

# DFT Analysis (8A)

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- X[1]
- X[2]
- X[3]
- X[4]
- X[5]
- X[6]
- X[7]

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# N=8 DFT

$$X[k] = \sum_{n=0}^7 W_8^{kn} x[n] \quad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} = \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\ W_8^0 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\ W_8^0 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\ W_8^0 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\ W_8^0 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\ W_8^0 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49} \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix}$$

# N=8 IDFT

$$x[n] = \frac{1}{N} \sum_{k=0}^7 W_8^{-kn} X[k] \quad W_8^{-kn} = e^{+j\left(\frac{2\pi}{8}\right)kn}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \frac{1}{N} \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^0 & W_8^{-1} & W_8^{-2} & W_8^{-3} & W_8^{-4} & W_8^{-5} & W_8^{-6} & W_8^{-7} \\ W_8^0 & W_8^{-2} & W_8^{-4} & W_8^{-6} & W_8^{-8} & W_8^{-10} & W_8^{-12} & W_8^{-14} \\ W_8^0 & W_8^{-3} & W_8^{-6} & W_8^{-9} & W_8^{-12} & W_8^{-15} & W_8^{-18} & W_8^{-21} \\ W_8^0 & W_8^{-4} & W_8^{-8} & W_8^{-12} & W_8^{-16} & W_8^{-20} & W_8^{-24} & W_8^{-28} \\ W_8^0 & W_8^{-5} & W_8^{-10} & W_8^{-15} & W_8^{-20} & W_8^{-25} & W_8^{-30} & W_8^{-35} \\ W_8^0 & W_8^{-6} & W_8^{-12} & W_8^{-18} & W_8^{-24} & W_8^{-30} & W_8^{-36} & W_8^{-42} \\ W_8^0 & W_8^{-7} & W_8^{-14} & W_8^{-21} & W_8^{-28} & W_8^{-35} & W_8^{-42} & W_8^{-49} \end{bmatrix} \begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix}$$

# N=8 DFT Matrix (1)

$$X[k] = \sum_{n=0}^7 W_8^{kn} x[n] \quad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} = \begin{bmatrix} e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} \\ e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 1} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 3} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 5} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 7} \\ e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 6} \\ e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 3} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 1} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 7} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 5} \\ e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 4} \\ e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 5} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 7} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 1} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 3} \\ e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 2} \\ e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 7} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 5} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 3} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 1} \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix}$$

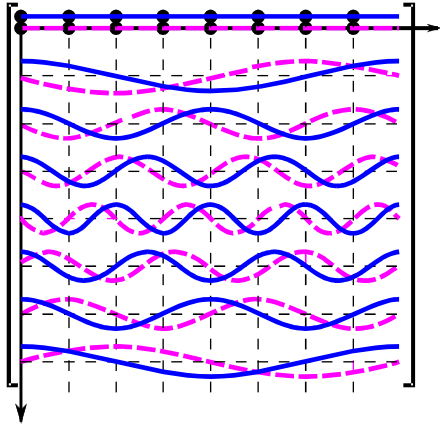
# N=8 IDFT Matrix (1)

$$x[n] = \frac{1}{N} \sum_{k=0}^7 W_8^{-kn} X[k] \quad W_8^{-kn} = e^{+j\left(\frac{2\pi}{8}\right)kn}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \begin{bmatrix} e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 1} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 3} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 5} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 7} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 6} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 3} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 1} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 7} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 5} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 4} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 5} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 7} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 1} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 3} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 2} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 7} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 5} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 3} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 1} \end{bmatrix} \begin{bmatrix} \frac{X[0]}{N} \\ \frac{X[1]}{N} \\ \frac{X[2]}{N} \\ \frac{X[3]}{N} \\ \frac{X[4]}{N} \\ \frac{X[5]}{N} \\ \frac{X[6]}{N} \\ \frac{X[7]}{N} \end{bmatrix}$$

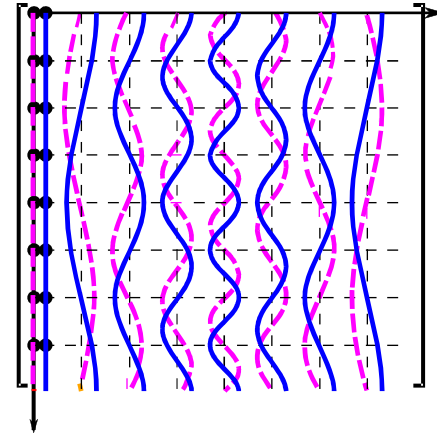
# Symmetric Matrices

*DFT Matrix in the row-wise view*

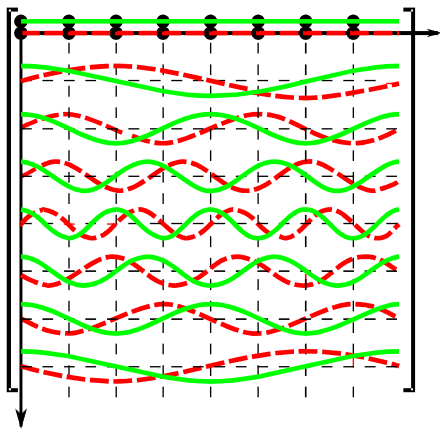


$$\mathbf{A} = \mathbf{A}^T$$

*DFT Matrix in the column-wise view*

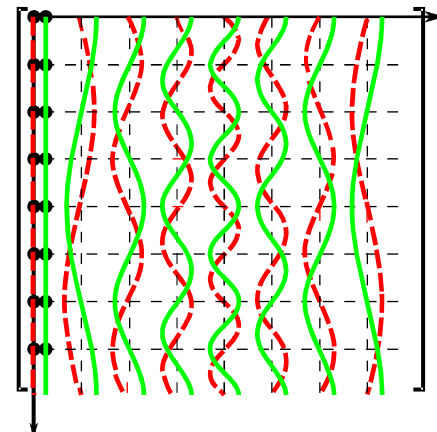


*IDFT Matrix in the row-wise view*



$$\mathbf{B} = \mathbf{B}^T$$

*IDFT Matrix in the column-wise view*



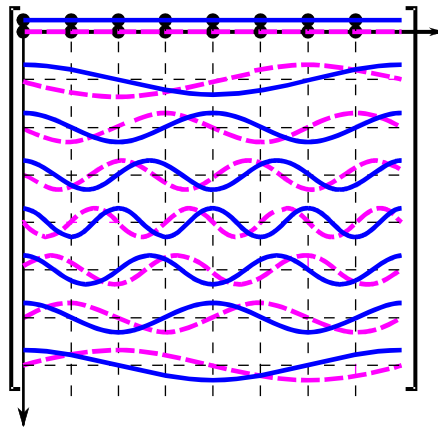
# Conjugate Transpose Matrices

$$A = B^*$$

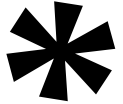
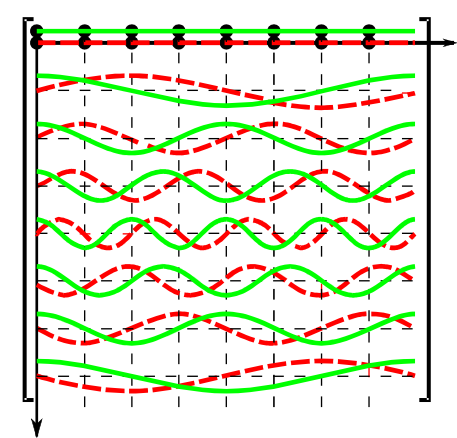
— Real    - - Imaginary

