Baseband (3A)

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This document was produced by using OpenOffice and Octave.

Pulse & Waveform

Bit Time Slot

Codeword Time Slot

Bits / PCM Word

L : number of quantization levels $L = 2^l$

Bits / Symbol

M: size of a set of message symbols $M = 2^k$

M-ary Pulse Modulation Waveforms

PAM (Pulse Amplitude Modulation)

PPM (Pulse Position Modulation)

PDM (Pulse Duration Modulation)

PWM (Pulse Width Modulation)

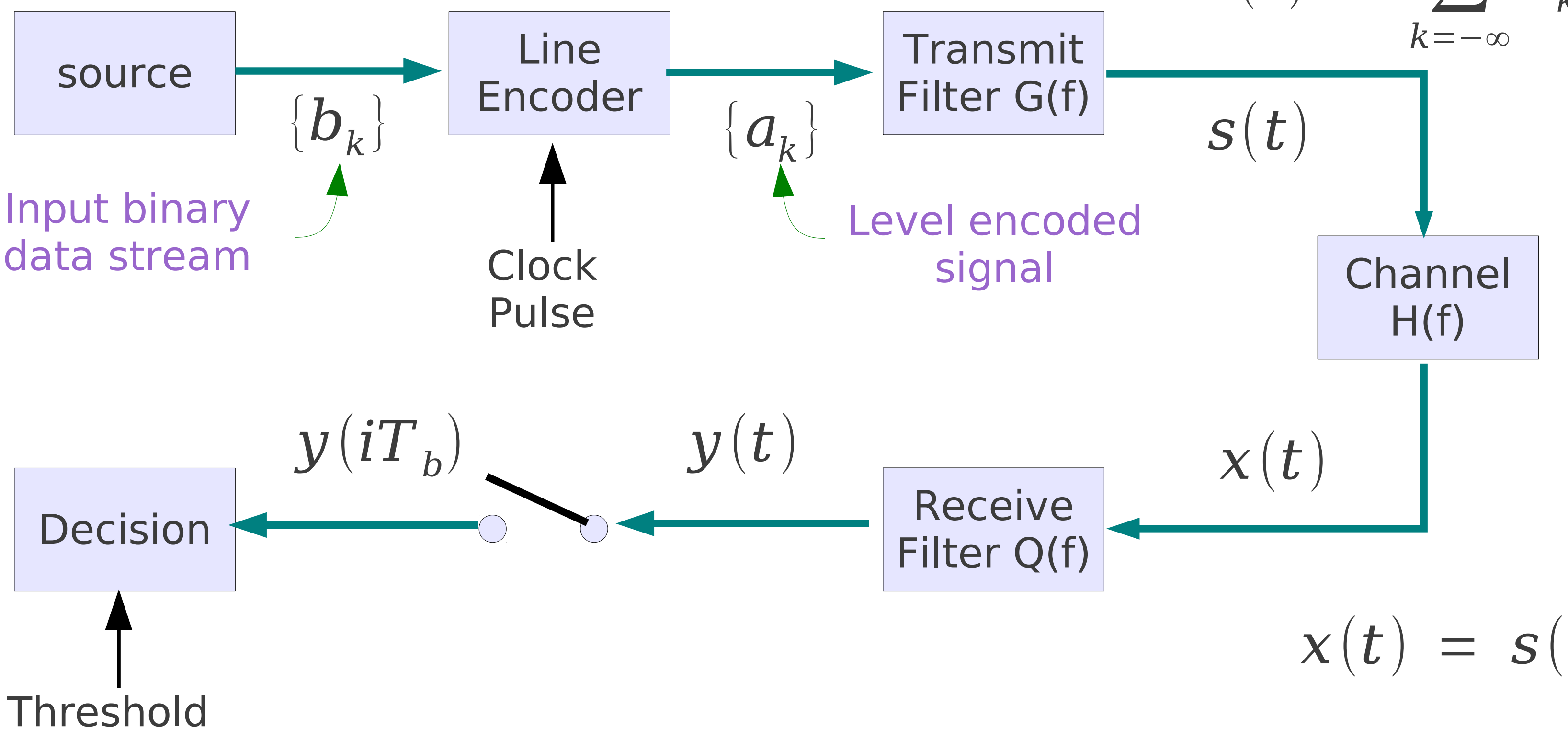
M-ary Pulse Modulation M-ary alphabet set

M-ary PAM : M allowable amplitude levels are assigned to each of the M possible symbol values.

PAM

The amplitude of transmitted pulses is varied in a discrete manner in accordance with an input stream of digital data

$$s(t) = \sum_{k=-\infty}^{+\infty} a_k g(t - kT_b)$$

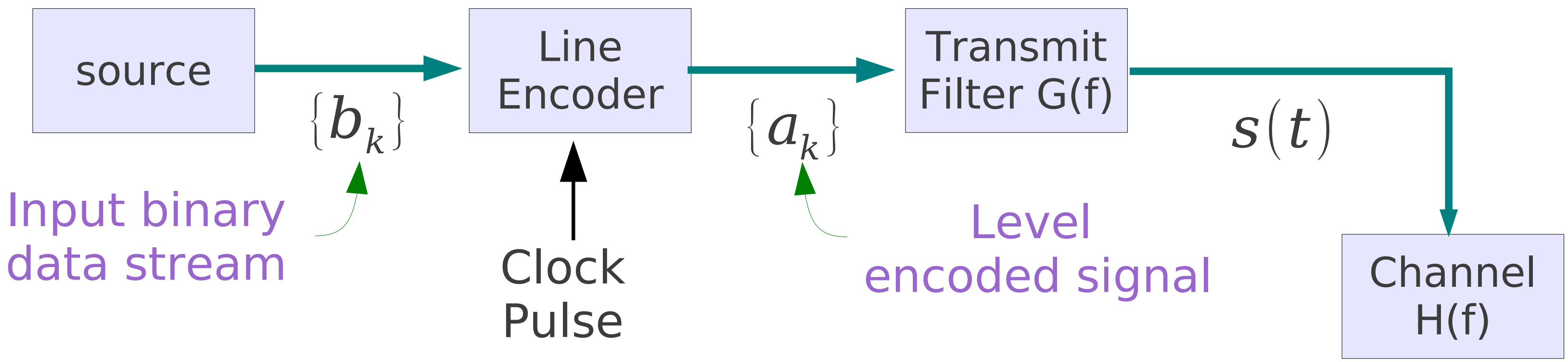


$$x(t) = s(t) * h(t)$$

$$y(t) = x(t) * q(t)$$

M-ary PAM

The amplitude of transmitted pulses is varied in a discrete manner in accordance with an input stream of digital data



