Fourier Transforms (0A)

- CTFS: Continuous Time Fourier Series
- CTFT: Continuous Time Fourier Transform
- DTFT: Discrete Time Fourier Transform
- DFT: Discrete Fourier Transform

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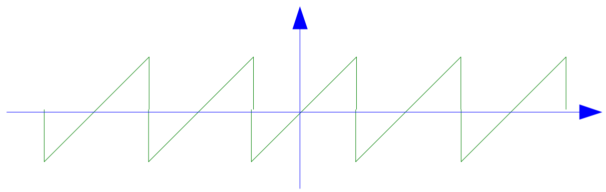
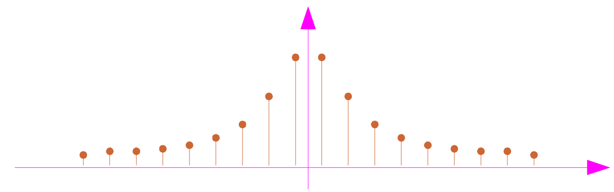
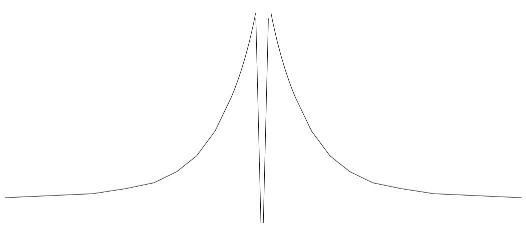
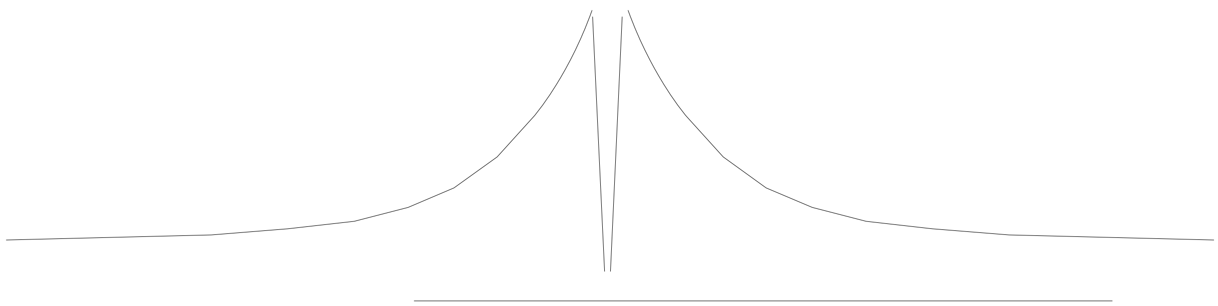
CTFS: Fourier Series

