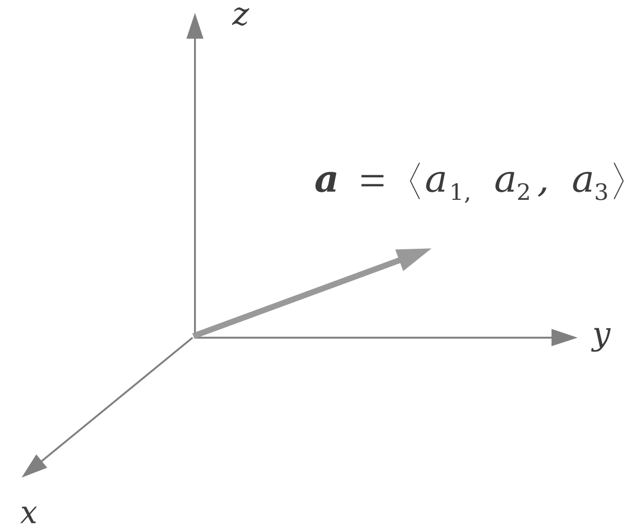
$$\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k} = \langle b_1, b_2, b_3\rangle$$

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

$$= (a_2 b_3 - a_3 b_2)\mathbf{i} - (a_1 b_3 - a_3 b_1)\mathbf{i} + (a_1 b_2 - a_2 b_1)\mathbf{i}$$

Cross Product (2)





$$\boldsymbol{i} \times \boldsymbol{j} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \boldsymbol{k}$$

⇐ normal to ***i*** & ***j*** ⇒

$$\boldsymbol{j} \times \boldsymbol{i} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = -\boldsymbol{k}$$

$$\boldsymbol{j} \times \boldsymbol{k} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \boldsymbol{i}$$

⇐ normal to ***j*** & ***k*** ⇒

$$\boldsymbol{k} \times \boldsymbol{j} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -\boldsymbol{i}$$

$$\boldsymbol{k} \times \boldsymbol{i} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = \boldsymbol{j}$$

⇐ normal to ***k*** & ***i*** ⇒

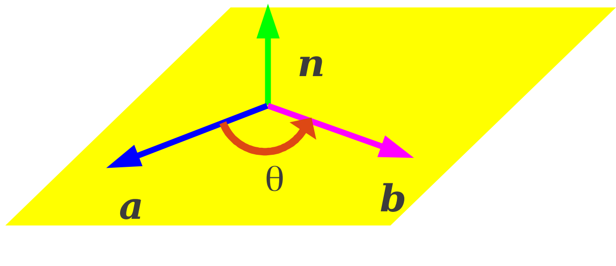
$$\boldsymbol{i} \times \boldsymbol{k} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -\boldsymbol{j}$$

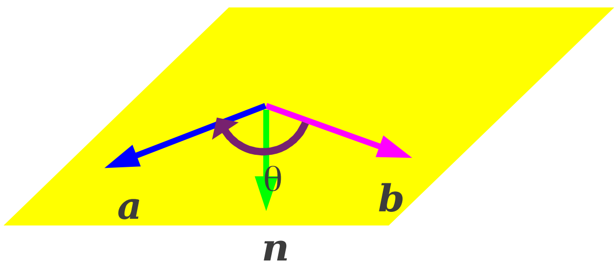
Right Hand Rule

$$\boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

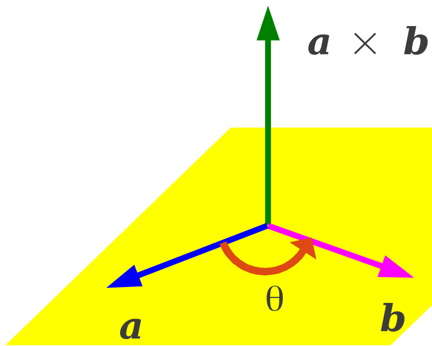
$$\boldsymbol{b} \times \boldsymbol{a} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$$