DFT

$$A =$$



=

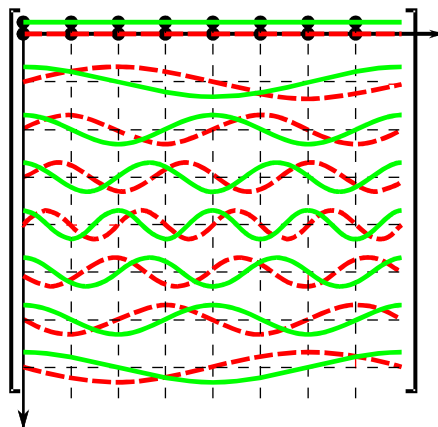


$$B = A^*$$

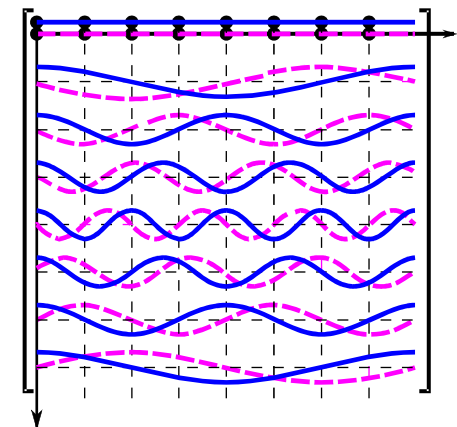
— Real    - - Imaginary

IDFT

$$B =$$



=





# Product AB

DFT

IDFT

$A$

$\cdot$

$B$

$=$

$C$

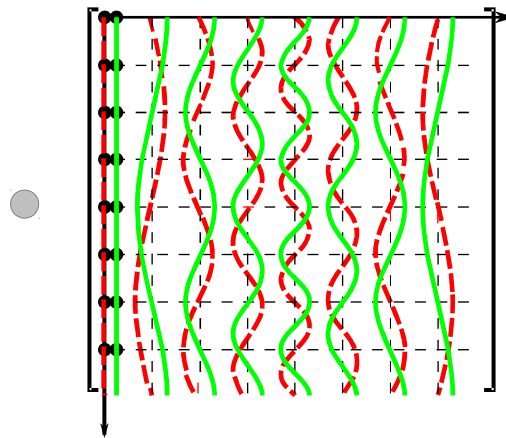
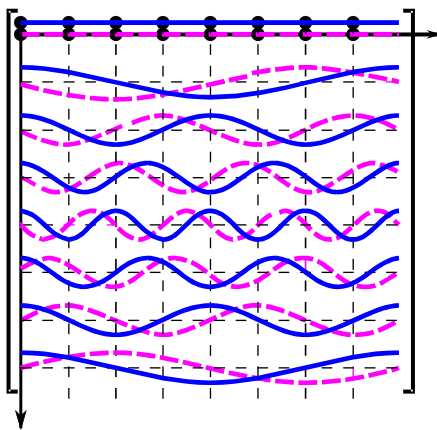
$A$

