Convolution (1A)

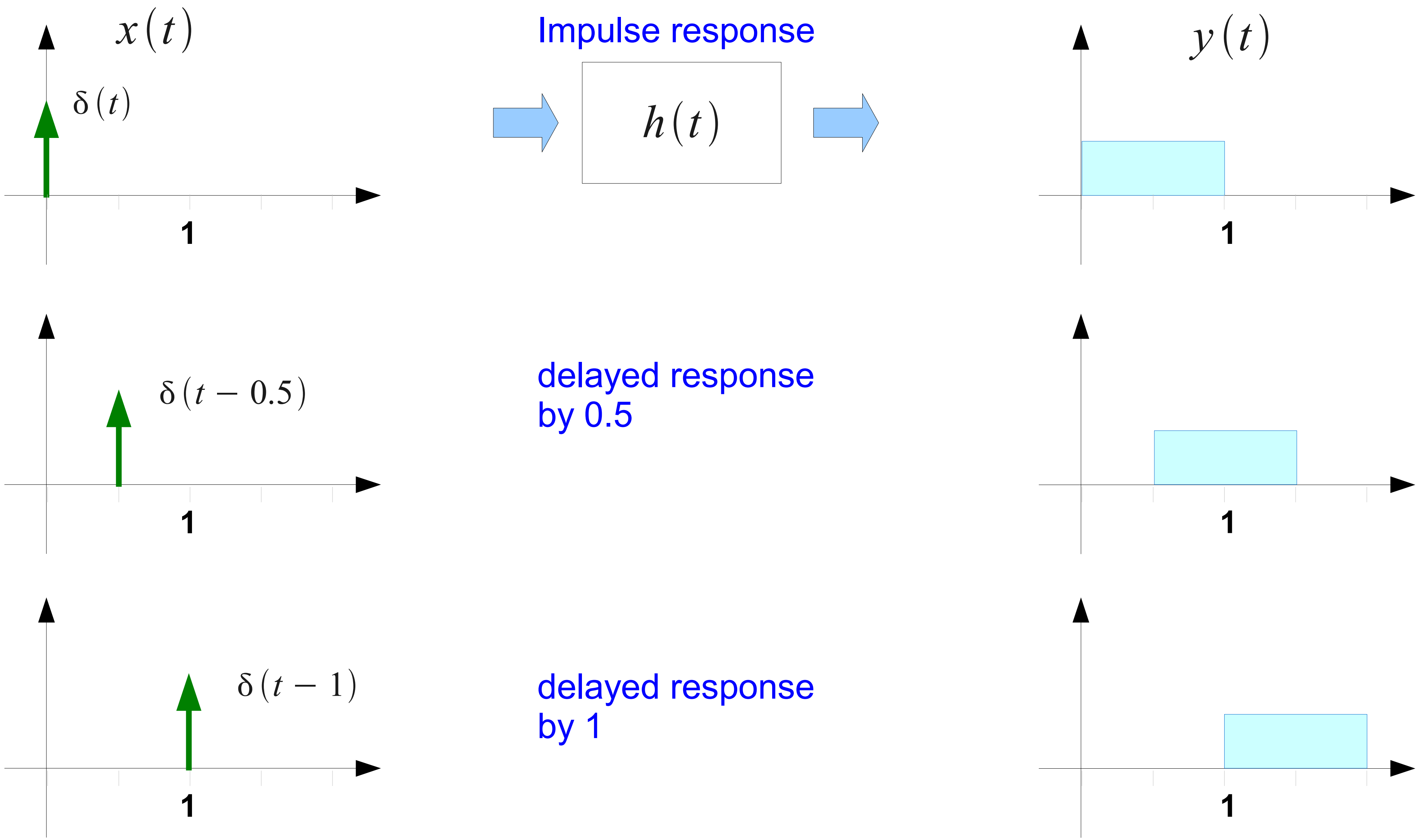
•

Young Won Lim
4/18/11

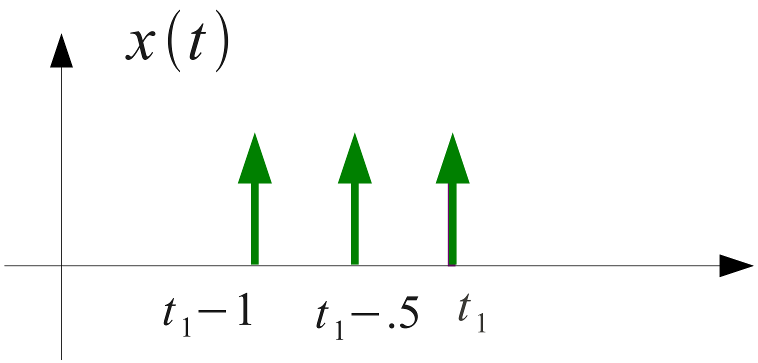
Please send corrections (or suggestions) to youngwlim@hotmail.com.

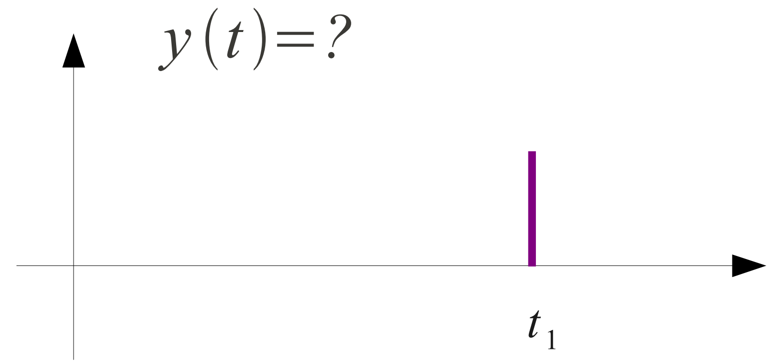
This document was produced by using OpenOffice and Octave.

Impulse Response



LTI System





$\delta(t-t_1+1) \longrightarrow \boxed{h(t)} \longrightarrow h(t-t_1+1)$