M-ary PAM Bit Rate

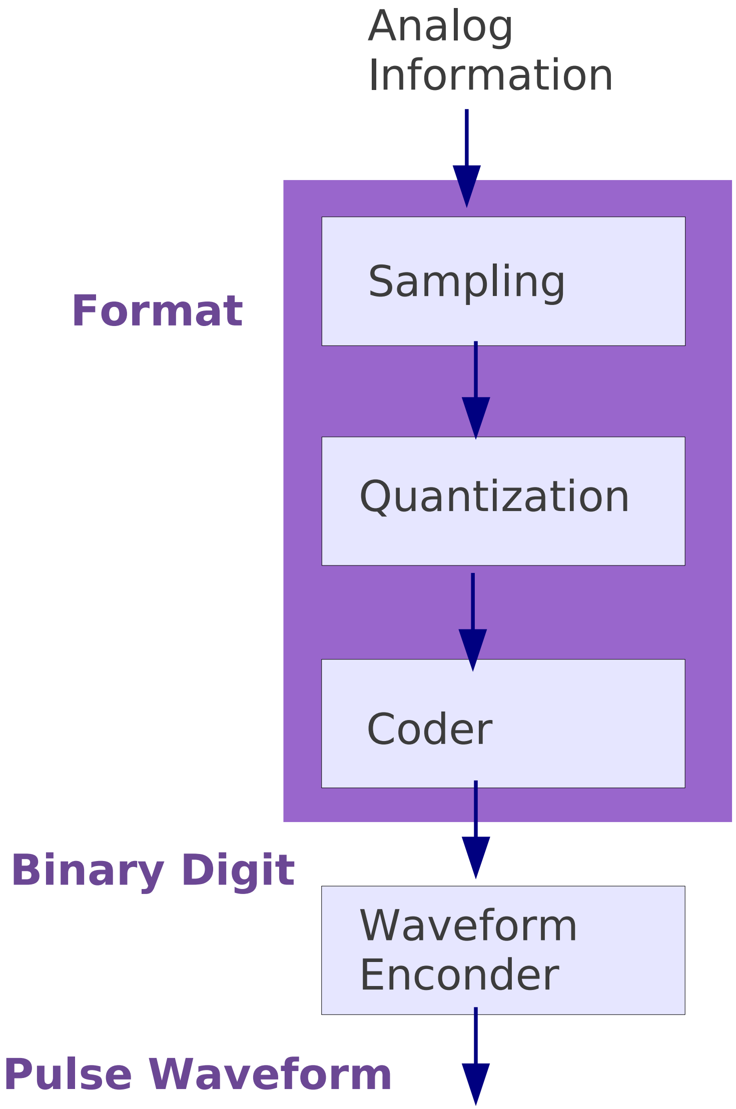
$$T = T_b \log_2 M$$

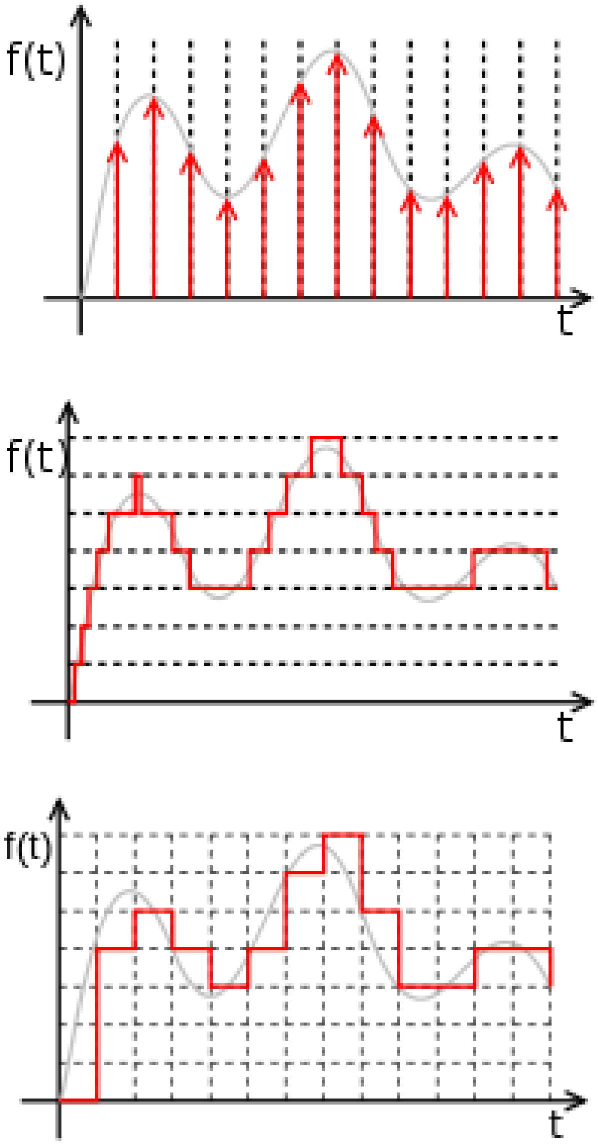
M possible amplitude level (M>2)
M symbols
Transmits sequence of symbols