$\cdot$

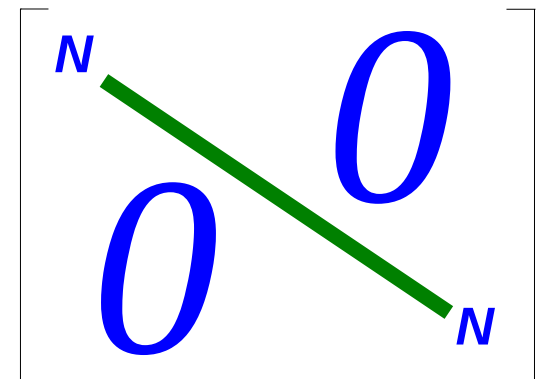
$A^*$

$=$

$C$

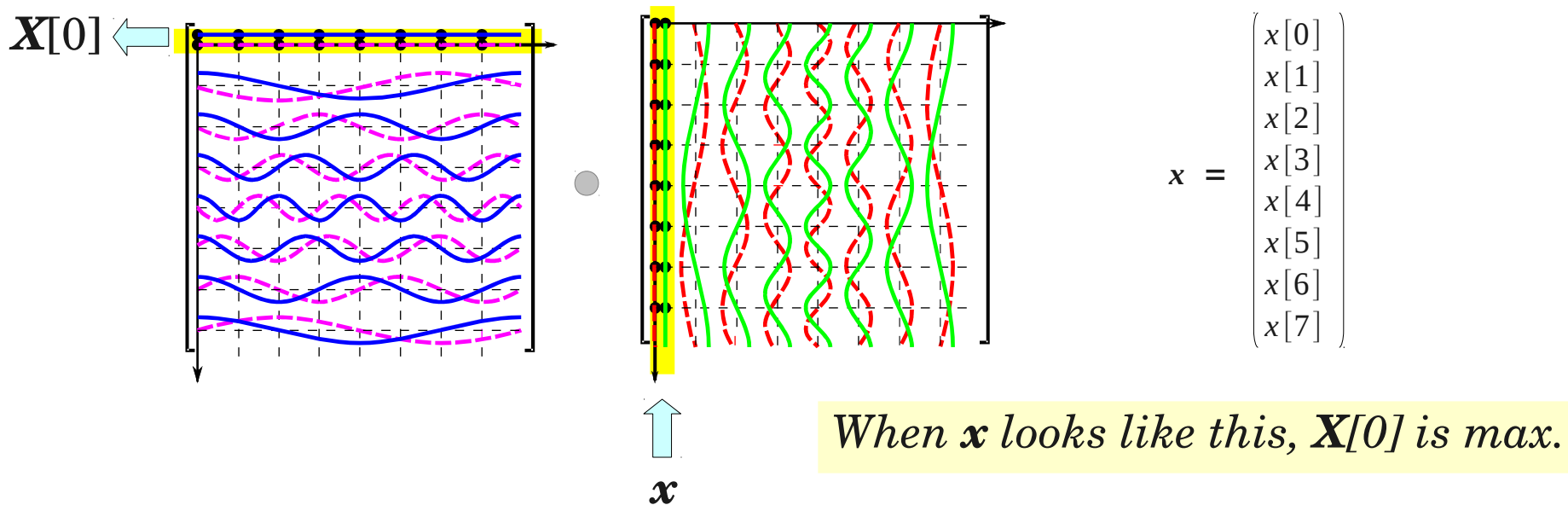


$=$



# N=8 DFT : Inner Product X[0]

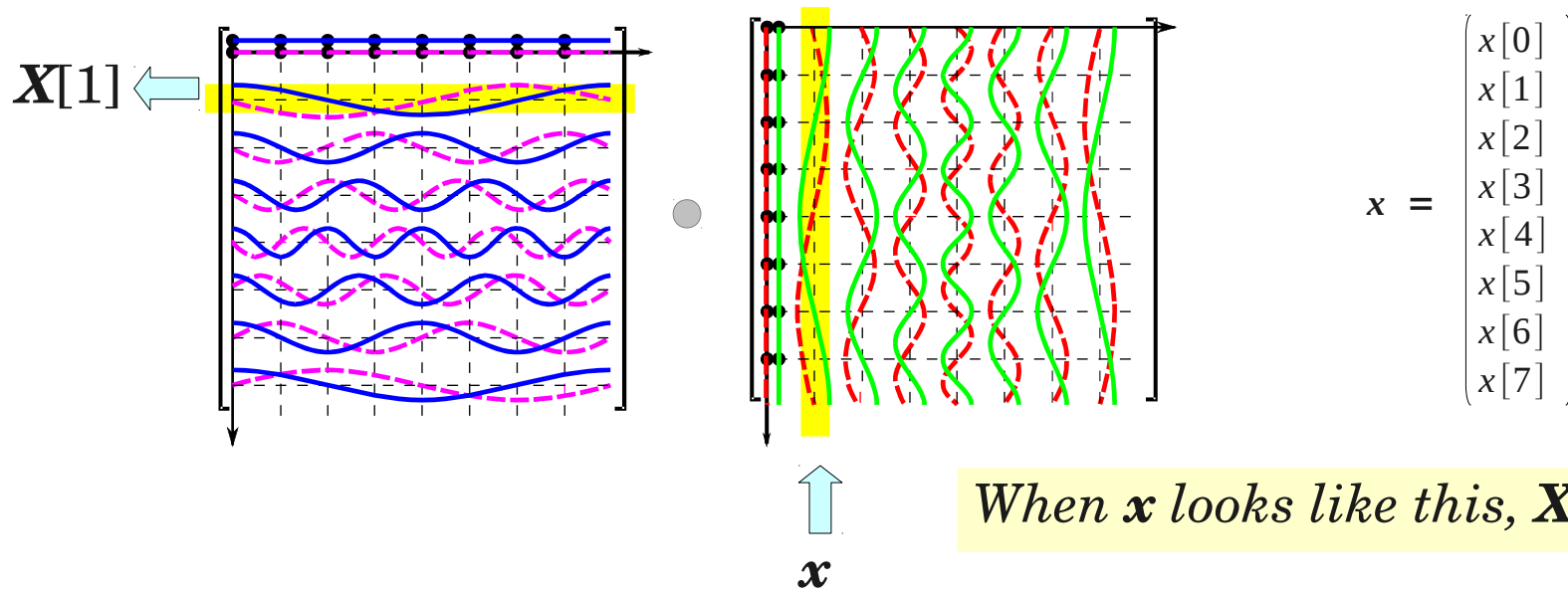
$$X[k] = \sum_{n=0}^{7} W_8^{kn} x[n] \quad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$



$$X[0] = \begin{pmatrix} e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} \end{pmatrix} \bullet \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix}^T$$

# N=8 DFT : Inner Product X[1]

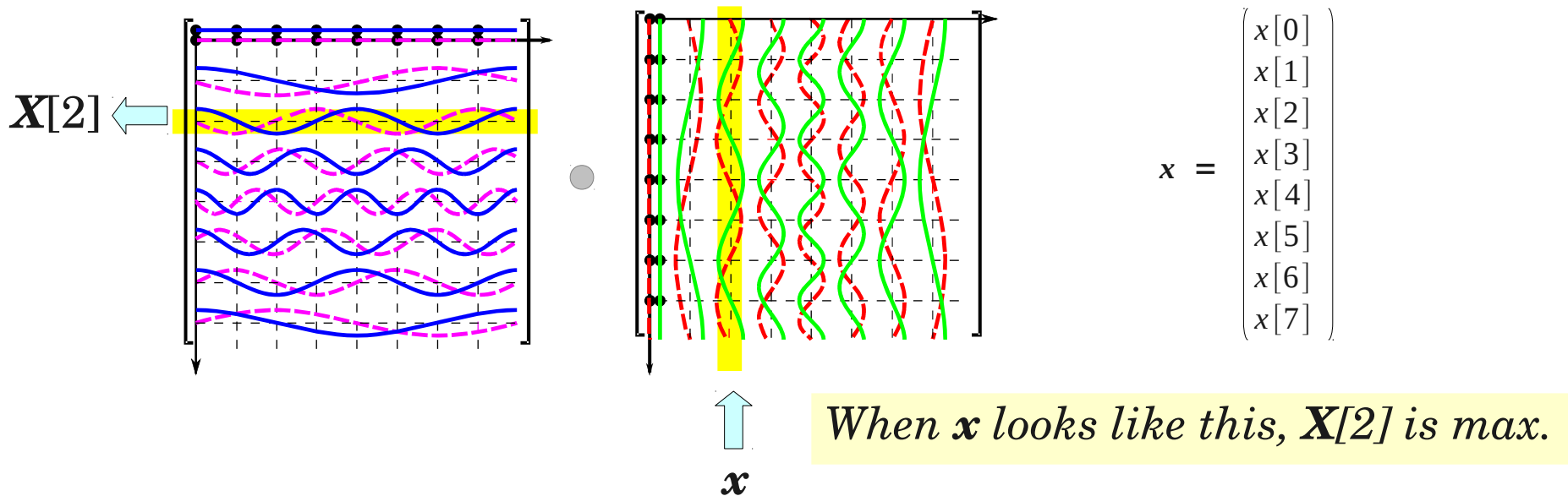
$$X[k] = \sum_{n=0}^7 W_8^{kn} x[n] \quad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$



$$X[1] = \begin{pmatrix} e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 1} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 3} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 5} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 7} \end{pmatrix} \bullet \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix}^T$$

# N=8 DFT : Inner Product X[2]

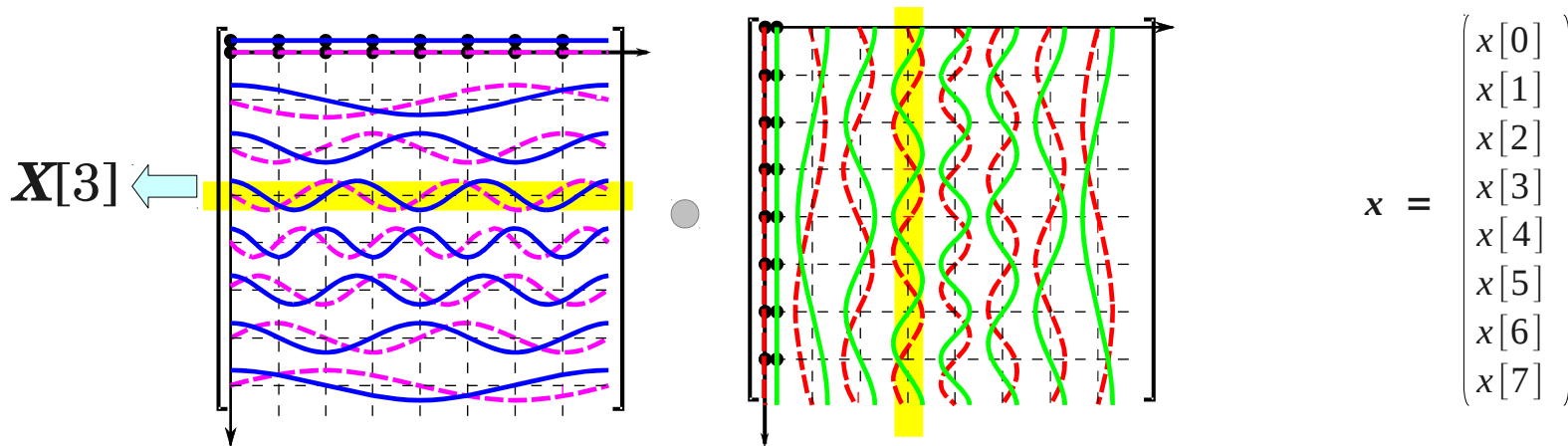
$$X[k] = \sum_{n=0}^7 W_8^{kn} x[n] \quad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$



$$X[2] = \begin{pmatrix} e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 6} \end{pmatrix} \bullet \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix}^T$$

# N=8 DFT : Inner Product X[3]

$$X[k] = \sum_{n=0}^7 W_8^{kn} x[n] \quad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$