$\delta(t-t_1+.5) \longrightarrow \boxed{h(t)} \longrightarrow h(t-t_1+.5)$

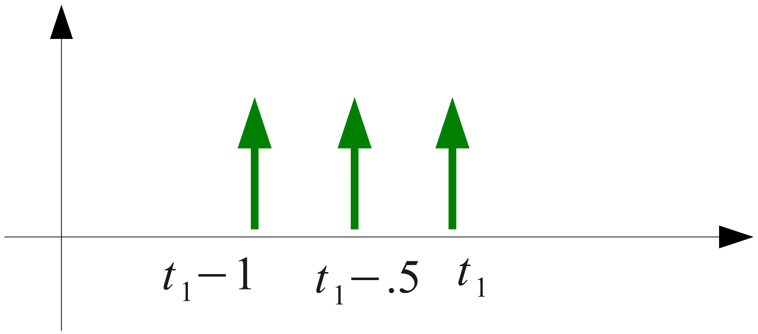
$\delta(t-t_1) \longrightarrow \boxed{h(t)} \longrightarrow h(t-t_1)$

$\longrightarrow \boxed{h(t)} \longrightarrow$

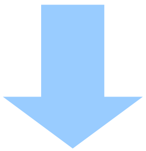
$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$

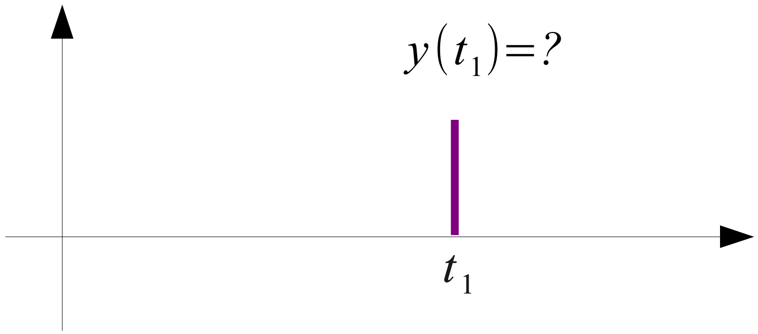
$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$

