

Downsampling (4B)

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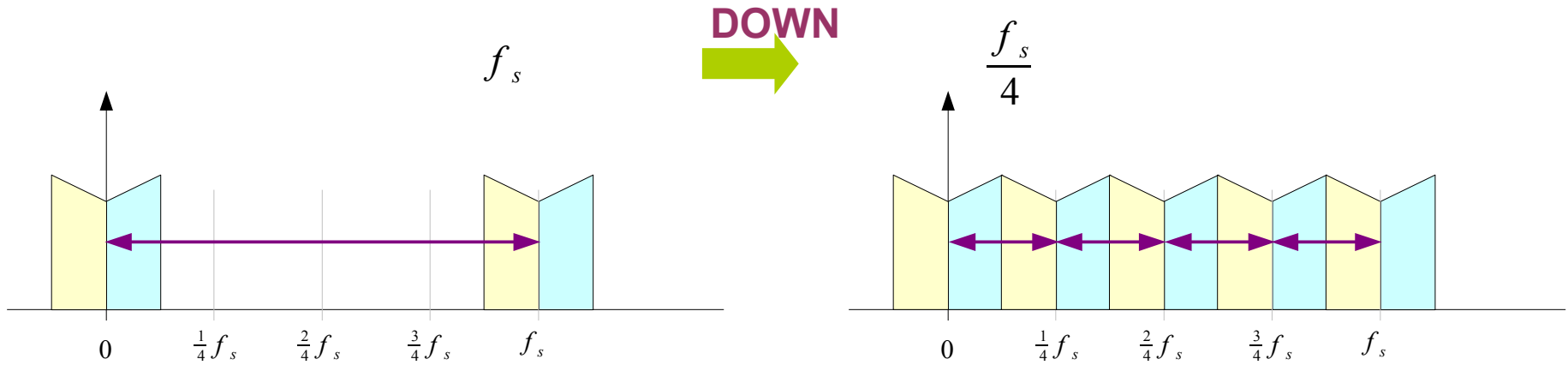
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Band-limited Signal

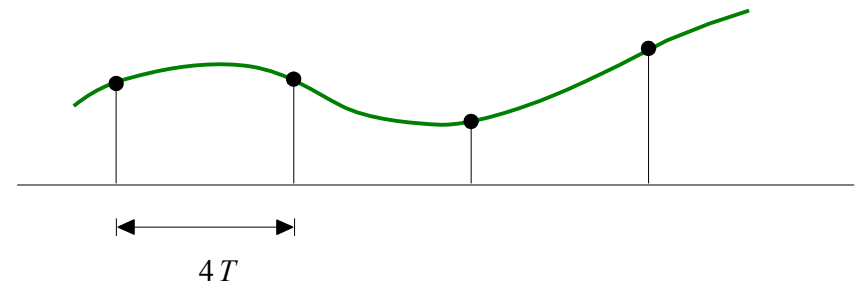
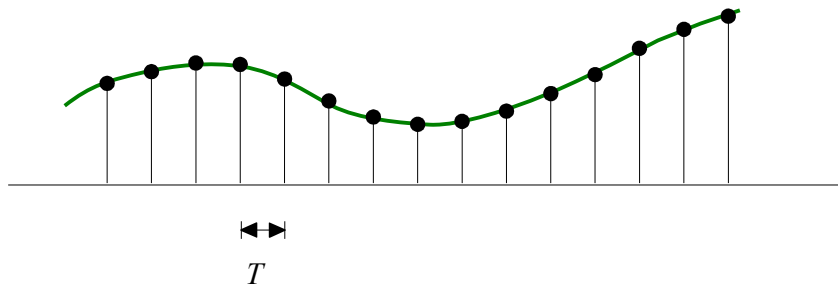


Sampling Frequency f_s

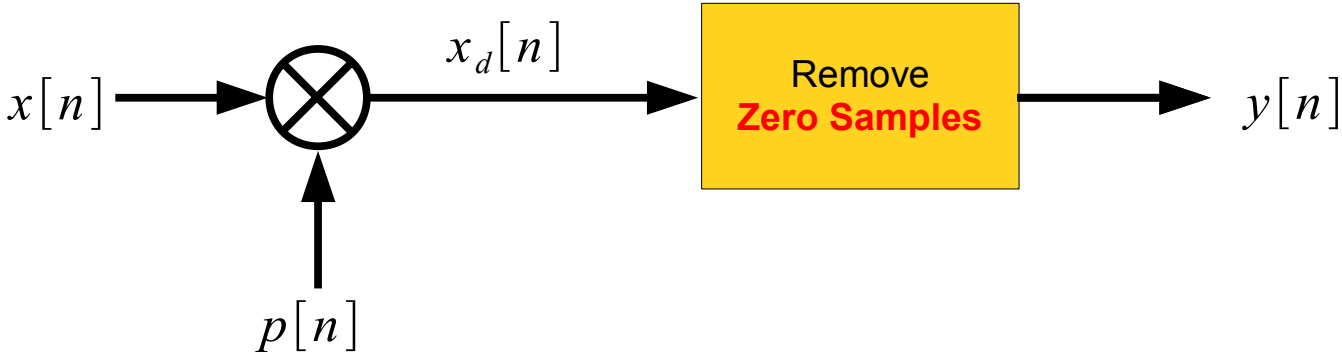
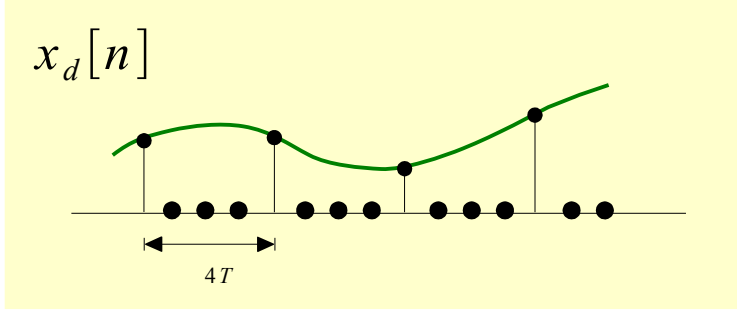
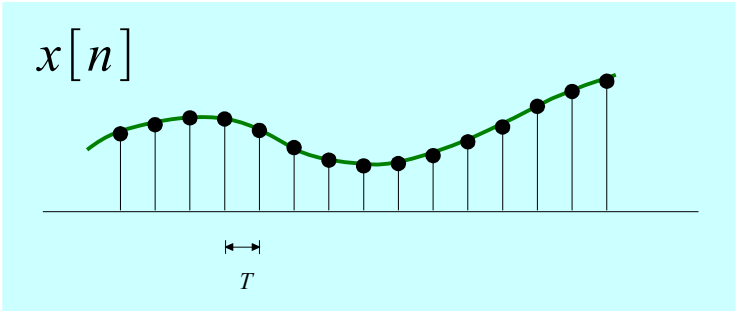
Sampling Time $T = \frac{1}{f_s}$