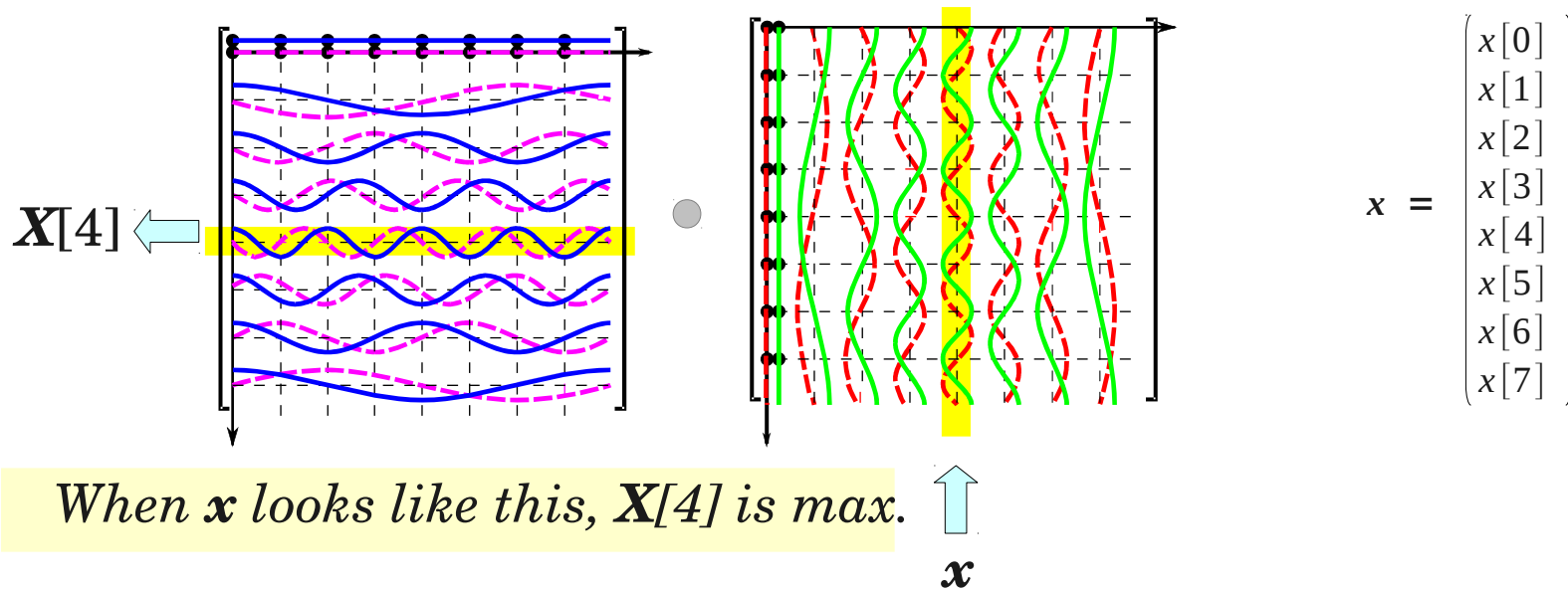


↑  
 $x$  When  $x$  looks like this,  $X[3]$  is max.

$$X[3] = \begin{pmatrix} e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 3} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 1} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 7} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 5} \end{pmatrix} \bullet \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix}^T$$

# N=8 DFT : Inner Product X[4]

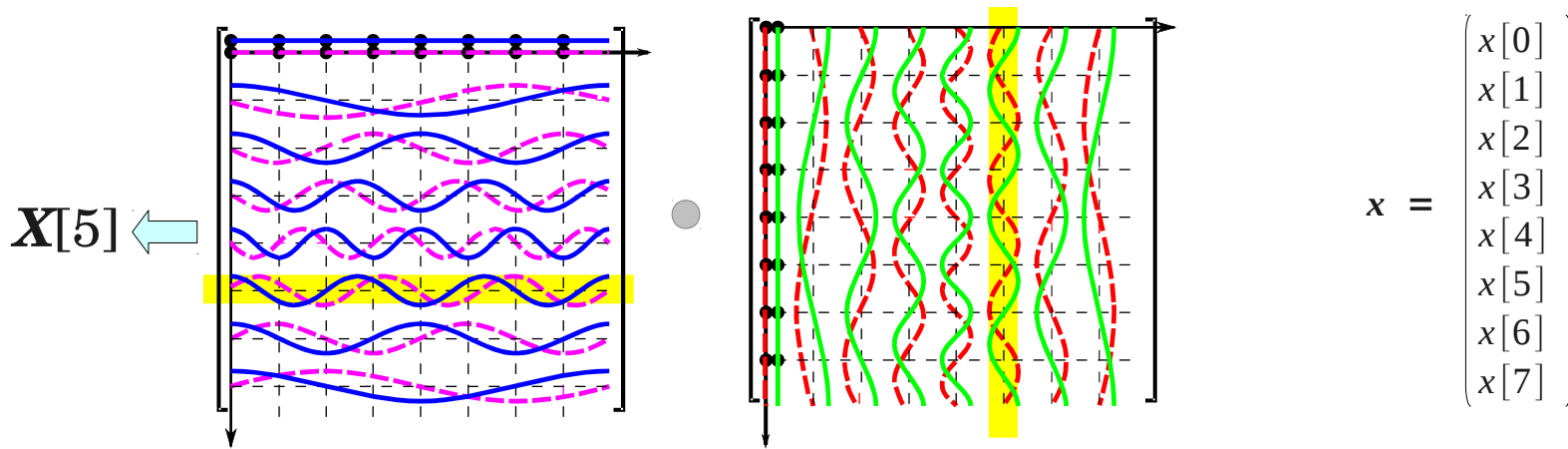
$$X[k] = \sum_{n=0}^7 W_8^{kn} x[n] \quad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$



$$X[4] = \begin{pmatrix} e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 4} \end{pmatrix} \cdot \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix}^T$$

# N=8 DFT : Inner Product X[5]

$$X[k] = \sum_{n=0}^7 W_8^{kn} x[n] \quad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$



$$\mathbf{x} = \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix}$$

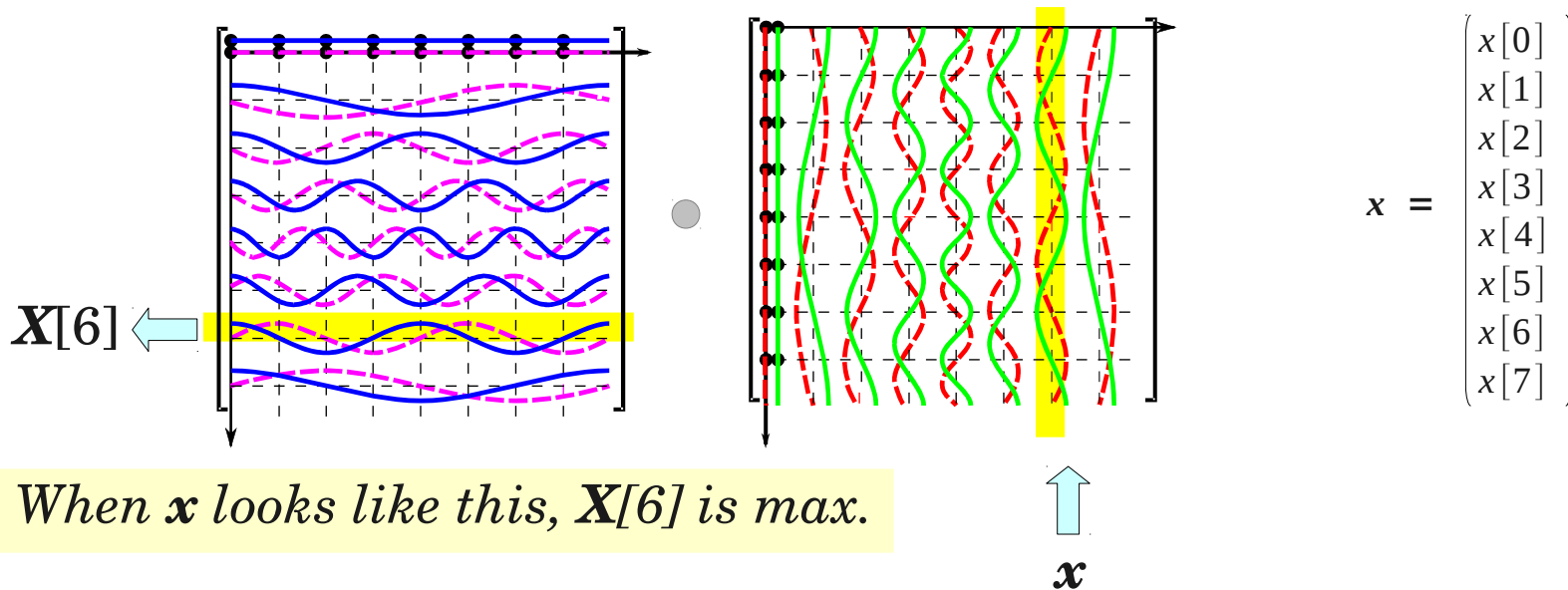
When  $\mathbf{x}$  looks like this,  $\mathbf{X}[5]$  is max.

