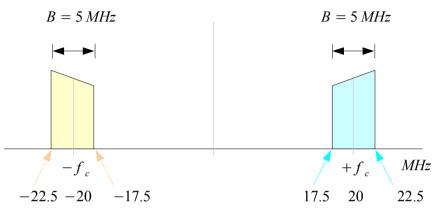
Bandpass Sampling (2B)

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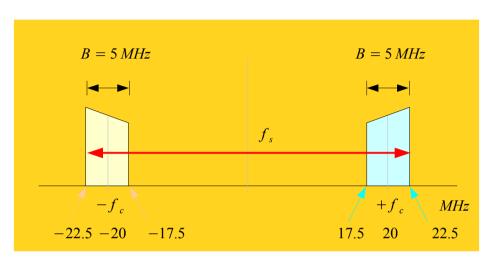
Band-limited Signal



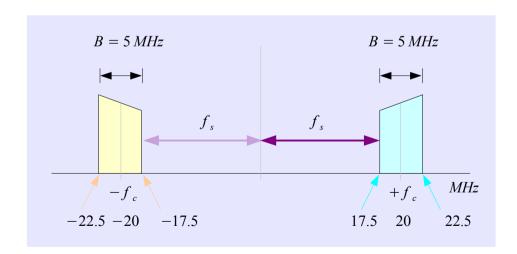




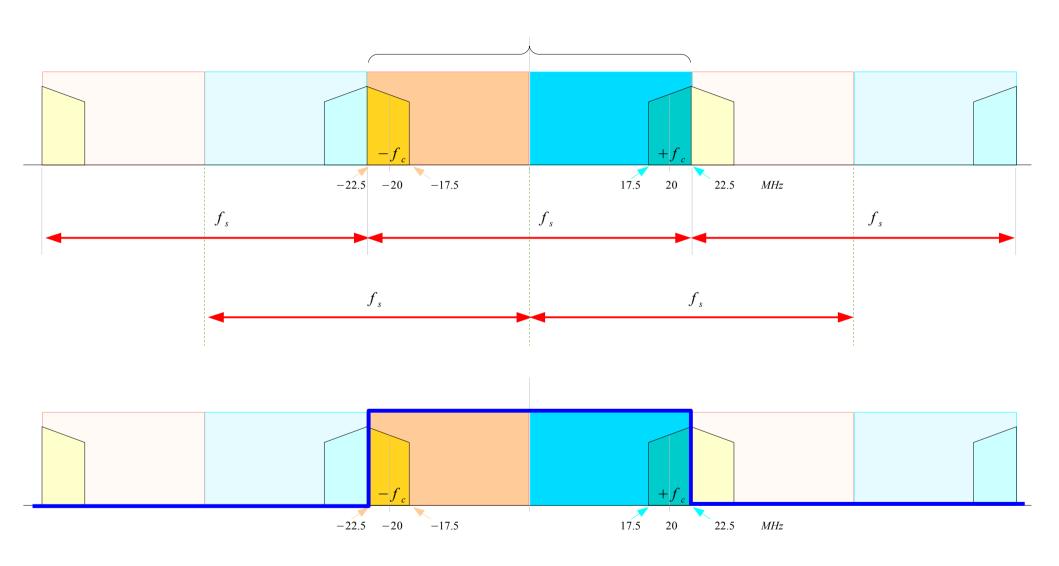
- IF filtering
- Harmonic Sampling
- Sub-Nyquist Sampling



