Anti-aliasing Prefilter (6B)

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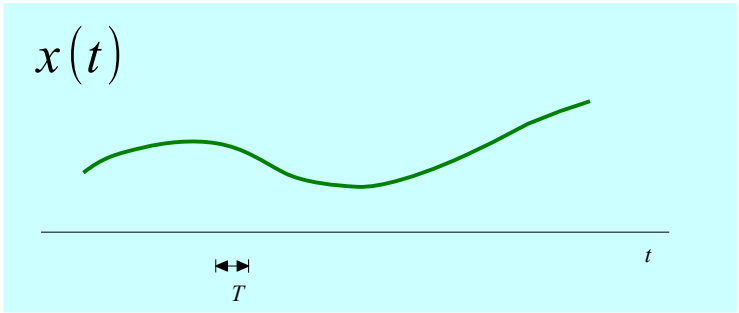
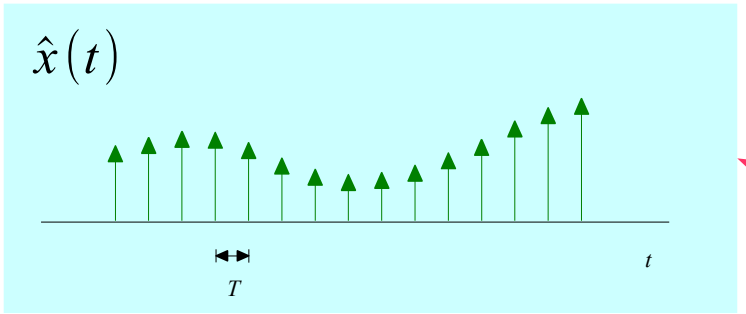
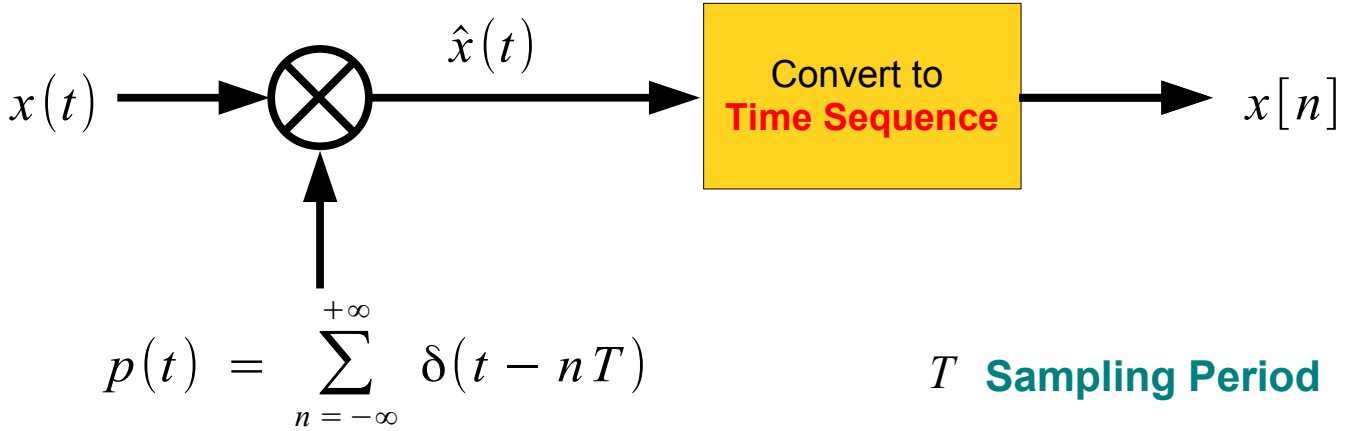
Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice and Octave.

