Anti-aliasing Prefilter (6B)

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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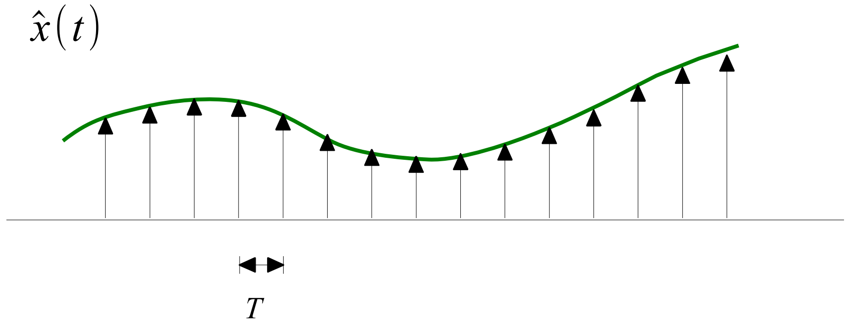
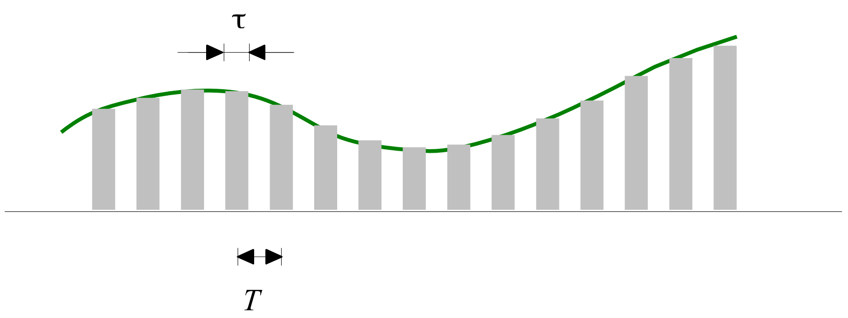
Sampler



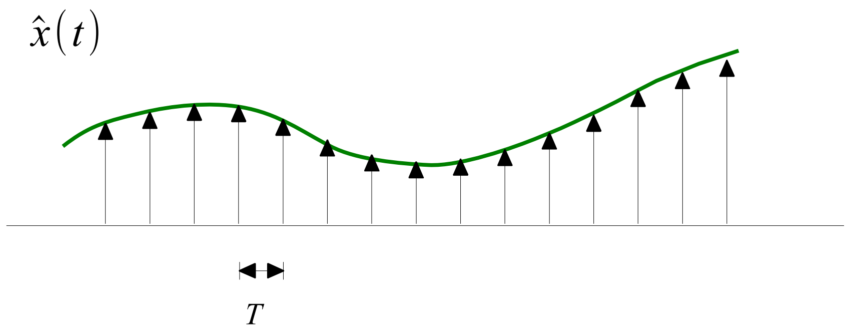


$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

$$\hat{x}(t) \approx \sum_{n=-\infty}^{+\infty} x(nT)\,p(t-nT)$$

$$\hat{X}(f) = \int_{-\infty}^{+\infty} \hat{x}(t)\,e^{-j2\pi f t}\,dt$$

Discrete Time Fourier Transform



$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

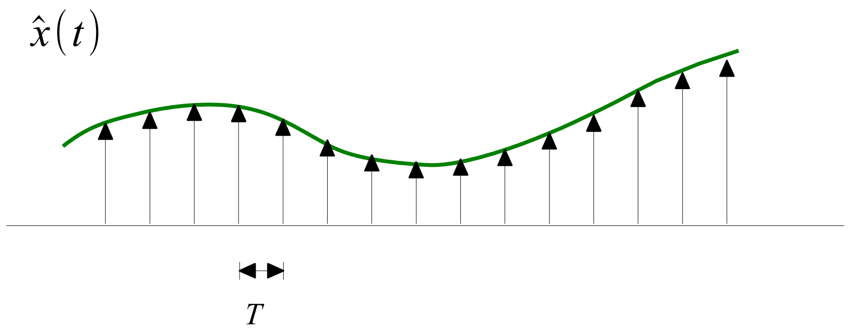
$$\hat{X}(f) = \int_{-\infty}^{+\infty} \hat{x}(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)\, e^{-j2\pi f t}\, dt$$

$$= \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n}\, dt$$

$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n}\, dt$$

Fourier Series



$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT)\,e^{-j2\pi f T n}\,dt$$

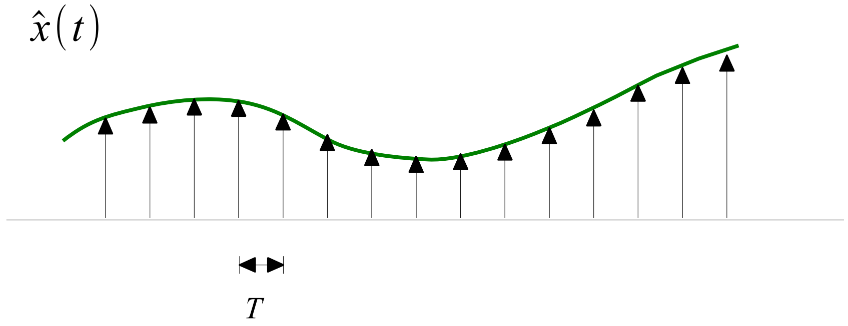
$$x(nT) = \frac{1}{f_s}\int_{-f_s/2}^{+f_s/2} \hat{X}(f)\,e^{+j2\pi f T n}\,df$$

