p. 36-3 (cont'd)

3) Back to superposition:

$$\underbrace{q_{ij}}_{\text{true shear flow}} = q + \tilde{q}_{ij}$$

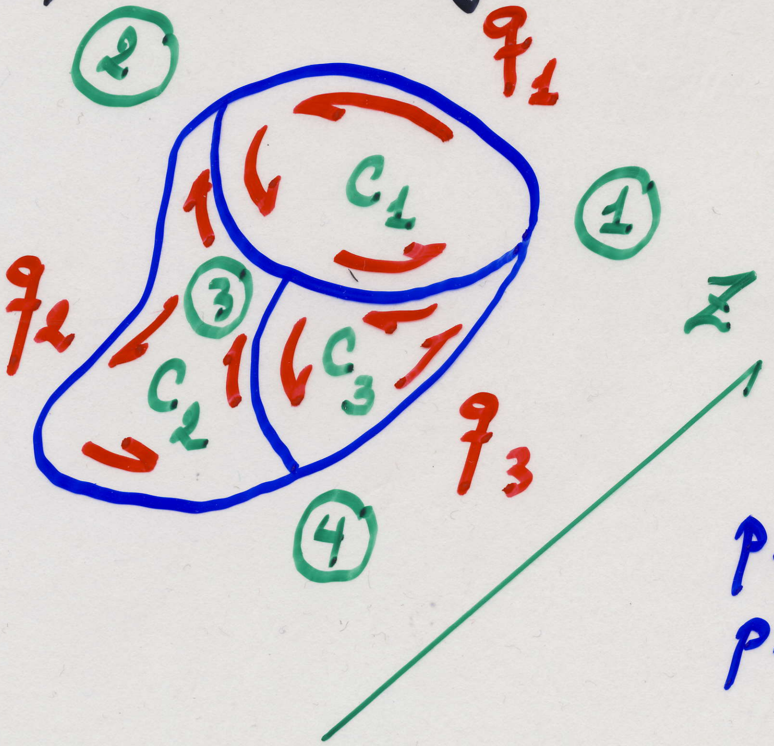
q ← closed cell const. shear flow

$\tilde{q}_{ij}$ ← open cell piecewise const. shear flow

(end algo)

M) Multi-cell:

M.1) w/o stringers:



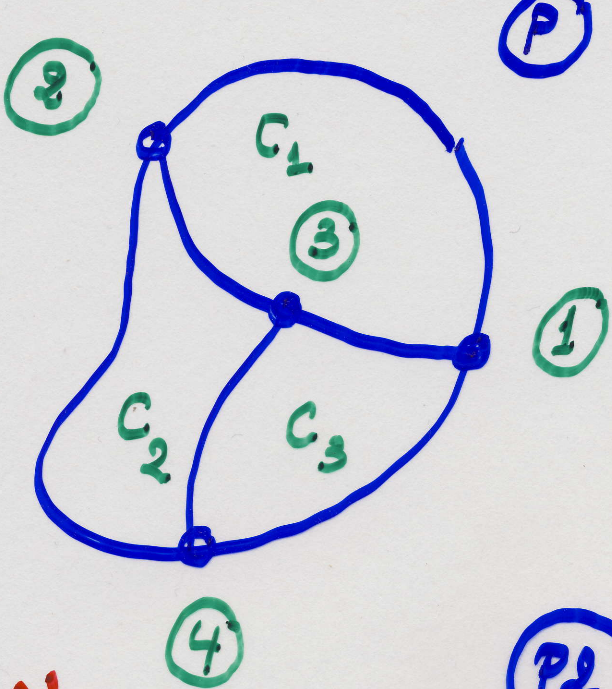
$$R^z = \sum_{i=1}^{ncells} R_i^z$$

ncells = numb. of cells

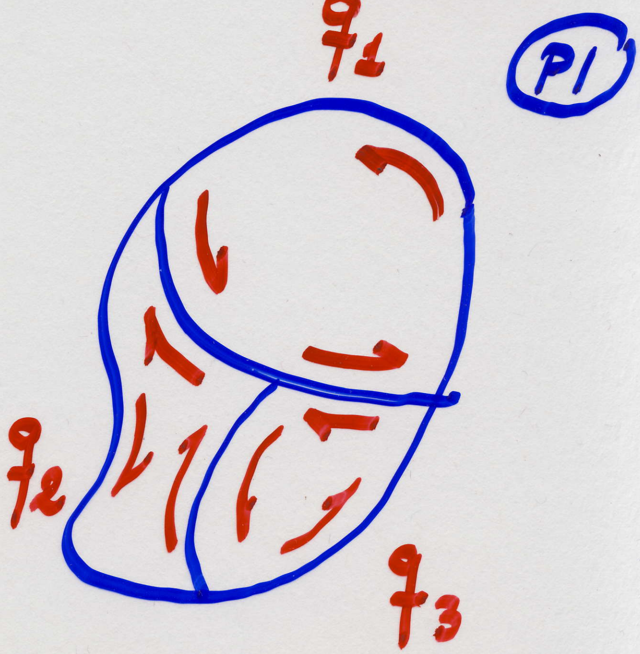
p. 12-2
p. 36-1 } $R_i^z = 0$

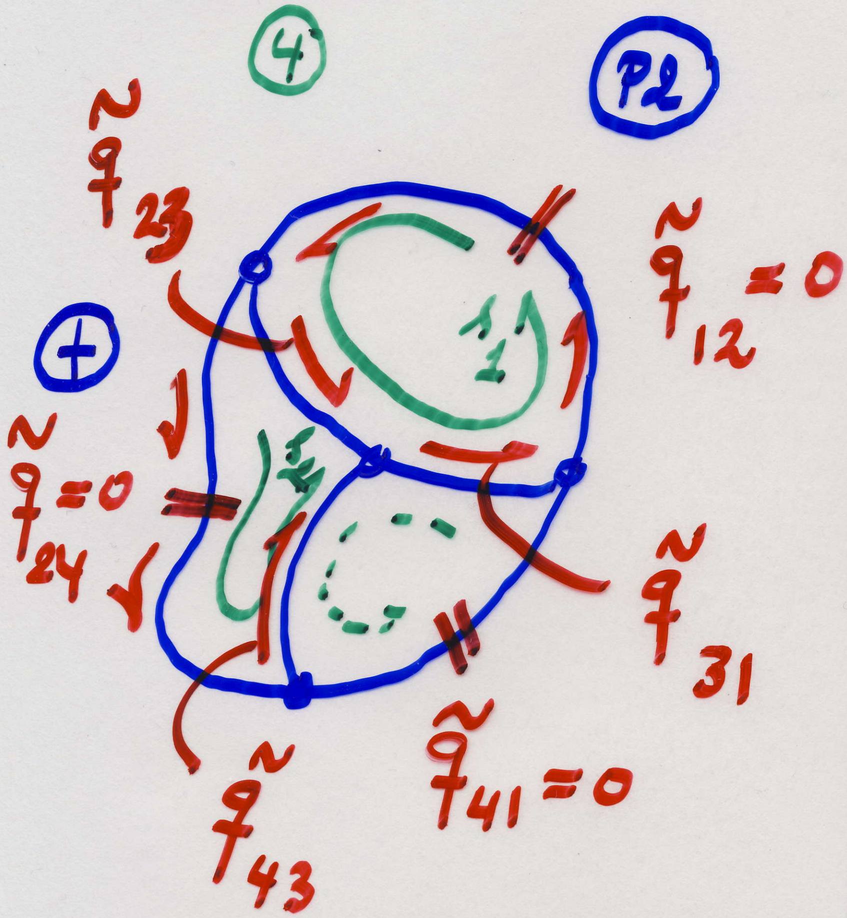
$$\boxed{R^z = 0}$$

M.2) w/ stringers



=





$$R^{Z} = R^{Z1} + R^{Z2}$$

result. of P

result. of P1

result. of P2

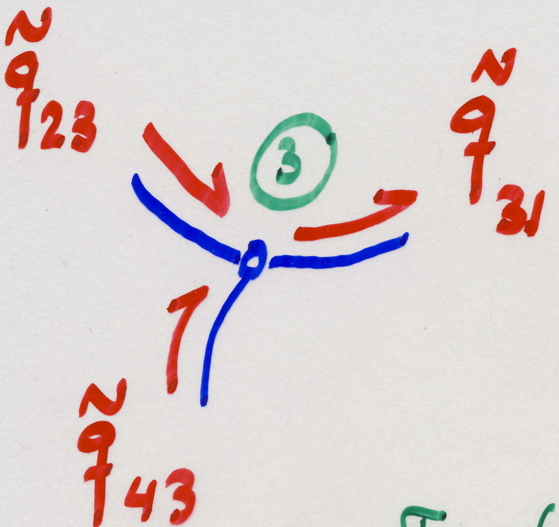
Recall $R^{Z1} = \sum_i \underbrace{R_i^{Z1}}_{=0} = 0$