$\mathbf{x}$

$$\mathbf{X}[5] = \begin{pmatrix} e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 5} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 7} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 1} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 3} \end{pmatrix} \bullet \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix}^T$$

# N=8 DFT : Inner Product X[6]

$$X[k] = \sum_{n=0}^7 W_8^{kn} x[n] \quad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$

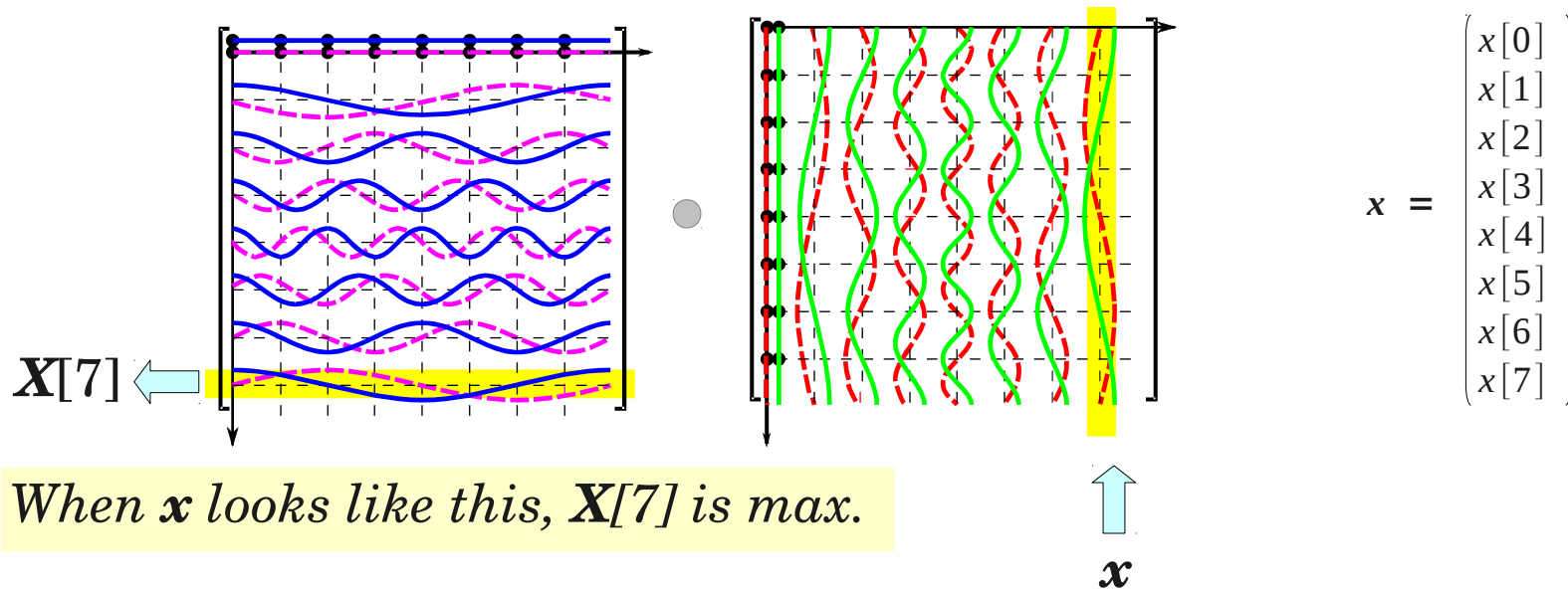


$$\mathbf{X}[6] = \begin{pmatrix} e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 2} \end{pmatrix} \bullet \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix}^T$$



# N=8 DFT : Inner Product X[7]

$$X[k] = \sum_{n=0}^{7} W_8^{kn} x[n] \quad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$

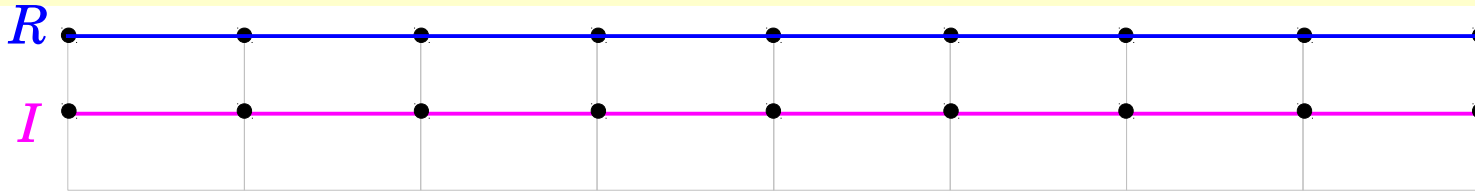


$$\mathbf{X}[7] = \begin{pmatrix} e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 7} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 5} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 3} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 1} \end{pmatrix} \bullet \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix}^T$$

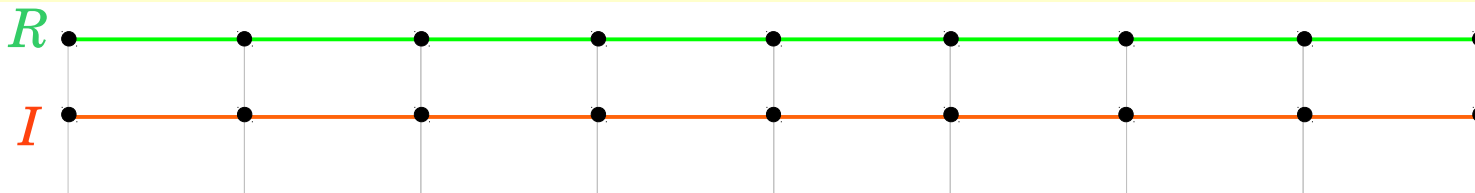


# N=8 DFT : X[0]

$$\left( e^{-j \cdot \frac{\pi}{4} \cdot 0} \quad e^{-j \cdot \frac{\pi}{4} \cdot 0} \quad e^{-j \cdot \frac{\pi}{4} \cdot 0} \quad e^{-j \cdot \frac{\pi}{4} \cdot 0} \quad e^{-j \cdot \frac{\pi}{4} \cdot 0} \quad e^{-j \cdot \frac{\pi}{4} \cdot 0} \quad e^{-j \cdot \frac{\pi}{4} \cdot 0} \quad e^{-j \cdot \frac{\pi}{4} \cdot 0} \right) \bullet$$



$$\left( x[0] \quad x[1] \quad x[2] \quad x[3] \quad x[4] \quad x[5] \quad x[6] \quad x[7] \right)^T$$



