Note: Open intervals

$[a,b]$: closed interv.
end pts $a, b \in [a,b]$

$]a,b[\equiv (a,b)$: open interv.
end pts $a, b \notin]a,b[$
i.e., excluded from $\int$

HW: Runge phenomenon S+M p. 208

1) $I = \int_{-5}^{+5} \frac{1}{1+x^2}\, dx$
using Newton-Cotes for $n = 1, 2, \ldots, 15$.

2) Plot f, f_n, $n = 1, 2, 3, 8, 12$.

3) Plot I_n vs. n and obs. that I_n does not converge as $n \to \infty$.

4) Obs. weights $w_{i,n} := \int_a^b \ell_{i,n}(x)\, dx$ are not all positive for $n \geq 8$. Plot $\ell_{i,n}$ for $\begin{matrix} i = 1,2,\ldots,8 \\ n = 8 \end{matrix}$

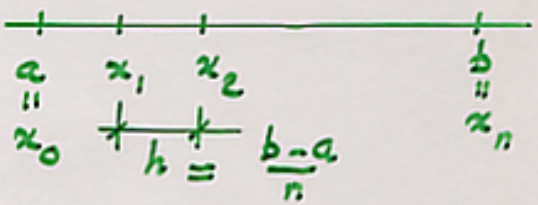
Motivation for Comp. rules:

Runge pheno. $\Rightarrow$ keep n small (i.e., $n = 1, 2$), and subdivide $[a, b]$ into smaller sub interv.

$n = 1$ Trap. simple

$n = 2$ simple Simpson

Error for Comp. Trap:



Trap.

$$E_n^1 := I - I_n$$

$$= \int_a^b f(x)\,dx - h\left[\tfrac{1}{2}f(x_0) + f(x_1) + \cdots + f(x_{n-1}) + \tfrac{1}{2}f(x_n)\right]$$

$$= \sum_{i=1}^{n}\left[\int_{x_{i-1}}^{x_i} f(x)\,dx - \frac{h}{2}\left[f(x_{i-1}) + f(x_i)\right]\right]$$

$$|E_n^1| \leq \frac{h^3}{12} \underbrace{\sum_{i=1}^{n} \left(\max_{\xi \in [x_{i-1}, x_i]} |f^{(2)}(\xi)| \right)}_{\overline{M}_2}$$

M_2

$$\Rightarrow \overline{M}_2 = \max_{\xi \in [a,b]} |f^{(2)}(\xi)|$$

p.13-2

$$\overline{M}_2 \leq n \cdot M_2$$

$$\Rightarrow |E_n^1| \leq \frac{(b-a)^3}{12\, n^3}\, n M_2$$

$$= \frac{(b-a)^3}{12\, n^2} M_2 = \underbrace{\frac{(b-a)h^2}{12} M_2}$$

A. p.253.