Time Sequence



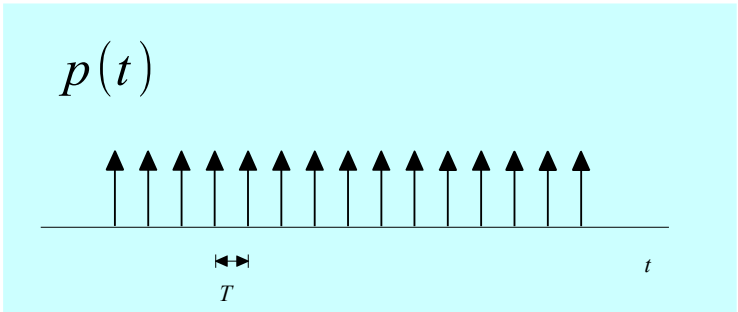


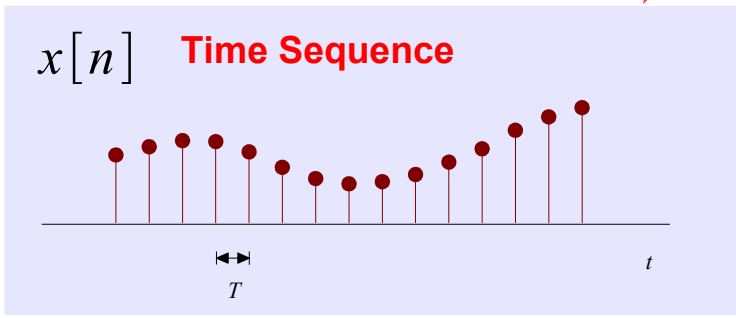


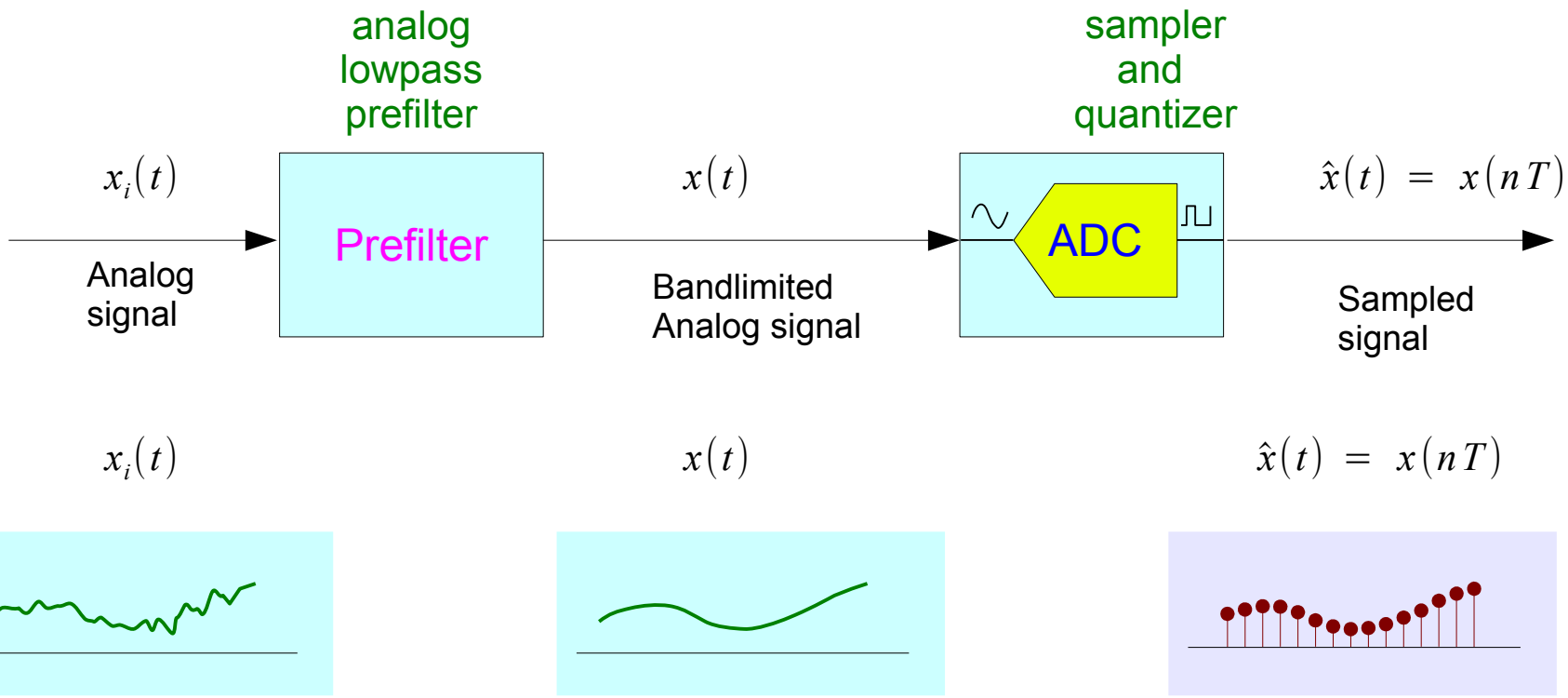
Ideal Sampling

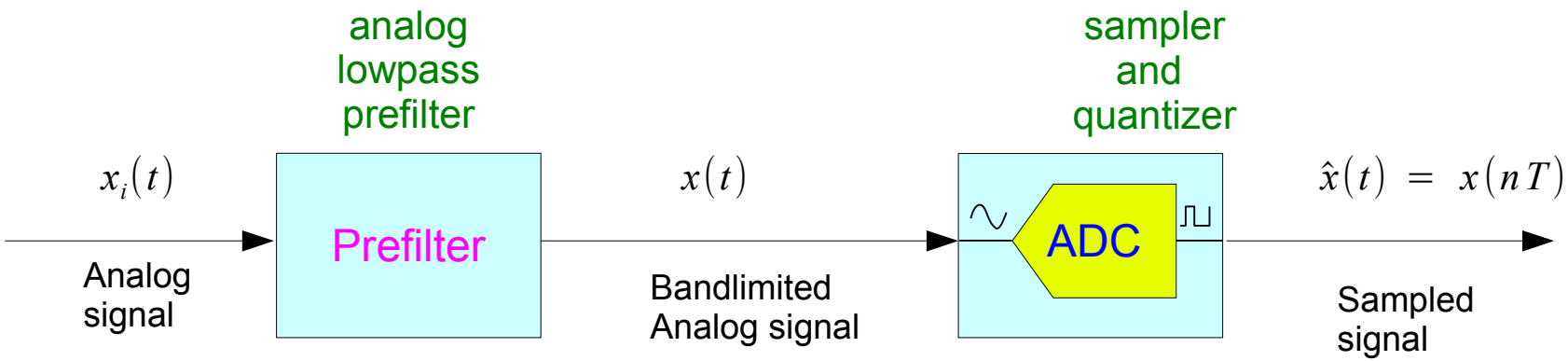
$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