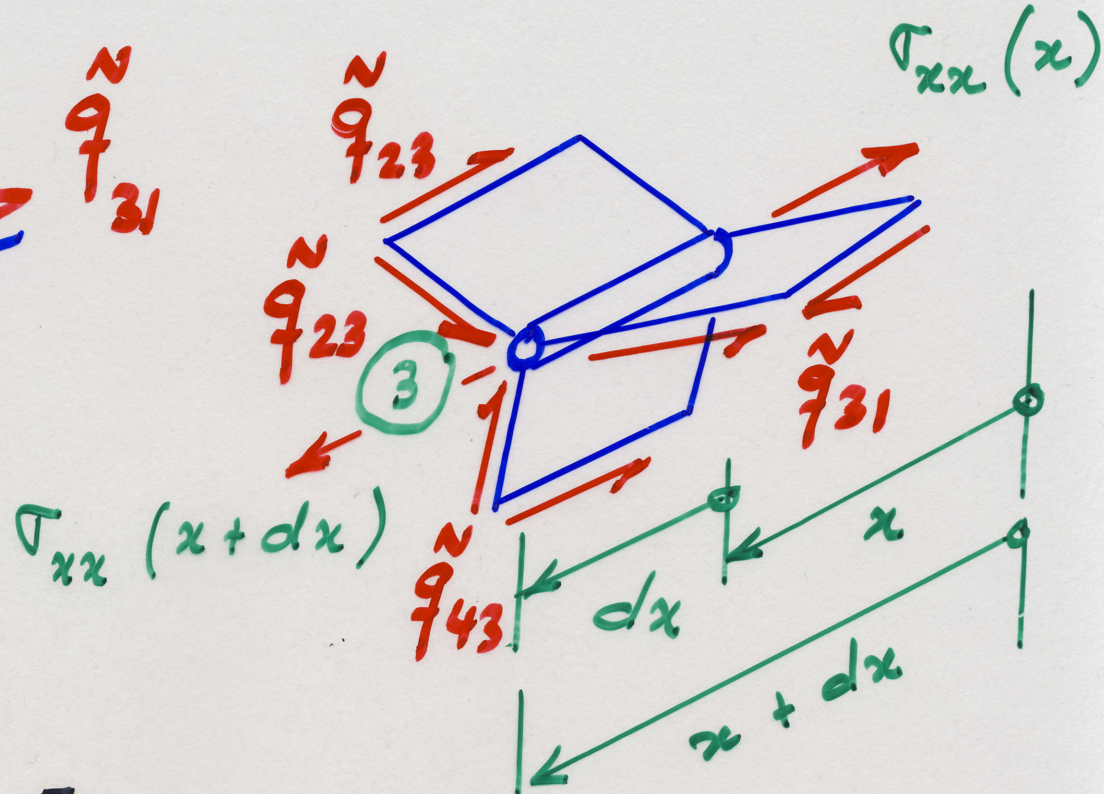
$$\Rightarrow \boxed{R^{Z} = R^{Z2} = V_{Z}} \neq 0$$

Mtg 38: Mon, 24 Nov 08. EAS 4200C

Solving P2: Equil of each stringer (Euler cut princ)

stringer 3





$$\sum F_x = 0 = \int_{A_3} [\underbrace{\sigma_{xx}(x+dx) - \sigma_{xx}(x)}] dA_3$$

$$\left[-\tilde{q}_{23} - \tilde{q}_{43} + \tilde{q}_{31} \right] dx$$

Taylor Series: $\frac{d\sigma_{xx}}{dx} dx + h.o.t.$

$$\boxed{\tilde{q}_{31} = \tilde{q}_{23} + \tilde{q}_{43} + q^{(3)}}$$

$$\boxed{q^{(3)} = -\int_{A_3} \frac{d\sigma_{xx}}{dx}\, dA_3}$$

stringer 3

Contrib. to shear flow by stringer 3.

Recall $V_y = \dfrac{dM_z}{dx}$, $V_z = \dfrac{dM_y}{dx}$

(1)

p. 34-3:

$$\boxed{\begin{aligned} q^{(3)} &= -\left(k_y V_y - k_{yz} V_z\right) Q_z^{(3)} \\ &\quad -\left(k_z V_z - k_{yz} V_y\right) Q_y^{(3)} \\ Q_z^{(3)} &= \int_{A_3} y \, dA_3 , \quad Q_y^{(3)} = \text{HW 7} \end{aligned}}$$

<u>string 2</u>:

$$\tilde{q}_{23} = \underbrace{\cancel{\tilde{q}_{12}}^{\,0}}_{\text{flow in}} - \underbrace{\cancel{\tilde{q}_{24}}^{\,0}}_{\text{flow out}} + q^{(2)}$$

because of cuts

$q^{(2)}$ comp. as in (1)

$$Q_z^{(2)} = y_2 \, A_2$$

area of stringer 2

y coord of stringer 2

$$Q_y^{(2)} = \text{HW7}$$

Stringer 4: $\tilde{q}_{43} = \cancel{\tilde{q}_{24}}^{\,0} - \cancel{\tilde{q}_{41}}^{\,0} + q^{(4)}$

$q^{(4)}$ comp. as above for $q^{(2)}$.

Hence deduce $\tilde{q}_{31}$ (p. 38-1).

Superposition (again)

$$q_{ij} = \tilde{q}_{ij} + q_k$$

$$q_{12} = \underbrace{\tilde{q}_{12}}_{\text{known}} + \underbrace{q_1}_{\text{unknown}}$$

$$q_{23} = \tilde{q}_{23} + q_1 - q_2$$

$$q_{31} = \tilde{q}_{31} + q_1 - q_3$$

$$q_{24} = \underbrace{\tilde{q}_{24}}_{\text{known}} + \underbrace{q_2}_{\text{unknown.}}$$

