

# Vector Functions (1A)

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- Vector Functions
- Motion
- Curvature

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# Vector Valued Functions

Set of points  $(x, y, z)$

Parametric functions  $x = f(t)$   $y = g(t)$   $z = h(t)$

$$(x, y, z) \rightarrow (f(t), g(t), h(t))$$

Vector Valued Function  $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$

$$= f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$$

# Limit of a Vector Function

Vector Valued Function  $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$

Limit of a Vector Valued Function

$$\lim_{t \rightarrow a} \mathbf{r}(t) = \left\langle \lim_{t \rightarrow a} f(t), \lim_{t \rightarrow a} g(t), \lim_{t \rightarrow a} h(t) \right\rangle$$

Limit of a Vector Valued Function

$\mathbf{r}(a)$  is defined

$\lim_{t \rightarrow a} \mathbf{r}(t)$  exists

$$\mathbf{r}(a) = \lim_{t \rightarrow a} \mathbf{r}(t)$$

# Derivative of a Vector Function

Vector Valued Function  $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$

Derivative of a Vector Valued Function

$$\mathbf{r}'(t) = \lim_{\Delta t \rightarrow 0} \frac{\mathbf{r}(t + \Delta t) - \mathbf{r}(t)}{\Delta t}$$

$$\mathbf{r}'(t) = \langle f'(t), g'(t), h'(t) \rangle$$

# Chain Rule of a Vector Function

Vector Valued Function

$$\mathbf{r}(s) = \langle f(s), g(s), h(s) \rangle$$

Scalar Function

$$s = u(t)$$

$$\mathbf{r}(u(t)) = \langle f(u(t)), g(u(t)), h(u(t)) \rangle$$

Derivative of a Vector Valued Function

$$s = u(t) \quad \longrightarrow \quad \frac{ds}{dt} = \frac{du(t)}{dt}$$

$$\frac{d\mathbf{r}}{dt} = \frac{d\mathbf{r}}{ds} \frac{ds}{dt} = \mathbf{r}'(s) u'(t)$$

# Integration of a Vector Function

Vector Valued Function

$$\begin{aligned}\mathbf{r}(t) &= \langle f(t), g(t), h(t) \rangle \\ &= f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}\end{aligned}$$

Limit of a Vector Valued Function

$$\begin{aligned}\int \mathbf{r}(t) dt &= \left\langle \int f(t) dt, \int g(t) dt, \int h(t) dt \right\rangle \\ &= \int f(t) dt \mathbf{i} + \int g(t) dt \mathbf{j} + \int h(t) dt \mathbf{k}\end{aligned}$$

# Integration of a Vector Function

Vector Valued Function

$$\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$$

Displacement

$$\mathbf{v}(t) = \mathbf{r}'(t) = f'(t)\mathbf{i} + g'(t)\mathbf{j} + h'(t)\mathbf{k}$$

acceleration

$$\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t) = f''(t)\mathbf{i} + g''(t)\mathbf{j} + h''(t)\mathbf{k}$$

Speed

$$\begin{aligned}\|\mathbf{v}(t)\| &= \left\| \frac{d\mathbf{r}(t)}{dt} \right\| = \|f'(t)\mathbf{i} + g'(t)\mathbf{j} + h'(t)\mathbf{k}\| \\ &= \sqrt{(f'(t))^2 + (g'(t))^2 + (h'(t))^2} \\ &= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\end{aligned}$$



# Unit Tangent of a Vector Function

Vector Valued Function

$$\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$$

Displacement

$$\mathbf{v}(t) = \mathbf{r}'(t) = f'(t)\mathbf{i} + g'(t)\mathbf{j} + h'(t)\mathbf{k}$$

Unit Tangent

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$$

Arc length

$$s \quad \Rightarrow \quad \frac{d\mathbf{r}}{dt} = \frac{d\mathbf{r}}{ds} \frac{ds}{dt}$$

Unit Tangent

$$\Rightarrow \quad \frac{d\mathbf{r}}{ds} = \frac{\frac{d\mathbf{r}}{dt}}{\frac{ds}{dt}} = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|} = \mathbf{T}(t)$$

# Curvature of a Vector Function

Vector Valued Function

$$\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$$

Unit Tangent

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|} = \frac{d\mathbf{r}}{ds}$$