$$= \int_{-\pi}^{+\pi} \hat{X}(\omega)\,e^{+j\omega n}\,\frac{d\omega}{2\pi}$$

$$\omega = 2\pi f / f_s$$

$$\frac{d\omega}{2\pi} = \frac{df}{f_s}$$

Numerical Approximation



$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n}\, dt$$

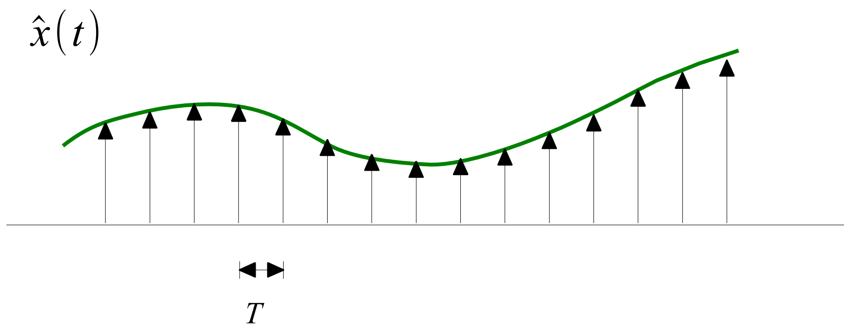
$$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{+j2\pi f t}\, dt$$

$$\approx \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n} \cdot T$$

$$X(f) \approx T\,\hat{X}(f)$$

$$X(f) = \lim_{T\to 0} T\,\hat{X}(f)$$

Spectrum Replication (1)



$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

$$= x(t) \sum_{n=-\infty}^{+\infty} \delta(t-nT) = x(t)\,s(t)$$

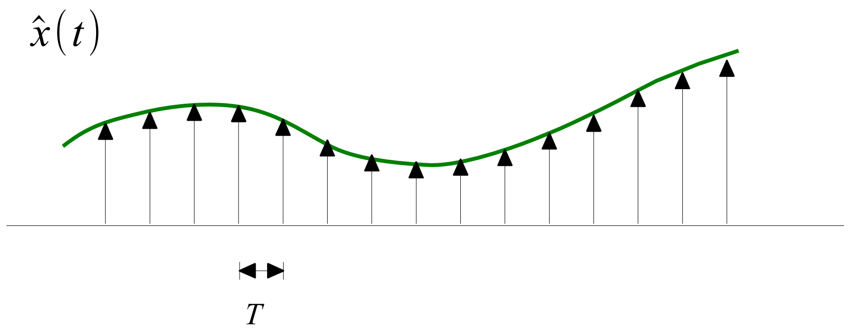
$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n}\, dt$$

$$s(t) = \sum_{n=-\infty}^{+\infty} \delta(t-nT) = \frac{1}{T} \sum_{m=-Infintiy}^{+\infty} e^{+j2\pi m f_s t}$$

$$\hat{x}(t) = x(t)\,s(t) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} x(t)\, e^{+j2\pi m f_s t}$$

$$\hat{X}(f) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} X(f - m f_s)$$

Spectrum Replication (2)



$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n}\, dt$$

$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

$$= x(t) \sum_{n=-\infty}^{+\infty} \delta(t-nT) = x(t)\, s(t)$$

$$\hat{X}(f) = \int_{-\infty}^{+\infty} X(f-f')\, S(f')\, df'$$

$$= \frac{1}{T} \sum_{m=-\infty}^{+\infty} \int_{-\infty}^{+\infty} X(f-f')\,\delta(f'-m f_s)\, df'$$

$$s(t) = \sum_{n=-\infty}^{+\infty} \delta(t-nT) = \frac{1}{T} \sum_{m=-Infintiy}^{+\infty} e^{+j2\pi m f_s t}$$

$$S(f) = \frac{1}{T} \sum_{m=-\infty}^{+\infty} \delta(f-m f_s)$$

$$\hat{x}(t) = x(t)\, s(t) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} x(t)\, e^{+j2\pi m f_s t}$$

$$\hat{X}(f) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} X(f-m f_s)$$

References

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[4] R. G. Lyons, Understanding Digital Signal Processing, 1997
[5] AVR121: Enhancing ADC resolution by oversampling
[6] S.J. Orfanidis, Introduction to Signal Processing
www.ece.rutgers.edu/~orfanidi/intro2sp