Exploratory Factor Analysis Overview 1. What is factor analysis? 2. Assumptions 3. Steps / process 4. Examples 5. Summary 2



A galaxy is like a "factor" within the universe.



The variance of many variables, may be largely explained by some underlying factors (due to the co-relations (or clustering together)) of many variables. 5



Factor analysis...

- is used to identify clusters of intercorrelated variables (called 'factors').
- is a *family* of multivariate statistical techniques for examining correlations amongst variables.
- empirically tests theoretical data structures.
- is commonly used in psychometric instrument development. 7

Purposes

- There are two main applications of factor analytic techniques:
 - **1. Theory development**: Detect structure in the relationships between variables, that is, to classify variables.
 - **2. Data reduction**: Reduce the number of variables to a smaller number of factors.

Purposes: Theory development

- Investigates the underlying correlational pattern shared by the variables in order to test theoretical models e.g., How many personality factors are there? Is intelligence general or multiple?
- The goal is to address a theoretical question (as opposed to calculating factor scores).

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Purposes: Data reduction

- Simplifies data structure by revealing a smaller number of underlying factors (part of psychometrics)
- Helps to eliminate or identify items for improvement:
 - redundant variables
 - unclear variables
 - irrelevant variables
- Leads to calculating factor scores

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History of factor analysis

- Invented by Charles Spearman (1904)
- Usage hampered by onerousness of hand calculation
- Since the advent of computers, usage has thrived, esp. to develop:
 - Theory e.g., determining the structure of personality or intelligence
 - Practice e.g., 10,000s+ of psychological screening & measurement tests

Two main types of FA: Exploratory vs. confirmatory factor analysis

EFA = Exploratory Factor Analysis

 explores & summarises underlying correlational structure for a data set

CFA = Confirmatory Factor Analysis

 tests the correlational structure of a data set against a hypothesised structure and rates the "goodness of fit"













Example: Personality

How many dimensions of personality are there – and what are they?

How could you decide between 3 or 5 personality factors?

Eysenck's 3?

- Extraversion
- Neuroticism
- Psychoticism
- AgreeablenessOpenness

Neuroticism

Extraversion

"Big 5"?

Conscientiousness 20



Example: Essential facial features

Six orthogonal (independent) factors, represent 76.5% of the total variability in facial recognition (in order of importance) (Ivancevic, 2003):

- upper-lip
- eyebrow-position
- nose-width
- eye-position
- eye/eyebrow-length
- face-width

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EFA assumptions

- 1. Garbage-In-Garbage-Out
- 2. Sample size
- 3. Levels of measurement
- 4. Normality
- 5. Linearity
- 6. Outliers
- 7. Factorability

· Use variables that theoretically "go together"

Assumption testing: Sample size

Some guidelines:

- Min.: N > 5 cases per variable • e.g., 20 variables, should have > 100 cases (1:5)
- Ideal: N > 20 cases per variable
 e.g., 20 variables, ideally have > 400 cases (1:20)
- Total *N* > 200 preferable

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Assumption testing: Sample size

Comrey and Lee's (1992) guidelines:

50 = very poor, 100 = poor, 200 = fair, 300 = good, 500 = very good 1000+ = excellent

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	Jour Perso and Psyce	nal of onality Social hology	Journal of Applied Psychology	
Variable Ratio of variable to factors	N	%	N	%
Less than 3:1	1	0.6	I	1.7
3:1	28	17.6	9	15.5
4:1	26	16.4	10	17.2
5:1	14	8.8	10	17.2
6:1	13	8.2	6	10.3
More than 6:1	74	46.5	18	31.0
Unknown	2	1.3	4	6.9

Assumption testing: Sample size

Assumption testing: Level of measurement

• All variables must be suitable for correlational analysis

i.e., they should be ratio/metric data or at least Likert data with several interval levels.

Assumption testing: Normality

- FA is generally robust to minor violation of assumptions of normality.
- If the variables are normally distributed then the solution is enhanced.

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Assumption Testing: Linearity

 Because FA is based on correlations between variables, it is important to check there are linear relations amongst the variables (i.e., check scatterplots)

Assumption testing: Outliers

- FA is sensitive to outlying cases
 - -Bivariate outliers (e.g., check scatterplots)
 - -Multivariate outliers (e.g., Mahalanobis' distance)
- Identify outliers, then remove or transform

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Example factor analysis: Classroom behaviour

- 15 classroom behaviours of highschool children were rated by teachers using a 5-point Likert scale.
- Task: Identify groups of variables (behaviours) that are strongly inter-related & represent underlying factors.