Curvature

$$\kappa = \left\| \frac{d\mathbf{T}}{ds} \right\|$$

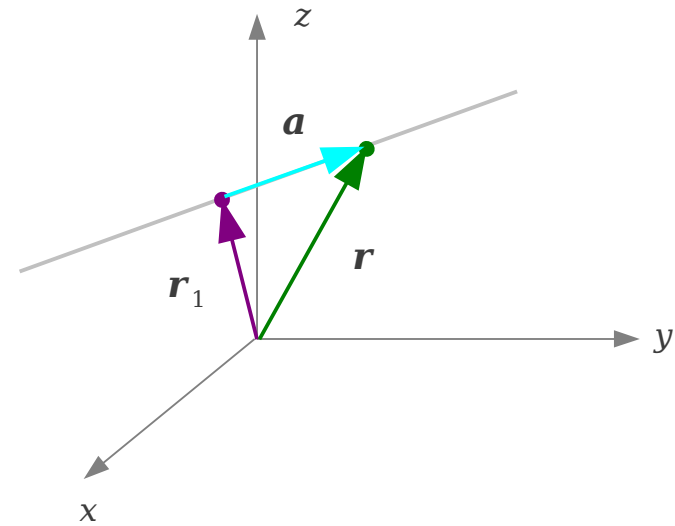
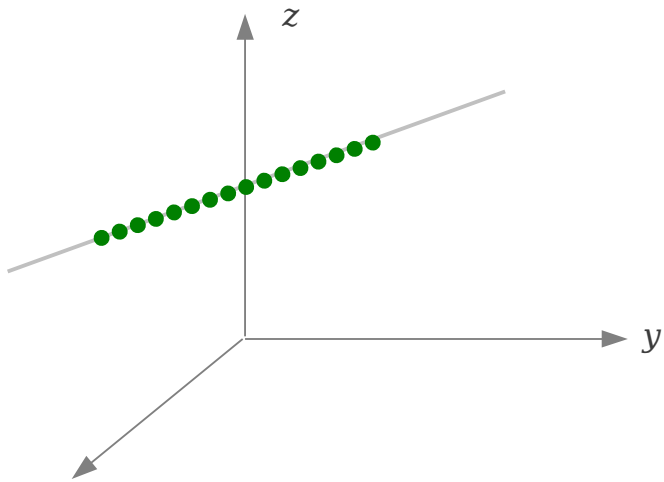
Arc length

$s$

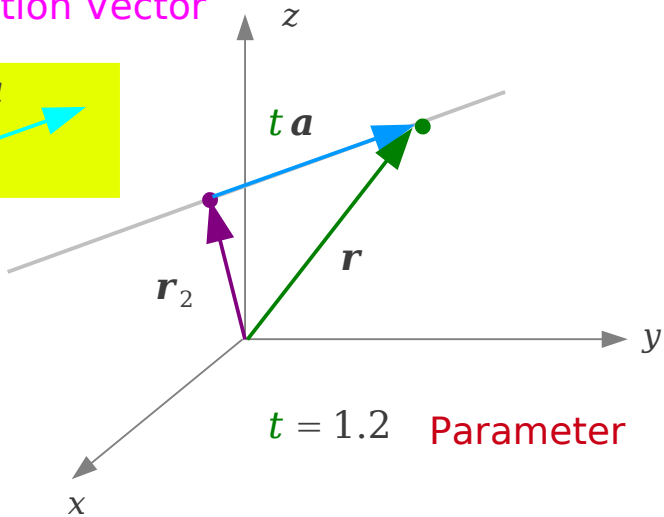
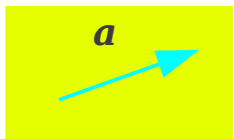
$$\frac{d\mathbf{T}}{dt} = \frac{d\mathbf{T}}{ds} \frac{ds}{dt}$$

$$\frac{d\mathbf{T}}{ds} = \frac{\frac{d\mathbf{T}}{dt}}{\frac{ds}{dt}} = \frac{\mathbf{T}'(t)}{\|\mathbf{r}'(t)\|} = \kappa(t)$$

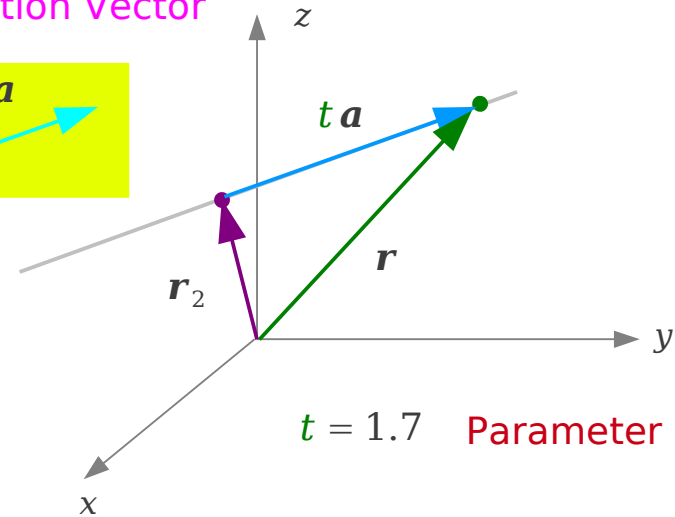
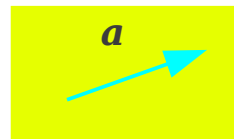
# Line Equations (2)



Direction Vector



Direction Vector



## References

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- [3] M.L. Boas, “Mathematical Methods in the Physical Sciences”
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