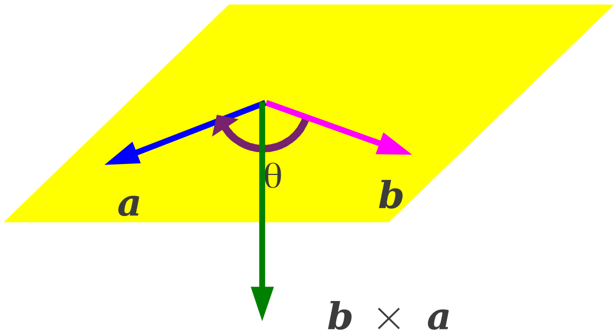
Normal direction $\boldsymbol{n}$





Magnitude = $\|\boldsymbol{a} \times \boldsymbol{b}\| = \|\boldsymbol{a}\| \, \|\boldsymbol{b}\| \, \sin\theta$





Line Equations (1)

Vector Equation

$$\boldsymbol{r} = \boldsymbol{r}_2 + t\,\boldsymbol{a}$$

Parameter

Direction Vector

$$\boldsymbol{r}_2 = \langle x_2, y_2, z_2 \rangle$$
$$\boldsymbol{a} = \langle a_1, a_2, a_3 \rangle$$

Parametric Equation

Component

$$x = x_2 + t\,a_1$$
$$y = y_2 + t\,a_2$$
$$z = z_2 + t\,a_3$$

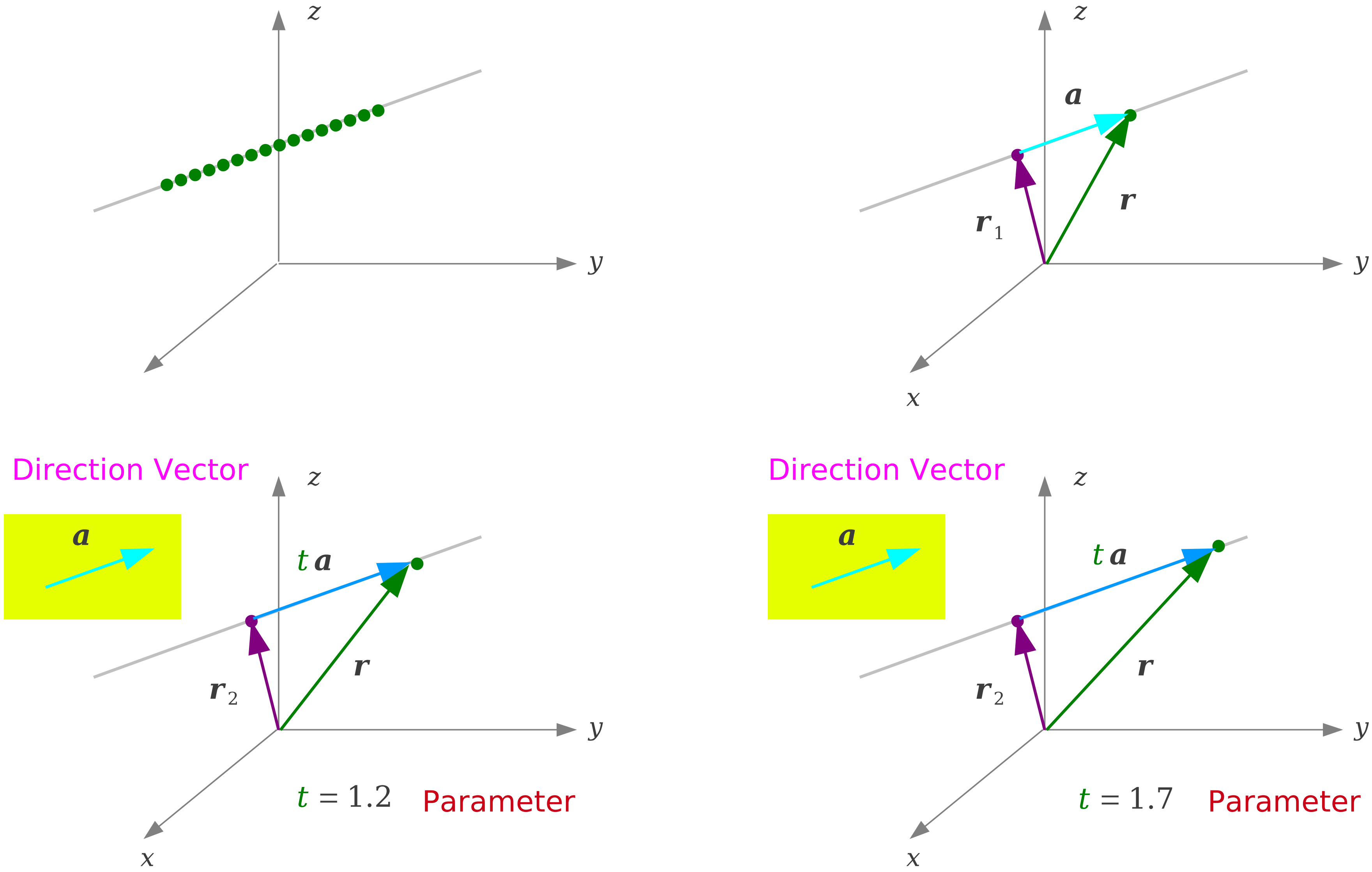
$$t\,a_1 = x - x_2$$
$$t\,a_2 = y - y_2$$
$$t\,a_3 = z - z_2$$