Sampling Frequency $f'_s = \frac{1}{4}f_s$

Sampling Time $T' = \frac{4}{f_s}$

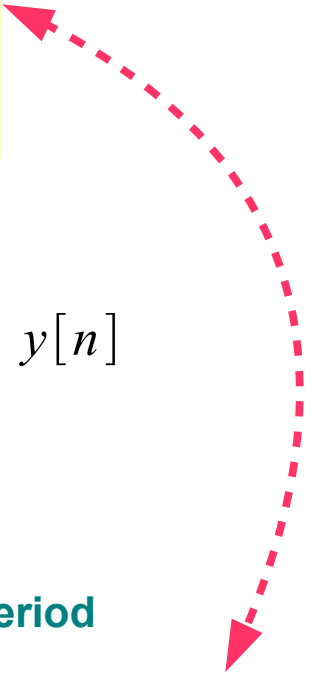
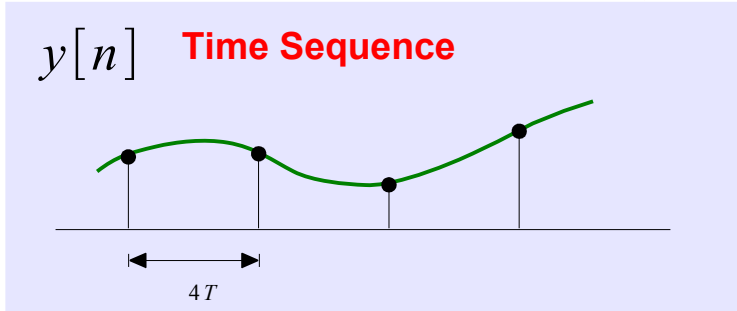
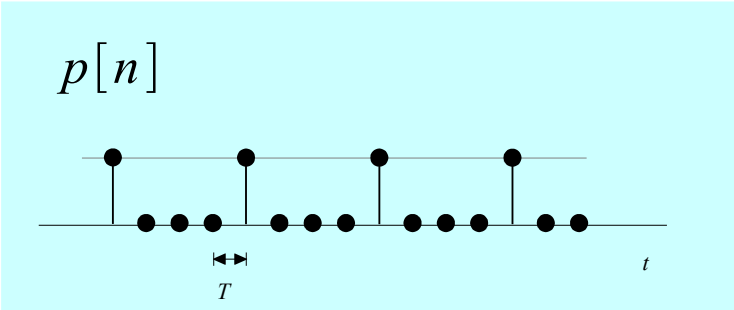


Time Sequence

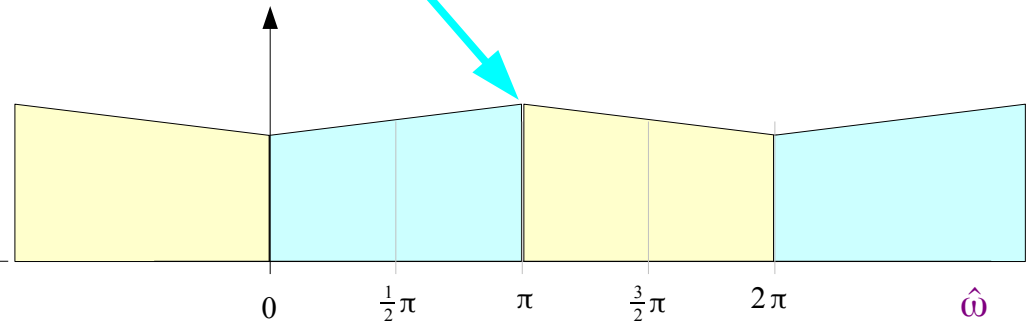
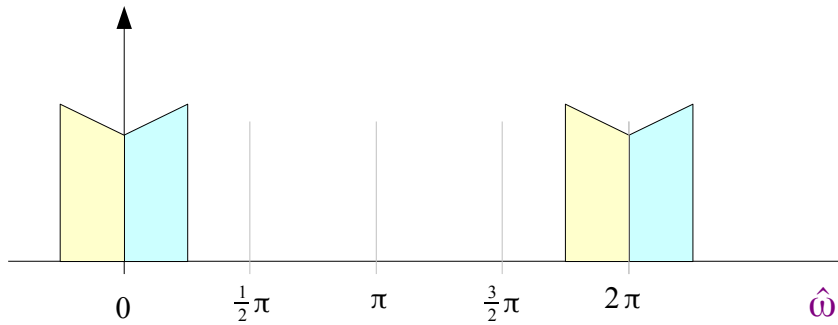
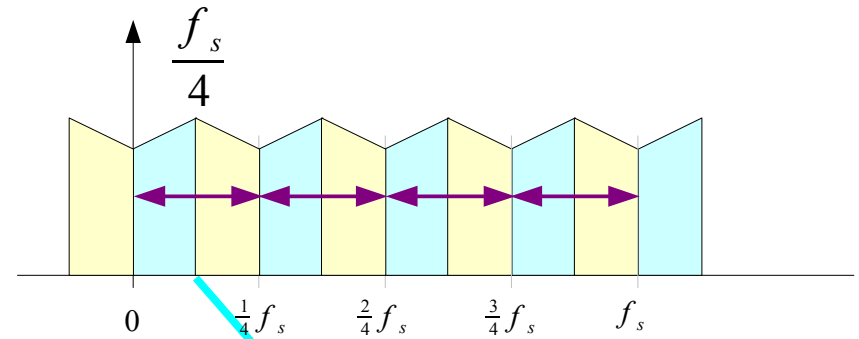
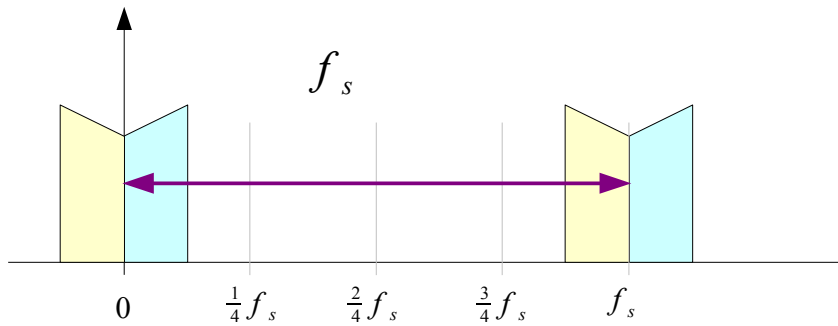
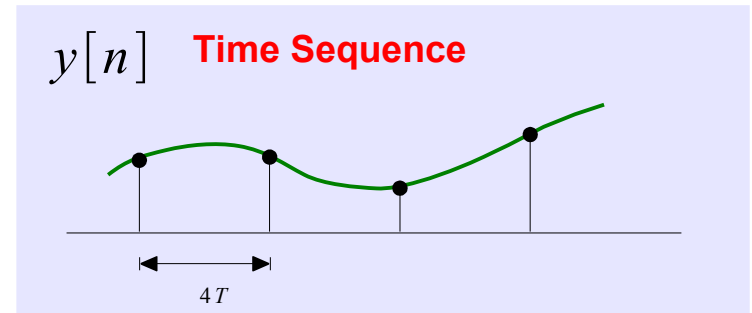
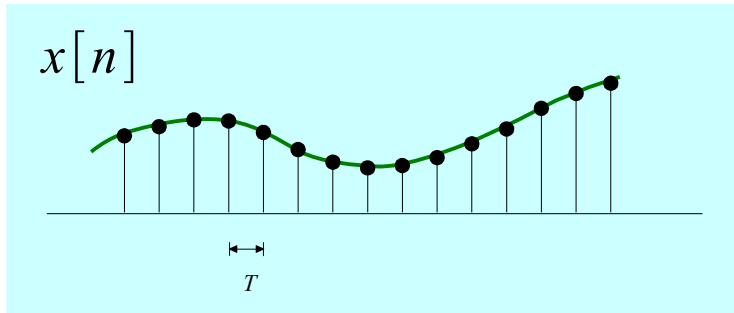


