Section 2.2

Problem #3

$(P \Rightarrow Q) \neq (Q \Rightarrow P)$

P	Q	(P⇒Q)	(Q⇒P)
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

If-Then Statements:

Before you can really understand the above problem, you must understand what exactly an if-then statement is. Using P and Q, you can think of If-Then statements as "P implies Q" or in other words, "If P happens, then Q will happen."

So let's look at the above problem. We'll focus first on (P⇒Q). This is saying, "P implies Q." Think of this in terms of an actual problem. First, let's assign conditions to both P and Q:

P: The traffic light is red.

Q: You will stop your car.

Read using If-Then, this statement would say, "**If** the traffic light is red, **then** you will stop your car." Now let's address the following true-false situations:

P	Q	(P⇒Q)
T	T	T: If the traffic light is red, then you will stop your car.

This is true because you must always stop at red lights, so that you do not break the law.

P	Q	(P⇒Q)
T	F	F: If the traffic light is red, then you will not stop your car.

This is false because you must stop at a red light. If the light is red, and you do not stop your car you are breaking the law.

P	Q	(P⇒Q)
F	T	T: If the traffic light is not red, then you will stop your car.

This is true because you can stop your car for a number of reasons, not just for a red light.

P	Q	(P⇒Q)
F	F	T: If the traffic light is not red, then you will not stop your car.

This is true because if the light is not red, that means it is green or yellow, in which case you can keep going.

Now let's examine the opposite situations by reversing P and Q. We will now see why statements of "Q implies P" may be true or false:

P	Q	(Q⇒P)
T	T	T: If you stop your car, then the traffic light is red.

This is true because a reason for stopping your car could be because the traffic light is red.

P	Q	(Q⇒P)
T	F	T: If you stop your car, then the traffic light is not red.

This is true because you could have stopped your car for some other reason than the traffic light being red.

P	Q	(Q⇒P)
F	T	F: If you do not stop your car, then the traffic light is red.

This is false because if you do not stop your car when the light is red, you have broken the law.

P	Q	(Q⇒P)
F	F	T: If you do not stop your car, then the traffic light is not red.

This is true because if you do not stop your car then the traffic light could not have been red.

Looking at these situations and their explanations for why they are true or false, we can see that (P⇒Q) does not equal (Q⇒P). We can see this visually by looking at the truth table and comparing the respective columns:

P	Q	(P⇒Q)		(Q⇒P)
T	T	T		T
T	F	F	≠	T
F	T	T		F
F	F	T		T