Correlation (1A)

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Moments of a Random Process

The n-th Moment

$$E[\,X^n(t)\,] = \int_{-\infty}^{+\infty} x^n\, f(x,t)\, dx$$

1st Moment

$$E[\,X(t)\,] = \int_{-\infty}^{+\infty} x\, f(x,t)\, dx = \mu(t)$$

2nd Moment

$$E[\,X^2(t)\,] = \int_{-\infty}^{+\infty} x^2\, f(x,t)\, dx$$

The n-th Central Moment

$$E[\,(X(t)-\mu(t))^n\,] = \int_{-\infty}^{+\infty} (x-\mu(t))^n\, f(x,t)\, dx$$

1st Central Moment

$$E[\,(X(t)-\mu(t))\,] = \int_{-\infty}^{+\infty} (x-\mu(t))\, f(x,t)\, dx$$

2nd Central Moment

$$E[\,(X(t)-\mu(t))^2\,] = \int_{-\infty}^{+\infty} (x-\mu(t))^2\, f(x,t)\, dx$$

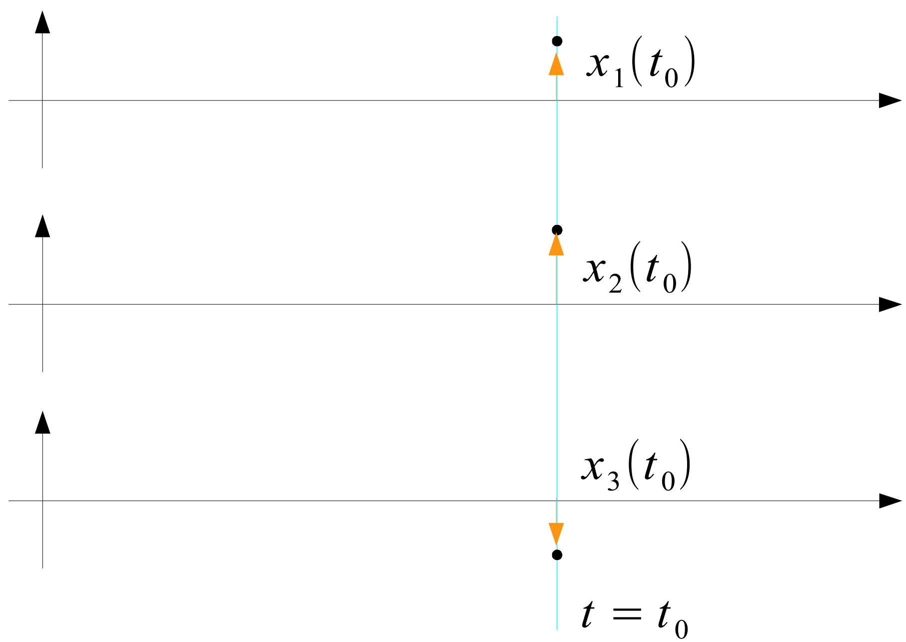
$$= \sigma^2(t)$$

Moments of a Random Process

$X(t)$ Random Variable at a given time t

$x_i(t)$ outcome of i[th] realization at a given time t

Ensemble



$$E[\,X(t_0)\,] = \int_{-\infty}^{+\infty} x\, f(x, t_0)\, dx = \mu(t_0)$$

Ensemble Average

$$\bar{x}(t) = \frac{1}{N} \sum_{n=1}^{N} X_n(t)$$

$N \to \infty$

$$E[\,X(t)\,] = \int_{-\infty}^{+\infty} x\, f(x, t)\, dx = \mu(t)$$

Moments of a Random Process

The n-th Moment

$$E[\,X^n(t)\,] = \int_{-\infty}^{+\infty} x^n\, f(x,t)\, dx$$

$$E[\,X^n(t)\,] = E[\,X^n\,]$$

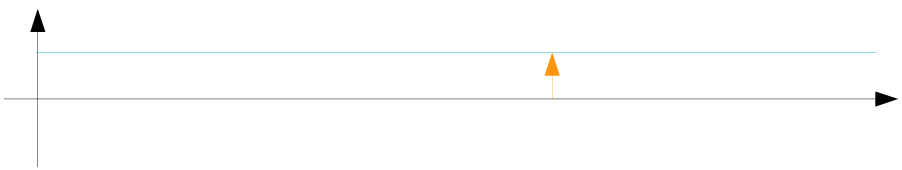
The n-th Central Moment

$$E[\,(X(t)-\mu(t))^n\,] = \int_{-\infty}^{+\infty} (x-\mu(t))^n\, f(x,t)\, dx$$

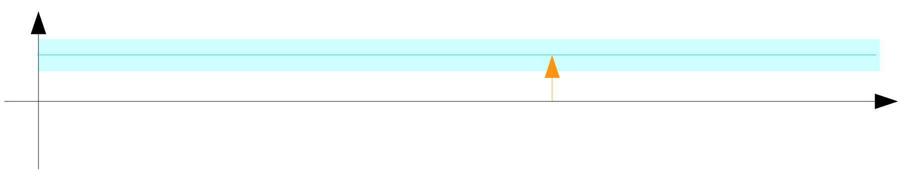
$$E[\,(X(t)-\mu(t))^n\,] = E[\,(X-\mu)^n\,]$$