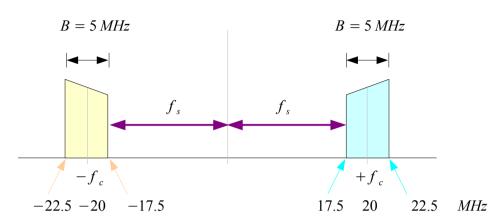
Lowpass Sampling

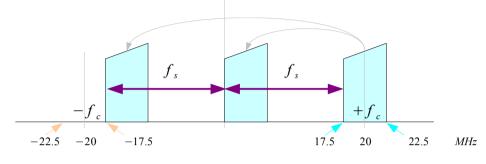


Low-pass Signal Sampling



Band-pass Signal Sampling

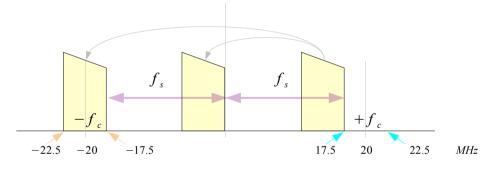


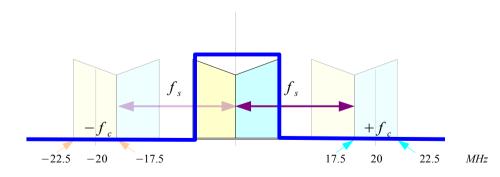




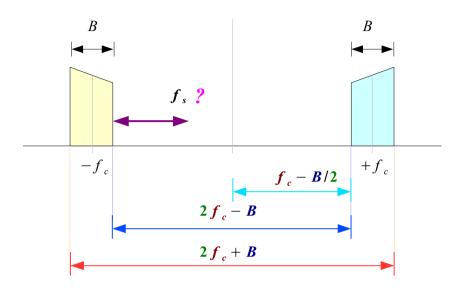


- IF filtering
- Harmonic Sampling
- Sub-Nyquist Sampling





Sampling Frequency f_s (1)



- Bandpass Sampling
- IF filtering
- Harmonic Sampling
- Sub-Nyquist Sampling

Assume there are m multiples of f_s

$$2f_c - B = m \cdot f_s$$

l

Given an integer m

 $Max f_s$ condition

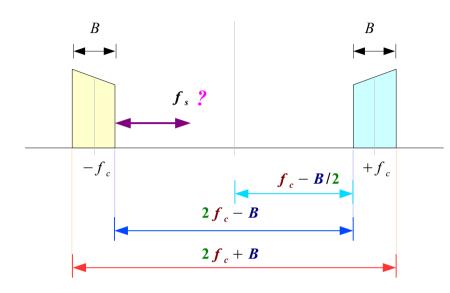
 f_s can be decreased according to the following condition without introducing aliasing problems

$$2f_c + B = (m+1) \cdot f_s$$



 $Min f_s$ condition

Sampling Frequency f_s (2)



When
$$m = 6$$

$$max f_s = \frac{2 f_c - B}{6}$$

$$min f_s = \frac{2 f_c + B}{7}$$

$$\min f_{s} \quad \frac{2f_{c} + B}{7} \leq f_{s} \leq \frac{2f_{c} - B}{6} \quad \max f_{s}$$

Assume there are $\frac{m}{s}$ multiples of $\frac{f}{s}$

$$2f_c - B = m \cdot f_s$$

Given an integer m

 $Max f_s$ condition

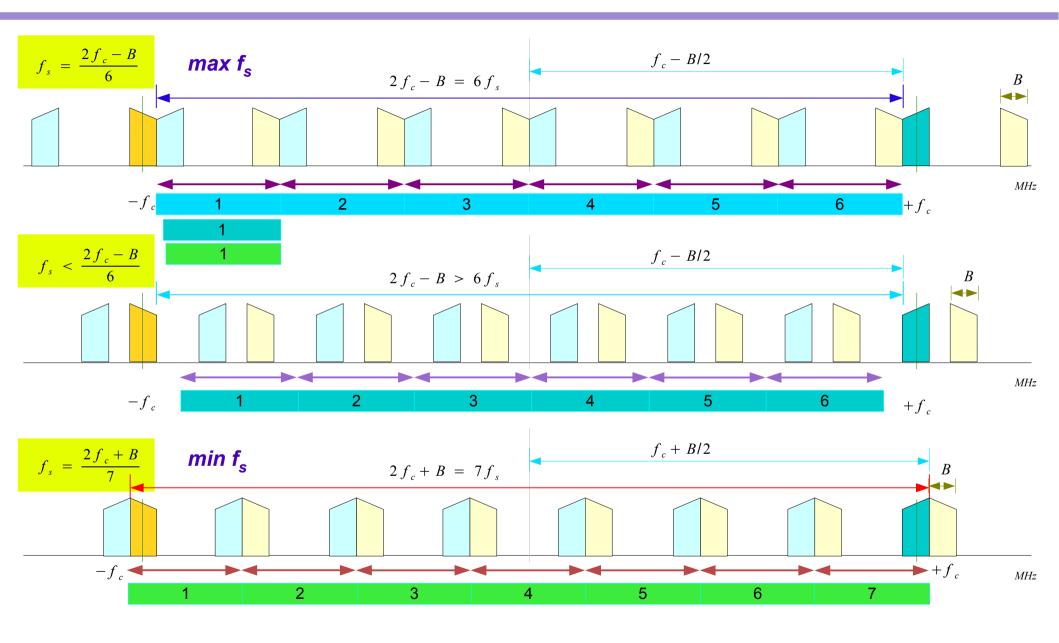
 f_s can be decreased according to the following condition without introducing aliasing problems