Ideal Sampling

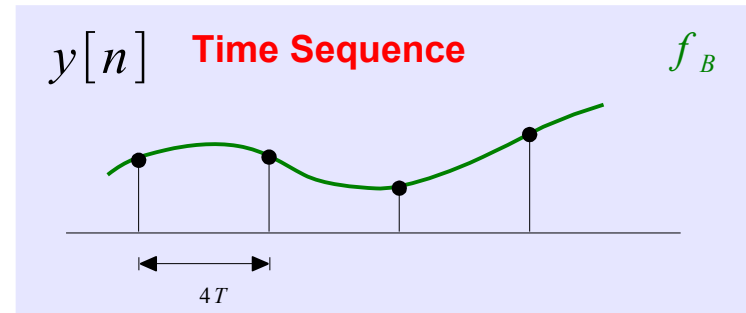
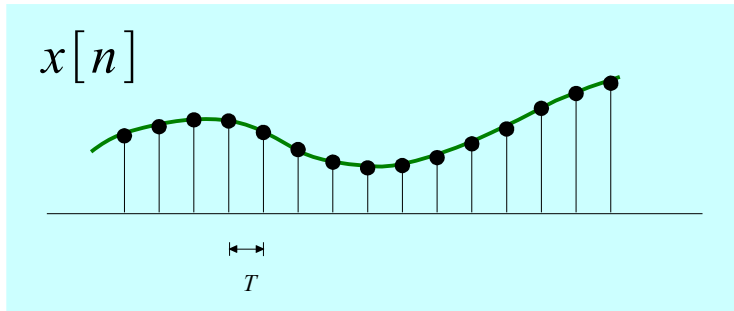
T Sampling Period



Time Sequence



Time Sequence



$$\hat{\omega} = \omega \cdot T_s = \frac{\omega}{1/T_s}$$

$$\hat{\omega} = \frac{\omega}{f_s} = 2\pi \frac{f}{f_s}$$

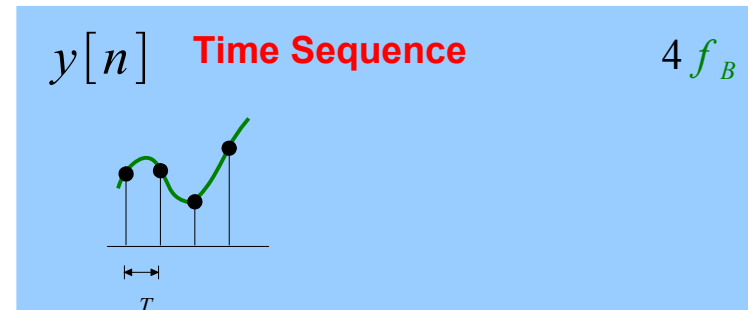


Normalized to f_s

Normalized Radian Frequency



The Same Time Sequence

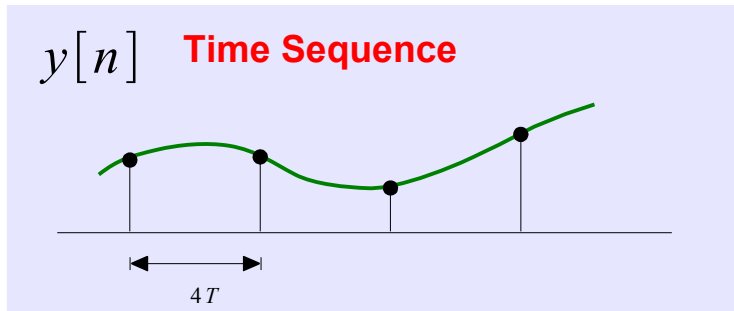
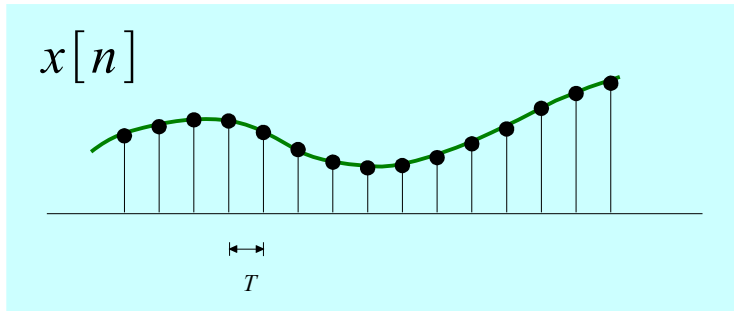


The Same Normalized Radian Frequency

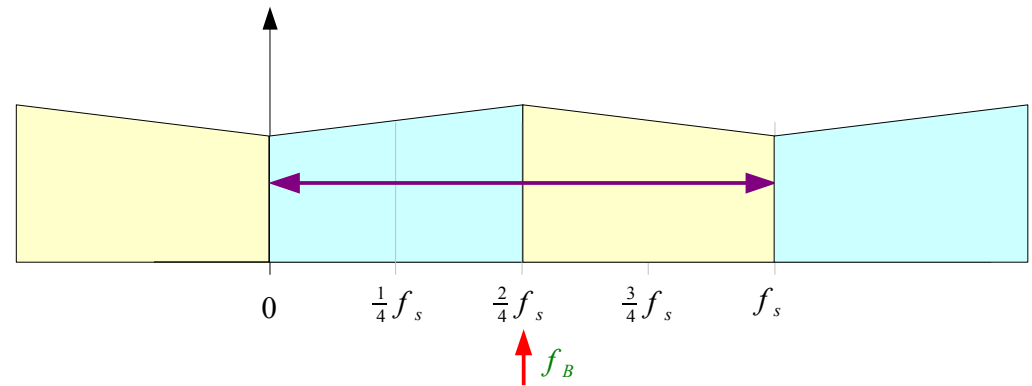
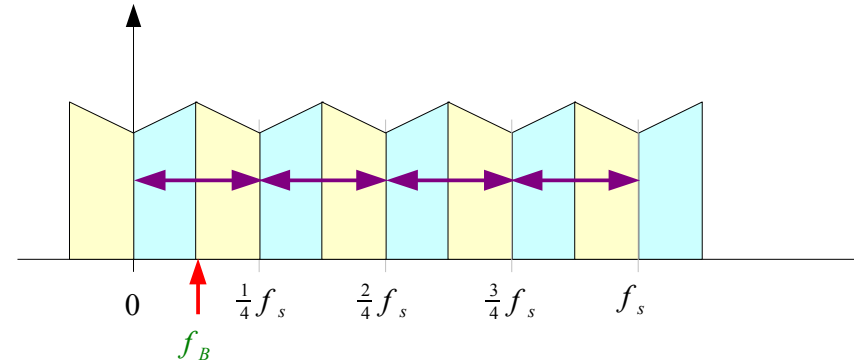
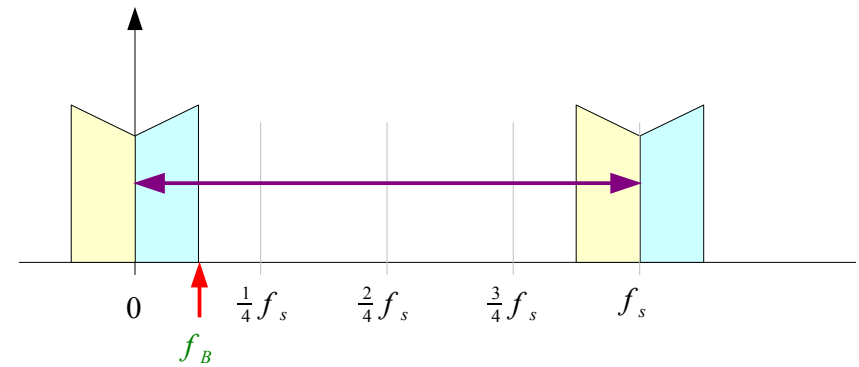
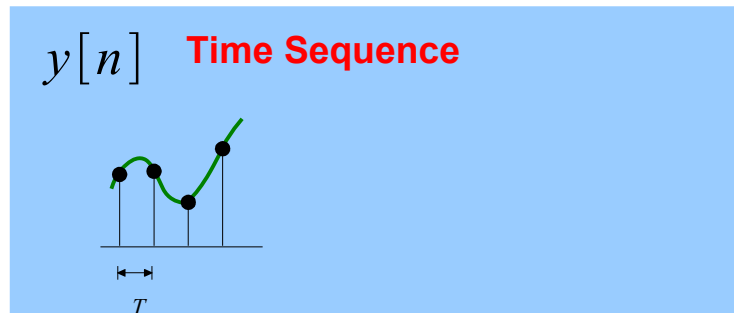
Normalized by the Sampling Frequency