T: Symbol duration
1/T : Symbol rate

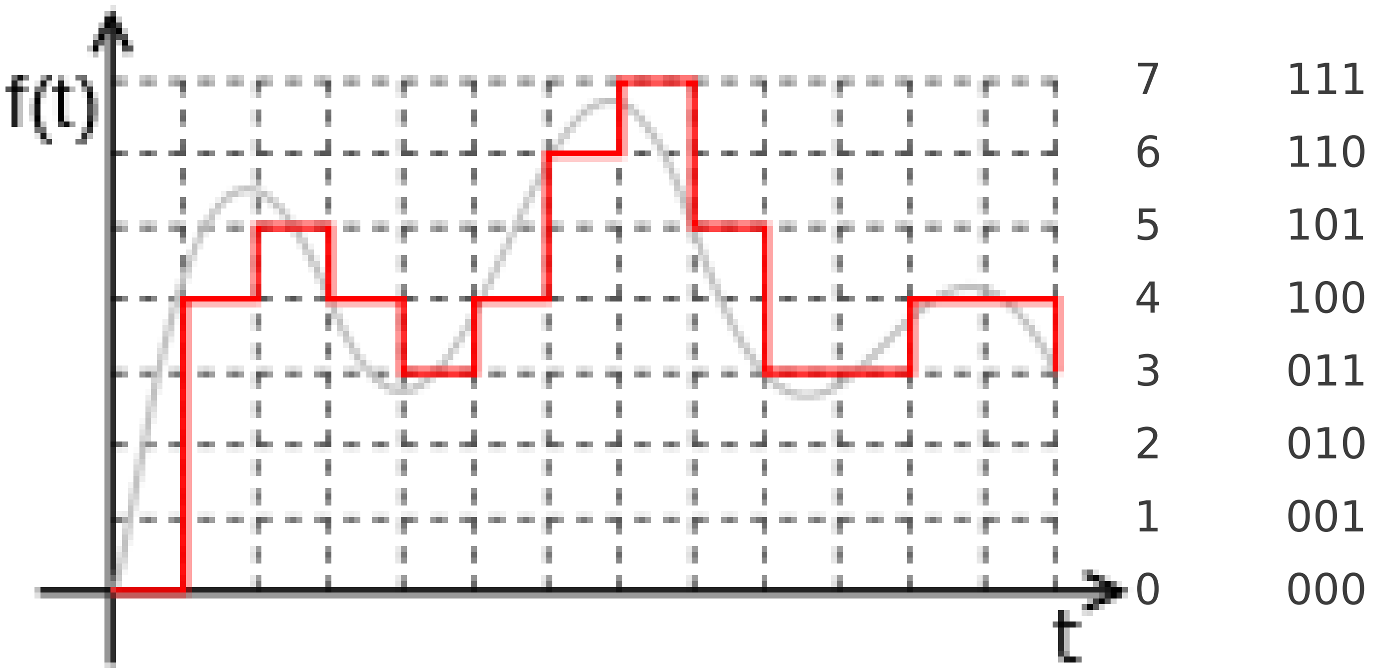
Binary PAM
Tb : Bit duration
1/Tb : Bit rate

Sampling and Quantization





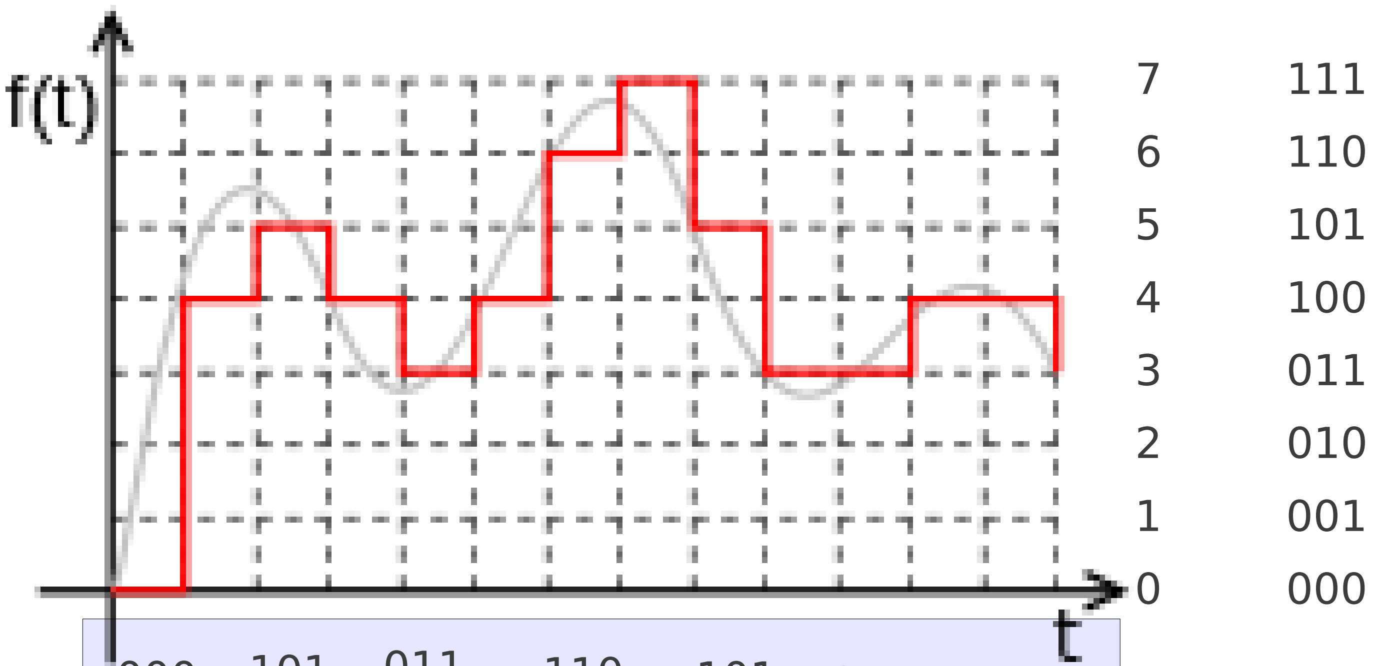
PAM (Pulse Amplitude Modulation)



4-ary PAM **2-bit modulator** **4 levels :** -3, -1, +1, +3 volts

8-ary PAM **3-bit modulator** **8 levels :** -7,-5,-3,-1,+1,+3,+5,+7

PCM (Pulse Coded Modulation)