Symmetric Equation

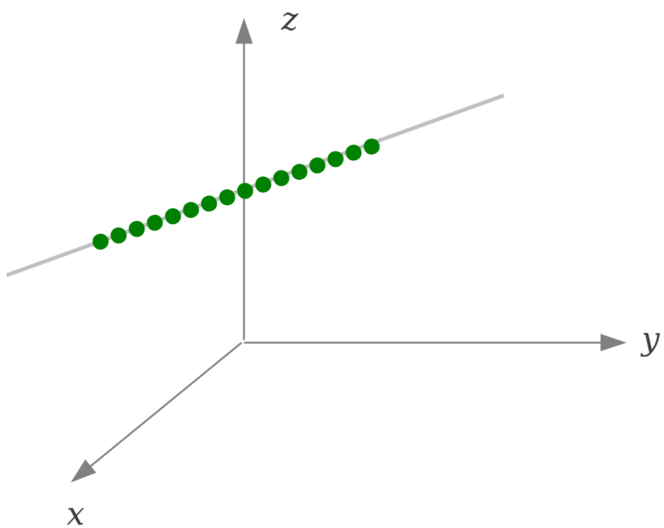
$$t = \frac{x - x_2}{a_1} = \frac{y - y_2}{a_2} = \frac{z - z_2}{a_3}$$

