

Young Won Lim 12/3/12 Copyright (c) 2012 Young W. Lim.

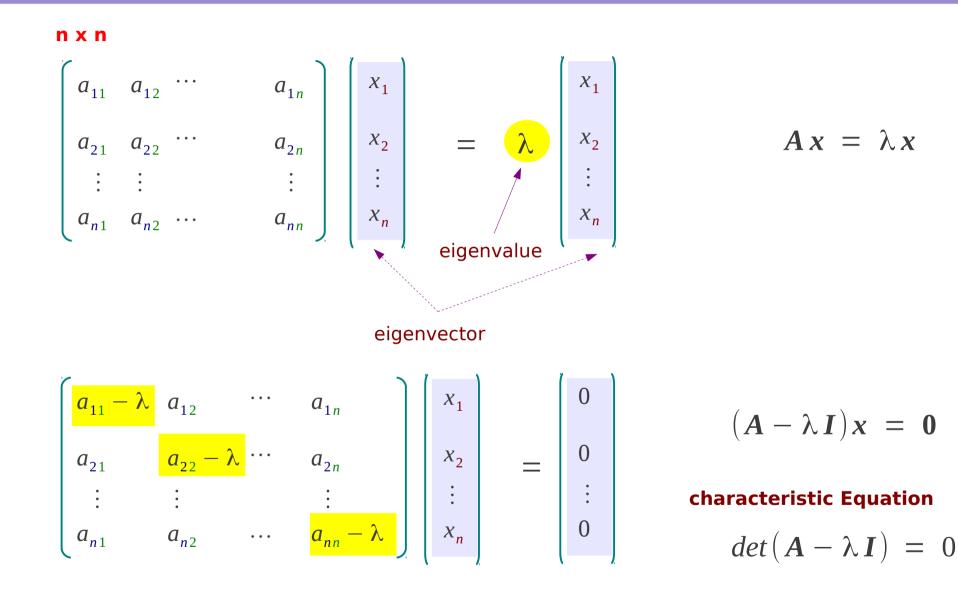
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EigenValues and EigenVectors



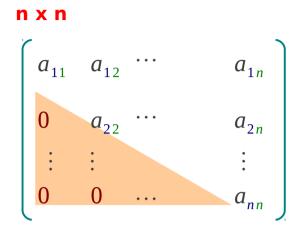
Characteristic Equation

n x n

$$\begin{bmatrix} a_{11} - \lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} - \lambda & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} - \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$
(A - \lambda I)x = 0
characteristic Equation
det (A - \lambda I) = 0
$$\begin{bmatrix} \lambda - a_{22} & -a_{12} & \cdots & -a_{1n} \\ -a_{21} & \lambda - a_{22} & \cdots & -a_{2n} \\ \vdots & \vdots & & \vdots \\ -a_{n1} & -a_{n2} & \cdots & \lambda - a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$
(\lambda I - A)x = 0
characteristic Equation
det (\lambda I - A) = 0
$$det (A - \lambda I) = 0 \\ det (\lambda I - A) = 0 \end{bmatrix}$$

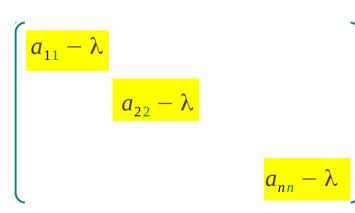
EigenSpaces (5A)

Triangular Matrix

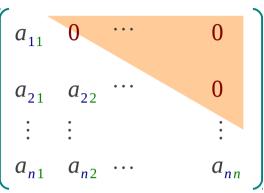




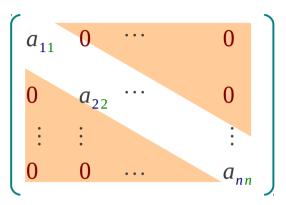
 $A - \lambda I$







n x n



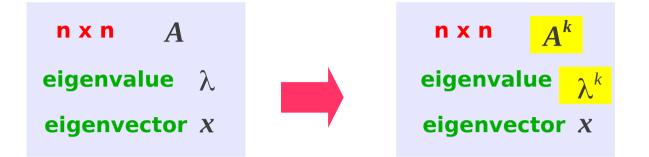
Lower Triangular

Diagonal

characteristic Equation

$$det(\mathbf{A} - \lambda \mathbf{I}) = 0 \quad det(\lambda \mathbf{I} - \mathbf{A}) = 0$$
$$(\lambda - a_{11})(\lambda - a_{22}) \cdots (\lambda - a_{nn}) = 0$$
$$\lambda = a_{11}, \quad \lambda = a_{22}, \cdots, \quad \lambda = a_{nn}$$

