# Time Domain Techniques (3B)

## for Noisy Signals

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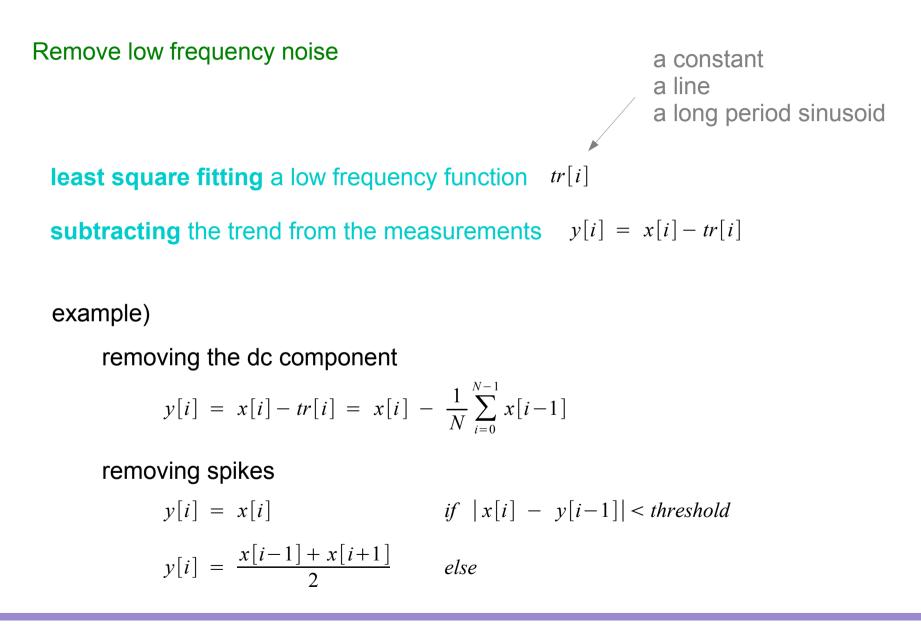
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### Signal Detrending / Spike Removal



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### Stacking

Remove background noise

Assume noise has a zero mean

Measuring multiple times (ensemble)

Averaging across the ensemble

Reduces the noise level in the averaged signal Increases the SNR of the correlated component (signal)

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$$\boldsymbol{E}[X[i]] \to x[i]$$

$$E[N[i]] \to 0$$
  
$$E[N^{2}[i]] \to \frac{\sigma_{n}}{\sqrt{M}} \qquad SNR \propto \sqrt{M}$$

#### **3A Time Domain Techniques**

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### Moving Average Filter

L-point Running Averager

$$y[n] = \frac{1}{L} \sum_{k=0}^{L-1} x[n-k] = \frac{1}{L} \{x[n] + x[n-1] + \dots + x[n-L+1]\}$$

$$Y(e^{j\hat{\omega}}) = \frac{1}{L} \sum_{k=0}^{L-1} e^{-j\hat{\omega}k} X(e^{j\hat{\omega}}) = \frac{1}{L} \{X(e^{j\hat{\omega}}) + X(e^{j\hat{\omega}})e^{-j\hat{\omega}1} + \dots + X(e^{j\hat{\omega}})e^{-j\hat{\omega}(L-1)}\}$$

$$H(e^{j\hat{\omega}}) = \frac{1}{L} \sum_{k=0}^{L-1} e^{j\hat{\omega}k} = \frac{1}{L} \{1 + e^{-j\hat{\omega}1} + \dots + e^{-j\hat{\omega}(L-1)}\} = \frac{1}{L} \left(\frac{1 - e^{-j\hat{\omega}L}}{1 - e^{-j\hat{\omega}}}\right)$$
$$= \frac{1}{L} \left(\frac{e^{-j\hat{\omega}L/2}}{e^{-j\hat{\omega}/2}}\right) \left(\frac{e^{+j\hat{\omega}L/2} - e^{-j\hat{\omega}L/2}}{e^{+j\hat{\omega}/2} - e^{-j\hat{\omega}/2}}\right) = \frac{1}{L} \left(\frac{e^{+j\hat{\omega}L/2} - e^{-j\hat{\omega}L/2}}{e^{+j\hat{\omega}/2} - e^{-j\hat{\omega}/2}}\right) e^{-j\hat{\omega}(L-1)/2}$$
$$= \left(\frac{\sin(\hat{\omega}L/2)}{L\sin(\hat{\omega}/2)}\right) e^{-j\hat{\omega}(L-1)/2}$$
Dirichlet Function
$$H(e^{j\hat{\omega}}) = D_L(e^{j\hat{\omega}}) e^{-j\hat{\omega}(L-1)/2} \qquad D_L(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}L/2)}{L\sin(\hat{\omega}/2)}$$

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### Correlation: Identifying Similarities

#### References

- [1] http://en.wikipedia.org/
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] J.C. Santamarina, D Fratta, "Discrete Signals and Inverse Problems: An Introduction for Engineers and Scientists", 2005