$$2f_c + B = (m+1) \cdot f_s$$

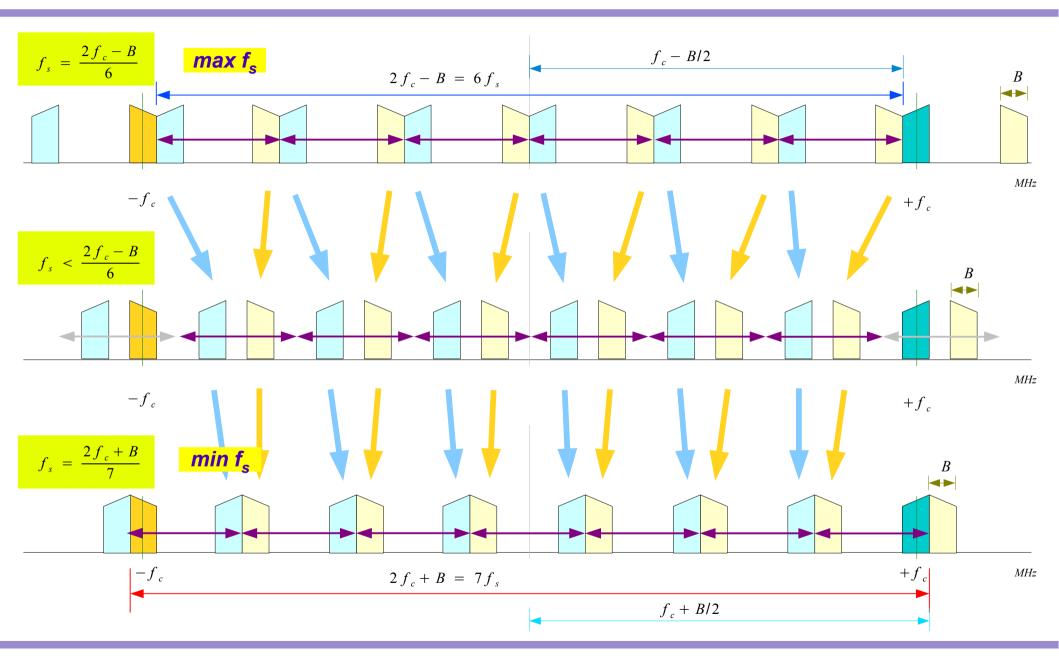


Min f_s condition

Sampling Frequency f_s (3)



Sampling Frequency f_s (4)



Range of f_s (1)

$$\frac{2f_c + B}{m+1} \leq f_s \leq \frac{2f_c - B}{m}$$

Nyquist Criterion

$$2B \leq f_s$$

$$f_c = 20 \, MHz$$

$$B = 5 MHz$$



max f_s

Optimum Sampling Frequency

$$m=1$$

$$m = 1$$
 \longrightarrow $\frac{2 \cdot 20 + 5}{1 + 1} = 22.5 \le f_s \le \frac{2 \cdot 20 - 5}{1} = 35$ \longrightarrow $f_s = 22.5 \, MHz \quad (10 \le f_s)$

