Fourier (1A)

- Fourier Series
- Discrete Time Fourier Transform
- Continuous Time Fourier Transform
- Discrete Time Fourier Transform

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Fourier Series

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{+\pi} f(x)\, dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \cos nx\, dx \qquad n = 1, 2, \ldots$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \sin nx\, dx \qquad n = 1, 2, \ldots$$

Trigonometric Orthogonality

$$\int_{-\pi}^{+\pi} \cos nx \cos mx \; dx = 0 \qquad (n \neq m)$$

$$\int_{-\pi}^{+\pi} \sin nx \sin mx \; dx = 0 \qquad (n \neq m)$$

$$\int_{-\pi}^{+\pi} \sin nx \cos mx \; dx = 0$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{+\pi} f(x) \; dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \cos nx \; dx \qquad n = 1, 2, \ldots$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \sin nx \; dx \qquad n = 1, 2, \ldots$$

Any Period $p = 2L$

$$g(v) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nv + b_n \sin nv)$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{+\pi} g(v)\, dv$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} g(v) \cos nv\, dv \qquad n = 1, 2, \ldots$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} g(v) \sin nv\, dv \qquad n = 1, 2, \ldots$$

$$v = kx$$

$$v: 2\pi \qquad x: 2L$$

$$k = \pi / L$$

$$v = \frac{\pi}{L} x$$

$$dv = \frac{\pi}{L} dx$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{L} x + b_n \sin \frac{n\pi}{L} x \right)$$

$$a_0 = \frac{1}{2L} \int_{-L}^{+L} f(x)\, dx$$

$$a_n = \frac{1}{L} \int_{-L}^{+L} f(x) \cos \frac{n\pi x}{L}\, dx \qquad n = 1, 2, \ldots$$

$$b_n = \frac{1}{L} \int_{-L}^{+L} f(x) \sin \frac{n\pi x}{L}\, dx \qquad n = 1, 2, \ldots$$

Time and Frequency

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{L} x + b_n \sin \frac{n\pi}{L} x \right)$$

$$a_0 = \frac{1}{2L} \int_{-L}^{+L} f(x)\, dx$$

$$a_n = \frac{1}{L} \int_{-L}^{+L} f(x) \cos \frac{n\pi x}{L}\, dx \qquad n = 1,\ 2,\ \ldots$$

$$b_n = \frac{1}{L} \int_{-L}^{+L} f(x) \sin \frac{n\pi x}{L}\, dx \qquad n = 1,\ 2,\ \ldots$$

Continuous Time Periodic Signal $x(t)$

$$2L = T$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2\pi n}{T} t + b_n \sin \frac{2\pi n}{T} t \right)$$

$$a_0 = \frac{1}{T} \int_{0}^{T} x(t)\, dt$$

$$a_n = \frac{2}{T} \int_{0}^{T} x(t) \cos \frac{2\pi n t}{T}\, dt \qquad n = 1,\ 2,\ \ldots$$

$$b_n = \frac{2}{T} \int_{0}^{T} x(t) \sin \frac{2\pi n t}{T}\, dt \qquad n = 1,\ 2,\ \ldots$$

Harmonic Frquency

$$\textit{resolution frequency} \quad f_0 \quad (\textit{the smallest frequency})$$

$$n-th \ \textit{harmonic frequency} \quad f_n = n f_0 = n \frac{1}{T}$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(n 2\pi f_0 t) + b_n \sin(n 2\pi f_0 t) \right)$$

$$a_0 = \frac{1}{T} \int_0^T x(t) \, dt$$

$$a_n = \frac{2}{T} \int_0^T x(t) \cos(n 2\pi f_0 t) \, dt \qquad n = 1, 2, \ldots$$

$$b_n = \frac{2}{T} \int_0^T x(t) \sin(n 2\pi f_0 t) \, dt \qquad n = 1, 2, \ldots$$