T **Sampling Period**






analog
lowpass
prefilter
sampler
and
quantizer
$x_i(t)$
Prefilter
$x(t)$
ADC
$\hat{x}(t) = x(nT)$
Analog
signal
Bandlimited
Analog signal
Sampled
signal
$x_i(t)$
$x(t)$
$\hat{x}(t) = x(nT)$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lowpass
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$x_i(t)$
Prefilter
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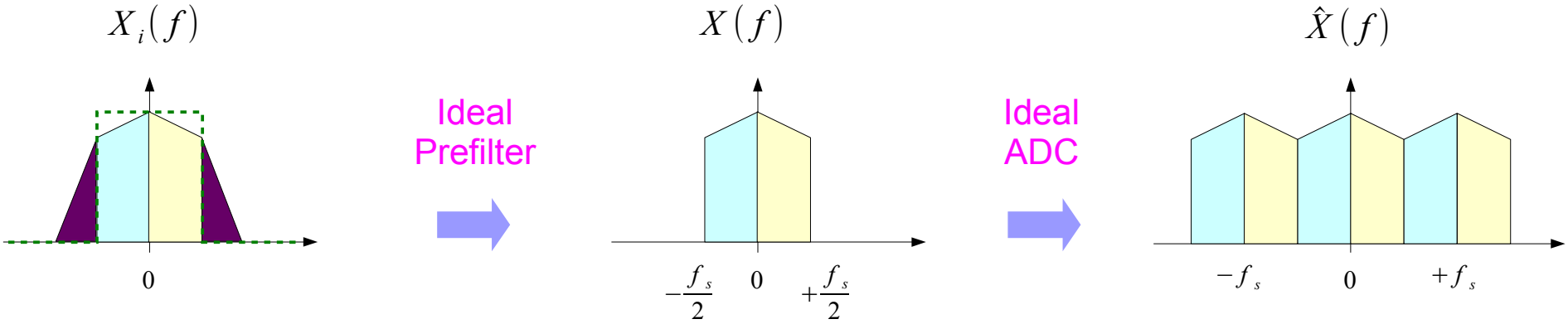
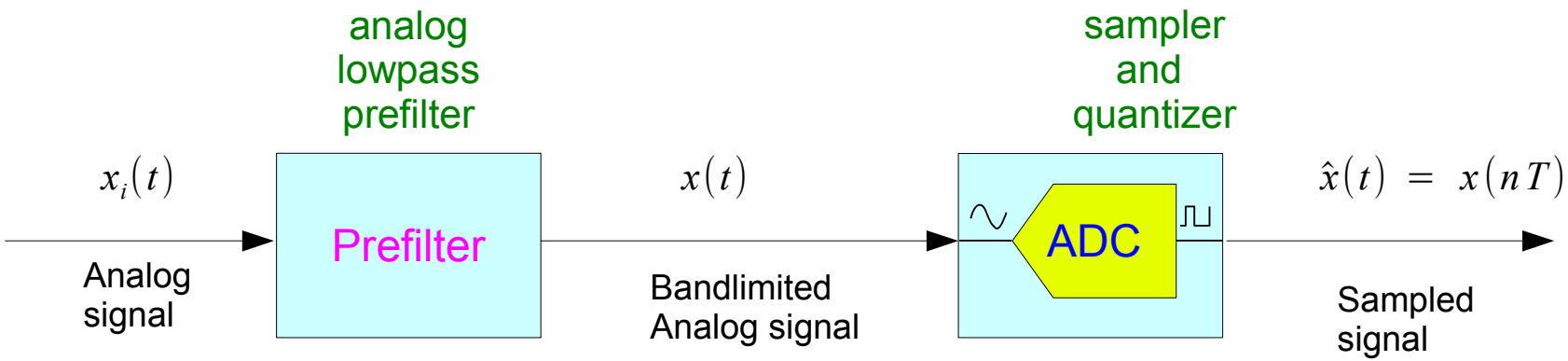
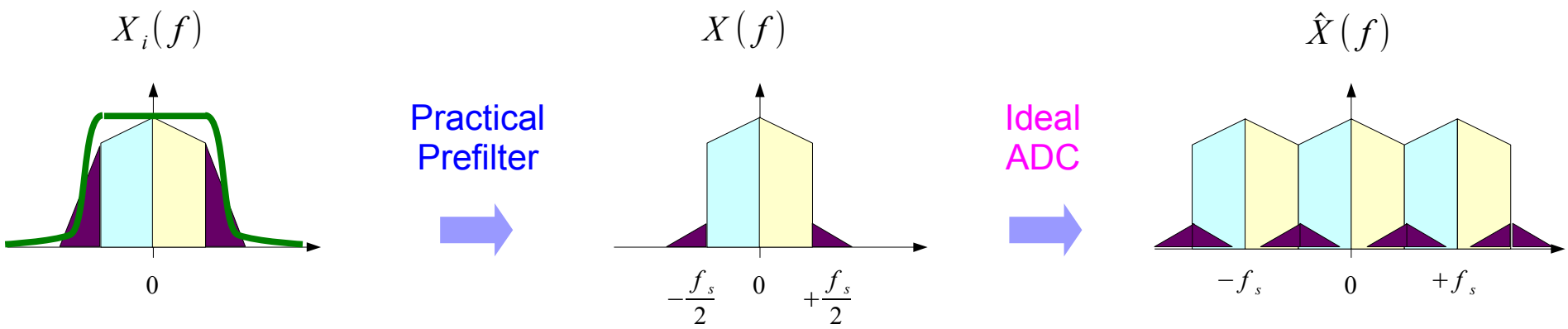
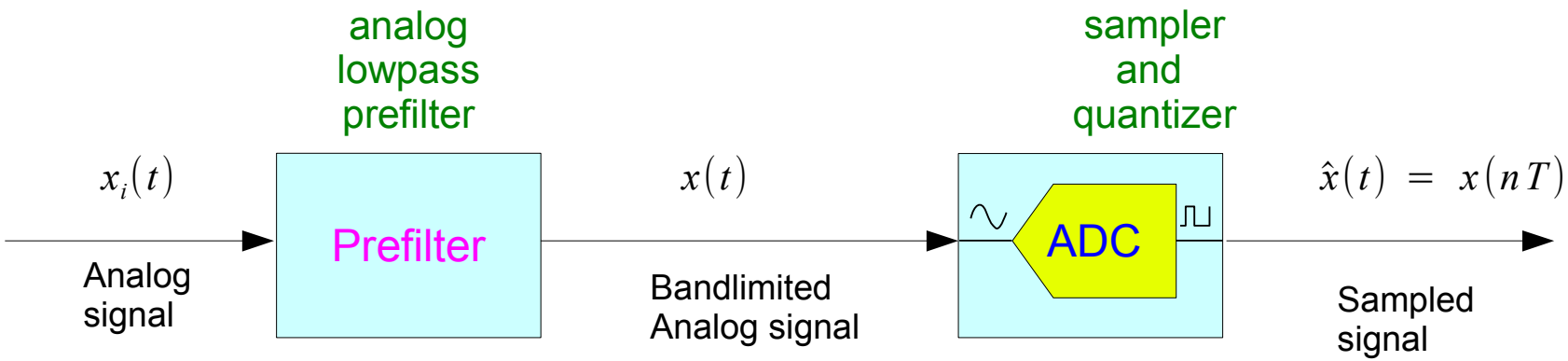
Analog
signal
Bandlimited
Analog signal
Sampled
signal