$$\frac{20-5}{1} = 35$$

$$f_s = 22$$

$$(10 \leq f_s)$$

$$m=2$$

$$\frac{2 \cdot 20 + 5}{2 + 1} = 15$$

$$f_s \leq \frac{2 \cdot 20 - 5}{2} =$$

$$m = 2$$
 \Rightarrow $\frac{2 \cdot 20 + 5}{2 + 1} = 15$ $\leq f_s \leq \frac{2 \cdot 20 - 5}{2} = 17.5$ \Rightarrow $f_s = 17.5 MHz$ $(10 \leq f_s)$

$$m=3$$

$$\frac{2 \cdot 20 + 5}{3 + 1} = 11.25 \le f_s$$

$$\frac{2 \cdot 20 - 5}{3} = 11.67$$

$$m = 3$$
 \Rightarrow $\frac{2 \cdot 20 + 5}{3 + 1} = 11.25 \le f_s \le \frac{2 \cdot 20 - 5}{3} = 11.67$ \Rightarrow $f_s = 11.25 \, MHz \, (10 \le f_s)$

$$m=4$$

$$\frac{2 \cdot 20 + 5}{4 + 1} = 9$$

$$m = 4$$
 $\Rightarrow \frac{2 \cdot 20 + 5}{4 + 1} = 9$ $\geq \frac{2 \cdot 20 - 5}{4} = 8.75$ \Rightarrow X



$$f_c = 20 MHz$$

$$m=5$$

$$\frac{2 \cdot 20 + 5}{5 + 1} = 7.5$$

$$m = 5$$
 $\Rightarrow \frac{2 \cdot 20 + 5}{5 + 1} = 7.5$ $\geq \frac{2 \cdot 20 - 5}{5} = 7.0$ \Rightarrow X

$$B = 5 MHz$$

Range of f_s (2)

$$\frac{2f_c + B}{m+1} \leq f_s \leq \frac{2f_c - B}{m}$$

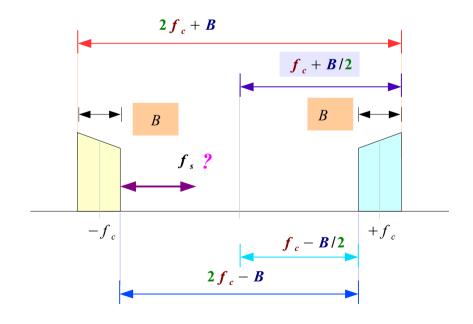
$$\frac{f_c + B/2}{B} = R$$
highest signal frequency
bandwidth B

$$\frac{2 f_c + B}{(m+1)B} = f(m,R)$$

$$\frac{minimum \ sampling \ rate}{bandwidth \ B}$$

$$\frac{2(f_c + B/2)}{(m+1)B} = \frac{2R}{m+1} = f(m,R)$$

$$m = 1$$
 $f(1,R) = R$ $m = 5$ $f(5,R) = \frac{1}{3}R$
 $m = 2$ $f(2,R) = \frac{2}{3}R$ $m = 6$ $f(6,R) = \frac{2}{7}R$
 $m = 3$ $f(3,R) = \frac{1}{2}R$ $m = 7$ $f(7,R) = \frac{1}{4}R$
 $m = 4$ $f(4,R) = \frac{2}{5}R$ $m = 8$ $f(8,R) = \frac{2}{9}R$



Range of f_s (3)

$$\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}$$

$$\frac{f_c + B/2}{B} = R$$



highest signal frequency

bandwidth B

$$\frac{2 f_c + B}{(m+1)B} = f(m,R)$$

minimum sampling rate

bandwidth B

$$\frac{2 f_c + B}{(m+1)B} = f(m,R) = \frac{2R}{m+1}$$

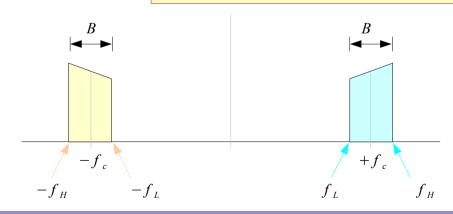
$$m+1=k$$

$$f_H = f_c + B/2$$

$$R = f_H / B$$

$$f_{s,min} = \frac{2f_c + B}{m+1} = \frac{2f_H}{k}$$

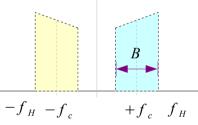
$$f(m,R) = \frac{2f_H}{kB} = \frac{2R}{k}$$