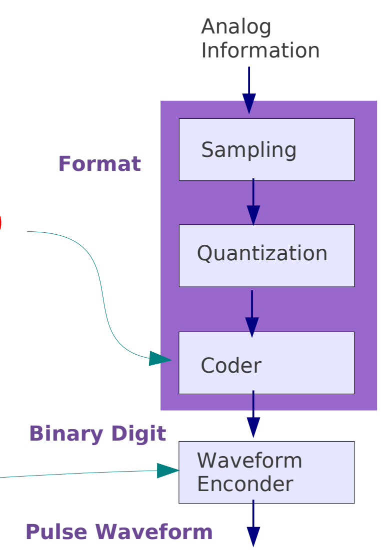


000 101 011 110 101 011 100

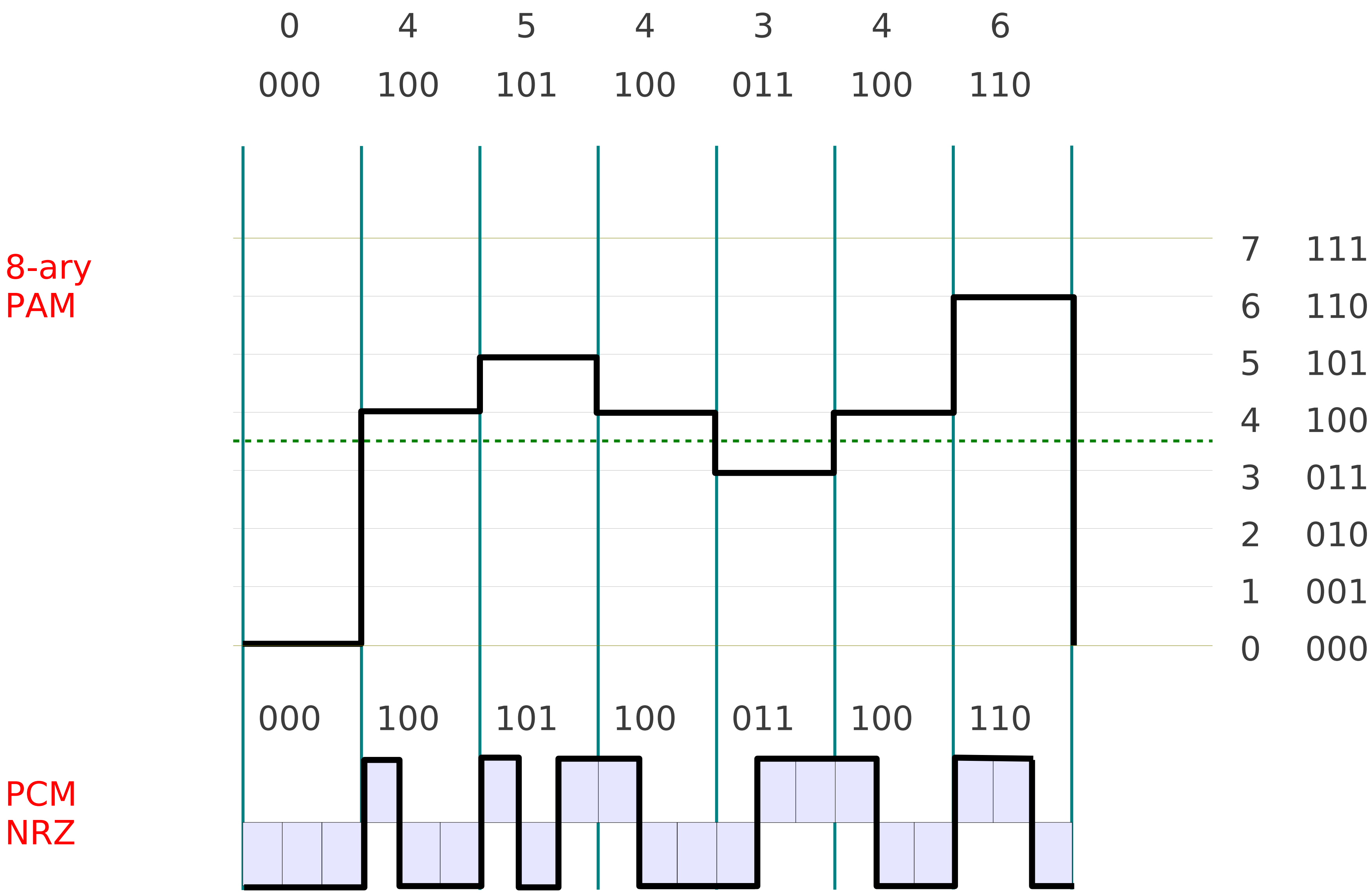
100 100 100 111 011 100

CODED

Line Encoder : NRZ, ...



8-ary PAM vs PCM



Line Encode

Digital BaseBand Modulation

NRZ-L

NRZ-M

NRZ-S

Unipolar RZ

Bipolar RZ

RZ-AMI

Bi-Phi-L

Bi-Phi-M

Bi-Phi-S

Delay Modulation

Dicode NRZ

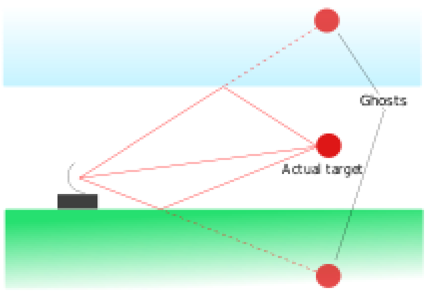
Dicod RZ

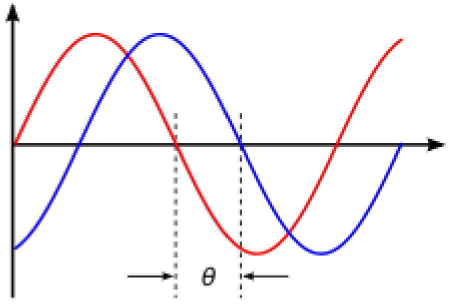
- DC component
- Self-Clocking
- Error Detection
- Bandwidth Compression
- Differential Encoding
- Noise Immunity