$X_i(f)$
$X(f)$
$\hat{X}(f)$
Ideal
Prefilter
Ideal
ADC
0
$-\frac{f_s}{2}$
0
$+\frac{f_s}{2}$
$-f_s$
0
$+f_s$


analog
lowpass
prefilter
sampler
and
quantizer
$x_i(t)$
Prefilter
$x(t)$
ADC
$\hat{x}(t) = x(nT)$
Analog
signal
Bandlimited
Analog signal
Sampled
signal


$X_i(f)$
$X(f)$
$\hat{X}(f)$
Practical
Prefilter
Ideal
ADC
0
$-\frac{f_s}{2}$
0
$+\frac{f_s}{2}$
$-f_s$
0
$+f_s$


analog
lowpass
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Analog
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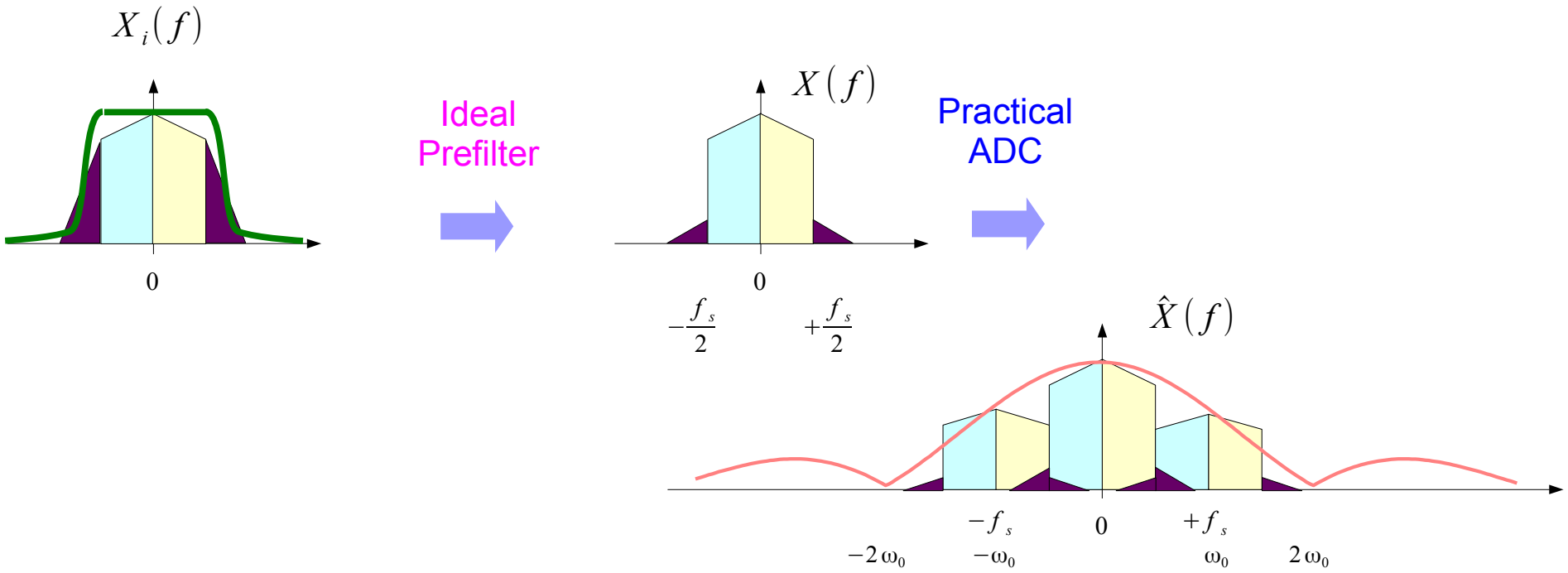

$X_i(f)$
0
Ideal
Prefilter
$X(f)$
Practical
ADC
0
$-\frac{f_s}{2}$
$+\frac{f_s}{2}$
$\hat{X}(f)$
$-f_s$
0
$+f_s$
$-2\omega_0$
$-\omega_0$
ω_0
$2\omega_0$

Not bandlimited signal

Lowpass Filter before sampling

Anti-aliasing Prefilter

Sampling Rate fs

Cutoff Frequency $f_{max} \le \frac{f_s}{2}$

Bandlimited to max freq f_{max}

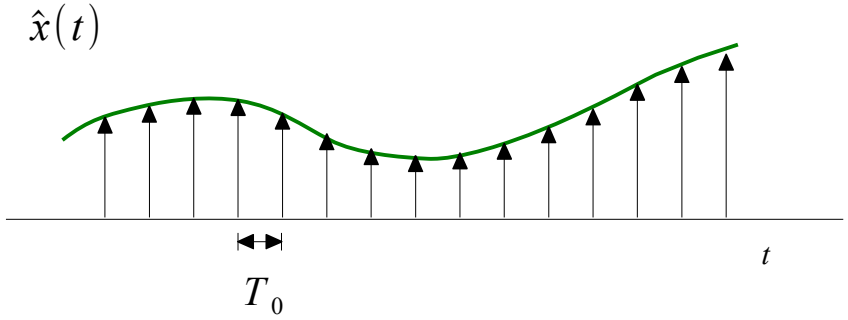
Spectrum replication by fs

Fs must be chosen high enough so that after prefiltering operation
the surviring signal spectrum in the Nyquist interval
contain all the significant frequency component

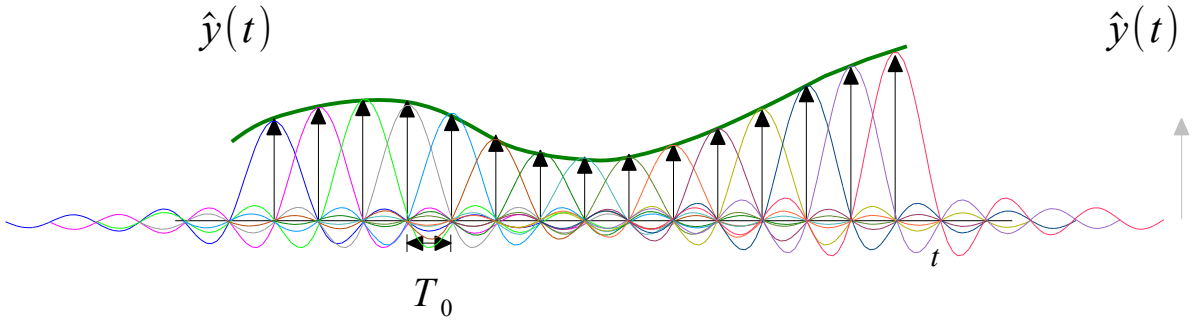
$$\left[-\frac{f_s}{2}, \; +\frac{f_s}{2} \right]$$

