

# Fourier Transforms (0A)

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- CTFS: Continuous Time Fourier Series
- CTFT: Continuous Time Fourier Transform
- DTFT: Discrete Time Fourier Transform
- DFT: Discrete Fourier Transform

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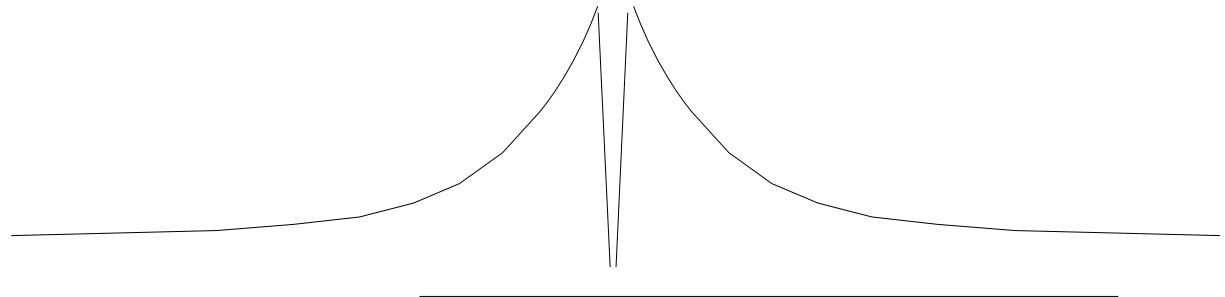
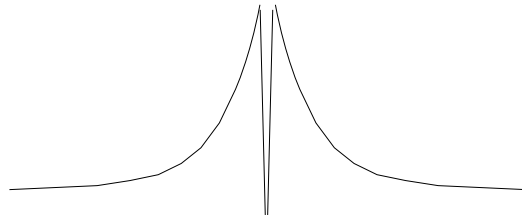
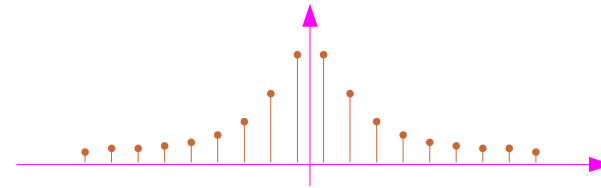
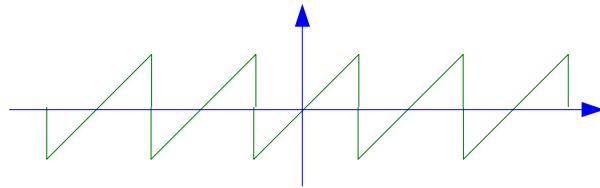
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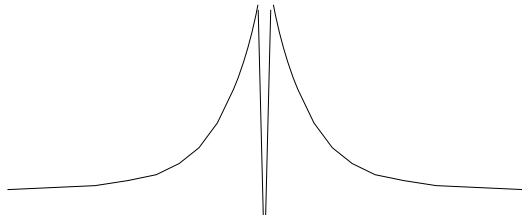
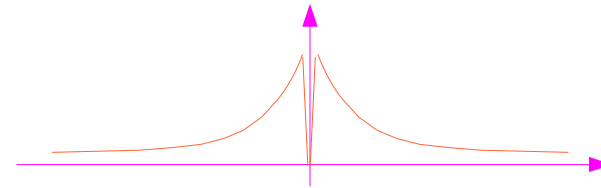
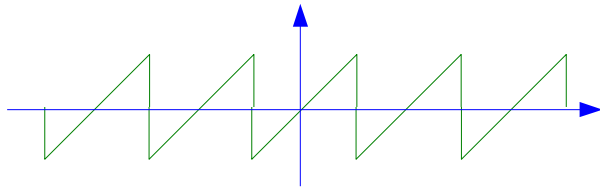
# CTFS: Fourier Series