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Classroom behaviour items

- 1. Cannot concentrate
- 2. Curious & enquiring
- 3. Perseveres
- 4. Irritable
- 5. Easily excited
- 6. Patient
- 7. Easily upset
- \leftrightarrow can concentrate
- ↔ little curiousity
- \leftrightarrow lacks perseverance
- ↔ even-tempered
- ↔ not easily excited
- \leftrightarrow demanding
- \leftrightarrow contented
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Classroom behaviour items

8. Control

11. Difficult

13. Lively

12. Restless

14. Purposeful

15. Cooperative

- \leftrightarrow no control
- 9. Relates warmly to others ↔ disruptive 10. Persistent
 - ↔ frustrated
 - ↔ easv
 - \leftrightarrow relaxed
 - \leftrightarrow settled
 - \leftrightarrow aimless
 - \leftrightarrow disputes

	Teachers, for each of the follo over the dot (e.g., ↔) whi TYPIC/	owing ch is i AL beł	paired neares navior (behavi t the st of THIS	oral st taterne S stude	ateme nt thai nt at s	nts, please mark a cross t best describes the school
1.	Cannot concentrate on any particular task; easily distracted	0	0	o	0	0	Can concentrate on any task; not easily distracted
2.	Perseveres in the face of difficult or challenging tasks	0	0	o	0	0	Lacks perseverance; is impatient with difficult or challenging tasks
7.	Persistent, sustained attention span	0	0	o	0	0	Easily frustrated; short attention span
0.	Purposeful activity	0	0	o	0	0	Aimless; impulsive activity

Assumption testing: Factorability

Check the factorability of the correlation matrix (i.e., how suitable is the data for factor analysis?) by one or more of the following methods:

- Correlation matrix correlations > .3?
- Anti-image matrix diagonals > .5?
- Measures of sampling adequacy (MSAs)?
 Bartlett's sig.?

- KMO > .5 or .6?

Assumption testing: Factorability (Correlations)

Are there SOME correlations over .3? If so, proceed with FA

Correlation Matrix

		CONCEN		PERSEV	EVEN-TE		
		TRATES	CURIOUS	ERES	MPERED-	PLACID	
Correlation	CONCENTRATES	1.000	.717	.751	.554	.429	
	CURIOUS	.717 (1.000	.826	.472	.262	
	PERSEVERES	.751	.826	1.000	.507	.311	
	EVEN-TEMPERED	.554	472	.507	1.000	.610	
	PLACID	.429	.262	.311	.610	1.000	
Та	Takes some effort with a large number of						

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variables, but accurate

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Assumption testing: Factorability: Anti-image correlation matrix

- Examine the diagonals on the antiimage correlation matrix
- Consider variables with correlations less than .5 for exclusion from the analysis – they lack sufficient correlation with other variables
- Medium effort, reasonably accurate

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Assumption testing: Factorability: Measures of sampling adequacy

- Global diagnostic indicators correlation matrix is factorable if:
 - -Bartlett's test of sphericity is significant and/or
 - Kaiser-Mayer Olkin (KMO) measure of sampling adequacy > .5 or .6
- Quickest method, but least reliable



Summary: Measures of sampling adequacy

Draw on one or more of the following to help determine the factorability of a correlation matrix:

- 1. Several correlations > .3?
- 2. Anti-image matrix diagonals > .5?
- 3. Bartlett's test significant?
- 4. KMO > .5 to .6? (depends on whose rule of thumb)

Steps / process

- Test assumptions
 Select type of analysis
 Determine no. of factors
- (Eigen Values, Scree plot, % variance explained) 4. Select items

(check factor loadings to identify which items belong in which factor; drop items one by one; repeat)

- 5. Name and define factors
- 6. Examine correlations amongst factors
- 7. Analyse internal reliability
- 8. Compute composite scores

Type of EFA: Extraction method: PC vs. PAF

Two main approaches to EFA:

- Analyses **shared** variance: Principle Axis Factoring (PAF)
- Analyses **all** variance: Principle Components (PC)

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Principal axis factoring (PAF)

- Used to uncover the structure of an underlying set of *p* original variables
- More theoretical
- Analyses only shared variance (i.e. leaves out unique variance)

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Principal components (PC)

- More common
- More practical
- Used to reduce data to a set of factor scores for use in other analyses
- Analyses all the variance in each variable