$$\frac{f_B}{1/4T} = f_B(4T) \quad \frac{4f_B}{1/T} = f_B(4T)$$

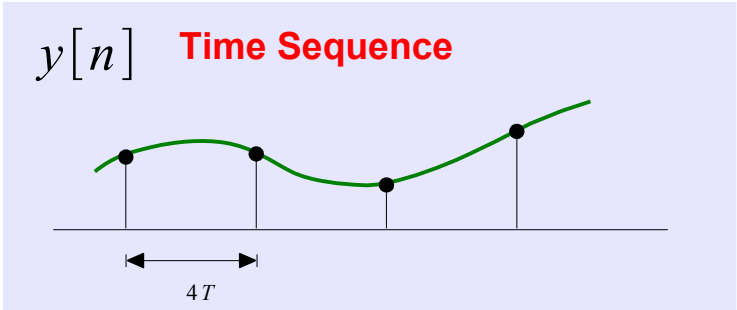
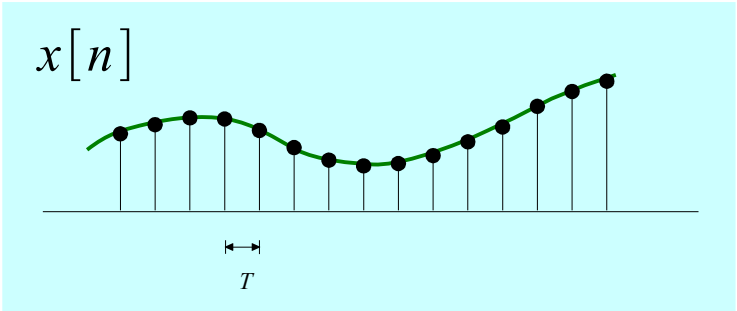
Time Sequence



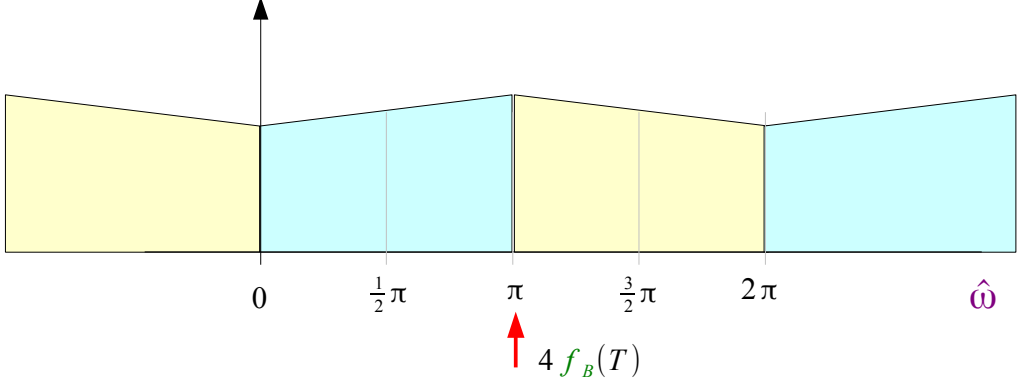
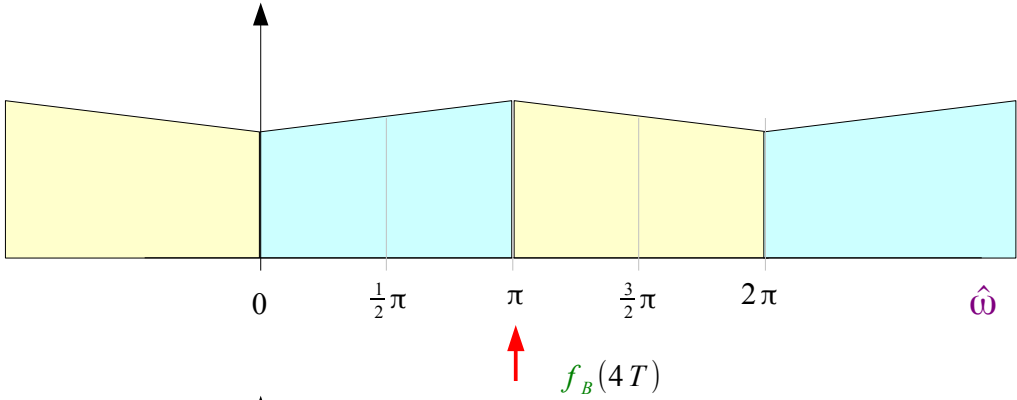
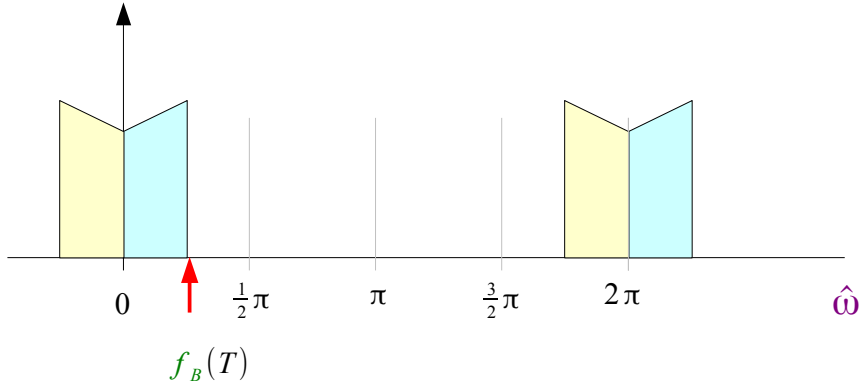
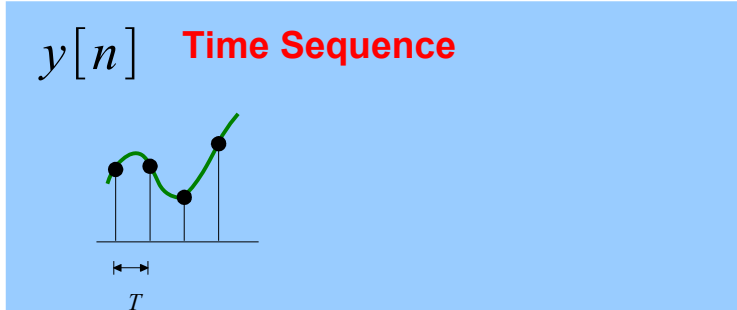
|| The Same Time Sequence