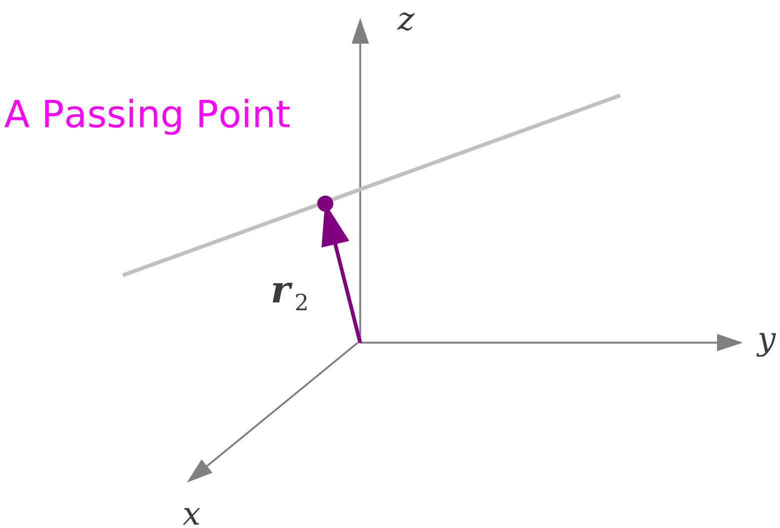
Elimination of parameter

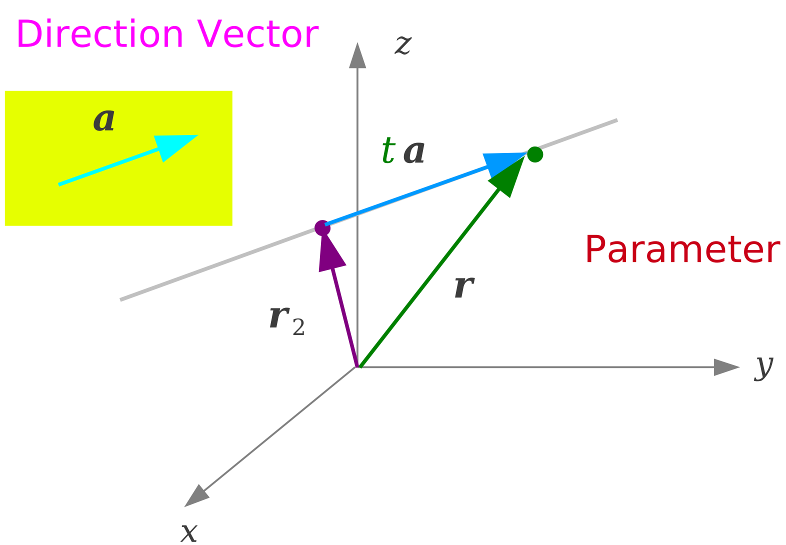
Line Equations (2)



Line Equations (3)







$$\boldsymbol{r} = \boldsymbol{r}_2 + t\,\boldsymbol{a}$$

$$\boldsymbol{r}_2 = \langle x_2, y_2, z_2 \rangle$$

$$\boldsymbol{a} = \langle a_1, a_2, a_3 \rangle$$

Plane Equations (1)

Vector equation

$$\boldsymbol{n} \cdot (\boldsymbol{r} - \boldsymbol{r}_1) = 0$$

Normal Vector

$$\boldsymbol{r} = \langle x, y, z \rangle$$

$$\boldsymbol{r}_1 = \langle x_1, y_1, z_1 \rangle$$

$$\boldsymbol{n} = \langle a, b, c \rangle$$

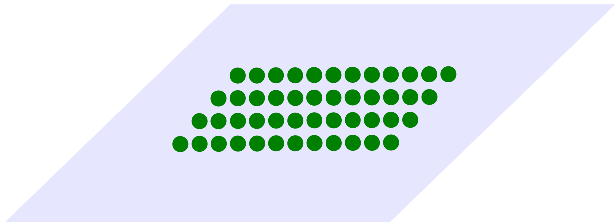
$$\boldsymbol{r} - \boldsymbol{r}_1 = \langle x - x_1, \; y - y_1, \; z - z_1 \rangle$$

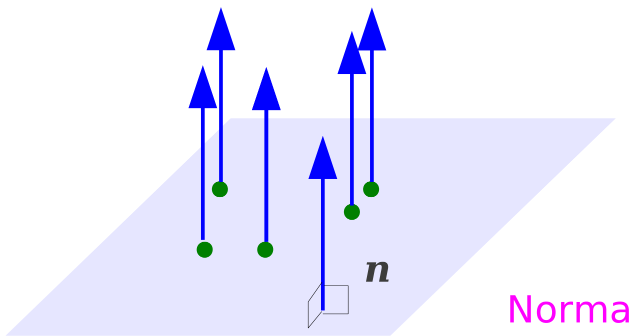
$$\boldsymbol{n} = \langle \quad a, \quad b, \quad c \rangle$$