Sampler

Ideal Sampling

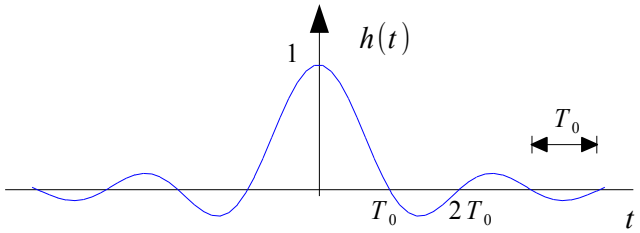


Ideal Reconstructor



CTFT of Reconstructors (1)

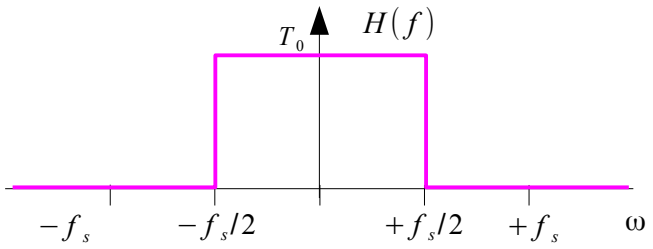
$$t = \pm T_0,\ \pm 2T_0,\ \pm 3T_0,\ \cdots \quad \Rightarrow \quad h(t) = 0$$



$$\frac{1}{T_0} \equiv f_s$$

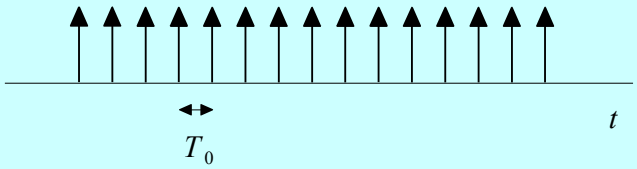
$$h(t) = \frac{\sin(\pi t/T_0)}{\pi t/T_0} = \frac{\sin(\pi f_s t)}{\pi f_s t}$$

CTFT



$$H(f) = \begin{cases} T_0, & |f| \le f_s/2 \\ 0, & otherwise \end{cases}$$

X



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CTFT

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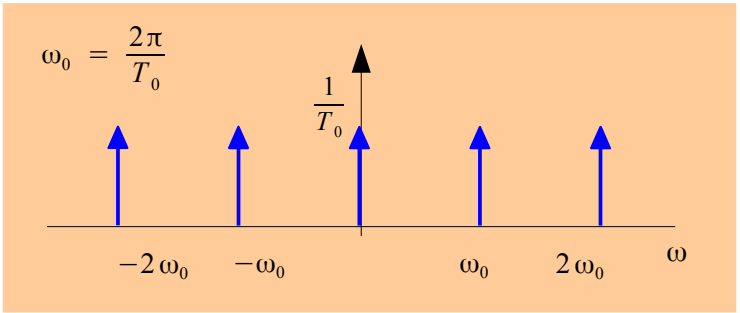

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Sampling (2)

Effect of sampling

$$f, \quad f \pm f_s, \quad f \pm 2f_s, \quad f \pm 3f_s, \quad \cdots$$

Replace the original frequency f
With the replicated set of

Ideal reconstructor

Extracts from a sampled signal
All the frequency components
That lie within Nyquist interval

Removes all frequencies outside that interval

Lowpass filter

$$\left[-\frac{f_s}{2}, \quad +\frac{f_s}{2} \right]$$

Cutoff frequency

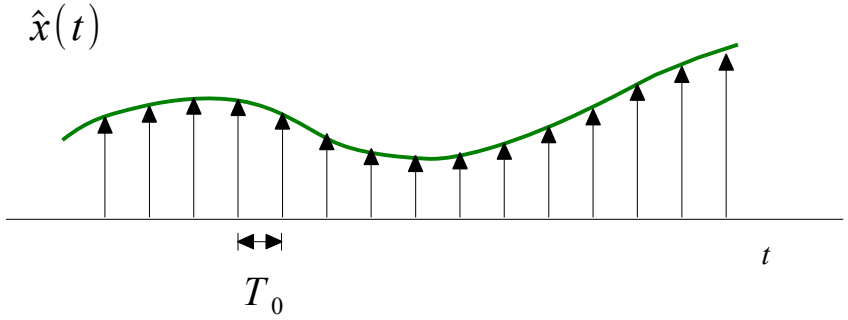
Sampling (2)