PC vs. PAF

- Often there is little difference in the solutions for the two procedures.
- If unsure, check your data using both techniques
- If you get different solutions for the two methods, try to work out why and decide on which solution is more appropriate

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Communalities

- Each variable has a communality =

 the proportion of its variance explained by the extracted factors
- Ranges between 0 and 1
- If communality for a variable is low (e.g., < .5, consider extracting more factors or removing the variable)

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Communalities

- High communalities (> .5): Extracted factors explain most of the variance in the variables being analysed
- Low communalities (< .5): A variable has considerable variance unexplained by the extracted factors
 - May then need to extract MORE factors to explain the variance or remove these items from the EFA 51

Communalities		
	Initial	Extraction
behav1 CONCENTRATES	.713	.746
behav2 CURIOUS	.743	.788
behav3 PERSEVERES	.766	.811
behav4 EVEN-TEMPERED	.729	.747
behav5 PLACID	.609	.664
behav6 COMPLIANT	.687	.710
behav7 SELF-CONTROLLED	.730	.749
behav8 RELATES-WARMLY	.605	.660
behav9 SUSTAINED ATTENTION	.776	.803
behav10 COMMUNICATIVE	.657	.674
behav11 RELAXED	.786	.820
behav12 CALM	.737	.786
behav13 PURPOSEFUL ACTIVITY	.764	.798
behav14 COOPERATIVE	.626	.647
behav15 CONTENTED	.595	.621/
Extraction Method: Principal Axis Factoring.		$\overline{}$

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Explained variance

- A good factor solution is one that explains the most variance with the fewest factors
- Realistically, researchers are happy with 50-75% of the variance explained

	1	Initial Eigenva	lues	Extraction	Sums of Squ	ared Loadings	Rotation	
Factor	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	
1	9.355	62.366	62.366	9.094	60.628	60.628	7.80	
2	1.532	10.216	72.583	1.294	8.625	69.253	7.26	
3	.933	6.220	78.802	.635	4.232	73.485	5.73	
4	.467	3.113	81.915			\sim		
5	.378	2.519	84.434	3 fac	tore o	volain 7	3 50/	
6	.344	2.295	86.729	Jiac		Aplalit 1	5.570	
7	.305	2.032	88.761	of the	e varia	ance in t	he	
8	.285	1.902	90.663	itom	- vo	ny ucofu		
9	.262	1.745	92.408	nem	s - ve	ly useiu	1:	
10	.229	1.525	93.933					
11	.219	1.459	95.392					
12	.201	1.340	96.732					
13	.184	1.227	97.959					
14	.159	1.059	99.018					
15	.147	.982	100.000					

Eigen values (EVs)

- Each factor has an EV which indicates the amount of variance each factor accounts for.
- EVs for successive factors have lower values.
- Rule of thumb: Eigen values over 1 are 'stable' (Kaiser's criterion).
- EVs can also be expressed as %s.
- Total of all EVs is the number of variables. Each variable contributes a variance of one. EVs are then allocated to factors according to amount of variance explained.

	Ir	itial Eigenva	lues	Extraction	Sums of Squ	ared Loadings	Rotation
-		% of			% of		
Factor	Totat	Variance	Cumulative %	Total	Variance	Cumulative %	Total
1	9.355	62.366	62.366	9.094	60.628	60.628	7.80
2	1.532	10.216	72.583	1.294	8.625	69.253	7.26
3	.933	6.220	78.802	.635	4.232	73.485	5.73
4	.467	3.113	81.915	The F	No ro	nand	
5	.378	2.519	84.434	The c	20518	ngeu	
6	.344	2.295	86.729	betwe	en 1	6 and 9	35
7	.305	2.032	88.761	-			
8	.285	1.902	90.663	I WO I	actors	s satisfie	d
9	.262	1.745	92.408	Kaise	r's crit	tarion (F	:\/e ~
10	.229	1.525	93.933	T also			
11	.219	1.459	95.392	1) bui	t the tl	nird EV i	is .93
12	.201	1.340	96.732	and it	alen	annoare	to h
13	.184	1.227	97.959	anun	ai30 (appears	10 0
14	.159	1.059	99.018	a use	ful fac	ctor.	
15	147	.982	100.000				



- A line graph of EVs.
- Depicts amount of variance explained by each factor.
- Cut-off: Look for where additional factors fail to add appreciably to the cumulative explained variance.
- 1st factor explains the most variance.
- Last factor explains the least amount of variance.





How many factors?