Cartesian equation

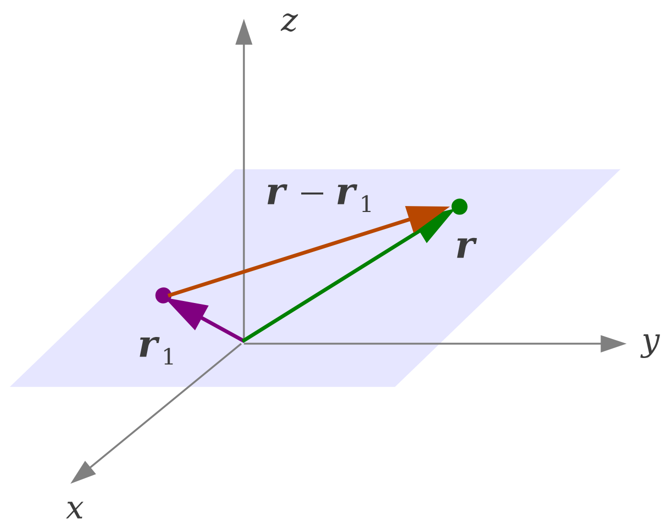
$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

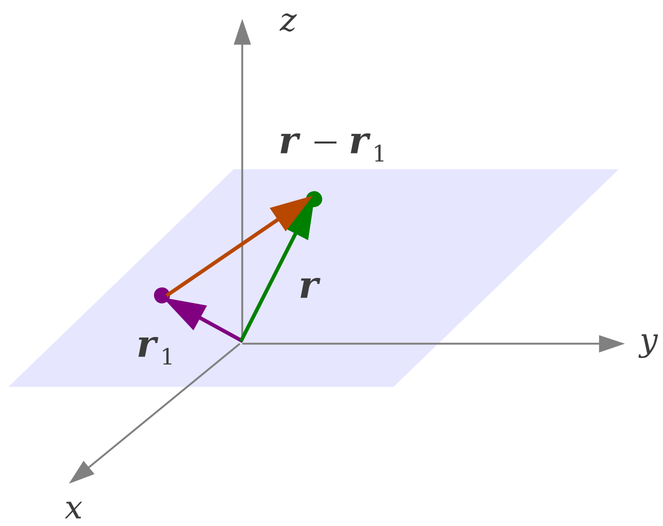
Plane Equations (2)