Powers of Matrix



$$A^{2}x = A(Ax) = A(\lambda I)x = \lambda Ax = \lambda^{2}x$$

 $A^{2}x = \lambda^{2}x$

EigenValue 0

n x n

$$\begin{bmatrix} a_{11} - \lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} - \lambda & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} - \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$(\boldsymbol{A} - \lambda \boldsymbol{I})\boldsymbol{x} = \boldsymbol{0}$$

characteristic Equation $det(\boldsymbol{A} - \lambda \boldsymbol{I}) = 0$

$$det(\lambda I - A) = \lambda^{n} + c_{1}\lambda^{n-1} + \cdots + c_{n-1}\lambda + c_{n}$$

$$c_{n} = 0 \iff \lambda = 0 \iff det(-A) = c_{n} \iff \text{Non-invertible } A$$

$$(-1)^{n}det(A) = c_{n}$$

$$det(A) = 0$$

A nxn Matrix A (1)

- 1. A is invertible
- 2. **Ax = 0** has only the **trivial** solution
- 3. The $RREF(A) = I_n$
- 4. A can be written as a product of elementary matrix
- 5. Ax = b is consistent for every n x 1 b
- 6. **Ax** = **b** has exactly one solution for every n x 1 **b**

7. **det(A**) ≠ 0

- 8. The column vectors are linearly independent
- 9. The row vectors are linearly independent
- 10. The column vectors span Rⁿ
- 11. The row vectors span Rⁿ
- 12. The column vectors form a basis for Rⁿ
- 13. The row vectors form a basis for Rⁿ
- 14. rank(**A**) = n
- 15. **nullity**(**A**) = 0
- 16. The orthogonal complement of the null space is \mathbb{R}^n
- 17. The orthogonal complement of the row space is **{0**}

A nxn Matrix A (2)

- 18. The range of T_A is R^n
- 19. T_A is one-to-one
- 20. λ =0 is not the eigenvalue of **A**

Diagonalizable

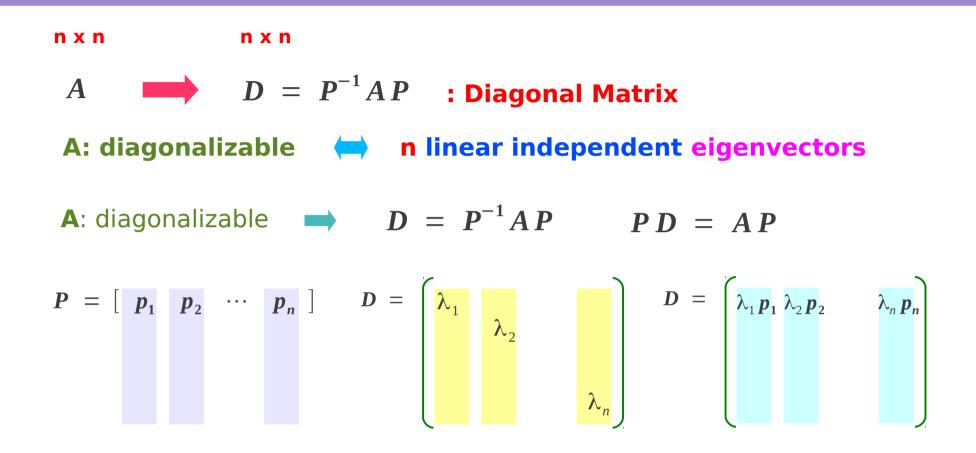
nxn nxn

 $A \qquad \Longrightarrow \qquad B = P^{-1}AP$

$$det(\mathbf{B}) = det(\mathbf{P}^{-1}\mathbf{A}\mathbf{P}) = det(\mathbf{P}^{-1})det(\mathbf{A})det(\mathbf{P})$$
$$= \frac{1}{det(\mathbf{P})}det(\mathbf{A})det(\mathbf{P}) = det(\mathbf{A})$$

$$rank(\mathbf{B}) = rank(\mathbf{A})$$
$$nullity(\mathbf{B}) = nullity(\mathbf{A})$$
$$(\lambda \mathbf{I} - \mathbf{A}) = 0 \qquad (\lambda \mathbf{I} - \mathbf{B}) = 0$$

Similarity Transform



References

- [1] http://en.wikipedia.org/
- [2] Anton, et al., Elementary Linear Algebra, 10th ed, Wiley, 2011
- [3] Anton, et al., Contemporary Linear Algebra,