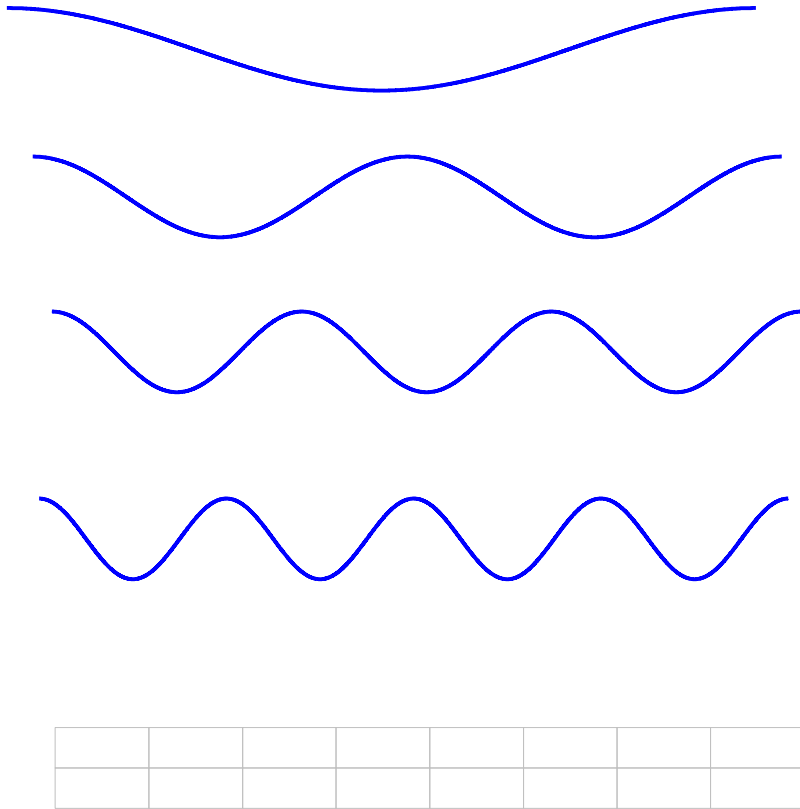
Time Sequence



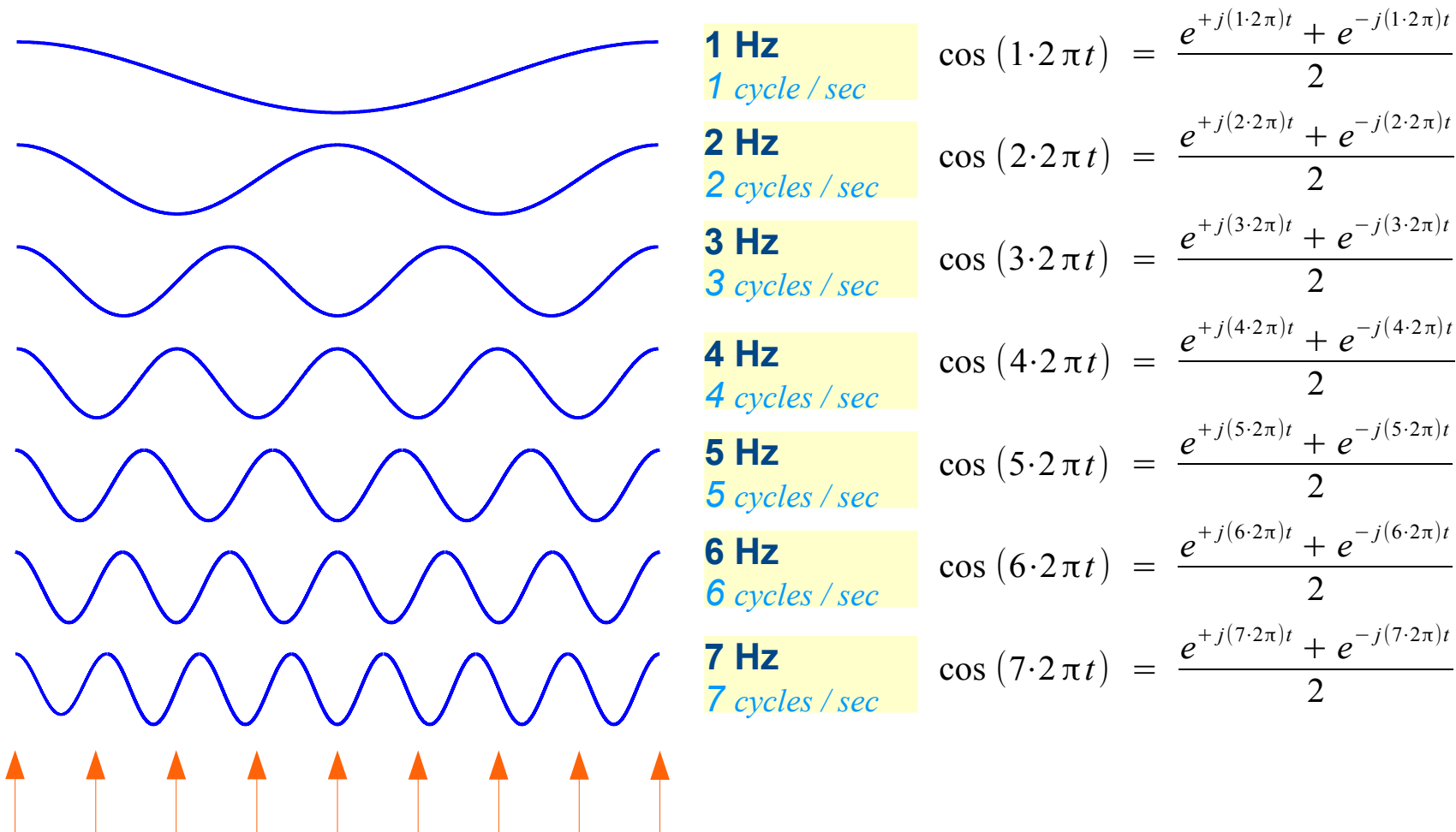
|| The Same Time Sequence



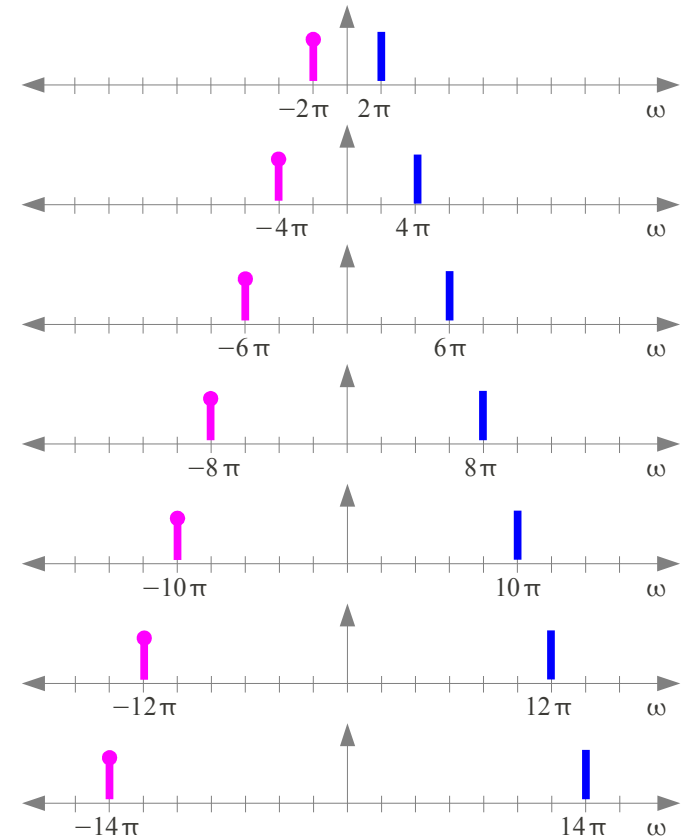
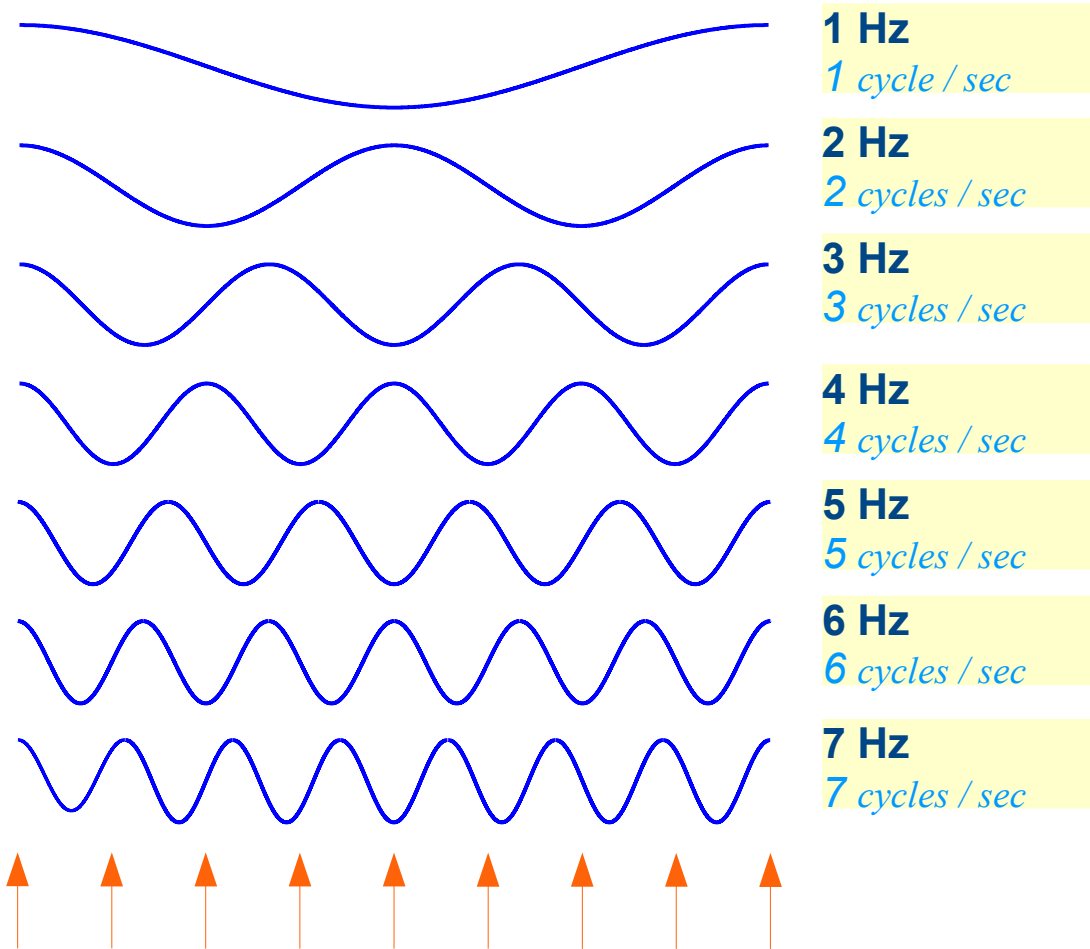