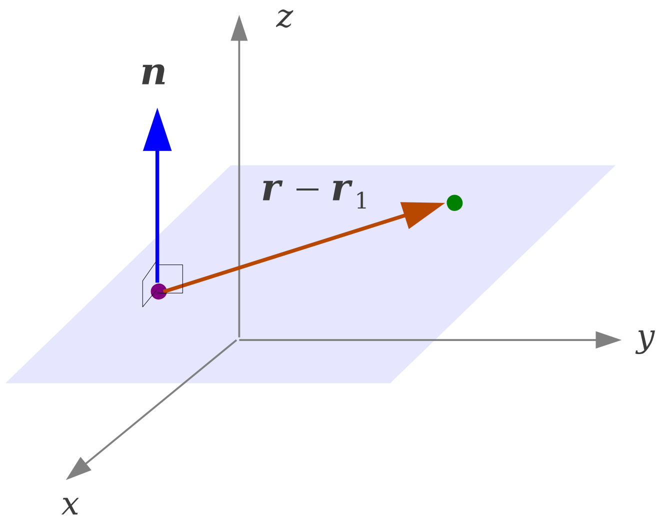


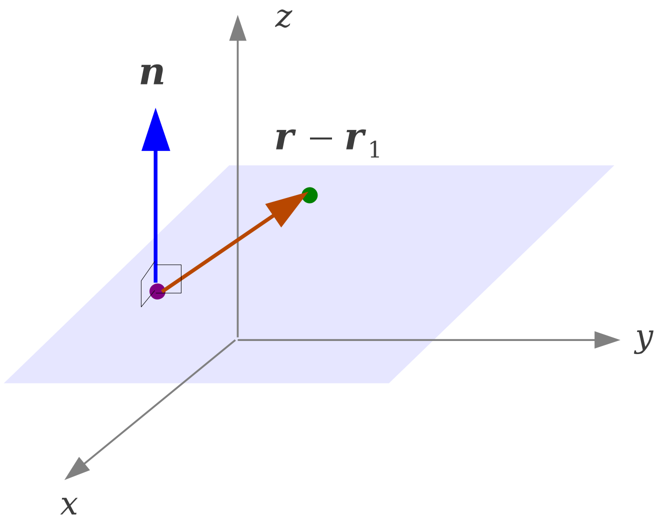
No Parameter

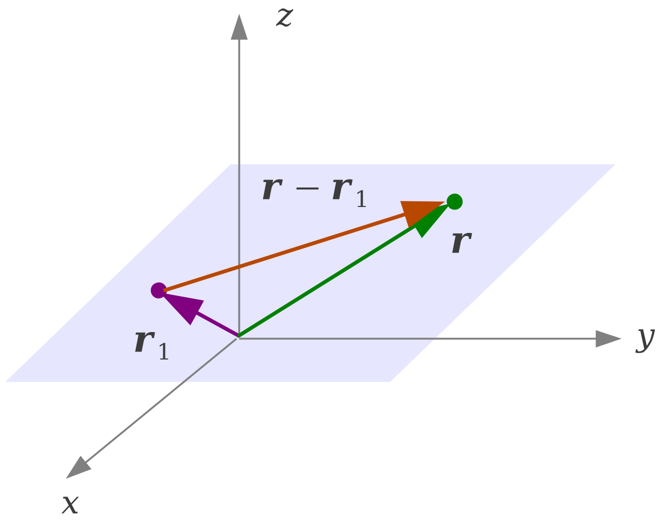




Plane Equations (3)







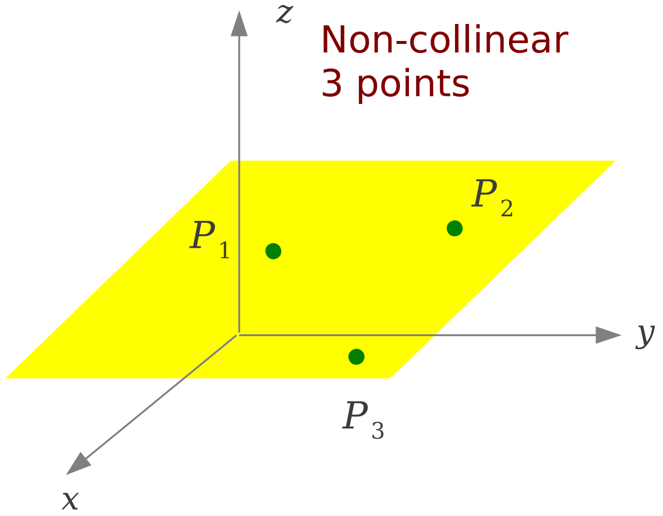
$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_1) = 0$$

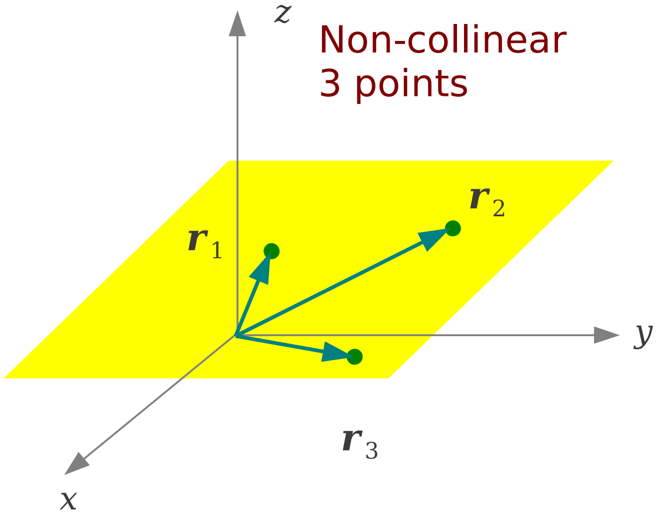
$$\mathbf{r} = \langle x, y, z \rangle$$