## Continuous Time Fourier Series



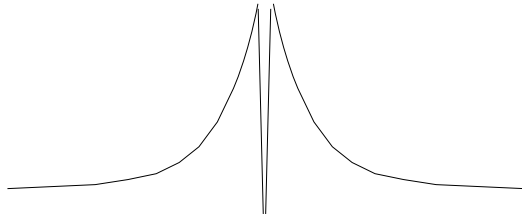
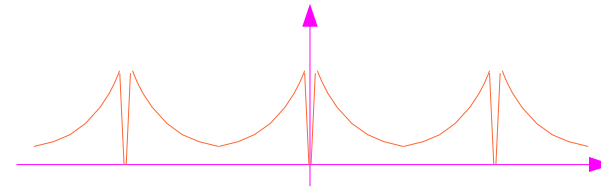
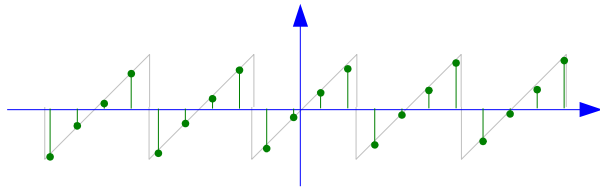
# CTFT: Fourier Integral

## Continuous Time Fourier Transform



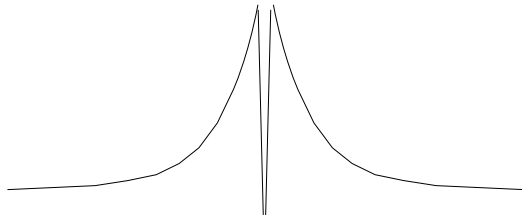
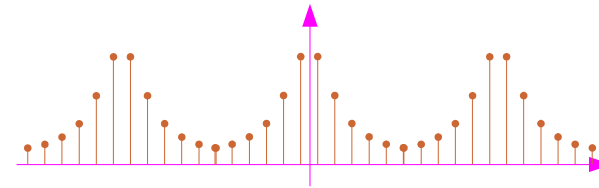
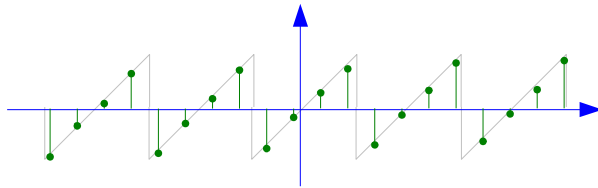
# DTFT: Discrete Time Fourier Transform

## Discrete Time Fourier Transform



# DTFT: Discrete Time Fourier Transform

## Discrete Time Fourier Transform



# Fourier Transform Types

## Continuous Time Fourier Series

$$C_n = \frac{1}{T} \int_0^T x(t) e^{-jn\omega_0 t} dt \quad \longleftrightarrow \quad x(t) = \sum_{n=0}^{\infty} C_n e^{+jn\omega_0 t}$$

## Continuous Time Fourier Transform

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt \quad \longleftrightarrow \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{+j\omega t} d\omega$$

## Discrete Time Fourier Transform

$$X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\hat{\omega}n} \quad \longleftrightarrow \quad x[n] = \frac{1}{2\pi} \int_{-\pi}^{+\pi} X(e^{j\hat{\omega}}) e^{+j\hat{\omega}n} d\hat{\omega}$$

## Discrete Fourier Transform

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j(2\pi/N)kn} \quad \longleftrightarrow \quad x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{+j(2\pi/N)kn}$$

# Continuous Time

## Continuous Time Fourier Series

$$C_n = \frac{1}{T} \int_0^T x(t) e^{-jn\omega_0 t} dt \quad \longleftrightarrow \quad x(t) = \sum_{n=-\infty}^{\infty} C_n e^{+jn\omega_0 t}$$

**Aperiodic**  
**Discrete Frequency Spectrum**

**Periodic**  
**Continuous Time Signal**

## Continuous Time Fourier Transform

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt \quad \longleftrightarrow \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{+j\omega t} d\omega$$

**Aperiodic**  
**Discrete Frequency Spectrum**

**Aperiodic**  
**Continuous Time Signal**



# Discrete Time

## Discrete Time Fourier Transform

$$X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\hat{\omega}n} \iff x[n] = \frac{1}{2\pi} \int_{-\pi}^{+\pi} X(e^{j\hat{\omega}}) e^{+j\hat{\omega}n}$$

**Periodic**  
Continuous Frequency Spectrum

**Aperiodic**  
Discrete Time Signal

## Discrete Fourier Transform

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j(2\pi/N)kn} \iff x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{+j(2\pi/N)kn}$$

**Periodic**  
Discrete Frequency Spectrum

**Periodic**  
Discrete Time Signal

## References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003