

Correlation (1A)

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Correlation Functions (1)

auto-covariance function

$$C_{xx}(t_1, t_2) = E \left[\underbrace{(X(t_1) - \mu_x(t_1))}_{\text{green}} \underbrace{(X(t_2) - \mu_x(t_2))}_{\text{blue}} \right]$$

$$C_{xx}(t_2 - t_1) = E \left[(X(t_1) - \mu_x(t_1)) (X(t_2) - \mu_x(t_2)) \right]$$

$$C_{xx}(\tau) = E \left[(X(t) - \mu_x) (X(t + \tau) - \mu_x) \right]$$

auto-correlation function

$$R_{xx}(t_1, t_2) = E \left[\underbrace{X(t_1)}_{\text{green}} \underbrace{X(t_2)}_{\text{blue}} \right]$$

$$R_{xx}(\tau) = E \left[X(t) X(t + \tau) \right]$$

Correlation Functions (2)

cross-covariance function

$$C_{xy}(t_1, t_2) = E \left[\underbrace{(X(t_1) - \mu_x(t_1))}_{\text{green}} \underbrace{(Y(t_2) - \mu_y(t_2))}_{\text{blue}} \right]$$

$$C_{xy}(\tau) = E \left[(X(t) - \mu_x) (Y(t + \tau) - \mu_y) \right]$$

cross-correlation function

$$R_{xy}(t_1, t_2) = E \left[\underbrace{X(t_1)}_{\text{green}} \underbrace{Y(t_2)}_{\text{blue}} \right]$$

$$R_{xy}(\tau) = E \left[X(t) Y(t + \tau) \right]$$

Ergodicity

$$\mu_x = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x(t) dt$$

$$\hat{\mu}_x = \frac{1}{T} \int_0^T x(t) dt = \bar{x}$$

$$\bar{\Psi}_x^2 = \frac{1}{T} \int_0^T x^2(t) dt = \bar{x}^2$$

$$C_{xy}(\tau) = E[(X(t) - \mu_x)(Y(t+\tau) - \mu_y)]$$

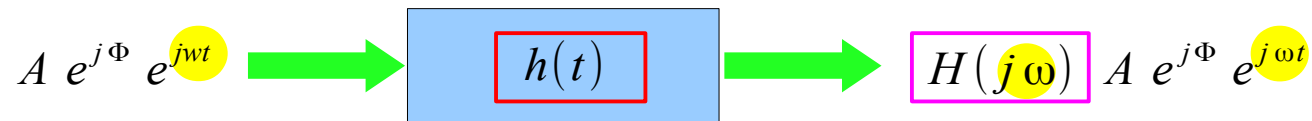
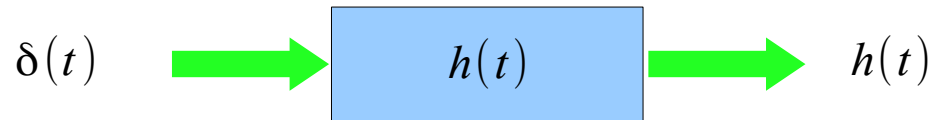
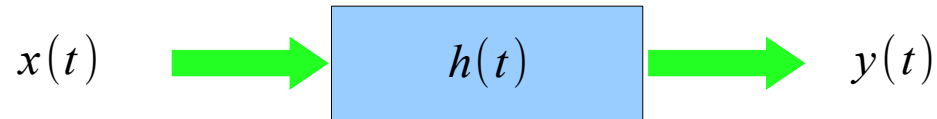
$$C_{xy}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T (x(t) - \mu_x)(y(t+\tau) - \mu_y) dt$$

$$\hat{C}_{xy}(\tau) = \frac{1}{T - \tau} \int_0^{T - \tau} (x(t) - \bar{x})(y(t+\tau) - \bar{y}) dt \quad 0 \leq \tau < T$$

$$\hat{C}_{xy}(\tau) = \frac{1}{T - |\tau|} \int_0^{T - |\tau|} (x(t) - \bar{x})(y(t+\tau) - \bar{y}) dt \quad -T < \tau \leq 0$$

Frequency Response

$$y(t) = h(t) * x(t) = \int_{-\infty}^{+\infty} h(\tau) x(t - \tau) d\tau$$



single frequency
component : ω

single frequency
component : ω

$$H(j\omega) = \int_{-\infty}^{+\infty} h(\tau) e^{-j\omega\tau} d\tau$$

References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] K.H. Shin, J.K. Hammond, Fundamentals of Signal Processing for Sound and Vibration Engineers, Wiley, 2008