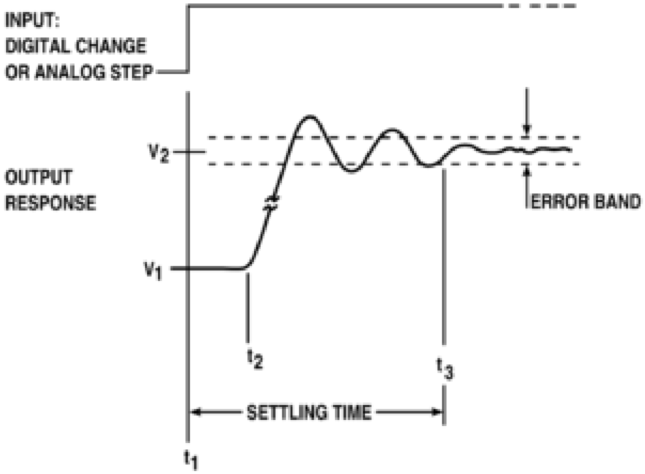
Inter-Symbol Interference

distortion of a signal
in which one symbol interferes with subsequent symbols.
multipath propagation
inherent non-linear filter → long tail, smear, blur …

- adaptive equalization
- error correcting codes

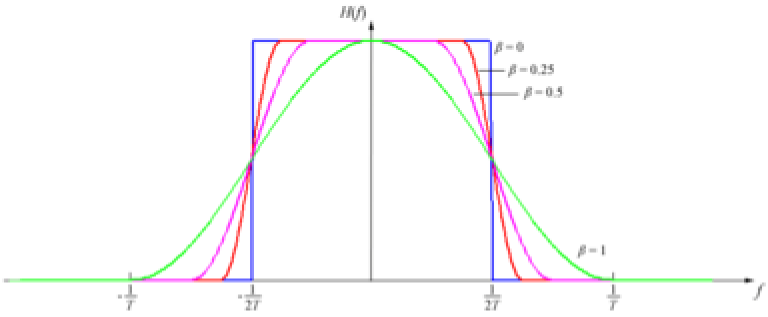




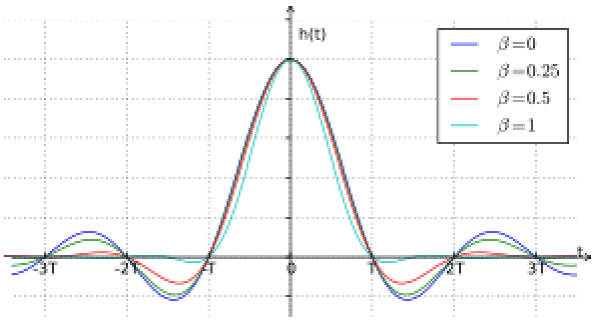


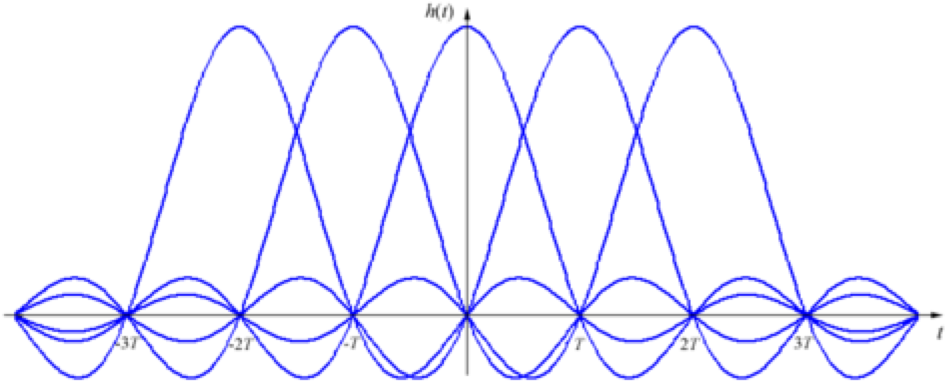
Pulse Shaping

Changing the waveform of transmitted p
Bandwidth constraints
Control ISI (inter-Symbol Interference)



- Sinc Filter
- Raised Cosine Filter
- Gaussian Filter





Signal Space

N-dim orthogonal space

Characterized by a set of N linearly independent functions

Basis functions $\Psi_j(t)$

Independent → not interfering in detection

$$\int_0^T \Psi_j(t)\Psi_k(t)\,dt = K_j\delta_{jk} \qquad 0 \le t \le T \qquad j,k = 1, \cdots, N$$

Kronecker delta functions

$$\delta_{jk} = \begin{cases} 1 & for\; j = k \\ 0 & otherwise \end{cases}$$

N-dim orthonormal space $K_j = 1$

$$E_j = \int_0^T \Psi_j^2(t)\,dt = K_j$$

Linear Combination

Any finite set of waveform $\{s_i(t)\}$ $\quad i = 1, \cdots, M$

Characterized by a set of N linearly independent functions

$$
\begin{array}{lcl}
s_1(t) & = & a_{11}\,\Psi_1(t) + a_{12}\,\Psi_2(t) + \cdots + a_{1n}\,\Psi_N(t) \\
s_2(t) & = & a_{21}\,\Psi_1(t) + a_{22}\,\Psi_2(t) + \cdots + a_{2n}\,\Psi_N(t) \\
\vdots & & \vdots \qquad\qquad\quad \vdots \qquad\qquad\qquad\qquad \vdots \\
s_M(t) & = & a_{M1}\,\Psi_1(t) + a_{M2}\,\Psi_2(t) + \cdots + a_{MN}\,\Psi_N(t)
\end{array}
$$

$$
s_1(t) = \sum_{j=1}^{N} a_{ij}\Psi_j(t) \qquad i = 1, \cdots, M
$$

$$
N \le M
$$

Linear Combination

Any finite set of waveform $\{s_i(t)\}$ $i = 1, \cdots, M$

Characterized by a set of N linearly independent functions

$$s_i(t) = \sum_{j=1}^{N} a_{ij} \Psi_j(t) \qquad i = 1, \cdots, M$$

$$N \le M$$

$$a_{ij} = \frac{1}{K_j} \int_0^T s_i(t) \Psi_j(t)\, dt \qquad i = 1, \cdots, M \qquad 0 \le t \le T$$