Guard Band

$$\delta = f_s - 2 f_{max}$$

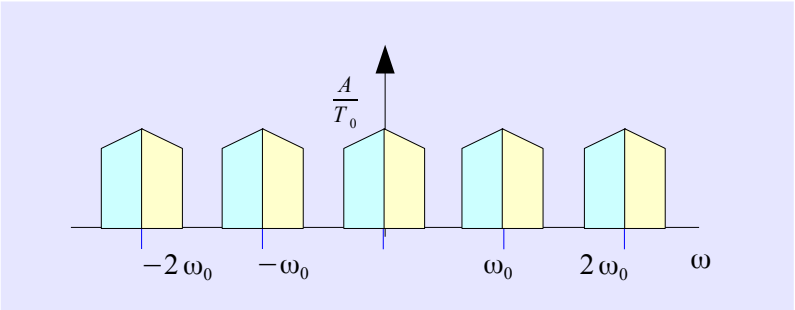
Sampling CTFT

Ideal Sampling

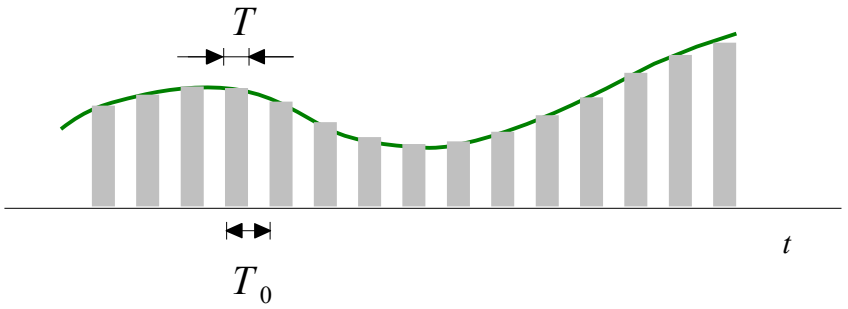


$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT_0)\,\delta(t-nT_0)$$

CTFT

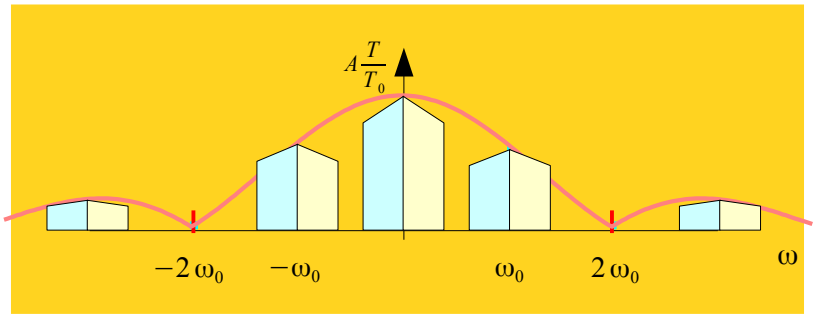


Practical Sampling



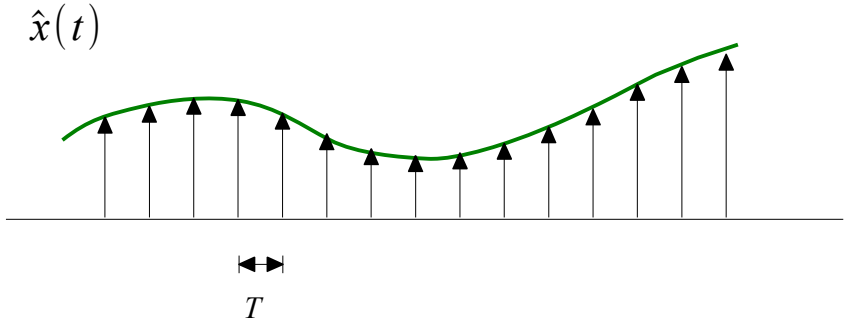
$$\hat{x}(t) \approx \sum_{n=-\infty}^{+\infty} x(nT_0)\,p(t-nT_0)$$