Radial Frequency

$$\begin{aligned} \textit{lienar} \quad \textit{frequency} \quad & f \\ \textit{angluar}(\textit{radial}) \quad \textit{frequency} \quad & \omega = 2\pi f \end{aligned}$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \right)$$

$$a_0 = \frac{1}{T} \int_0^T x(t)\, dt$$

$$a_n = \frac{2}{T} \int_0^T x(t) \cos(n\omega_0 t)\, dt \qquad n = 1, 2, \ldots$$

$$b_n = \frac{2}{T} \int_0^T x(t) \sin(n\omega_0 t)\, dt \qquad n = 1, 2, \ldots$$

Euler Equation (1)

$$e^{j\omega t} = \cos\omega t + j\sin\omega t$$

$$e^{-j\omega t} = \cos\omega t - j\sin\omega t$$

$$\cos\omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$\sin\omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

$$= a_n \frac{1}{2}\left(e^{jn\omega_0 t} + e^{-jn\omega_0 t}\right) + b_n \frac{1}{2j}\left(e^{jn\omega_0 t} - e^{-jn\omega_0 t}\right)$$

$$= \frac{1}{2}(a_n - jb_n)\, e^{jn\omega_0 t} + \frac{1}{2}(a_n + jb_n)\, e^{-jn\omega_0 t}$$

$$= \frac{1}{2}(a_n - jb_n)\, e^{jn\omega_0 t} + \frac{1}{2}(a_n + jb_n)\, e^{-jn\omega_0 t}$$

$$= A_n\, e^{jn\omega_0 t} + B_n\, e^{-jn\omega_0 t}$$

$$x(t) = A_0 + \sum_{n=1}^{\infty}\left(A_n\, e^{jn\omega_0 t} + B_n\, e^{-jn\omega_0 t}\right)$$

Euler Equation (2)

$$a_n = \frac{2}{T}\int_0^T x(t)\cos(n\omega_0 t)\,dt$$

$$b_n = \frac{2}{T}\int_0^T x(t)\sin(n\omega_0 t)\,dt$$

$$A_n = \frac{1}{2}(a_n - j b_n)$$

$$B_n = \frac{1}{2}(a_n + j b_n)$$

$$A_n = \frac{1}{T}\int_0^T x(t)\left(\cos(n\omega_0 t) - j\sin(n\omega_0 t)\right)dt$$

$$B_n = \frac{1}{T}\int_0^T x(t)\left(\cos(n\omega_0 t) + j\sin(n\omega_0 t)\right)dt$$

$$A_n = \frac{1}{T}\int_0^T x(t)\, e^{-jn\omega_0 t}\,dt$$

$$B_n = \frac{1}{T}\int_0^T x(t)\, e^{+jn\omega_0 t}\,dt$$

$$x(t) = A_0 + \sum_{n=1}^{\infty}\left(A_n e^{+jn\omega_0 t} + B_n e^{-jn\omega_0 t}\right) = \sum_{n=0}^{\infty}\left(A_n e^{+jn\omega_0 t} + B_n e^{-jn\omega_0 t}\right)$$

Complex Fourier Series

$$x(t) = A_0 + \sum_{n=1}^{\infty} \left(A_n e^{+jn\omega_0 t} + B_n e^{-jn\omega_0 t} \right)$$

$$= \sum_{n=0}^{\infty} \left(A_n e^{+jn\omega_0 t} + B_n e^{-jn\omega_0 t} \right)$$

$$A_n = \frac{1}{T} \int_0^T x(t)\, e^{-jn\omega_0 t}\, dt$$

$$B_n = \frac{1}{T} \int_0^T x(t)\, e^{+jn\omega_0 t}\, dt$$

$$x(t) = \sum_{n=0}^{\infty} C_n e^{+jn\omega_0 t}$$

$$C_n = \frac{1}{T} \int_0^T x(t)\, e^{-jn\omega_0 t}\, dt$$

References

[1] http://en.wikipedia.org/
[2] ...