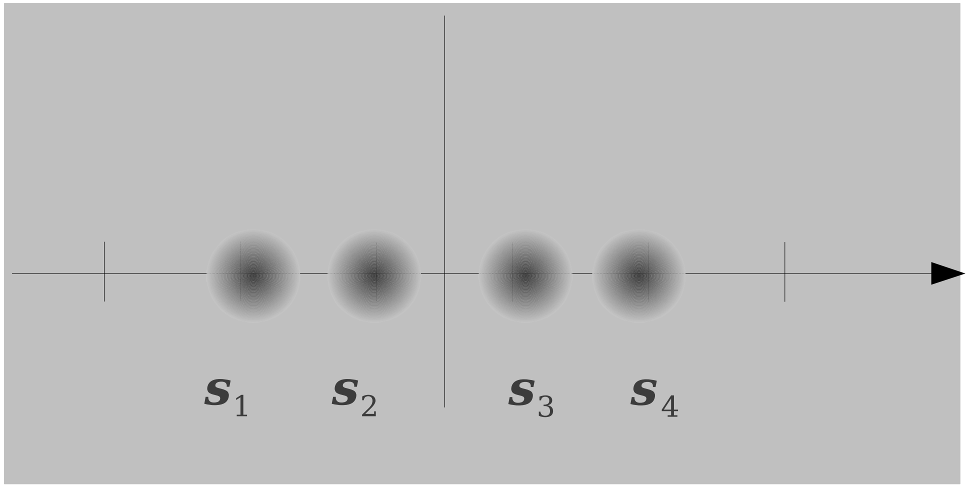
$$j = 1, \cdots, N$$

$$\{s_i(t)\} \Longleftrightarrow \{\mathbf{s}_i\} = \{a_{i1}, a_{i2}, \cdots, a_{iN}\} \qquad i = 1, \cdots, M$$

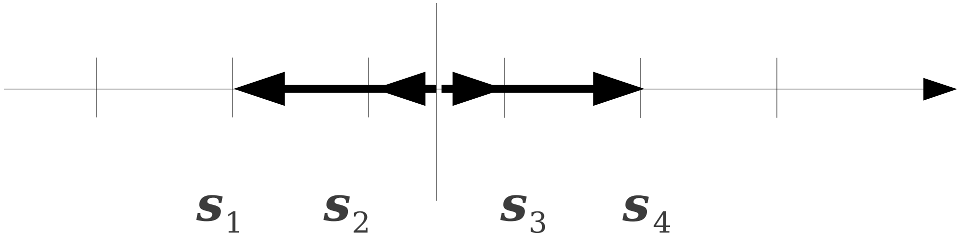
Any finite set of waveform $\{s_i(t)\}$ $i = 1, \cdots, M$

Characterized by a set of N linearly independent functions

$$\{s_i(t)\} \Leftrightarrow \{\mathbf{s}_i\} = \{a_{i1}, a_{i2}, \cdots, a_{iN}\} \quad i = 1, \cdots, M$$



4-ary PAM



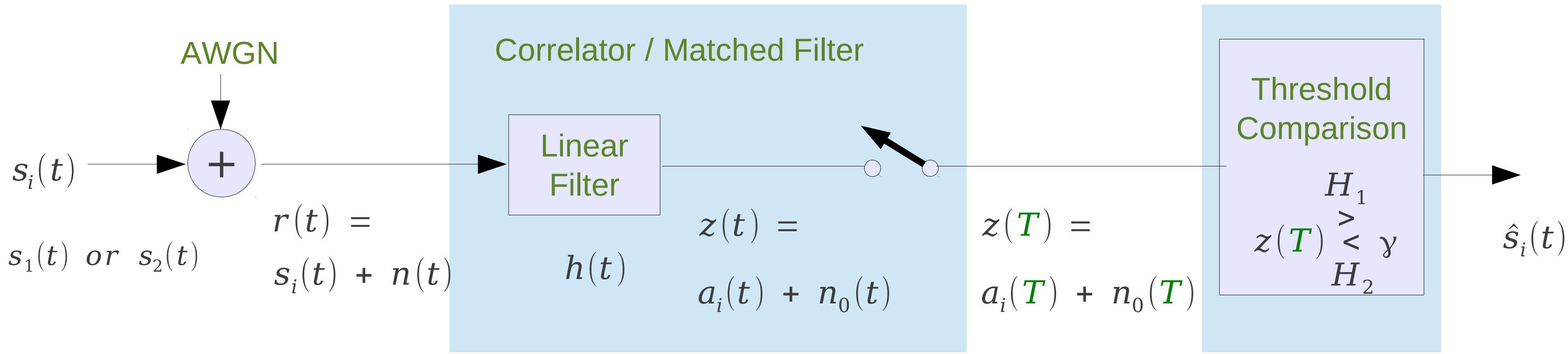
Detection of Binary Signals

Transmitted Signal

$$s_i(t) = \begin{cases} s_1(t) & 0 \le t \le T \quad \text{for a binary } 1 \\ s_2(t) & 0 \le t \le T \quad \text{for a binary } 0 \end{cases}$$