CTFT



Sampler

Ideal Sampling

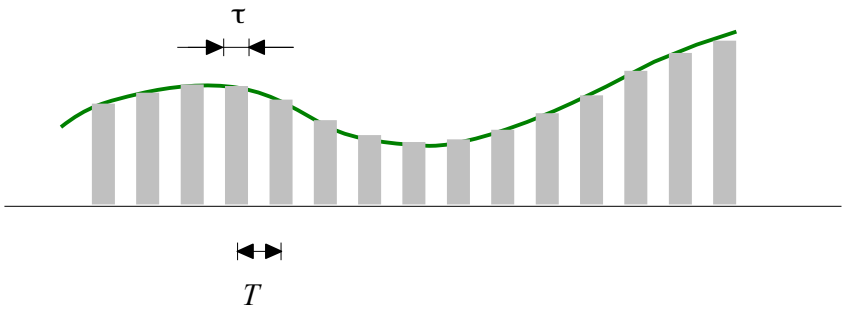


$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

CTFT

$$\hat{X}(f) = \int_{-\infty}^{+\infty} \hat{x}(t)\, e^{-j2\pi f t}\, dt$$

Practical Sampling

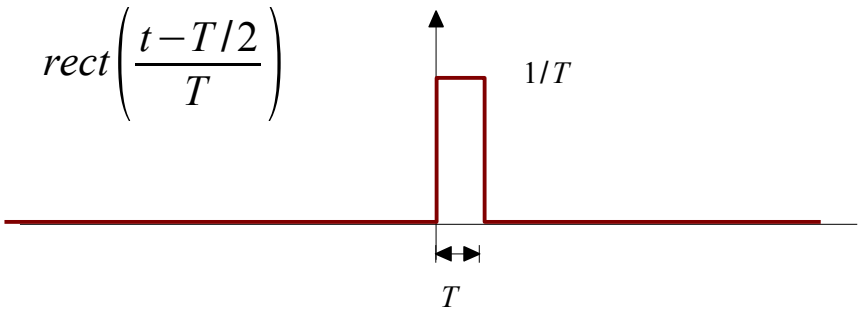


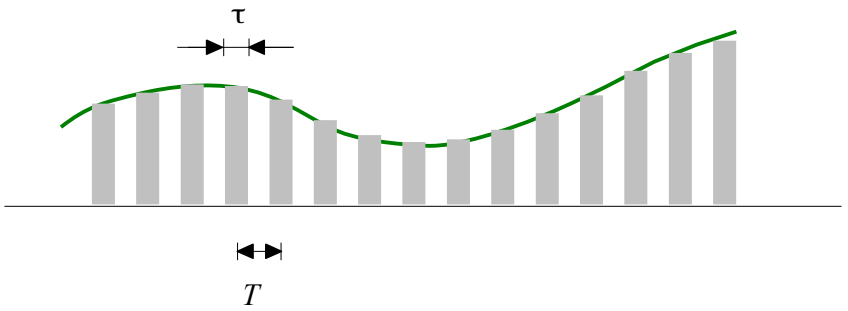
$$\hat{x}(t) \approx \sum_{n=-\infty}^{+\infty} x(nT)\,p(t-nT)$$

CTFT

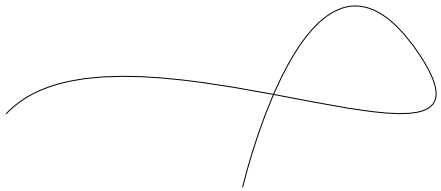
?

Zero Order Hold (ZOH)

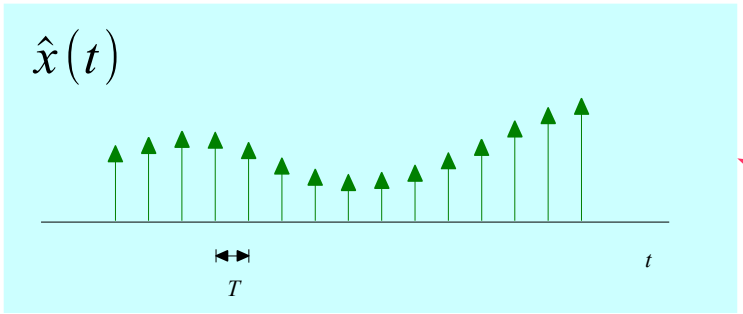


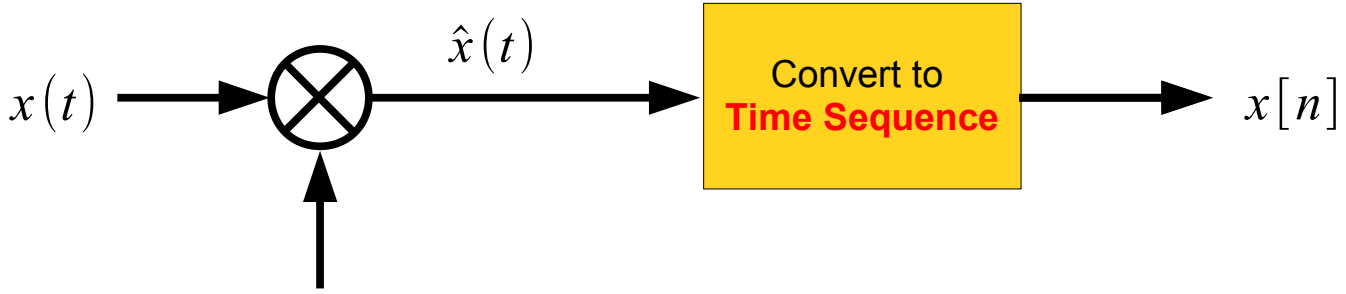


$$x_{ZOH}(t) = \sum_{n=-\infty}^{+\infty} x[n] \cdot rect\left(\frac{t-T/2-nT}{T}\right)$$



Time Sequence

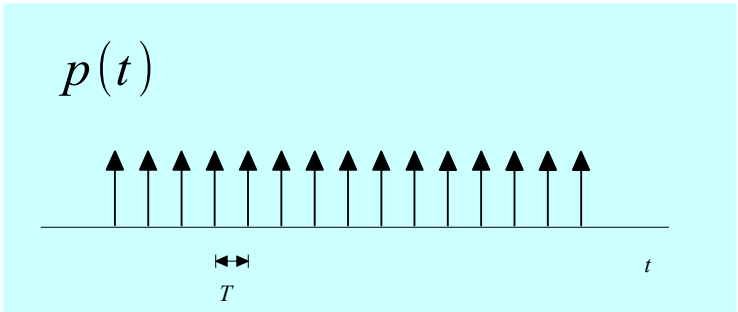




Ideal Sampling

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

T **Sampling Period**



References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A "graphical interpretation" of the DFT and FFT, by Steve Mann
[4] R. G. Lyons, Understanding Digital Signal Processing, 1997
[5] AVR121: Enhancing ADC resolution by oversampling
[6] S.J. Orfanidis, Introduction to Signal Processing
www.ece.rutgers.edu/~orfanidi/intro2sp