$$q_{43} = \text{HW}$$

$$q_{41} = \text{HW}$$

3 unknowns q_1, q_2, q_3

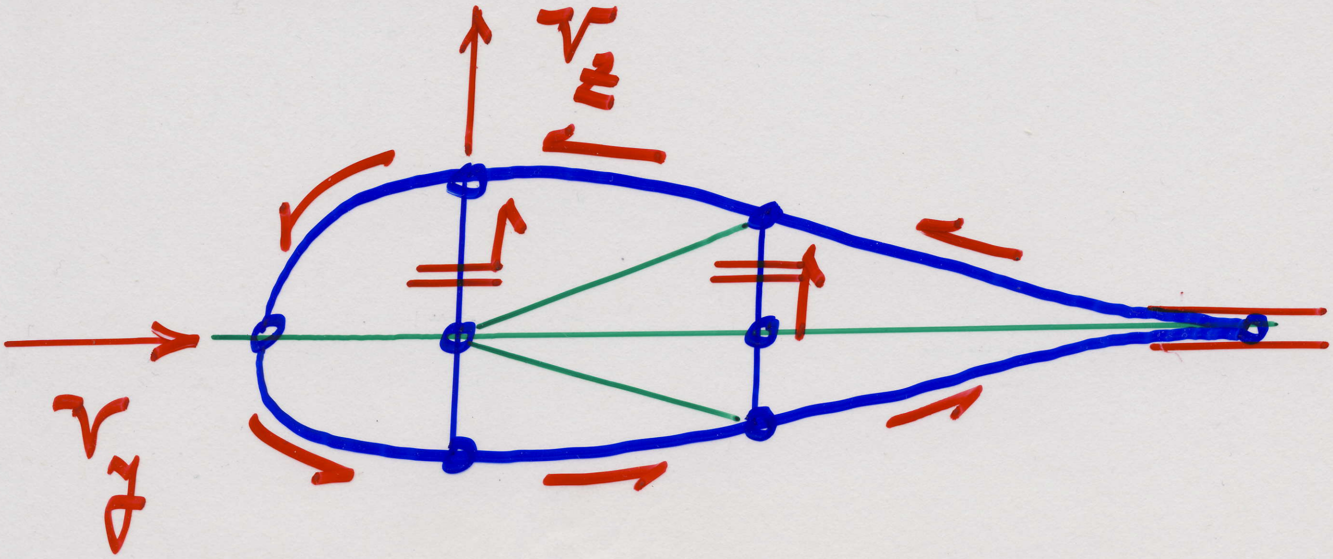
Need 3 eqs:

1) Mom. eq.: Take mom of V_y, V_z and $\{q_{12}, \dots, q_{41}\}$ about any convenient pt. (usually where lines of action of V_y and V_z intersect).

2) $\theta_1 = \theta_2$
3) $\theta_2 = \theta_3$

$\left.\begin{array}{l} \theta_1 = \theta_2 \\ \theta_2 = \theta_3 \end{array}\right\}$ Comp. eqs

NACA airfoil:



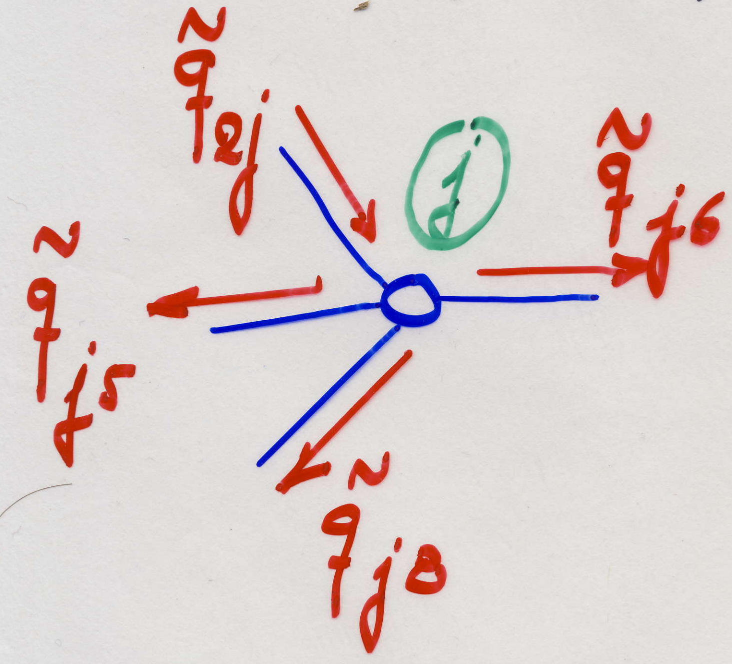
Solving P2 : For each cell,

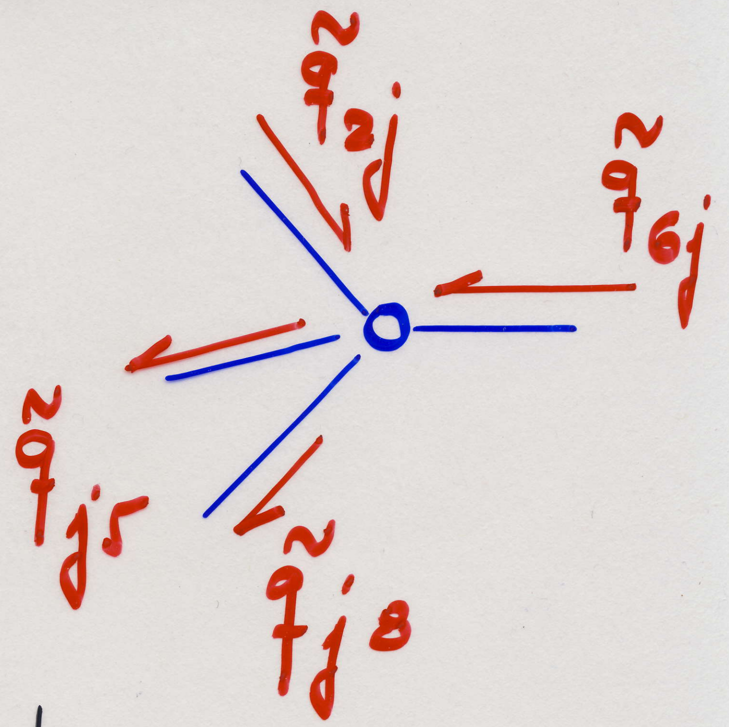
- Follow path "s_i"
- equil. of each stringer on path.