**X[0]** measures how much of the above signal component is present in **x**.



Sampling Time

$\tau$

Sampling Frequency  $f_s = \frac{1}{\tau}$

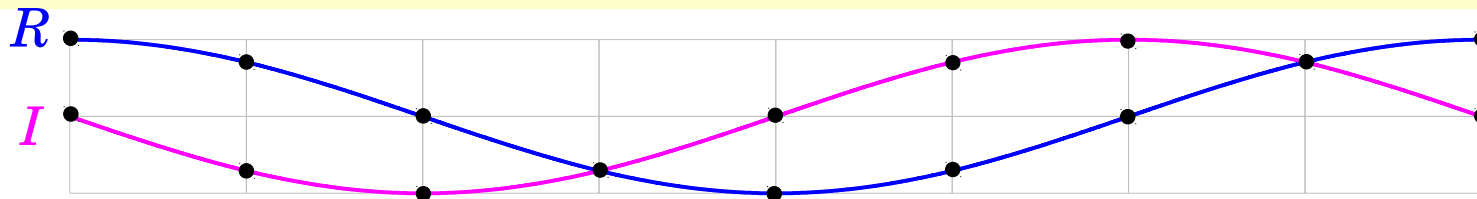
Sequence Time Length

$T = N\tau$

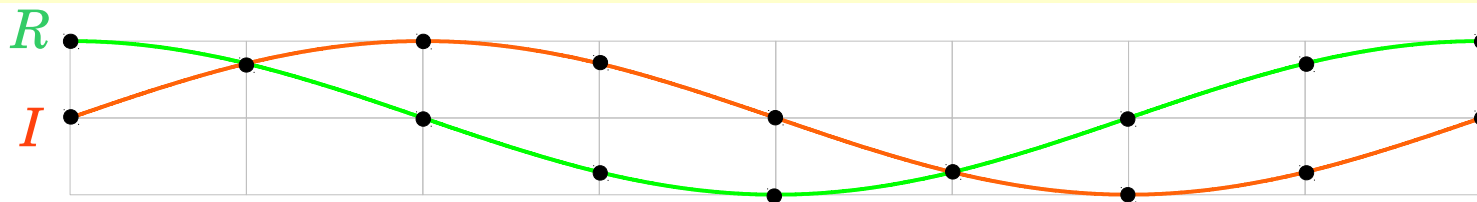
Zero Frequency

# N=8 DFT : X[1]

$$\left( e^{-j\frac{\pi}{4}\cdot 0} \quad e^{-j\frac{\pi}{4}\cdot 1} \quad e^{-j\frac{\pi}{4}\cdot 2} \quad e^{-j\frac{\pi}{4}\cdot 3} \quad e^{-j\frac{\pi}{4}\cdot 4} \quad e^{-j\frac{\pi}{4}\cdot 5} \quad e^{-j\frac{\pi}{4}\cdot 6} \quad e^{-j\frac{\pi}{4}\cdot 7} \right) \bullet$$



$$\left( x[0] \quad x[1] \quad x[2] \quad x[3] \quad x[4] \quad x[5] \quad x[6] \quad x[7] \right)^T$$



**X[1]** measures how much of the above signal component is present in **x**.



Sampling Time  $\tau$

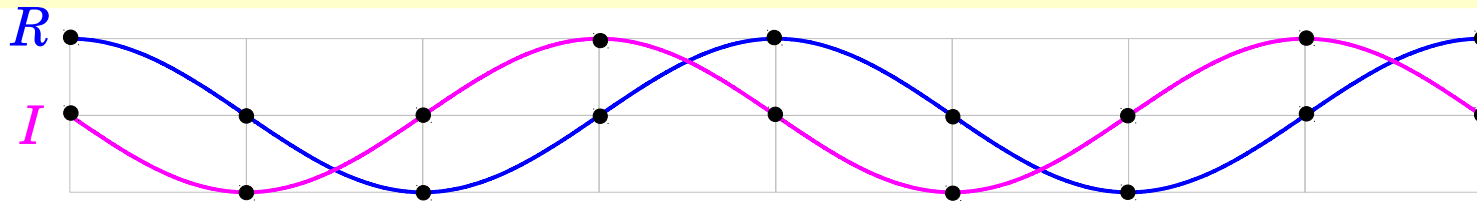
Sampling Frequency  $f_s = \frac{1}{\tau}$

Sequence Time Length  $T = N\tau$

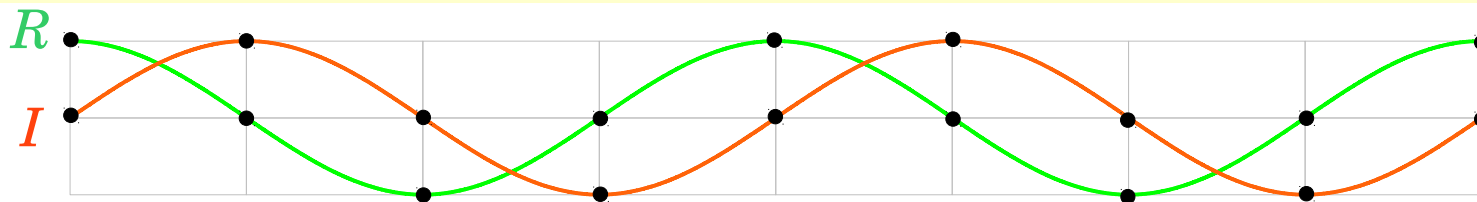
1<sup>st</sup> Harmonic Freq  $f_1 = \frac{1}{T} = \frac{1}{N\tau} = \frac{f_s}{N}$

# N=8 DFT : X[2]

$$\left( e^{-j\frac{\pi}{4}\cdot 0} \quad e^{-j\frac{\pi}{4}\cdot 2} \quad e^{-j\frac{\pi}{4}\cdot 4} \quad e^{-j\frac{\pi}{4}\cdot 6} \quad e^{-j\frac{\pi}{4}\cdot 0} \quad e^{-j\frac{\pi}{4}\cdot 2} \quad e^{-j\frac{\pi}{4}\cdot 4} \quad e^{-j\frac{\pi}{4}\cdot 6} \right) \bullet$$



$$\left( x[0] \quad x[1] \quad x[2] \quad x[3] \quad x[4] \quad x[5] \quad x[6] \quad x[7] \right)^T$$



**X[2]** measures how much of the above signal component is present in **x**.



Sampling Time  $\tau$

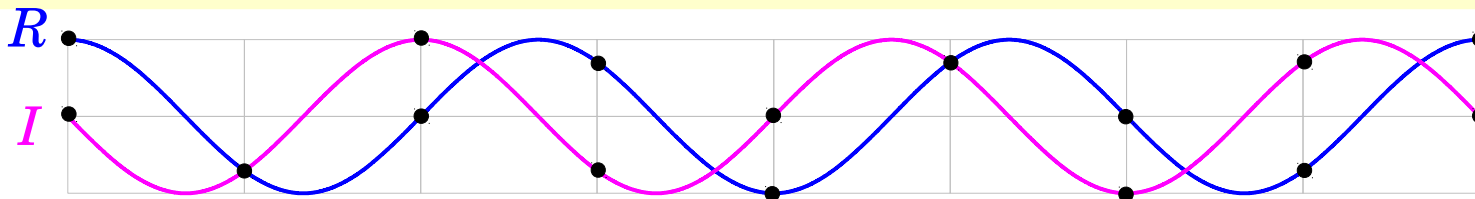
Sampling Frequency  $f_s = \frac{1}{\tau}$

