

Fourier (1A)

- Fourier Series
- Discrete Time Fourier Transform
- Continuous Time Fourier Transform
- Discrete Time Fourier Transform

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Fourier Series

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{+\pi} f(x) dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \cos nx dx \quad n = 1, 2, \dots$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \sin nx dx \quad n = 1, 2, \dots$$

Trigonometric Orthogonality

$$\int_{-\pi}^{+\pi} \cos nx \cos mx \, dx = 0 \quad (n \neq m)$$

$$\int_{-\pi}^{+\pi} \sin nx \sin mx \, dx = 0 \quad (n \neq m)$$

$$\int_{-\pi}^{+\pi} \sin nx \cos mx \, dx = 0$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{+\pi} f(x) \, dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \cos nx \, dx \quad n = 1, 2, \dots$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \sin nx \, dx \quad n = 1, 2, \dots$$

Any Period $p = 2L$

$$g(v) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nv + b_n \sin nv)$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{+\pi} g(v) dv$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} g(v) \cos nv dv \quad n = 1, 2, \dots$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} g(v) \sin nv dv \quad n = 1, 2, \dots$$

$$v = kx$$

$$v: 2\pi \quad x: 2L$$

$$k = \pi/L$$

$$v = \frac{\pi}{L} x$$

$$dv = \frac{\pi}{L} dx$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{L} x + b_n \sin \frac{n\pi}{L} x \right)$$

$$a_0 = \frac{1}{2L} \int_{-L}^{+L} f(x) dx$$

$$a_n = \frac{1}{L} \int_{-L}^{+L} f(x) \cos \frac{n\pi x}{L} dx \quad n = 1, 2, \dots$$

$$b_n = \frac{1}{L} \int_{-L}^{+L} f(x) \sin \frac{n\pi x}{L} dx \quad n = 1, 2, \dots$$

Time and Frequency

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{L} x + b_n \sin \frac{n\pi}{L} x \right)$$

$$a_0 = \frac{1}{2L} \int_{-L}^{+L} f(x) dx$$

$$a_n = \frac{1}{L} \int_{-L}^{+L} f(x) \cos \frac{n\pi x}{L} dx \quad n = 1, 2, \dots$$

$$b_n = \frac{1}{L} \int_{-L}^{+L} f(x) \sin \frac{n\pi x}{L} dx \quad n = 1, 2, \dots$$

Continuous Time Periodic Signal $x(t)$

$$2L = T$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2\pi n}{T} t + b_n \sin \frac{2\pi n}{T} t \right)$$

$$a_0 = \frac{1}{T} \int_0^T x(t) dt$$

$$a_n = \frac{2}{T} \int_0^T x(t) \cos \frac{2\pi n t}{T} dt \quad n = 1, 2, \dots$$

$$b_n = \frac{2}{T} \int_0^T x(t) \sin \frac{2\pi n t}{T} dt \quad n = 1, 2, \dots$$

Harmonic Frequency

resolution frequency f_0 (the smallest frequency)

n -th harmonic frequency $f_n = n f_0 = n \frac{1}{T}$

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(n2\pi f_0 t) + b_n \sin(n2\pi f_0 t))$$

$$a_0 = \frac{1}{T} \int_0^T x(t) dt$$

$$a_n = \frac{2}{T} \int_0^T x(t) \cos(n2\pi f_0 t) dt \quad n = 1, 2, \dots$$

$$b_n = \frac{2}{T} \int_0^T x(t) \sin(n2\pi f_0 t) dt \quad n = 1, 2, \dots$$

Radial Frequency

linear frequency f
angular (radial) frequency $\omega = 2\pi f$

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t))$$

$$a_0 = \frac{1}{T} \int_0^T x(t) dt$$

$$a_n = \frac{2}{T} \int_0^T x(t) \cos(n\omega_0 t) dt \quad n = 1, 2, \dots$$

$$b_n = \frac{2}{T} \int_0^T x(t) \sin(n\omega_0 t) dt \quad n = 1, 2, \dots$$

Euler Equation (1)

$$e^{j\omega t} = \cos\omega t + j\sin\omega t$$

$$e^{-j\omega t} = \cos\omega t - j\sin\omega t$$

$$\cos\omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$\sin\omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$\begin{aligned} & a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \\ &= a_n \frac{1}{2} (e^{jn\omega_0 t} + e^{-jn\omega_0 t}) + b_n \frac{1}{2j} (e^{jn\omega_0 t} - e^{-jn\omega_0 t}) \\ &= \frac{1}{2} (a_n - jb_n) e^{jn\omega_0 t} + \frac{1}{2} (a_n + jb_n) e^{-jn\omega_0 t} \\ &= \frac{1}{2} (a_n - jb_n) e^{jn\omega_0 t} + \frac{1}{2} (a_n + jb_n) e^{-jn\omega_0 t} \\ &= A_n e^{jn\omega_0 t} + B_n e^{-jn\omega_0 t} \end{aligned}$$

$$x(t) = A_0 + \sum_{n=1}^{\infty} (A_n e^{jn\omega_0 t} + B_n e^{-jn\omega_0 t})$$

Euler Equation (2)

$$a_n = \frac{2}{T} \int_0^T x(t) \cos(n\omega_0 t) dt$$

$$b_n = \frac{2}{T} \int_0^T x(t) \sin(n\omega_0 t) dt$$

$$A_n = \frac{1}{2} (a_n - jb_n)$$

$$B_n = \frac{1}{2} (a_n + jb_n)$$

$$A_n = \frac{1}{T} \int_0^T x(t) (\cos(n\omega_0 t) - j \sin(n\omega_0 t)) dt$$

$$B_n = \frac{1}{T} \int_0^T x(t) (\cos(n\omega_0 t) + j \sin(n\omega_0 t)) dt$$

$$A_n = \frac{1}{T} \int_0^T x(t) e^{-jn\omega_0 t} dt$$

$$B_n = \frac{1}{T} \int_0^T x(t) e^{+jn\omega_0 t} dt$$

$$x(t) = A_0 + \sum_{n=1}^{\infty} (A_n e^{+jn\omega_0 t} + B_n e^{-jn\omega_0 t}) = \sum_{n=0}^{\infty} (A_n e^{+jn\omega_0 t} + B_n e^{-jn\omega_0 t})$$

Complex Fourier Series

$$\begin{aligned}x(t) &= A_0 + \sum_{n=1}^{\infty} (A_n e^{+jn\omega_0 t} + B_n e^{-jn\omega_0 t}) \\ &= \sum_{n=0}^{\infty} (A_n e^{+jn\omega_0 t} + B_n e^{-jn\omega_0 t})\end{aligned}$$

$$A_n = \frac{1}{T} \int_0^T x(t) e^{-jn\omega_0 t} dt$$

$$B_n = \frac{1}{T} \int_0^T x(t) e^{+jn\omega_0 t} dt$$

$$x(t) = \sum_{n=0}^{\infty} C_n e^{+jn\omega_0 t}$$

$$C_n = \frac{1}{T} \int_0^T x(t) e^{-jn\omega_0 t} dt$$

References

- [1] <http://en.wikipedia.org/>
- [2] ...