$$f(x,t) = f(x)$$

*if **f** does not change with time*



$$E[\,X(t)\,] = E[\,X\,] = \mu$$



$$E[\,(X(t)-\mu(t))^2\,] = E[\,(X-\mu)^2\,] = \sigma^2$$

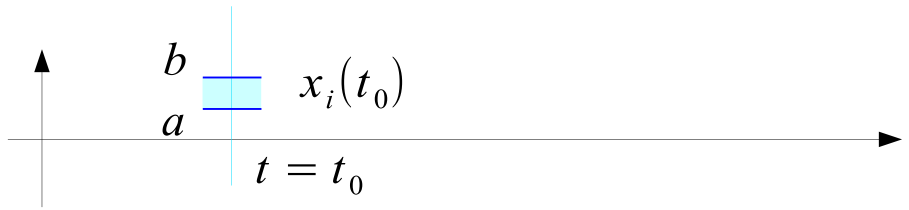
Probability Density Function

Probability Density Function $$f(x,t) = \frac{\partial}{\partial x} F(x,t)$$

$$P[\,a \le X(t) \le b\,] = \int_a^b f(x,t)\,dx$$

Cumulative Distribution Function $$F(x,t) = \int_{-\infty}^{x} f(x,t)\,dx$$

$$P[\,X(t) \le b\,] = \int_{-\infty}^{b} f(x,t)\,dx = F(b,t)$$



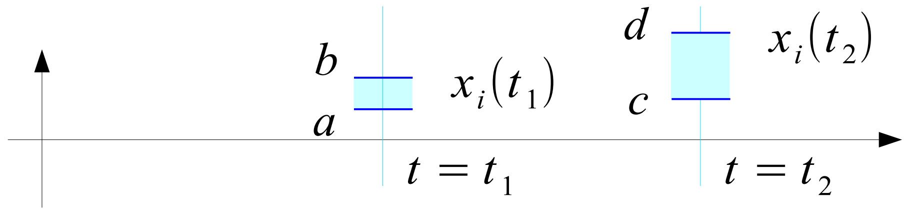
Joint Probability Density Function

Joint Probability Density Function $$f(x_1, t_1;\ x_2, t_2) = \frac{\partial^2}{\partial x_1 \partial x_2} F(x_1, t_1;\ x_2, t_2)$$

$$P\left[\{a \le X(t_1) \le b\} \cap \{c \le X(t_2) \le d\} \right] = \int_a^b \int_c^d f(x_1, t_1;\ x_2, t_2)\, dx_1\, dx_2$$

Joint Cumulative Distribution Function $$F(x_1, t_1;\ x_2, t_2) = \int_{-\infty}^{x_1} \int_{-\infty}^{x_2} f(x_1, t_1;\ x_2, t_2)\, dx_1\, dx_2$$

$$P\left[\{X(t_1) \le b\} \cap \{X(t_2) \le d\} \right] = \int_{-\infty}^{b} \int_{-\infty}^{d} f(x_1, t_1;\ x_2, t_2)\, dx_1\, dx_2 = F(b, t_1;\ d, t_2)$$



Stationarity

Correlation Functions (1)

auto-covariance function

$$C_{xx}(t_1, t_2) = E[\,(\underline{X(t_1) - \mu_x(t_1)})\ (\underline{X(t_2) - \mu_x(t_2)})\,]$$

$$C_{xx}(t_2 - t_1) = E[\,(X(t_1) - \mu_x(t_1))\ (X(t_2) - \mu_x(t_2))\,]$$

$$C_{xx}(\tau) = E[\,(X(t) - \mu_x)\ (X(t+\tau) - \mu_x)\,]$$

auto-correlation function

$$R_{xx}(t_1, t_2) = E[\,\underline{X(t_1)}\ \underline{X(t_2)}\,]$$

$$R_{xx}(\tau) = E[\,X(t)\ X(t+\tau)\,]$$

Correlation Functions (2)

cross-covariance function

$$C_{xy}(t_1, t_2) = E\left[\underline{(X(t_1) - \mu_x(t_1))} \; \underline{(Y(t_2) - \mu_y(t_2))} \right]$$