Sequence Time Length  $T = N\tau$

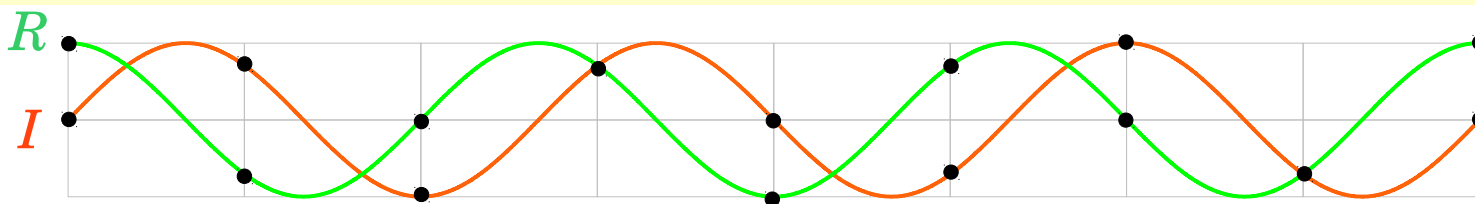
2<sup>nd</sup> Harmonic Freq  $f_2 = \frac{2}{T} = \frac{2}{N\tau} = \frac{2f_s}{N}$

# N=8 DFT : X[3]

$$\left( e^{-j\frac{\pi}{4}\cdot 0} \quad e^{-j\frac{\pi}{4}\cdot 3} \quad e^{-j\frac{\pi}{4}\cdot 6} \quad e^{-j\frac{\pi}{4}\cdot 1} \quad e^{-j\frac{\pi}{4}\cdot 4} \quad e^{-j\frac{\pi}{4}\cdot 7} \quad e^{-j\frac{\pi}{4}\cdot 2} \quad e^{-j\frac{\pi}{4}\cdot 5} \right) \bullet$$



$$\left( x[0] \quad x[1] \quad x[2] \quad x[3] \quad x[4] \quad x[5] \quad x[6] \quad x[7] \right)^T$$



**X[3]** measures how much of the above signal component is present in **x**.



$$T = N\tau$$

Sampling Time

$$\tau$$

Sampling Frequency  $f_s = \frac{1}{\tau}$

Sequence Time Length

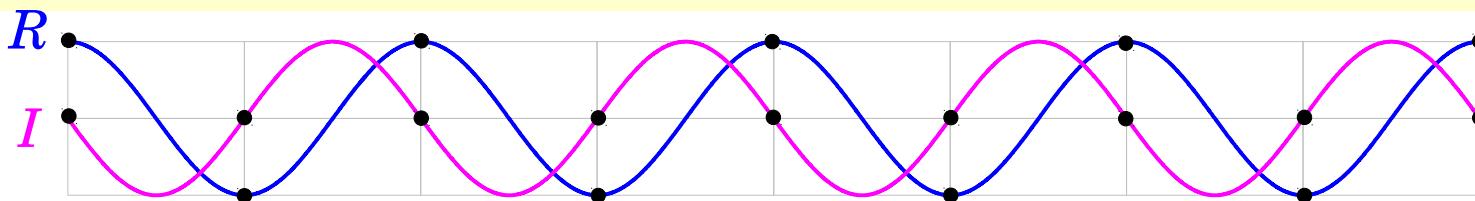
$$T = N\tau$$

3<sup>rd</sup> Harmonic Freq

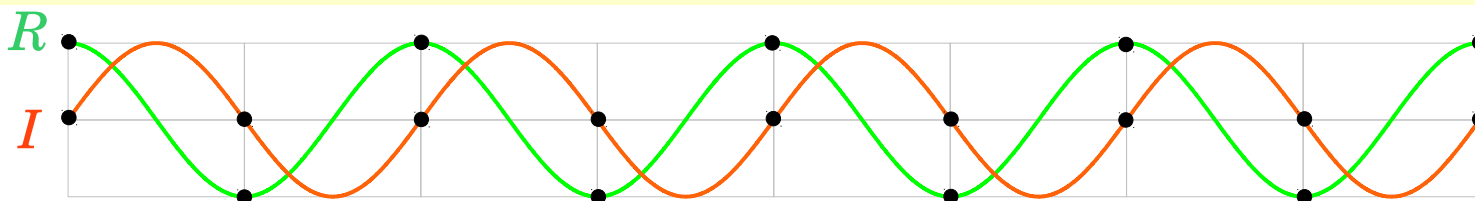
$$f_3 = \frac{3}{T} = \frac{3}{N\tau} = \frac{3f_s}{N}$$

# N=8 DFT : X[4]

$$\left( e^{-j\frac{\pi}{4}\cdot 0} \quad e^{-j\frac{\pi}{4}\cdot 4} \quad e^{-j\frac{\pi}{4}\cdot 0} \quad e^{-j\frac{\pi}{4}\cdot 4} \quad e^{-j\frac{\pi}{4}\cdot 0} \quad e^{-j\frac{\pi}{4}\cdot 4} \quad e^{-j\frac{\pi}{4}\cdot 0} \quad e^{-j\frac{\pi}{4}\cdot 4} \right) \bullet$$



$$\left( x[0] \quad x[1] \quad x[2] \quad x[3] \quad x[4] \quad x[5] \quad x[6] \quad x[7] \right)^T$$



**X[4]** measures how much of the above signal component is present in **x**.



Sampling Time

$\tau$

Sampling Frequency  $f_s = \frac{1}{\tau}$

Sequence Time Length

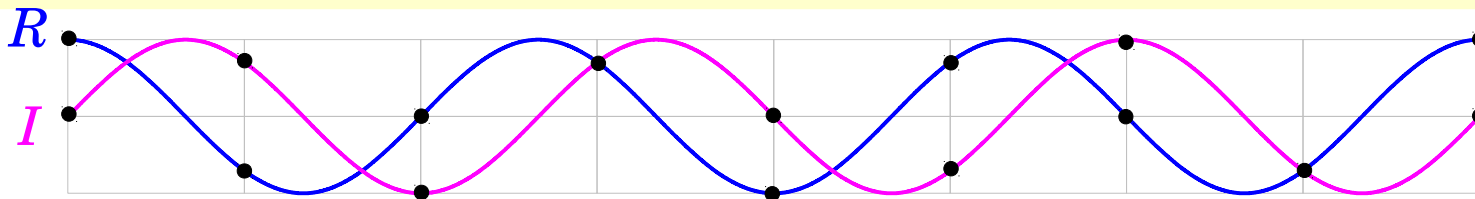
$T = N\tau$

4<sup>th</sup> Harmonic Freq

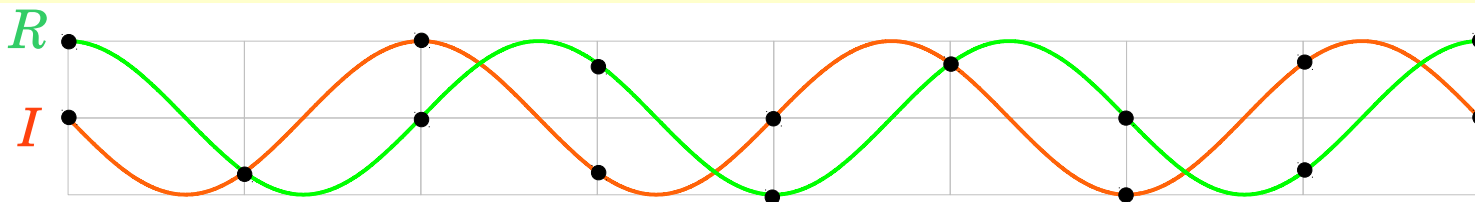
$f_4 = \frac{4}{T} = \frac{4}{N\tau} = \frac{4f_s}{N}$