Received Signal

$$r(t) = s_i(t) + n(t) \qquad i = 1,2; \qquad 0 \le t \le T$$



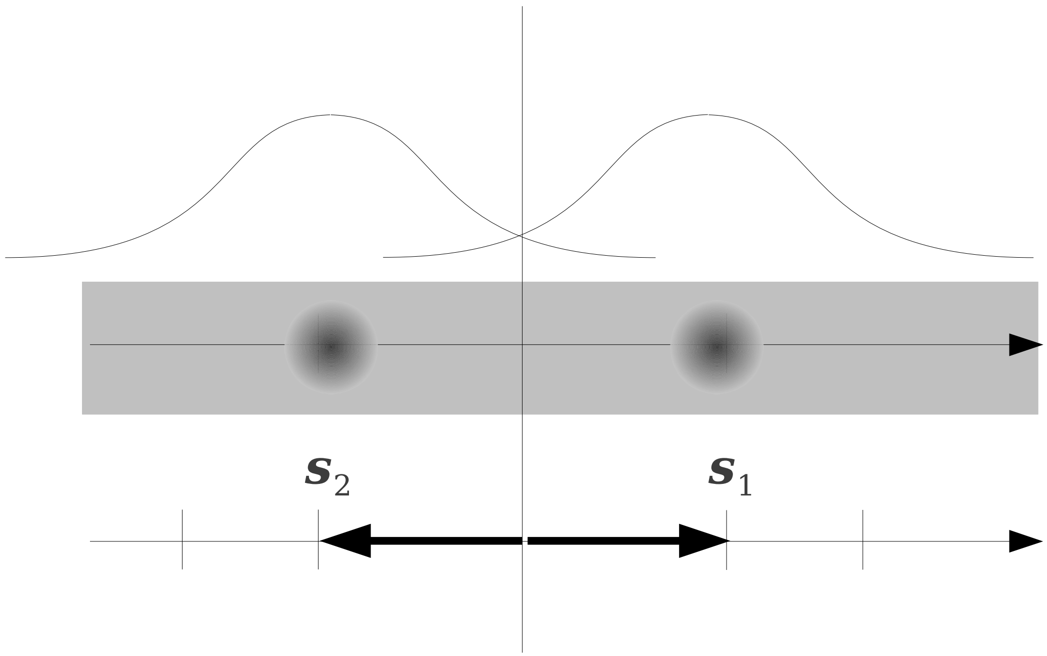
Detection of Binary Signals

$$z(T) = a_i(T) + n_0(T) \quad \Rightarrow \quad z = a_i + n_0$$

$$p(n_0) = \frac{1}{\sigma_0\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{n_0}{\sigma_0}\right)^2\right]$$

$$p(z|s_1) = \frac{1}{\sigma_0\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z-a_1}{\sigma_0}\right)^2\right]$$

$$p(z|s_2) = \frac{1}{\sigma_0\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z-a_2}{\sigma_0}\right)^2\right]$$



$$z(T) \underset{H_2}{\overset{H_1}{\gtrless}} \gamma$$

$$\frac{p(z|s_1)}{p(z|s_2)} \underset{H_2}{\overset{H_1}{\gtrless}} \frac{P(s_2)}{P(s_1)}$$

$$\frac{p(z|s_1)}{p(z|s_2)} \underset{H_2}{\overset{H_1}{\gtrless}} \frac{a_1+a_2}{2} = \gamma_0$$

Signals and Noise

ISI

Gaussian Random Process

$n($

Thermal Noise zero-mean white Gaussian random process

$n(t)$ random function
the value at time t is characterized by
Gaussian probability density function

$\Rightarrow$ $z(t) = a + n(t)$

$$p(n) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{n}{\sigma}\right)^2\right]$$

$\Rightarrow$ $$p(z) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z-a}{\sigma}\right)^2\right]$$

σ^2 variance of n

$\sigma = 1$ normalized (standardized)
Gaussian function

Central Limit Theorem sum of statistically independent random variables
approaches Gaussian distribution
regardless of individual distribution functions

White Gaussian Noise

$n($

Thermal Noise

power spectral density is the same for all frequencies

$$G_n(f) = \frac{N_0}{2}$$ watts / hertz

equal amount of noise power per unit bandwidth

uniform spectral density ➡ White Noise

average power

$$P_n = \int_{-\infty}^{+\infty} \frac{N_0}{2}\, d f = \infty$$

$$P_x^T = \frac{1}{T}\int_{-T/2}^{+T/2} x^2(t)\, dt = \int_{-\infty}^{+\infty} G_x(f)\, d f$$

$$R_n(t) = \frac{N_0}{2}\delta(t) \iff G_n(f) = \frac{N_0}{2}$$

$\delta(t)$ totally uncorrelated, noise samples are independent

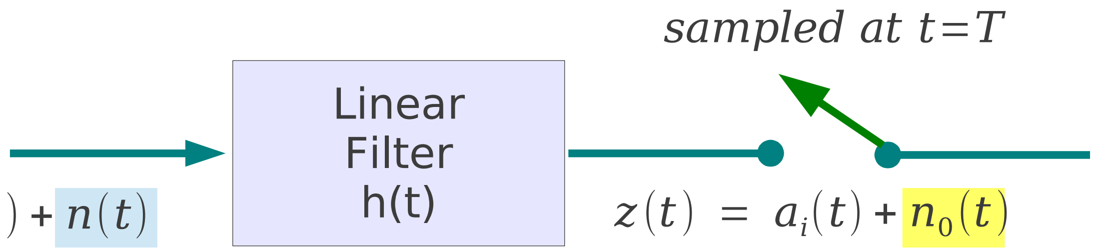
memoryless channel

additive and no multiplicative mechanism

Additive White Gaussian Noise (AWGN)

Matched Filter (1)

<u>to find</u> a filter h(t) that gives max signal-to-noise ratio



$r(t) = s_i(t) + n(t)$ $\quad$ $z(t) = a_i(t) + n_0(t)$

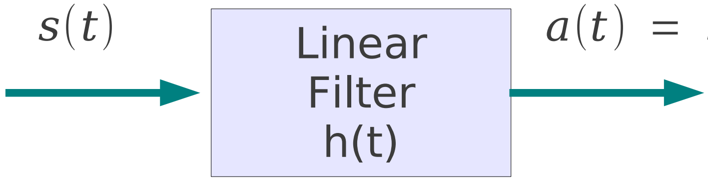
variance of $n_0(t)$ ➡ σ_0^2 avg noise power

$$\frac{\text{instantaneous signal power}}{\text{average noise power}} \Rightarrow \left(\frac{S}{N}\right)_T = \frac{a_i^2}{\sigma_0^2}$$

assume $H_0(f)$ a filter transfer function that maximizes $\left(\frac{S}{N}\right)_T$

Matched Filter (2)

$s(t)$

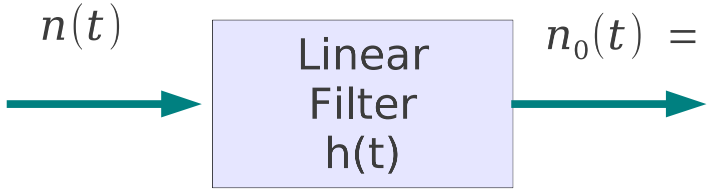


$$a(t) = s(t) * h(t)$$

$S(f)$

$$A(f) = S(f)H(f) \iff a(t) = \int_{-\infty}^{+\infty} S(f)H(f)e^{j2\pi f t}\, df$$