Continuous Time Fourier Series

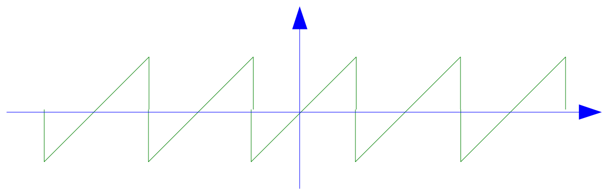


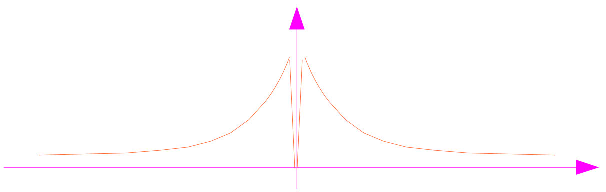


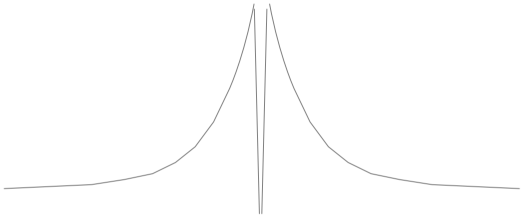




Continuous Time Fourier Transform

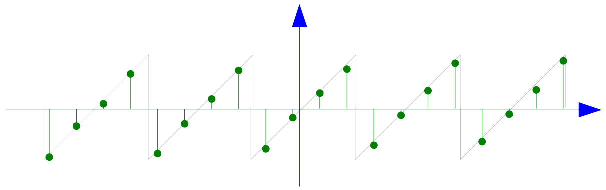


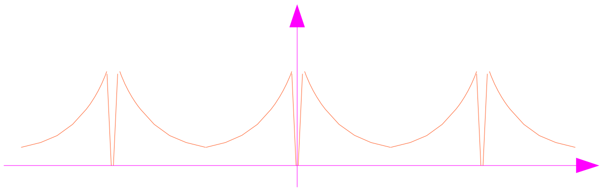


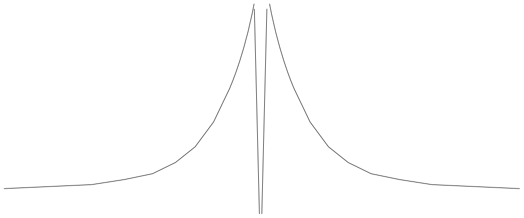


DTFT: Discrete Time Fourier Transform

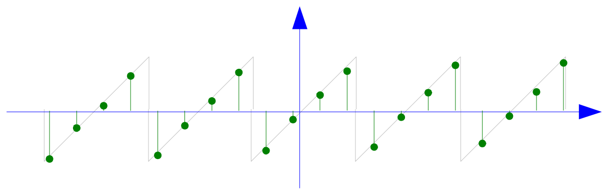
Discrete Time Fourier **Transform**

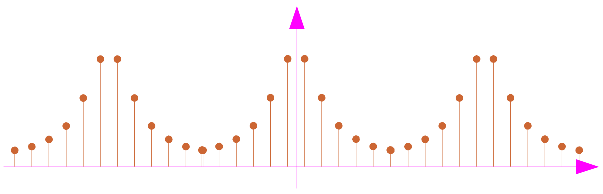


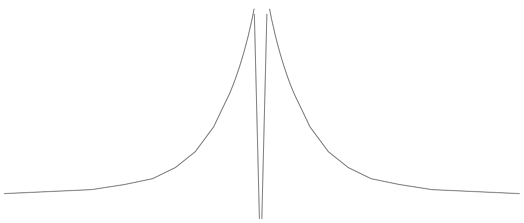




Discrete Time Fourier Transform







Fourier Transform Types

Continuous Time Fourier Series

$$C_n = \frac{1}{T}\int_0^T x(t)\, e^{-jn\omega_0 t}\, dt \quad \Leftrightarrow \quad x(t) = \sum_{n=0}^{\infty} C_n\, e^{+jn\omega_0 t}$$

Continuous Time Fourier Transform

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \quad \Leftrightarrow \quad x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

Discrete Time Fourier Transform

$$X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j\hat{\omega} n} \quad \Leftrightarrow \quad x[n] = \frac{1}{2\pi}\int_{-\pi}^{+pi} X(e^{j\hat{\omega}})\, e^{+j\hat{\omega} n}$$

Discrete Fourier Transform

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn} \quad \Leftrightarrow \quad x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{+j(2\pi/N)kn}$$

Continuous Time

Continuous Time Fourier Series

$$C_n = \frac{1}{T}\int_0^T x(t)\, e^{-jn\omega_0 t}\, dt \quad \Longleftrightarrow \quad x(t) = \sum_{n=0}^{\infty} C_n\, e^{+jn\omega_0 t}$$

Aperiodic
Discrete Frequency Spectrum

Periodic
Continuous Time Signal

Continuous Time Fourier Transform

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \quad \Longleftrightarrow \quad x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

Aperiodic
Discrete Frequency Spectrum

Aperiodic
Continuous Time Signal

Discrete Time

Discrete Time Fourier Transform

$$X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j\hat{\omega}n} \iff x[n] = \frac{1}{2\pi}\int_{-\pi}^{+pi} X(e^{j\hat{\omega}})\, e^{+j\hat{\omega}n}$$

Periodic
Continuous Frequency Spectrum

Aperiodic
Discrete Time Signal

Discrete Fourier Transform

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn} \iff x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{+j(2\pi/N)kn}$$

Periodic
Discrete Frequency Spectrum

Periodic
Discrete Time Signal

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003