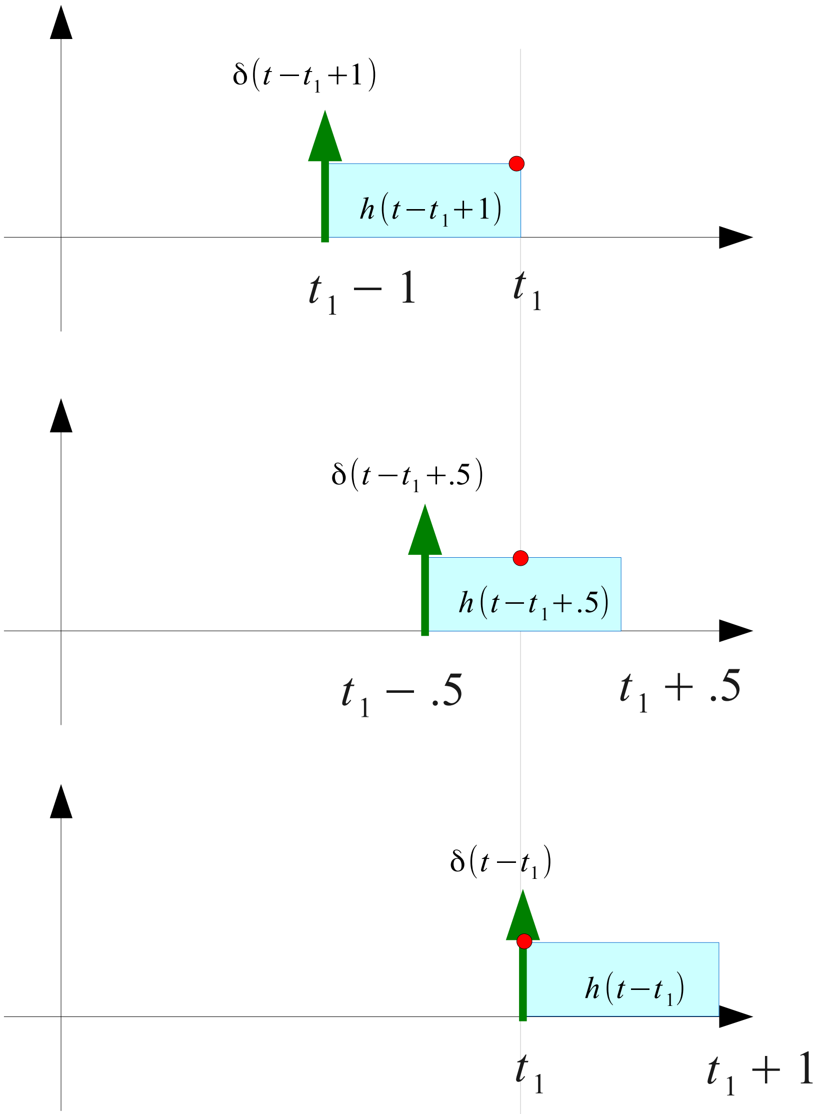
Output at t = t1



$$x(t) = \delta(t - t_1 + 1) + \delta(t - t_1 + .5) + \delta(t - t_1)$$

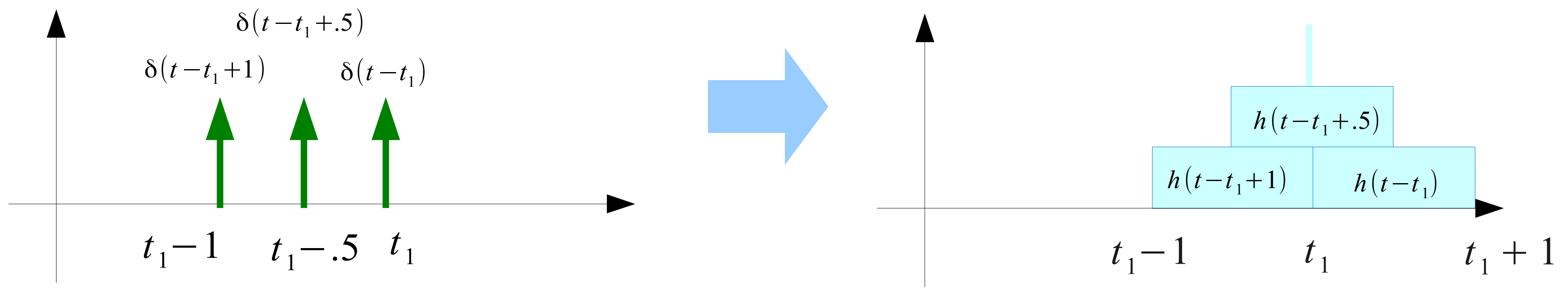






Using Convolution

$$x(t) = \delta(t - t_1 + 1) + \delta(t - t_1 + .5) + \delta(t - t_1) \qquad y(t) = h(t - t_1 + 1) + h(t - t_1 + .5) + h(t - t_1)$$



$$y(t) = \int x(v) h(t - v)\, dv$$