Measuring Rotation Rate



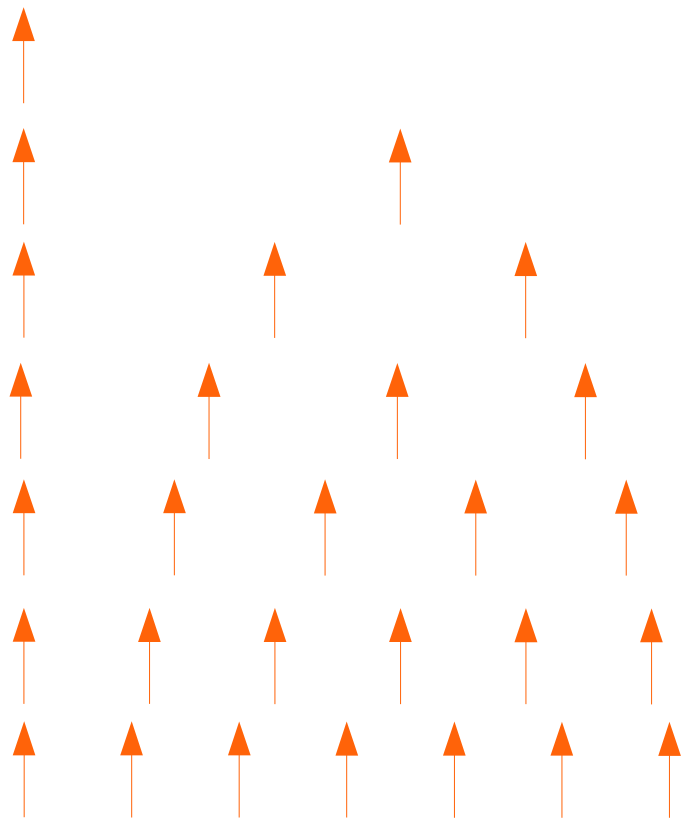
Signals with Harmonic Frequencies (1)



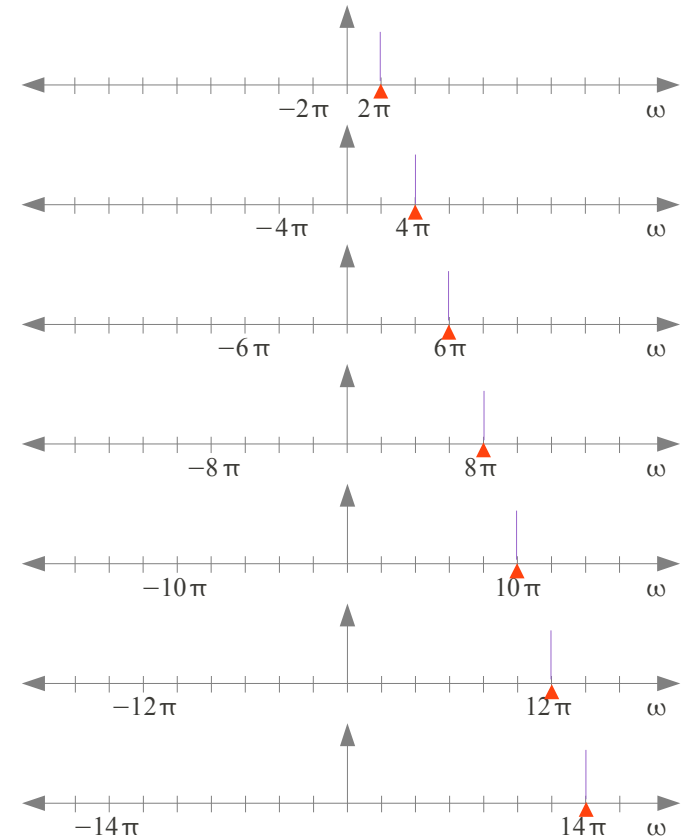
Signals with Harmonic Frequencies (2)