2 ways:

1) Complete method : FBD p. 38-1

2) Consequence of 1st method :



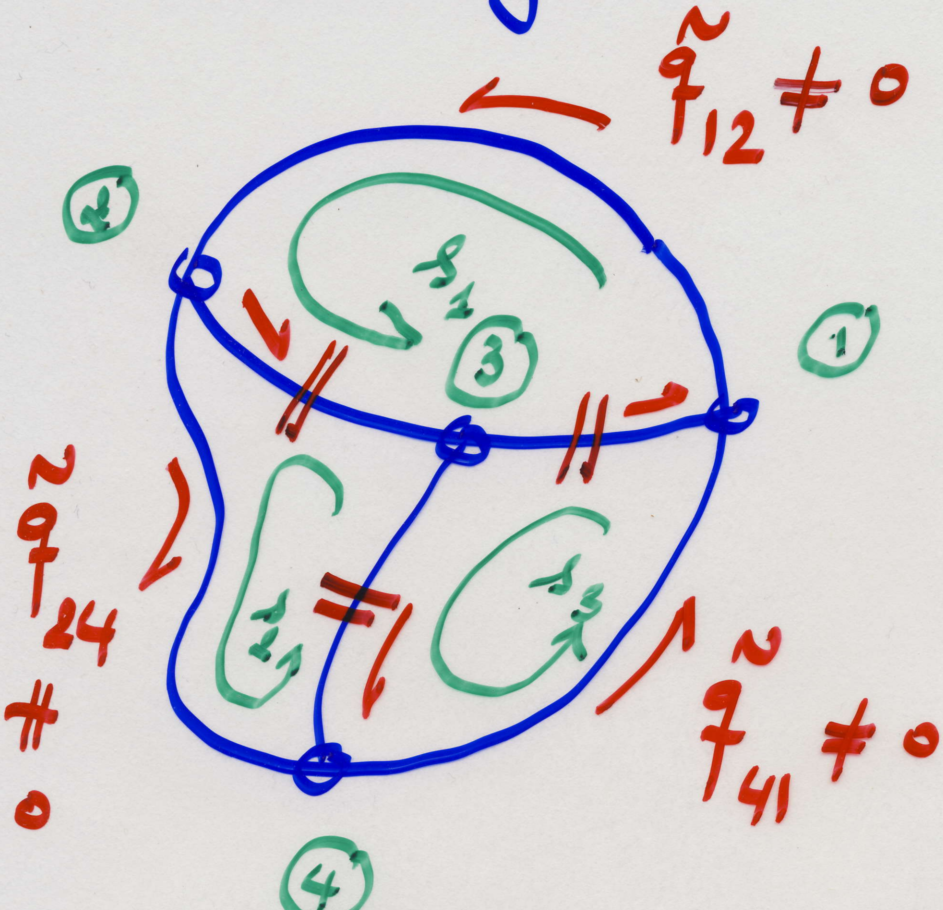


$$\tilde{q}_{j6} = \tilde{q}_{2j} - \tilde{q}_{j5} - \tilde{q}_{j8} + q^{(j)}$$

$$-\tilde{q}_{6j} = \tilde{q}_{2j} - \tilde{q}_{j5} - \tilde{q}_{j8} + q^{(j)}$$

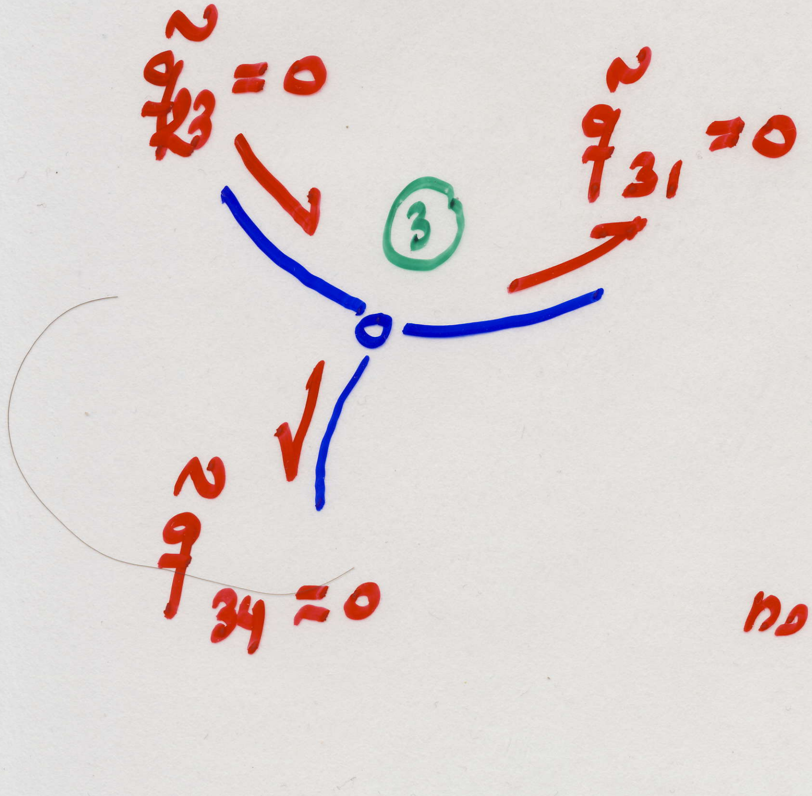
Note: (Jared)