$$= \int \delta(v - t_1 + 1) h(t - v)\, dv \quad \rightarrow \quad h(t - t_1 + 1)$$ impulse response delayed by $t_1 - 1$

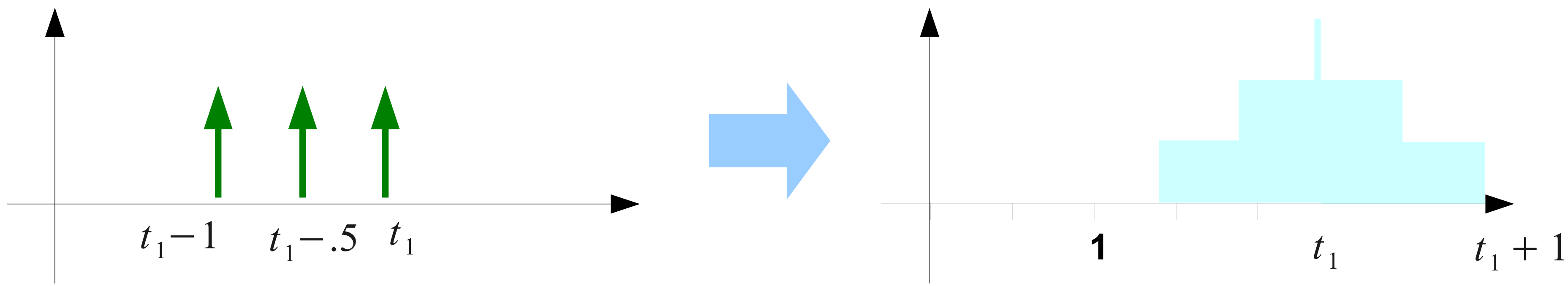
$$+ \int \delta(v - t_1 + .5) h(t - v)\, dv \quad \rightarrow \quad h(t - t_1 + .5)$$ impulse response delayed by $t_1 - .5$

$$+ \int \delta(v - t_1) h(t - v)\, dv \quad \rightarrow \quad h(t - t_1)$$ impulse response delayed by t_1