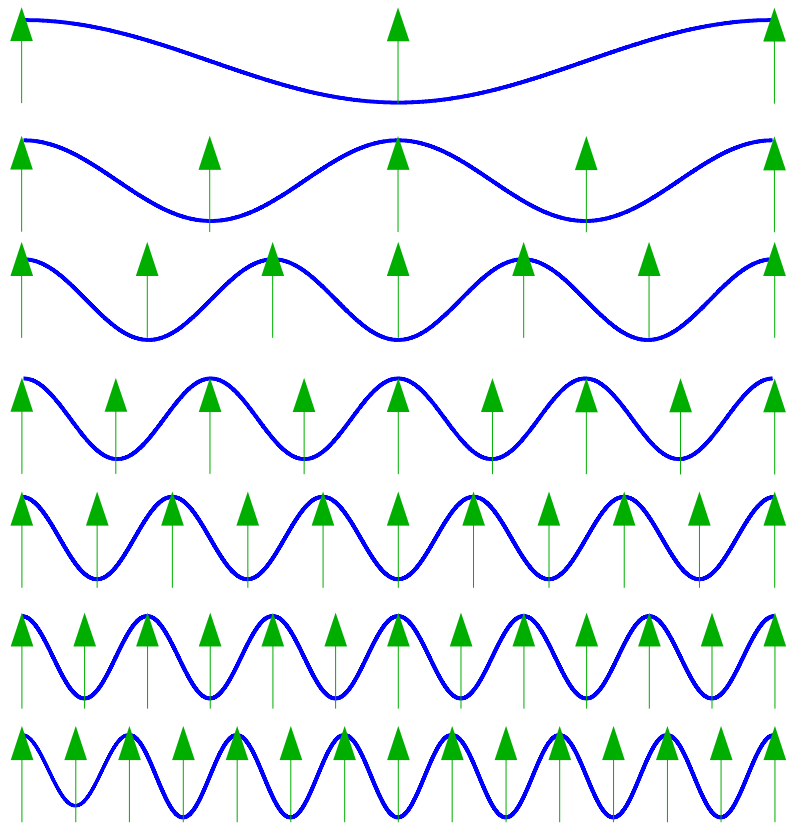
Sampling Frequency



- 1 Hz
1 sample / sec
- 2 Hz
2 samples / sec
- 3 Hz
3 samples / sec
- 4 Hz
4 samples / sec
- 5 Hz
5 samples / sec
- 6 Hz
6 samples / sec
- 7 Hz
7 samples / sec