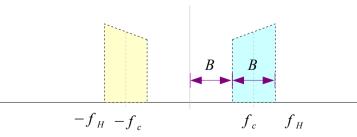
$$k = 1 \quad (m = 0)$$

$$-f_H - f_S + f_S f_H$$

$$k=1 \quad (m=0)$$



$$k = 1 \ (m = 0)$$



$$f_H = f_c + B/2 = 1B$$

$$f_H / B = R = 1$$

$$f_{s,min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = 2B$$

$$f_H = f_c + B/2 = 1.5B$$

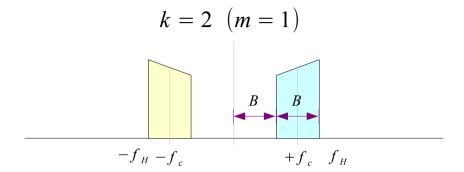
$$f_H / B = R = 1.5$$

$$f_{s,min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = 3B$$

$$f_H = f_c + B/2 = 2B$$

$$f_H / B = R = 2$$

$$f_{s,min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = 4B$$



$$k = 2 \quad (m = 1)$$

$$-f_H - f_C + f_H$$

$$k = 2 \quad (m = 3)$$

$$-f_H - f_c$$

$$f_c f_H$$

$$f_H = f_c + B/2 = 2B$$

$$f_H / B = R = 2$$

$$f_{s,min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = 2B$$

$$f_H = f_c + B/2 = 2.5B$$

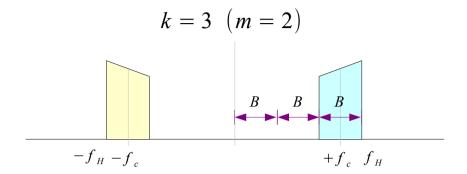
$$f_H / B = R = 2.5$$

$$f_{s,min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = 2.5B$$

$$f_H = f_c + B/2 = 3B$$

$$f_H / B = R = 3$$

$$f_{s,min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = 3B$$



$$k = 3 \quad (m = 2)$$

$$-f_H - f_C$$

$$+f_C f_H$$

$$k = 3 \quad (m = 2)$$

$$-f_H - f_c$$

$$f_c f_H$$

$$f_H = f_c + B/2 = 3B$$

$$f_H / B = R = 3$$

$$f_{s,min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = 2B$$

$$f_H = f_c + B/2 = 3.5 B$$

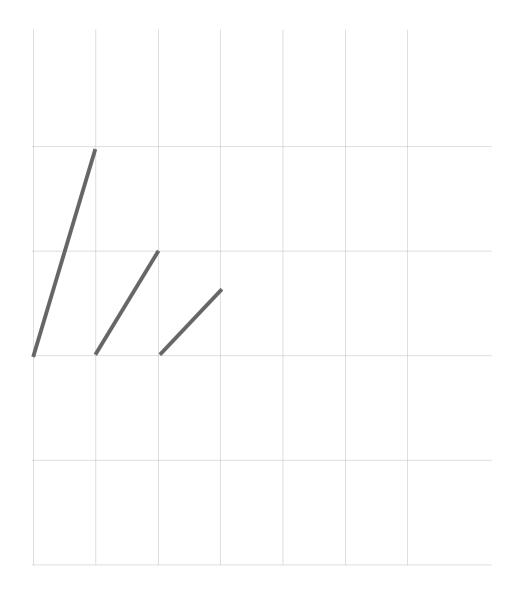
$$f_H / B = R = 3.5$$

$$f_{s,min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = \frac{7}{3} B$$

$$f_H = f_c + B/2 = 4B$$

$$f_H / B = R = 4$$

$$f_{s,min} = \frac{2f_H}{k} = \frac{2f_H}{m+1} = \frac{8}{3}B$$



References

- [1] http://en.wikipedia.org/
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] A "graphical interpretation" of the DFT and FFT, by Steve Mann
- [4] R. G. Lyons, Understanding Digital Signal Processing, 1997