- A *subjective* process ... Seek to explain maximum variance using fewest factors, considering:
- 1. Theory what is predicted/expected?
- 2. Eigen Values > 1? (Kaiser's criterion)
- 3. Scree Plot where does it drop off?
- 4. Interpretability of last factor?
- 5. Try several different solutions? (consider FA type, rotation, # of factors)
- Factors must be able to be meaningfully interpreted & make theoretical sense?

How many factors?

- Aim for 50-75% of variance explained by 1/4 to 1/3 as many factors as variables/items.
- Stop extracting factors when they no longer represent useful/meaningful clusters of variables.
- Keep checking/clarifying the meaning of each factor – make sure to examine the wording of each item.

Initial solution: Unrotated factor structure

- Factor loadings (FLs) indicate relative importance of each item to each factor.
 - In the initial solution, each factor tries "selfishly" to grab maximum unexplained variance.
 - All variables will tend to load strongly on the 1st factor

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Initial solution -Unrotated factor structure

- Factors are weighted combinations of variables
 A factor matrix
- shows variables in rows and factors in columns



Initial solution -Unrotated factor structure

1st factor extracted:

- Best possible line of best fit through the original variables
- Seeks to explain lion's share of all variance
- A single factor, best summary of the variance in the whole set of items

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Initial solution -Unrotated factor structure

- Each subsequent factor tries to explain the maximum amount of remaining unexplained variance.
- Second factor is orthogonal to first factor - seeks to maximise its own eigen value (i.e., tries to gobble up as much of the remaining unexplained variance as possible)

Vectors (Lines of best fit)



Initial solution: Unrotated factor structure

- Seldom see a simple unrotated factor structure
- Many variables load on 2 or more factors
- Some variables may not load highly on any factors (check: low communality)
- Until the FLs are rotated, they are difficult to interpret.
- Rotation of the FL matrix helps to find a more interpretable factor structure.

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Two basic factor ro	types of otation	Two basic types of factor rotation	
Orthogonal (SPSS Varimax)	Oblique (SPSS Oblimin)	 Orthogonal minimises factor covariation, produces factors which are uncorrelated Oblimin allows factors to covary, allows correlations between factors 	
(69		70

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Why rotate a factor loading matrix?

- After rotation, the vectors (lines of best fit) are rearranged to optimally go through clusters of shared variance
- Then the FLs and the factor they represent can be more readily interpreted

Orthogonal vs. oblique rotations

- Consider purpose of factor analysis
- If in doubt, try both
- Consider interpretability
- Look at correlations between factors in oblique solution
 - if >.3 then go with oblique rotation (>10% shared variance between factors)

Interpretability

- It is dangerous to be driven by factor loadings only – think carefully - be guided by theory and common sense in selecting factor structure.
- You must be able to understand and interpret a factor if you're going to extract it.

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Interpretability

- However, watch out for 'seeing what you want to see' when evidence might suggest a different, better solution.
- There may be more than one good solution! e.g., in personality -2 factor model
 - -5 factor model
 - -16 factor model



Rotated factor matrix - PC Oblimin						
				Component		
			1	2	3	
	RELATES-WARMLY		.920		.153	
0 1 11	CONTENTED		.845			
Sociability	COOPERATIVE		.784	108		
	EVEN-TEMPERED		.682		338	
	COMMUNICATIVE		.596	192	168	
	PERSEVERES		1	938		
Task	CURIOUS			933	.171	
1 uSK	PURPOSEFUL ACTIVITY			839		
Orientation	CONCENTRATES			831	201	
	SUSTAINED ATTENTION			788	181	
	PLACID			•	902	
~	CALM			131	841	
Settledness	RELAXED			314	686	
	COMPLIANT	1	.471		521	
	SELF-CONTROLLED		.400)209	433	
	Extraction Method: Principa	al Co	mponen	t Analysis.		_
	Rotation Method: Oblimin v	vith K	aiser N	ormalization.		7



How many items per factor?

- Bare min. = 2
- Recommended min. = 3
- Max. = unlimited
- More items:

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- \rightarrow \uparrow reliability

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- \rightarrow \uparrow 'roundedness'
- \rightarrow Law of diminishing returns
- Typically = 4 to 10 is reasonable

How do I eliminate items?

