

Triple Integrals (7A)

- Triple Integral
- Triple Integrals in Polar Coordinates

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Area and Volume

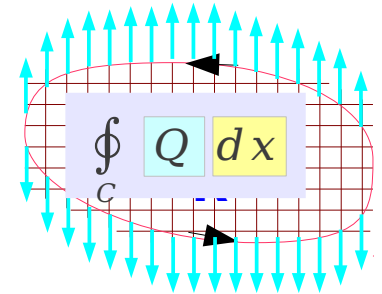
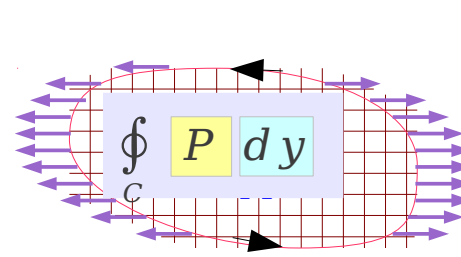
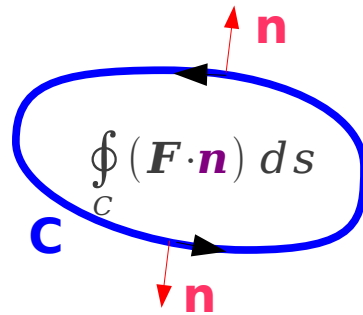
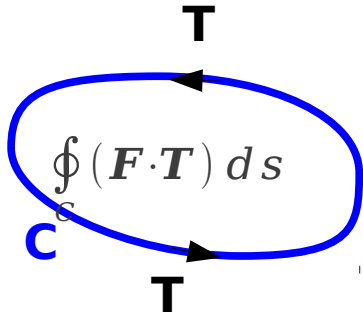
$$A = \iint_R dA$$

$$V = \iint_R f(x, y) dA$$

Vector Form of Green's Theorem – Div

C: a piecewise simple closed curve

bounding by a simply connected region **R**



Line Integral

$$\oint_C (\mathbf{F} \cdot \mathbf{T}) ds = \oint_C P dx + Q dy$$

$$\oint_C (\mathbf{F} \cdot \mathbf{n}) ds = \oint_C P dy - Q dx$$

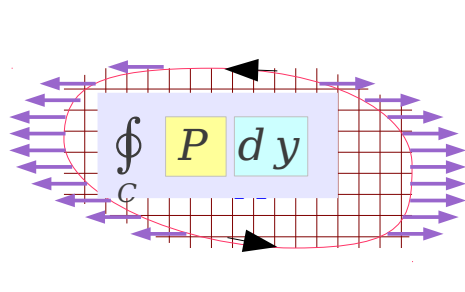
Double Integral

$$= \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \iint_R (\text{curl } \mathbf{F}) \cdot \mathbf{k} dA$$

$$= \iint_R \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} \right) dA = \iint_R (\text{div } \mathbf{F}) dA$$

Divergence Theorem

C: a piecewise simple closed curve
a simply connected region **R**



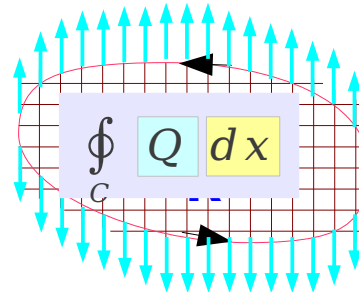
Line Integral

$$\oint_C (\mathbf{F} \cdot \mathbf{n}) \, ds$$

Surface Integral

$$\iint_S (\mathbf{F} \cdot \mathbf{n}) \, dS$$

D: a closed, bounded region
with a piecewise smooth boundary **S**

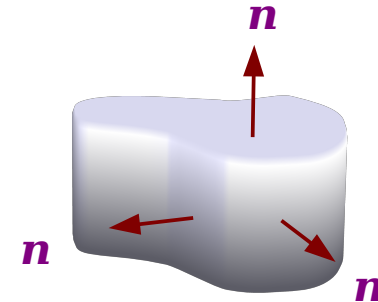


Double Integral

$$= \iint_R \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} \right) dA = \iint_R (\operatorname{div} \mathbf{F}) \, dA$$

Triple Integral

$$= \iiint_D \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \right) dV = \iiint_D (\operatorname{div} \mathbf{F}) \, dS$$



References

- [1] <http://en.wikipedia.org/>
- [2] <http://planetmath.org/>
- [3] M.L. Boas, “Mathematical Methods in the Physical Sciences”
- [4] D.G. Zill, “Advanced Engineering Mathematics”