$$y(t_1) = h(t_1 - t_1 + 1) + h(t_1 - t_1 + .5) + h(t_1 - t_1)$$

$$= h(1) + h(.5) + h(0)$$

N=8 DFT

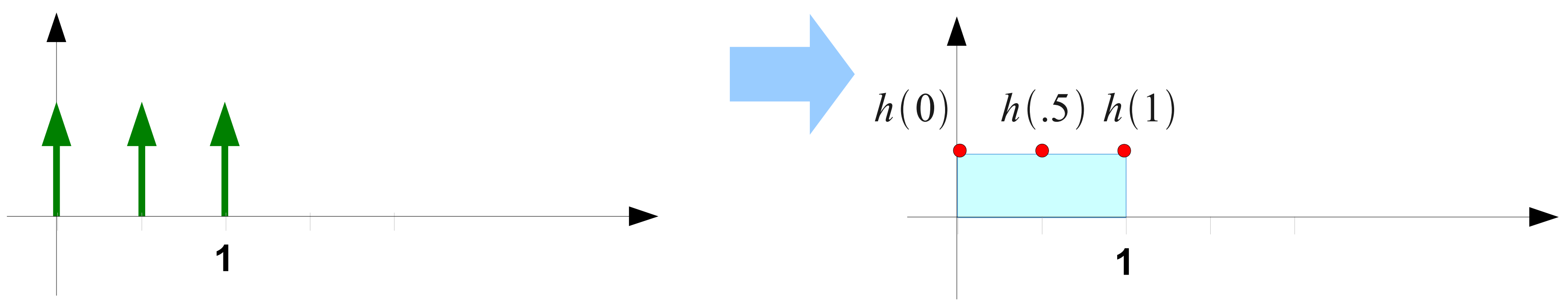


$$x(t) = \delta(t - t_1 + 1) + \delta(t - t_1 + .5) + \delta(t - t_1)$$

$$y(t) = h(t - t_1 + 1) + h(t - t_1 + .5) + h(t - t_1)$$

$$x(t) = \delta(t) + \delta(t - .5) + \delta(t - 1)$$

$$y(t_1) = h(1) + h(.5) + h(0)$$



The Computation of Convolution (1)

$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

Change of variables $t \rightarrow v$

$$x(v) = \delta(v-t_1+1) + \delta(v-t_1+.5) + \delta(v-t_1)$$

Flip around y axis and then shift to the right by t $v \rightarrow t-v$

$$x(t-v) = \delta(t-v-t_1+1) + \delta(t-v-t_1+.5) + \delta(t-v-t_1)$$

$$y(t) = \int x(t-v)\, h(v)\, dv$$

$$= \int \delta(t-v-t_1+1)\, h(v)\, dv \quad \rightarrow h(t-t_1+1)$$ impulse response delayed by t_1-1

$$+ \int \delta(t-v-t_1+.5)\, h(v)\, dv \quad \rightarrow h(t-t_1+.5)$$ impulse response delayed by $t_1-.5$

$$+ \int \delta(t-v-t_1)\, h(v)\, dv \quad \rightarrow h(t-t_1)$$ impulse response delayed by t_1

$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

$$\Rightarrow \quad y(t_1) = h(1) + h(.5) + h(0)$$

The Computation of Convolution (2)

$h(t)$

Change of variables $t \rightarrow v$

$h(v)$

Flip around y axis and then shift to the right by t $v \rightarrow t-v$

$h(t-v)$

$$y(t) = \int x(v)h(t-v)\,dv$$

$$= \int \delta(v-t_1+1)h(t-v)\,dv \rightarrow h(t-t_1+1)$$ impulse response delayed by t_1-1

$$+ \int \delta(v-t_1+.5)h(t-v)\,dv \rightarrow h(t-t_1+.5)$$ impulse response delayed by $t_1-.5$

$$+ \int \delta(v-t_1)h(t-v)\,dv \rightarrow h(t-t_1)$$ impulse response delayed by t_1