A subjective process; consider:

- 1. Size of main loading (min. = .4)
- 2. Size of cross loadings (max. = .3?)
- 3. Meaning of item (face validity)
- 4. Contribution it makes to the factor
- 5. Eliminate 1 variable at a time, then rerun, before deciding which/if any items to eliminate next
- 6. Number of items already in the factor 80

Factor loadings & item selection

Comrey & Lee (1992) guideline for primary (target) factor loadings:

- > .70 excellent
- > .63 very good
- > .55 good
- > .45 fair
- > .32 poor

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Factor loadings & item selection

Cut-off for acceptable loadings:

- Look for gap in loadings e.g., .8 .7
- .6
- .3 .2
- But also consider whether factor can be interpreted above but not below cut-off.

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Other considerations: Normality of items

- Check the item descriptives.
- The more normally distributed the item scores, the better the distribution of the composite scores.
 - e.g. if two items have similar Factor Loadings and Reliability analysis, consider selecting items which will have the least skew and kurtosis.

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Factor analysis in practice

- To find a good solution, consider: -PC/PAF
 - -Varimax/oblimin
- Range of possible factor structures, e.g., for 2, 3, 4, 5, 6, and 7 factors
- Thus, the researcher would normally conduct many initial EFAs before deciding on a probable structure.

Factor analysis in practice

- Eliminate poor items one at a time, retesting the possible solutions
- Check factor structure across subgroups (e.g., gender) if there is sufficient data
- You will probably come up with a different solution from someone else!
- Check/consider reliability analysis (next lecture)

Example: Condom use

- The Condom Use Self-Efficacy Scale (CUSES) was administered to 447 multicultural college students (Barkley & Burns, 2000).
- PC EFA with a varimax rotation.
- Three factors were extracted:
 - 1. Appropriation

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- 2. Sexually Transmitted Diseases
- 3. Partners' Disapproval

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Factor loadings & item selection

Factor 1: Appropriation - acquisition and use	
of a condom ($\alpha = .76$)	FL
I feel confident in my ability to put a condom on myself or my partner	.75
I feel confident I could purchase condoms without feeling embarrassed	.65
I feel confident I could remember to carry a condom with me should I need one	.61
I feel confident I could gracefully remove and dispose of a condom after sexual intercourse	.56
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Factor loadings & item selection

actor 2. Dexually Transmitted Diseases	Factor 2:	Sexually	Transmitted	Diseases
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Stigma associated with STDs ($\alpha = .83$)	FL
I would not feel confident suggesting using condoms with a new partner because I would be afraid he or she would think I've had a past homosexual experience	.72
I would not feel confident suggesting using condoms with a new partner because I would be afraid he or she would think I have a sexually transmitted disease	.86
I would not feel confident suggesting using condoms with a new partner because I would be afraid he or she would think I thought they had a sexually transmitted disease	.80
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Factor loadings & item selection

Factor 3: Partner's reaction - students'

partners' feelings about condoms ($\alpha = 1$	
66)	FL
If I were to suggest using a condom to a partner, I would feel afraid that he or she would reject me	.73
If I were unsure of my partner's feelings about using condoms I would not suggest using one	.65
If my partner and I were to try to use a condom and did not succeed, I would feel embarrassed to try to use one again (e.g. not being able to unroll condom, putting it on backwards or awkwardness)	.58
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Introduction: Summary

- Factor analysis is a family of multivariate correlational data analysis methods for summarising clusters of covariance.
- FA summarises correlations amongst items.
- The common clusters (called factors) are summary indicators of underlying fuzzy constructs.

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Assumptions: Summary

- Sample size
 - 5+ cases per variables (ideally 20+ cases per variable) - N > 200
- Bivariate & multivariate outliers
- Factorability of correlation matrix (Measures of Sampling Adequacy)
- Normality enhances the solution

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Summary: Steps / process

- 1. Test assumptions
- 2. Select type of analysis
- 3. Determine no. of factors (Eigen Values, Scree plot, % variance explained)
- 4. Select items (check factor loadings to identify which items belong in which factor; drop items one by one; repeat)
- 5. Name and define factors
- 6. Examine correlations amongst factors
- 7. Analyse internal reliability | Next
- 8. Compute composite scores | lecture 93

Summary: Types of FA

- PAF: Theoretical data exploration -uses shared variance
- PC: Data reduction -uses all variance
- Consider trying both ways -Are solutions different? Why?

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Summary: Rotation

- Orthogonal (varimax)
 perpendicular vectors
- Oblique (oblimin)
- - angled vectors
- Consider trying both ways
 - Are solutions different? Why?

Summary: Factor extraction

No. of factors to extract?

- Inspect EVs

 look for > 1 or sudden drop (inspect scree plot)
- % of variance explained
 - aim for 50 to 75%
- Interpretability / theory

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