3.4.6

$$f(x) = x^3\sqrt{x} = x^3 \cdot x^{\frac{1}{2}} = x^{7/2}$$

$$f'(x) = \frac{7}{2} \cdot x^{5/2}$$

$$\tilde{E}_n = -\frac{h^2}{12}(1-0)$$

$f'(b)$ $f'(a)$

Trap. Error estimate

$$f''(x) = \frac{7}{2} \cdot \frac{5}{2} \cdot x^{3/2}$$

$$f'''(x) = \frac{35}{4} \cdot \frac{3}{2} \cdot x^{1/2}$$

$$f''''(x) = \frac{105}{8} \cdot \frac{1}{2} x^{-1/2} = \frac{105}{16} \cdot \frac{1}{\sqrt{x}}$$

$f''''(x)$ as $x \to 0$ $\frac{1}{\sqrt{x}} \to \infty$

Singular

Can't use 5.1.17

$$E_n = -\frac{h^4}{180}(b-a) f^4(\eta)$$

$$\eta \in [a, b]$$

$$f'''(0) = 0$$
$$f'''(1) = \frac{105}{8}$$

Simpson's Using $\tilde{E}_n = -\frac{h^4}{180}\left(f^3(b) - f^3(a)\right)$

$$\int_0^1 x^{7/2}\,dx = 2\frac{x^{9/2}}{9}\Big|_0^1 = \frac{2}{9} = 0.2222\ldots$$

Although $f^{(4)}(x)$ is singular @ $x=0$ based on the $\tilde{E}_n$ Approx error should be smaller than actual but actual limit ratio rate stay 4 for Trap. & 16 for Simp.
See E_n vs $\tilde{E}_n$ in table HW5-4