$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

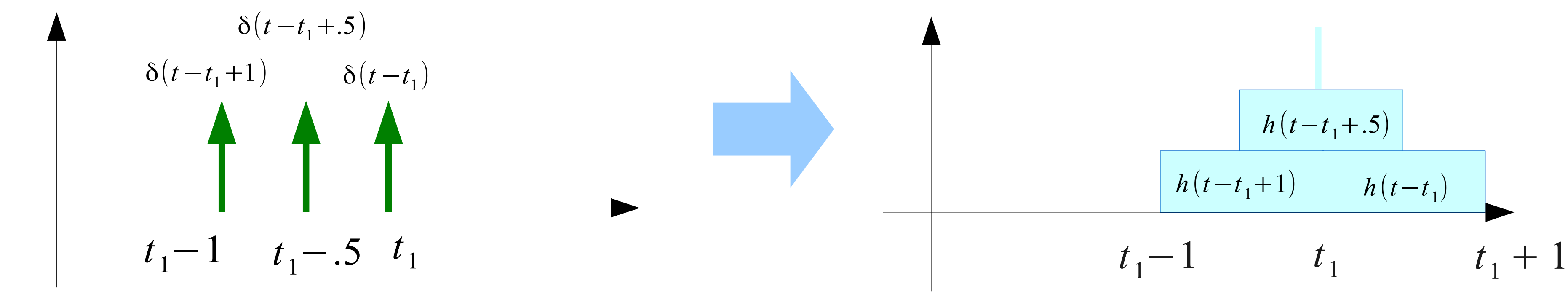
$$\Rightarrow y(t_1) = h(1) + h(.5) + h(0)$$

The Commutativity of Convolution (1)

$y(t) = \int x(v)h(t-v)\,dv$				$y(t) = \int x(t-v)h(v)\,dv$
$= \int \delta(v-t_1+1)h(t-v)\,dv$	➡	$h(t-t_1+1)$	⬅	$= \int \delta(t-v-t_1+1)h(v)\,dv$
$+ \int \delta(v-t_1+.5)h(t-v)\,dv$	➡	$h(t-t_1+.5)$	⬅	$+ \int \delta(t-v-t_1+.5)h(v)\,dv$
$+ \int \delta(v-t_1)h(t-v)\,dv$	➡	$h(t-t_1)$	⬅	$+ \int \delta(t-v-t_1)h(v)\,dv$

Sum of delayed impulse response

$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1) \qquad y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$



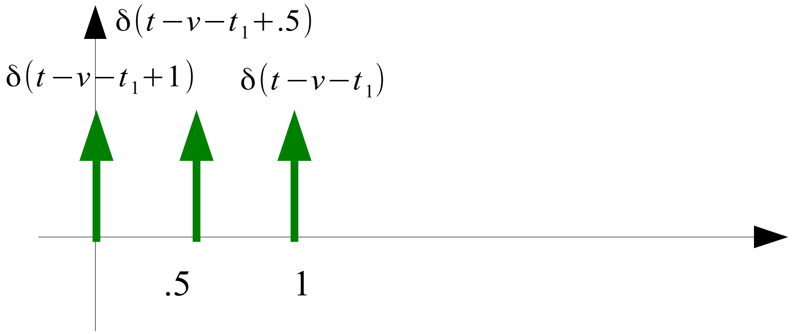
The Commutativity of Convolution (2)

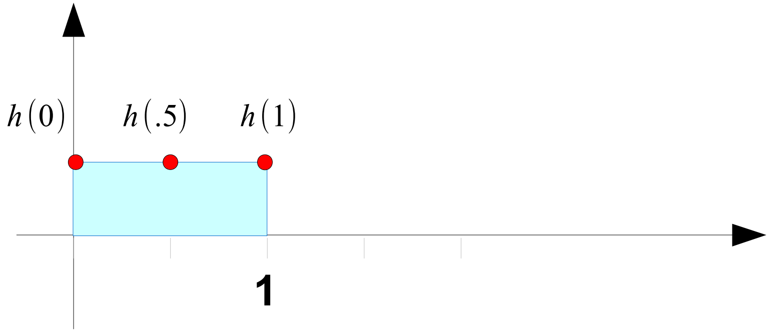
$y(t) = \int x(v)h(t-v)\,dv$			$y(t) = \int x(t-v)h(v)\,dv$
$= \int \delta(v-t_1+1)h(t-v)\,dv$	➡	$h(t-t_1+1)$ ⬅	$= \int \delta(t-v-t_1+1)h(v)\,dv$
$+ \int \delta(v-t_1+.5)h(t-v)\,dv$	➡	$h(t-t_1+.5)$ ⬅	$+ \int \delta(t-v-t_1+.5)h(v)\,dv$
$+ \int \delta(v-t_1)h(t-v)\,dv$	➡	$h(t-t_1)$ ⬅	$+ \int \delta(t-v-t_1)h(v)\,dv$