$$C_{xy}(\tau) = E\left[(X(t) - \mu_x) \; (Y(t+\tau) - \mu_y) \right]$$

cross-correlation function

$$R_{xy}(t_1, t_2) = E\left[\underline{X(t_1)} \; \underline{Y(t_2)} \right]$$

$$R_{xy}(\tau) = E\left[X(t) \, Y(t + \tau) \right]$$

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Ergodicity

$$\mu_x = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t)\, dt$$

$$\hat{\mu}_x = \frac{1}{T} \int_0^T x(t)\, dt = \bar{x}$$

$$\bar{\psi}_x^2 = \frac{1}{T} \int_0^T x^2(t)\, dt = \overline{x^2}$$

$$C_{xy}(\tau) = E\left[(X(t) - \mu_x)\ (Y(t+\tau) - \mu_y) \right]$$

$$C_{xy}(\tau) = \lim_{T \to 0} \frac{1}{T} \int_0^T (x(t) - \mu_x)\ (y(t+\tau) - \mu_y)\, dt$$

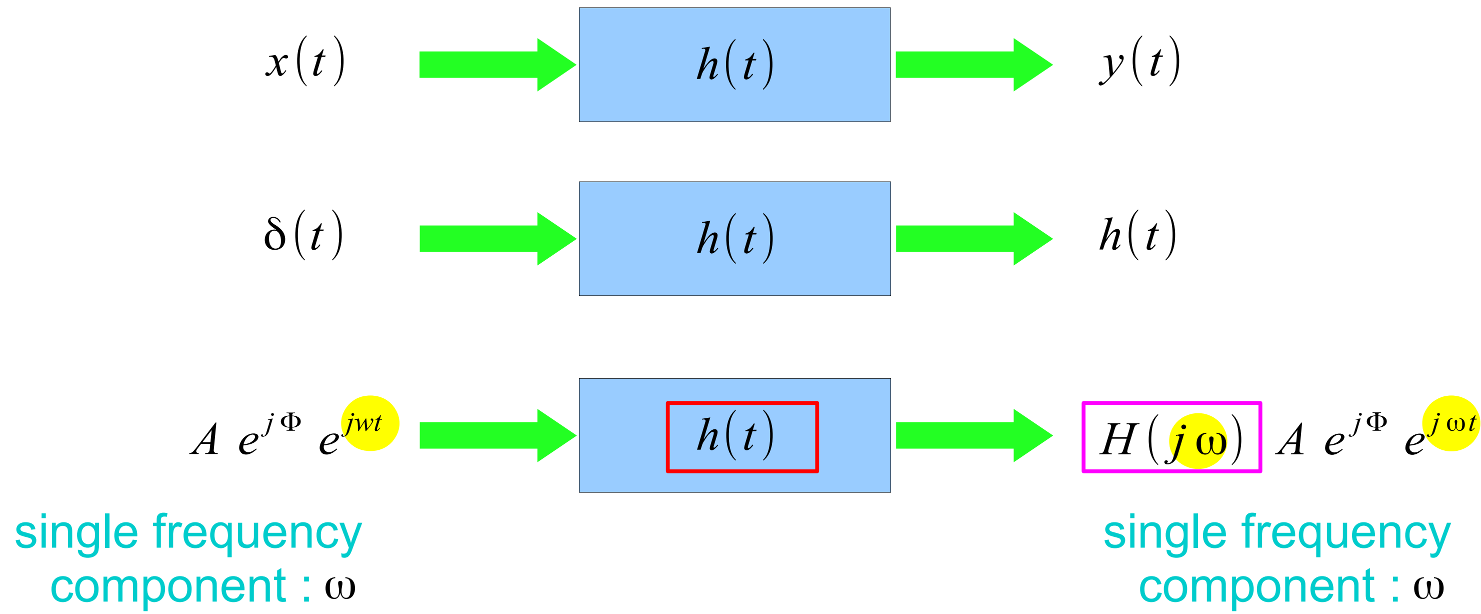
$$\hat{C}_{xy}(\tau) = \frac{1}{T-\tau} \int_0^{T-\tau} (x(t) - \bar{x})\ (y(t+\tau) - \bar{y})\, dt \qquad 0 \le \tau < T$$

$$\hat{C}_{xy}(\tau) = \frac{1}{T-|\tau|} \int_0^{T-|\tau|} (x(t) - \bar{x})\ (y(t+\tau) - \bar{y})\, dt \qquad -T < \tau \le 0$$

Time Average

Frequency Response

$$y(t) = h(t) * x(t) = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$$



$$H(j\omega) = \int_{-\infty}^{+\infty} h(\tau)\, e^{-j\omega\tau}\, d\tau$$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] K.H. Shin, J.K. Hammond, Fundamentals of Signal Processing for Sound and Vibration Engineers, Wiley, 2008