Nyquist Frequency



1 Hz
1 cycle / sec

2x1 sample / sec

2 Hz
2 cycles / sec

2x2 samples / sec

3 Hz
3 cycles / sec

2x3 samples / sec

4 Hz
4 cycles / sec

2x4 samples / sec

5 Hz
5 cycles / sec

2x5 samples / sec

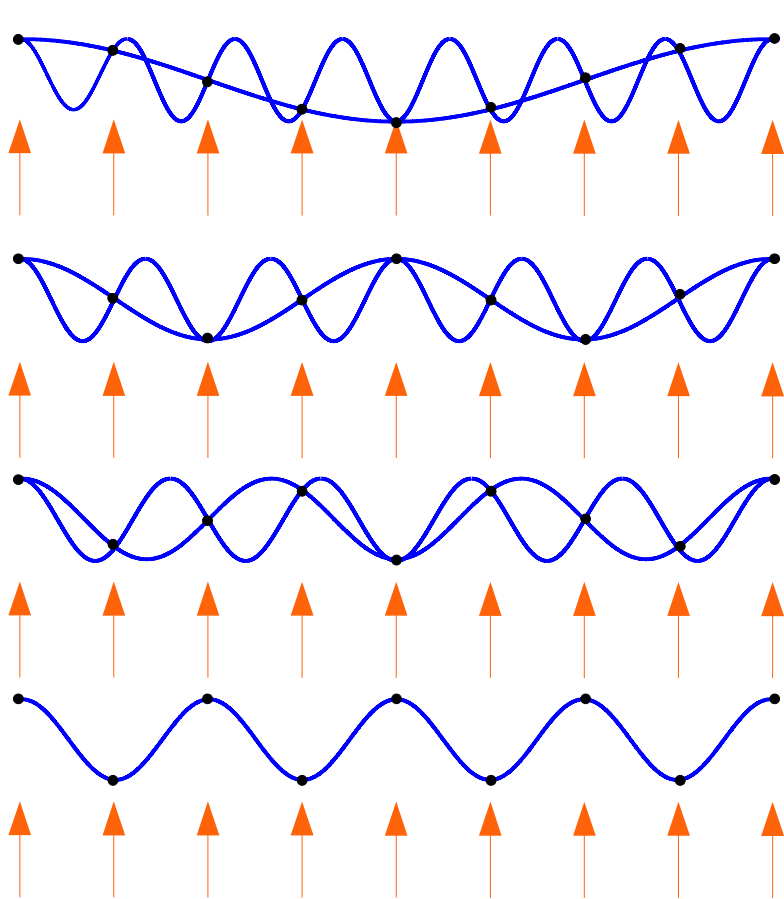
6 Hz
6 cycles / sec

2x6 samples / sec

7 Hz
7 cycles / sec

2x7 samples / sec

Aliasing



1 Hz
7 Hz

2x4 samples / sec

2 Hz
6 Hz

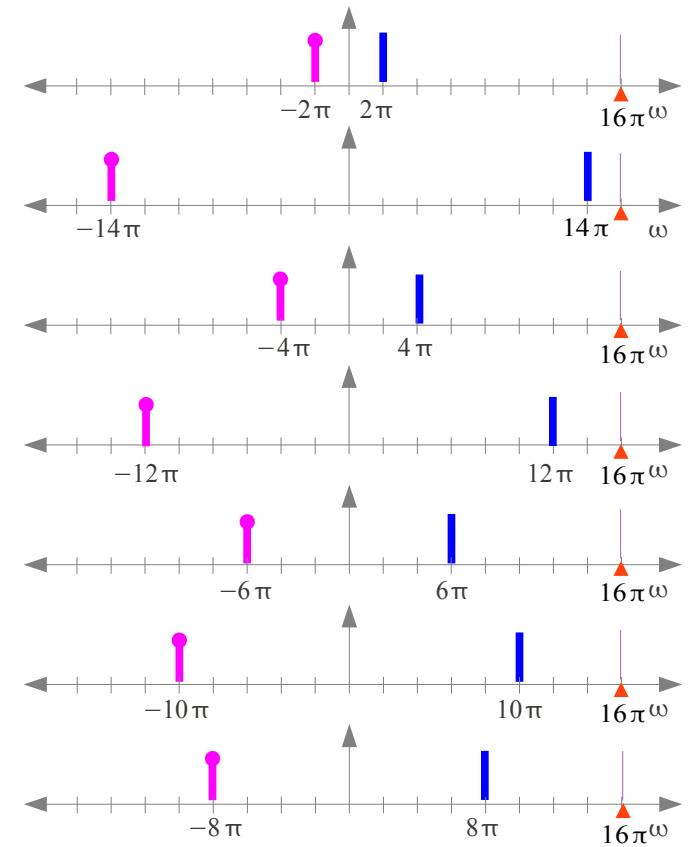
2x4 samples / sec

3 Hz
5 Hz

2x4 samples / sec

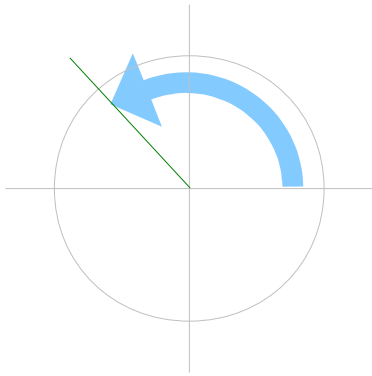
4 Hz

2x4 samples / sec

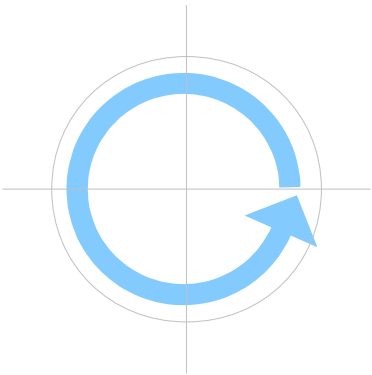


Sampling

$$\omega_s = 2\pi f_s \text{ (rad/sec)}$$



$$2\pi \text{ (rad)} / T_s \text{ (sec)}$$

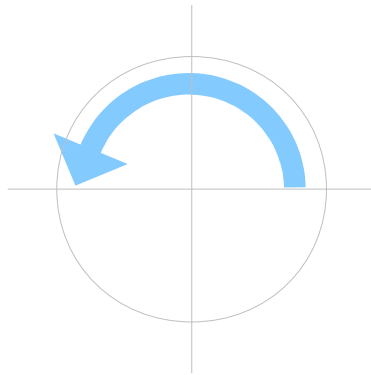


$$\omega_1 = 2\pi f_1$$

$$\omega_1 = \frac{\omega_s}{2} \text{ (rad/sec)}$$

$$f_1 = \frac{f_s}{2} \text{ (rad/sec)}$$

$$\pi \text{ (rad)} / T_s \text{ (sec)}$$

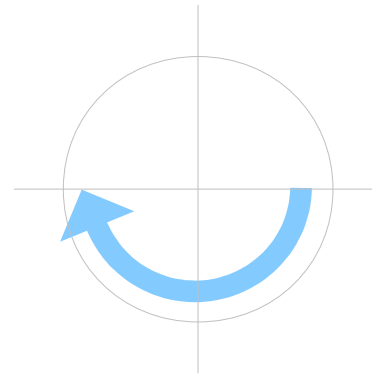


$$\omega_2 = 2\pi f_2$$

$$\omega_2 = -\frac{\omega_s}{2} \text{ (rad/sec)}$$

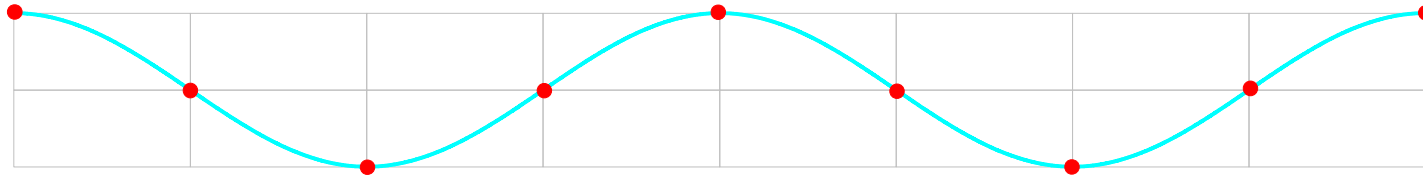
$$f_2 = -\frac{f_s}{2} \text{ (rad/sec)}$$

$$-\pi \text{ (rad)} / T_s \text{ (sec)}$$

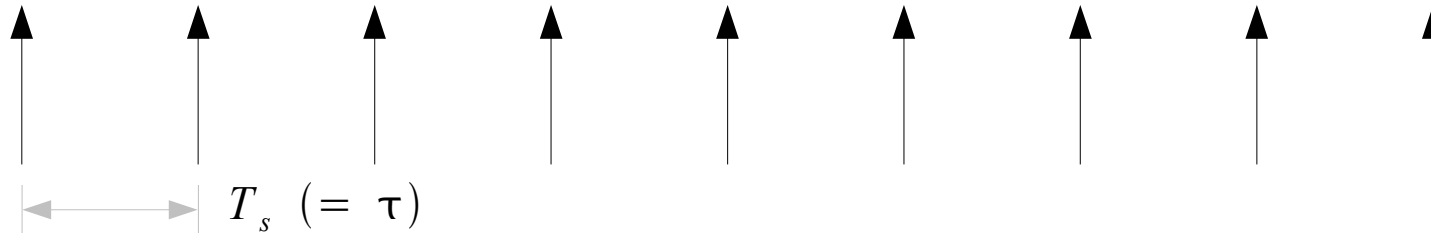


Sampling

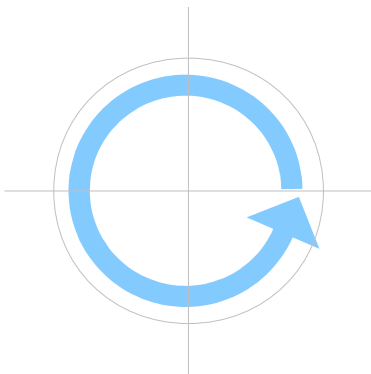
$$\omega_1 = 2\pi f_1 \text{ (rad/sec)}$$



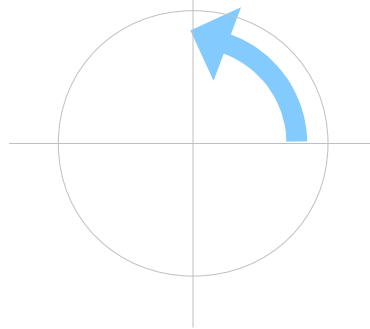
$$\omega_s = 2\pi f_s \text{ (rad/sec)}$$



$$2\pi \text{ (rad)} / T_s \text{ (sec)}$$



$$\frac{\pi}{2} \text{ (rad)} / T_s \text{ (sec)}$$



For the period of T_s
Angular displacement $\frac{\pi}{2}$ (rad)

$$\begin{aligned} \hat{\omega} &= \omega \cdot T_s \text{ (rad)} \\ &= 2\pi f_1 \cdot T_s \text{ (rad)} \\ &= 2\pi \frac{f_s}{4} \cdot T_s \text{ (rad)} \\ &= \frac{\pi}{2} \text{ (rad)} \end{aligned}$$

Angular Frequencies in Sampling

continuous-time signals

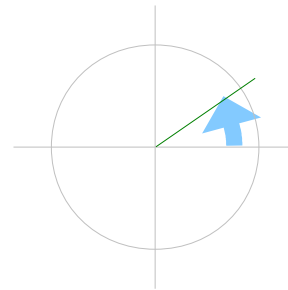
Signal Frequency

$$f_0 = \frac{1}{T_0}$$

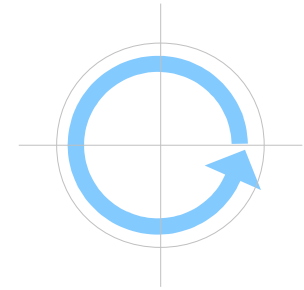
Signal Angular Frequency

$$\omega_0 = 2\pi f_0 \text{ (rad/sec)}$$

For 1 second
 $2\pi f_0 \text{ (rad/sec)}$



For 1 revolution
 $2\pi \text{ (rad)}$ $T_0 \text{ (sec)}$



sampling sequence

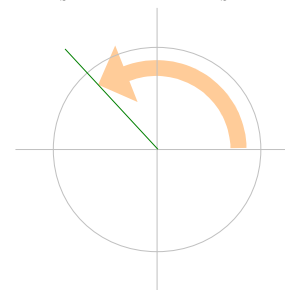
Sampling Frequency

$$f_s = \frac{1}{T_s}$$

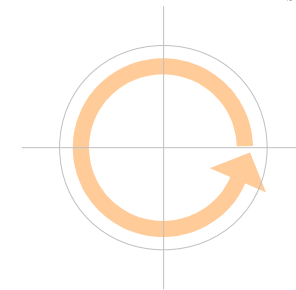
Sampling Angular Frequency

$$\omega_s = 2\pi f_s \text{ (rad/sec)}$$

For 1 second
 $2\pi f_s \text{ (rad/sec)}$



For 1 revolution
 $2\pi \text{ (rad)}$ $T_s \text{ (sec)}$



References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] A “graphical interpretation” of the DFT and FFT, by Steve Mann