Q: What if we cut cell walls at one stringer was isolated.



$$\tilde{q}_{23} = \tilde{q}_{31} = \tilde{q}_{34} = 0$$

due to cuts

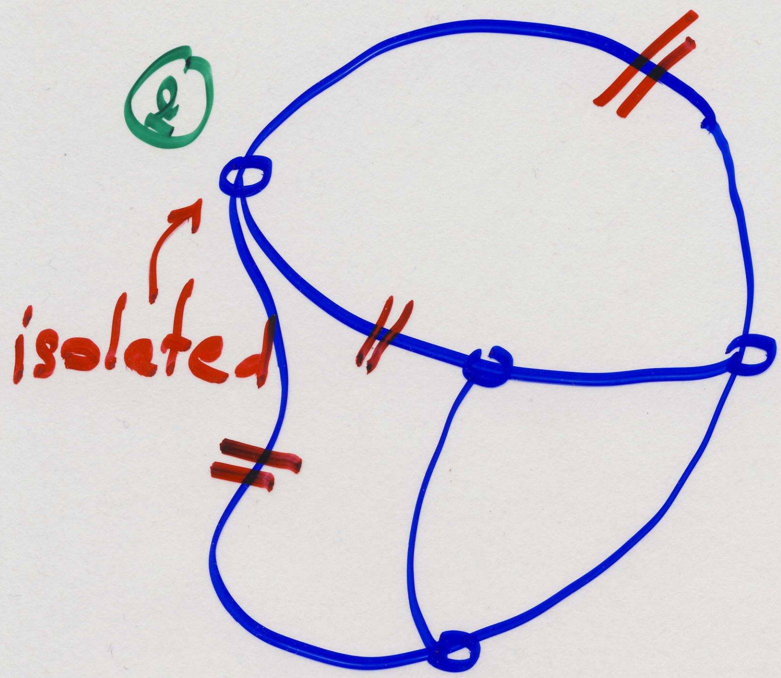


$$\cancelto{0}{\tilde{q}_{31}} = \cancelto{0}{\tilde{q}_{23}} - \cancelto{0}{\tilde{q}_{34}} + q^{(3)}$$

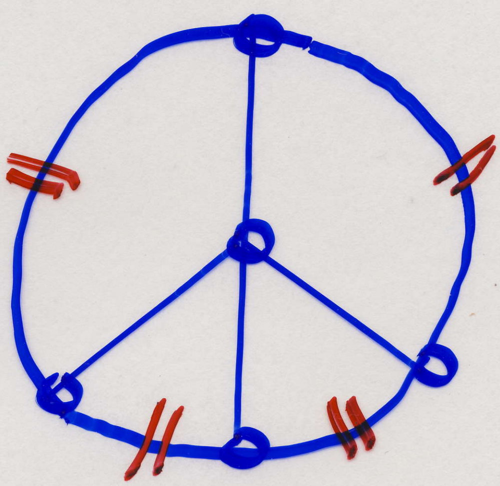
$$\Rightarrow q^{(3)} = 0 \quad !!$$

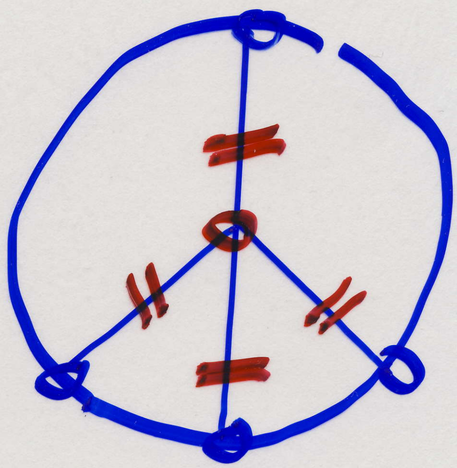
not true. (p. 38-2)

Another cut



HW7:





- shear buckling (HW7)
- Plotting buck. shape under shear

Expr. $\{C_{22}, C_{13}, C_{31}, C_{33}\}$ in terms of C_{11}, and for $\vartheta = 1.5$

\vartheta

1) Find λ for $\vartheta = 1.5$ Eq. (30) in wiki

2) Eval. num. $\underline{K}_{5\times5}$ in Eq (26)

3) $\underline{K} = [K_{ij}]$ row col.