# N=8 DFT : X[5]

$$\left( e^{-j\frac{\pi}{4}\cdot 0} \quad e^{-j\frac{\pi}{4}\cdot 5} \quad e^{-j\frac{\pi}{4}\cdot 2} \quad e^{-j\frac{\pi}{4}\cdot 7} \quad e^{-j\frac{\pi}{4}\cdot 4} \quad e^{-j\frac{\pi}{4}\cdot 1} \quad e^{-j\frac{\pi}{4}\cdot 6} \quad e^{-j\frac{\pi}{4}\cdot 3} \right) \bullet$$



$$\left( x[0] \quad x[1] \quad x[2] \quad x[3] \quad x[4] \quad x[5] \quad x[6] \quad x[7] \right)^T$$



**X[5]** measures how much of the above signal component is present in **x**.



Sampling Time

$\tau$

Sampling Frequency  $f_s = \frac{1}{\tau}$

Sequence Time Length

$T = N\tau$

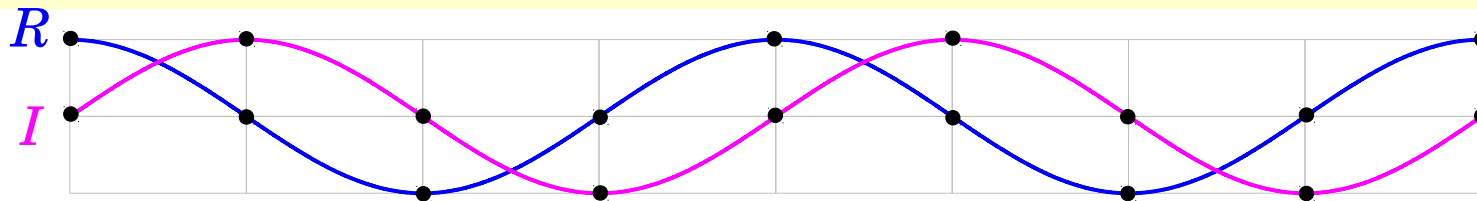
3<sup>rd</sup> Harmonic Freq

$f_3 = \frac{3}{T} = \frac{3}{N\tau} = \frac{3f_s}{N}$

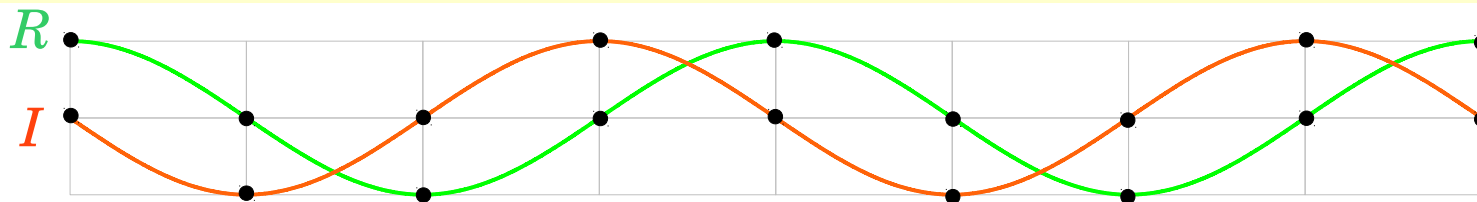


# N=8 DFT : X[6]

$$\left( e^{-j\frac{\pi}{4}\cdot 0} \quad e^{-j\frac{\pi}{4}\cdot 6} \quad e^{-j\frac{\pi}{4}\cdot 4} \quad e^{-j\frac{\pi}{4}\cdot 2} \quad e^{-j\frac{\pi}{4}\cdot 0} \quad e^{-j\frac{\pi}{4}\cdot 6} \quad e^{-j\frac{\pi}{4}\cdot 4} \quad e^{-j\frac{\pi}{4}\cdot 2} \right) \bullet$$



$$\left( x[0] \quad x[1] \quad x[2] \quad x[3] \quad x[4] \quad x[5] \quad x[6] \quad x[7] \right)^T$$



**X[6]** measures how much of the above signal component is present in **x**.



Sampling Time  $\tau$

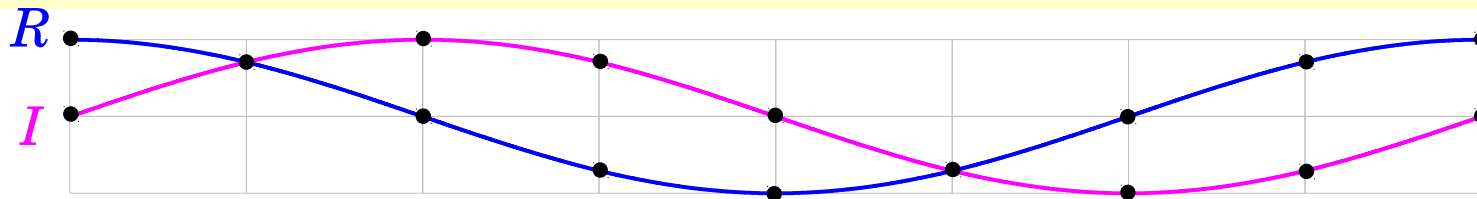
Sampling Frequency  $f_s = \frac{1}{\tau}$

Sequence Time Length  $T = N\tau$

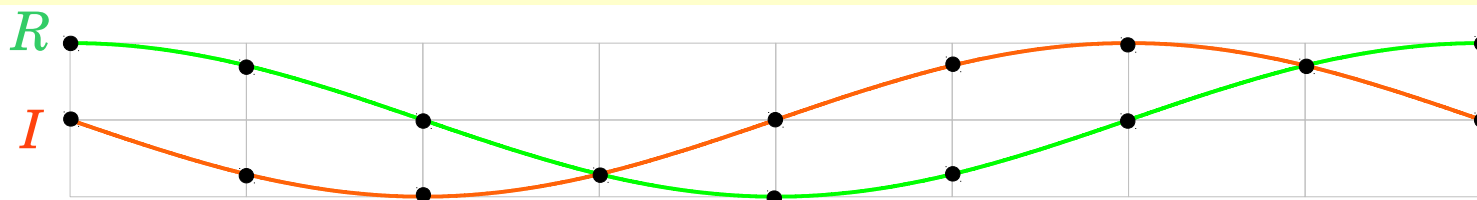
2<sup>nd</sup> Harmonic Freq  $f_2 = \frac{2}{T} = \frac{2}{N\tau} = \frac{2f_s}{N}$

# N=8 DFT : X[7]

$$\left( e^{-j\frac{\pi}{4}\cdot 0} \quad e^{-j\frac{\pi}{4}\cdot 7} \quad e^{-j\frac{\pi}{4}\cdot 6} \quad e^{-j\frac{\pi}{4}\cdot 5} \quad e^{-j\frac{\pi}{4}\cdot 4} \quad e^{-j\frac{\pi}{4}\cdot 3} \quad e^{-j\frac{\pi}{4}\cdot 2} \quad e^{-j\frac{\pi}{4}\cdot 1} \right) \bullet$$



$$\left( x[0] \quad x[1] \quad x[2] \quad x[3] \quad x[4] \quad x[5] \quad x[6] \quad x[7] \right)^T$$



$X[7]$  measures how much of the above signal component is present in  $x$ .



Sampling Time  $\tau$

Sampling Frequency  $f_s = \frac{1}{\tau}$

Sequence Time Length  $T = N\tau$

1<sup>st</sup> Harmonic Freq  $f_1 = \frac{1}{T} = \frac{1}{N\tau} = \frac{f_s}{N}$





## References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] A “graphical interpretation” of the DFT and FFT, by Steve Mann