$n(t)$



$$n_0(t) = n(t) * h(t)$$

$$G_n(f) = \frac{N_0}{2}$$

$$G_{n0}(f) = G_n(f)\,|H(f)|^2 = \begin{cases} \dfrac{N_0}{2}|H(f)|^2 & for\ |f| < f_u \\ 0 & otherwise \end{cases}$$

Average output noise power
$$\sigma_0 = \frac{N_0}{2} \int_{-\infty}^{+\infty} |H(f)^2|\, df$$

Matched Filter (3)

instantaneous signal power a_i^2 $\longleftarrow$ $a(t) = \int_{-\infty}^{+\infty} S(f)H(f)e^{j2\pi f t}\,df$

average output noise power $\sigma_0 = \dfrac{N_0}{2}\int_{-\infty}^{+\infty} |H(f)^2|\,df$

$$\left(\frac{S}{N}\right)_T = \frac{a_i^2}{\sigma_0^2} = \frac{\left|\int_{-\infty}^{+\infty} H(f)S(f)e^{+j2\pi f T}\,df\right|^2}{N_0/2\int_{-\infty}^{+\infty}|H(f)|^2\,df}$$

Does not depend on the particular shape of the waveform

Cauchy Schwarz's Inequality

$$\left|\int_{-\infty}^{+\infty} f_1(x)f_2(x)\,dx\right|^2 \le \int_{-\infty}^{+\infty}|f_1(x)|^2\,dx \int_{-\infty}^{+\infty}|f_2(x)|^2\,dx \qquad \text{'=' } \textit{holds when } f_1(x) = k f_2^*(x)$$

$$\left|\int_{-\infty}^{+\infty} H(f)S(f)e^{+j2\pi ft}\,dx\right|^2 df \le \int_{-\infty}^{+\infty}|H(f)|^2\,df \int_{-\infty}^{+\infty}|S(f)e^{+j2\pi f T}|^2\,df$$

$$\left(\frac{S}{N}\right)_T = \frac{a_i^2}{\sigma_0^2} = \frac{\left|\int_{-\infty}^{+\infty} H(f)S(f)e^{+j2\pi f T}\,df\right|^2}{N_0/2\int_{-\infty}^{+\infty}|H(f)|^2\,df} \le \frac{\boxed{\int_{-\infty}^{+\infty}|H(f)|^2\,df}\int_{-\infty}^{+\infty}|S(f)e^{+j2\pi f T}|^2\,df}{N_0/2\boxed{\int_{-\infty}^{+\infty}|H(f)|^2\,df}} = \frac{2}{N_0}\int_{-\infty}^{+\infty}|S(f)|^2\,df$$

Matched Filter (4)

Two-sided power spectral density of input noise ➡ $\frac{N_0}{2}$

Average noise power $\sigma_0 = \frac{N_0}{2} \int_{-\infty}^{+\infty} |H(f)^2| \, df$

$$\left(\frac{S}{N}\right)_T = \frac{a_i^2}{\sigma_0^2} = \frac{\left|\int_{-\infty}^{+\infty} H(f) S(f) e^{+j2\pi f T} \, df\right|^2}{N_0/2 \int_{-\infty}^{+\infty} |H(f)|^2 \, df}$$

Cauchy Schwarz's Inequality

$$\left(\frac{S}{N}\right)_T \leq \frac{2}{N_0} \int_{-\infty}^{+\infty} |S(f)|^2 \, df$$

input signal energy

$$max \left(\frac{S}{N}\right)_T = \frac{2}{N_0} \int_{-\infty}^{+\infty} |S(f)|^2 \, df = \frac{2E}{N_0}$$

power spectral density of input noise

does not depend on the particular shape of the waveform

Matched Filter (5)

$$\left| \int_{-\infty}^{+\infty} H(f)\, S(f)\, e^{+j2\pi ft}\, dx \right|^2 df \le \int_{-\infty}^{+\infty} |H(f)|^2\, df \int_{-\infty}^{+\infty} |S(f)\, e^{+j2\pi f T}|^2\, df$$

$$\left(\frac{S}{N}\right)_T \le \frac{2}{N_0} \int_{-\infty}^{+\infty} |S(f)|^2\, df$$

$$max \left(\frac{S}{N}\right)_T = \frac{2}{N_0} \int_{-\infty}^{+\infty} |S(f)|^2\, df = \frac{2E}{N_0}$$

when complex conjugate relationship exists

$$H(f) = H_0(f) = k\, S^*(f)\, e^{-j2\pi f T}$$

$$\Leftrightarrow \quad h(t) = h_0(t) = \begin{cases} k\, s(T-t) & 0 \le t \le T \\ 0 & elsewhere \end{cases}$$

$H_0(f)$ a filter transfer function that maximizes $\left(\frac{S}{N}\right)_T$

Matched Filter (6)

$H_0(f)$ a filter transfer function that maximizes $\left(\frac{S}{N}\right)_T$

$$H(f) = H_0(f) = k S^*(f) e^{-j2\pi f T}$$

$\Leftrightarrow$

$$h(t) = h_0(t) = \begin{cases} k s(T-t) & 0 \le t \le T \\ 0 & elsewhere \end{cases}$$

impulse response : delayed version of the mirror image of the signal waveform

$r(t)$ → Linear Filter h(t) → $z(t) = r(t) * h(t)$

$$z(t) = r(t) * h(t) = \int_0^t r(\tau) h(t-\tau)\, d\tau$$

$$= \int_0^t r(\tau) s(T-(t-\tau))\, d\tau$$

$$= \int_0^t r(\tau) s(T-t+\tau)\, d\tau$$

$$z(T) = \int_0^t r(\tau) s(\tau)\, d\tau$$

Power spectral density of input noise

Time Averaging and Ergodicity

Autocorrelation of Random and Power Signals

Time Averaging and Ergodicity

References

[1] http://en.wikipedia.org/
[2] http://planetmath.org/
[3] B. Sklar, "Digital Communications: Fundamentals and Applications"