$\overline{\underline{K}}_{4\times4}$

$$\overbrace{\begin{bmatrix} K_{22} & K_{23} & \cdots & K_{25} \\ \vdots & & & \vdots \\ K_{52} & \cdots\cdots & & K_{55} \end{bmatrix}}\begin{Bmatrix} C_{22} \\ \vdots \\ C_{33} \end{Bmatrix} = \begin{Bmatrix} -\frac{4}{9}C_{11} \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

Solve $\{C_{22}, C_{13}, C_{31}, C_{33}\}$ in terms of C_{11}

$\underline{\bar{K}}^{-1}$ inverse of $\underline{\bar{K}}$

$$\begin{Bmatrix} C_{22} \\ C_{13} \\ C_{31} \\ C_{33} \end{Bmatrix} = \underline{\bar{K}}^{-1} \begin{Bmatrix} -\frac{4}{9} C_{11} \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$u_z = C_{11} \sin\frac{\pi x}{a} \sin\frac{\pi}{b}y$$

$$+ C_{22} \sin\frac{2\pi x}{a} \sin\frac{2\pi}{b}y$$

$$+ C_{13} \sin\frac{\pi x}{a} \sin\frac{3\pi}{b}y$$

m

n

$$+ C_{31} \sin\frac{3\pi x}{a} \sin\frac{\pi}{b}y$$

$$+ C_{33} \sin\frac{3\pi x}{a} \sin\frac{3\pi}{b}y$$

Set $C_{11} = 1$, plot $u_z(x, y)$.

Note: Cont. from p. 39.2

Ans. in 2 parts.

Part 1: Equil. of isolated stringer (done)

Part 2: Closed cell, equil of stringers

Stringer 1:

$$\underbrace{\tilde{q}_{12}}_{\text{unknown}} = \cancel{\tilde{q}_{31}}^{\,0} + \tilde{q}_{41} + q^{(1)}$$

Stringer 2:

$$\tilde{q}_{24} = \tilde{q}_{12} - \cancel{\tilde{q}_{23}}^{\,0} + q^{(2)}$$

Stringer 4:

$$\cancel{\tilde{q}_{41}} = \tilde{q}_{24} + \cancel{\tilde{q}_{34}}^{\,0} + q^{(4)}$$

$$= \left(\cancel{\tilde{q}_{41}} + q^{(1)}\right) + q^{(2)} + \cancel{\tilde{q}_{34}}^{\,0} + q^{(4)}.$$

p. 40-3 cont'd

$$0 = q^{(1)} + q^{(2)} + q^{(4)} = -q^{(3)} \neq 0$$

Not true (or possible) since

$$0 = q^{(1)} + q^{(2)} + q^{(3)} + q^{(4)}$$

$$\boxed{0 = \sum_{e=1}^{4} q^{(e)}}$$

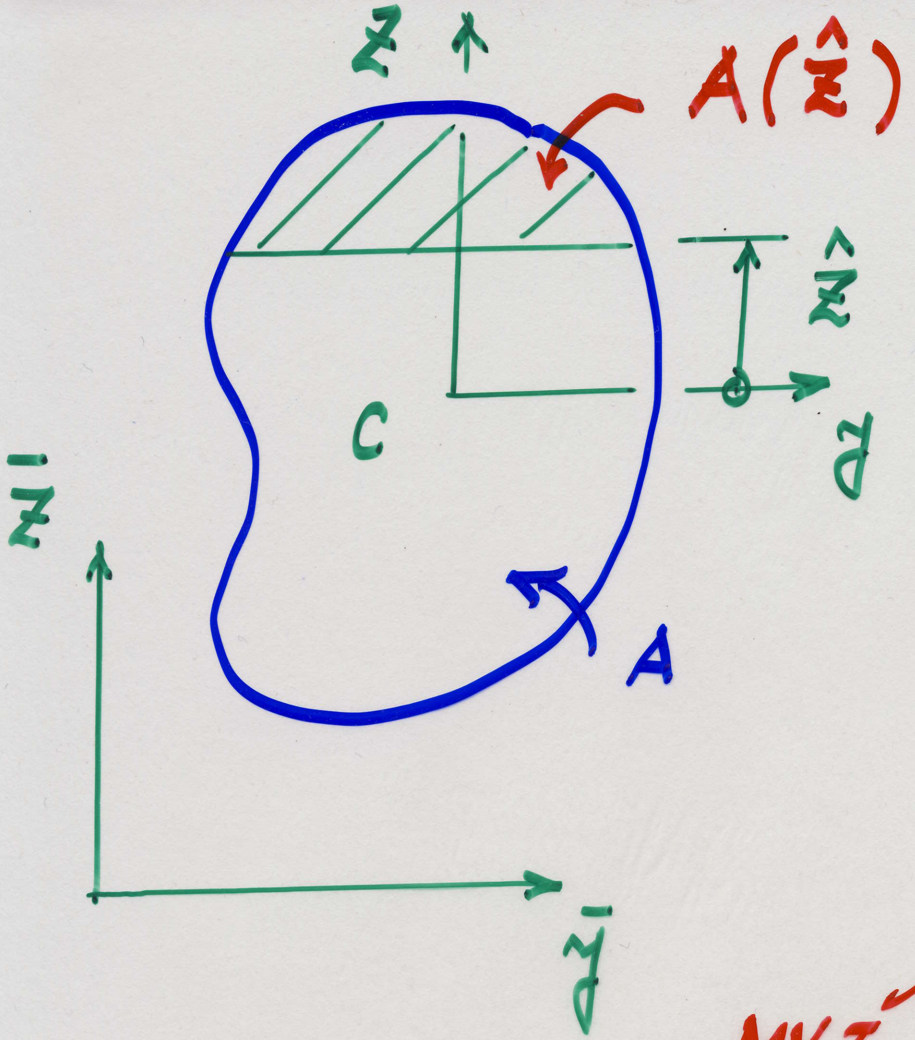
p. 38-2:

$$q^{(e)} = n_z Q_z^{(e)} + n_y Q_y^{(e)}$$

$$n_z := -(k_y V_y - k_{yz} V_z)$$

$n_y :=$ HW 7

$$\sum_{e=1}^{4} q^{(e)} = n_z \underbrace{\sum_{e=1}^{4} Q_z^{(e)}}_{=0} + n_y \underbrace{\sum_{e=1}^{4} Q_y^{(e)}}_{=0}$$

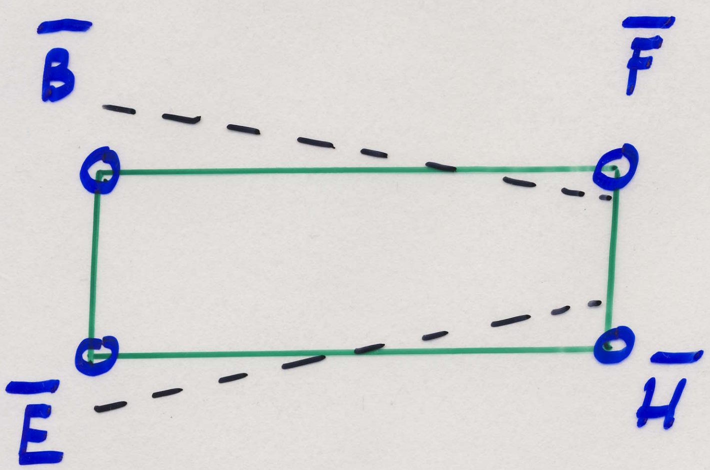


$$Q_y(\hat{z}) = \int_{A(\hat{z})} z \, dA$$

$$Q_y = \int_A z \, dA = 0$$

$$\underset{\text{MVT}}{=} \underbrace{z_c}_{=0} \int_A dA$$

HW7: NACA airfoil — "back of the envelope" verification



$$\bar{B}\bar{E} = \bar{F}\bar{H}$$

$$= \frac{1}{2}(BE + FH)$$

Verify magni. of : I_y, I_z, I_{yz}

angle β of N.A.

σ_{xx} @ each stringer

(C-section pb p. 34-4)

Bi.dir. bending (cont'd) p. 34-2 41-3

R&R Sec 4.2 (4.22a) → (4.28c)

$$M_y = -EI_{yz}\frac{d^2 v_0}{dx^2} - EI_y \frac{d^2 w_0}{dx^2} \qquad (4.26)$$

$$M_z = -EI_z \underbrace{\frac{d^2 v_0}{dx^2}}_{\chi_y} - EI_{yz}\underbrace{\frac{d^2 w_0}{dx^2}}_{\chi_z}$$

curvatures

Mat. form:

$$\begin{Bmatrix} M_y \\ M_z \end{Bmatrix} = \underbrace{\begin{bmatrix} I_y & I_{yz} \\ I_{yz} & I_z \end{bmatrix}}_{\underline{I}} \begin{Bmatrix} -E\chi_z \\ -E\chi_y \end{Bmatrix}$$

(4.25)

$$\varepsilon_{xx} = -y\,\chi_y - z\,\chi_z$$

$$= \lfloor z \quad y \rfloor \begin{Bmatrix} -\chi_z \\ -\chi_y \end{Bmatrix}$$

$$\boxed{\begin{aligned}\sigma_{xx} &= E\,\varepsilon_{xx} = E \lfloor z \quad y \rfloor \begin{Bmatrix} -\chi_z \\ -\chi_y \end{Bmatrix} \\ &= \cancel{E} \lfloor z \quad y \rfloor \frac{1}{\cancel{E}} \underline{I}^{-1} \begin{Bmatrix} M_y \\ M_z \end{Bmatrix}\end{aligned}}$$

$$\underline{I}^{-1} = \frac{1}{D} \begin{bmatrix} I_z & -I_{yz} \\ -I_{yz} & I_y \end{bmatrix}$$

$$D = I_y I_z - (I_{yz})^2$$

Shear flow: $$q = -\int_A \frac{d\sigma_{xx}}{dx}\, dA$$

$$\frac{d\sigma_{xx}}{dx} = \lfloor z \quad y \rfloor \underline{I}^{-1} \begin{Bmatrix} dM_y/dx \\ dM_z/dx \end{Bmatrix}$$

$$V_y = \frac{dM_y}{dx}, \quad V_z = \frac{dM_z}{dx}$$

$$Q_y = \int_A z\, dA, \quad Q_z = \int y\, dA$$