$$\mathbf{r}_1 = \langle x_1, y_1, z_1 \rangle$$

$$\mathbf{n} = \langle a, b, c \rangle$$

Normal Vector & 3 Points



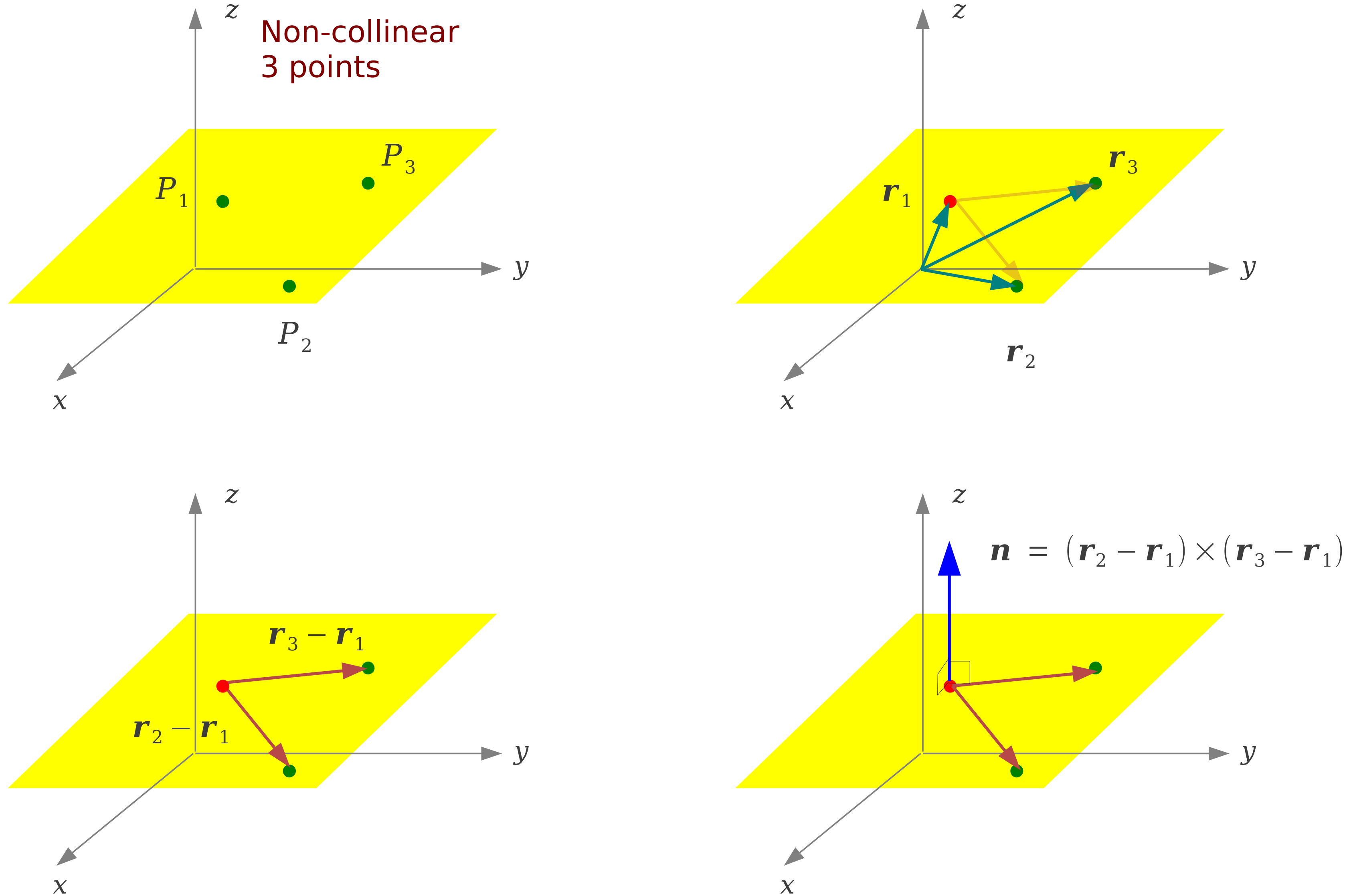


Graph of a plane

Line intersection of two planes

Point of intersection of a line and plane

Normal Vector & 3 Points



Vector Triple Product

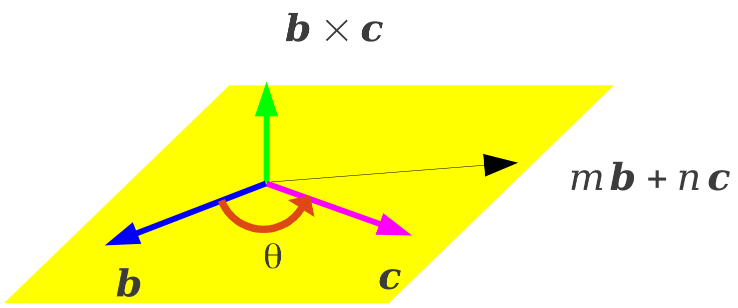
$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c})$$

⇨ Perpendicular to $\boldsymbol{b} \times \boldsymbol{c}$

⇨ Any vector perpendicular to $\boldsymbol{b} \times \boldsymbol{c}$ lies in the plane perpendicular to $\boldsymbol{b} \times \boldsymbol{c}$

⇨ lies in the plane of $\boldsymbol{b}$ and $\boldsymbol{c}$

Perpendicular to the plane of



$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) \Rightarrow m\boldsymbol{b} + n\boldsymbol{c}$$

$$(\boldsymbol{a} \cdot \boldsymbol{c})\boldsymbol{b} - (\boldsymbol{a} \cdot \boldsymbol{b})\boldsymbol{c}$$

Scalar Triple Product

$$\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c})$$

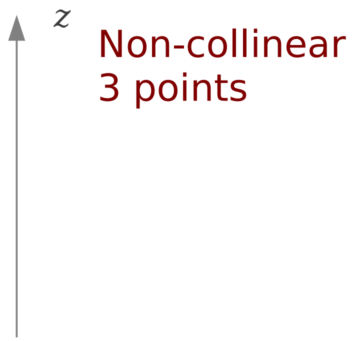
$$\boldsymbol{a} = a_1 \boldsymbol{i} + a_2 \boldsymbol{j} + a_3 \boldsymbol{k}$$
$$\boldsymbol{b} = b_1 \boldsymbol{i} + b_2 \boldsymbol{j} + b_3 \boldsymbol{k}$$
$$\boldsymbol{c} = c_1 \boldsymbol{i} + c_2 \boldsymbol{j} + c_3 \boldsymbol{k}$$

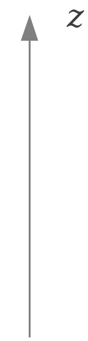
$$\boldsymbol{b} \times \boldsymbol{c} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \boldsymbol{i} \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - \boldsymbol{j} \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + \boldsymbol{k} \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$$\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c}) = (a_1 \boldsymbol{i} + a_2 \boldsymbol{j} + a_3 \boldsymbol{k}) \cdot \left(\boldsymbol{i} \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - \boldsymbol{j} \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + \boldsymbol{k} \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix} \right)$$

$$= \left(a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix} \right) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Normal Vector & 3 Points

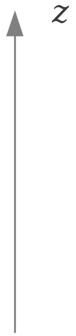




$$\boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \boldsymbol{i} \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \boldsymbol{j} \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \boldsymbol{k} \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

Normal Vector & 3 Points

Non-collinear
3 points



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[3] M.L. Boas, “Mathematical Methods in the Physical Sciences”
[4] D.G. Zill, “Advanced Engineering Mathematics”