Correlation (1A)

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Correlation Functions (1)

auto-covariance function

$$C_{xx}(t_{1,}\ t_2) \ = \ E\big[\ (X(t_1) - \mu_x(t_1))\ \ (X(t_2) - \mu_x(t_2))\ \big]$$

$$C_{xx}(t_2 \ - \ t_1) \ = \ E\big[\ (X(t_1) - \mu_x(t_1))\ \ (X(t_2) - \mu_x(t_2))\ \big]$$

$$C_{xx}(\tau) \ = \ E\big[\ (X(t) - \mu_x)\ \ (X(t+\tau) - \mu_x)\ \big]$$

auto-correlation function

$$R_{xx}(t_{1,}\ t_2) \ = \ E\big[\ X(t_1)\ X(t_2)\ \big]$$

$$R_{xx}(\tau) \ = \ E\big[\ X(t)\ X(t + \tau)\ \big]$$

Correlation Functions (2)

cross-covariance function

$$C_{xy}(t_1, t_2) = E\left[\underline{(X(t_1) - \mu_x(t_1))}\ \underline{(Y(t_2) - \mu_y(t_2))} \right]$$

$$C_{xy}(\tau) = E\left[(X(t) - \mu_x)\ (Y(t+\tau) - \mu_y) \right]$$

cross-correlation function

$$R_{xy}(t_1, t_2) = E\left[\underline{X(t_1)}\ \underline{Y(t_2)} \right]$$

$$R_{xy}(\tau) = E\left[X(t)\, Y(t + \tau) \right]$$

Ergodicity

$$\mu_x = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t)\, dt$$

$$\hat{\mu}_x = \frac{1}{T} \int_0^T x(t)\, dt = \bar{x}$$

$$\bar{\psi}_x^2 = \frac{1}{T} \int_0^T x^2(t)\, dt = \overline{x^2}$$

$$C_{xy}(\tau) = E\left[(X(t) - \mu_x)\ (Y(t+\tau) - \mu_y) \right]$$

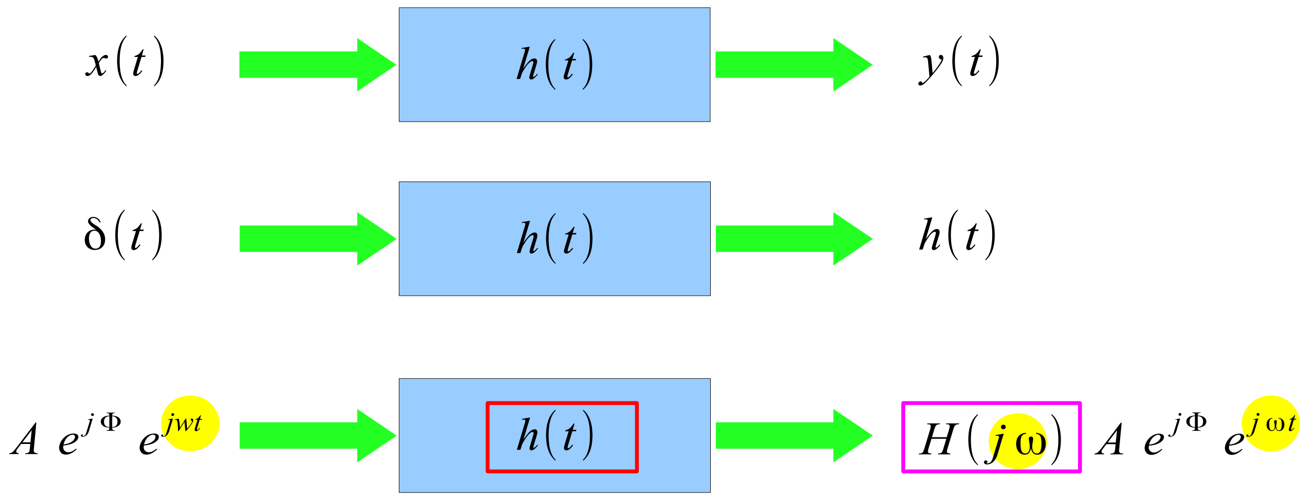
$$C_{xy}(\tau) = \lim_{T \to 0} \frac{1}{T} \int_0^T (x(t) - \mu_x)\ (y(t+\tau) - \mu_y)\, dt$$

$$\hat{C}_{xy}(\tau) = \frac{1}{T-\tau} \int_0^{T-\tau} (x(t) - \bar{x})\ (y(t+\tau) - \bar{y})\, dt \qquad 0 \le \tau < T$$

$$\hat{C}_{xy}(\tau) = \frac{1}{T-|\tau|} \int_0^{T-|\tau|} (x(t) - \bar{x})\ (y(t+\tau) - \bar{y})\, dt \qquad -T < \tau \le 0$$

Frequency Response

$$y(t) = h(t) * x(t) = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$$



single frequency component : ω

single frequency component : ω

$$H(j\omega) = \int_{-\infty}^{+\infty} h(\tau)\, e^{-j\omega\tau}\, d\tau$$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] K.H. Shin, J.K. Hammond, Fundamentals of Signal Processing for Sound and Vibration Engineers, Wiley, 2008