Flip and shift input

$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

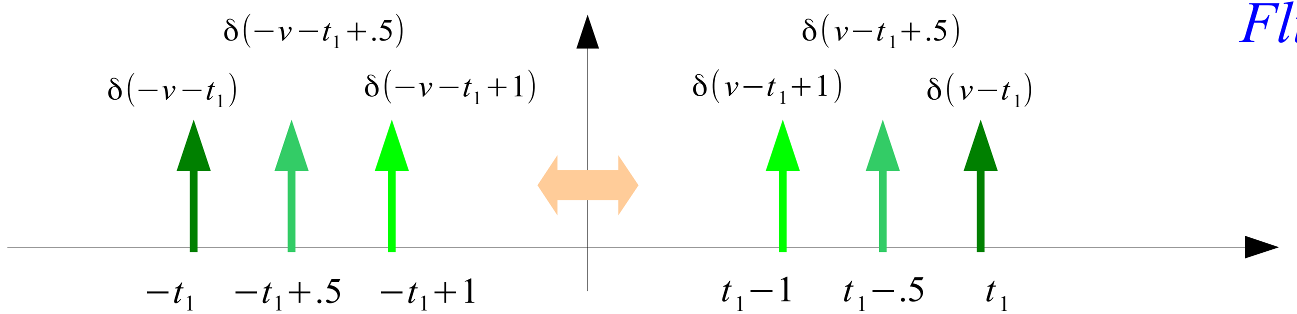
$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$



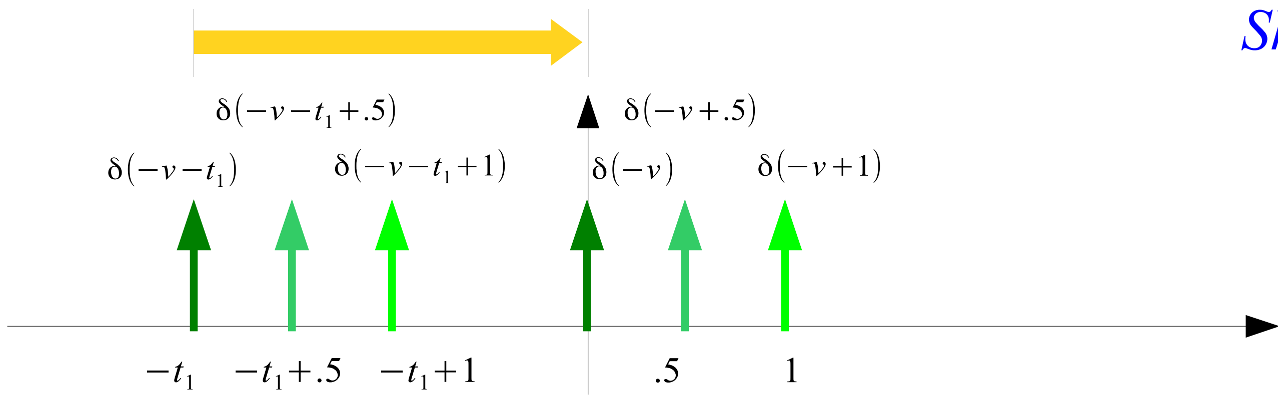


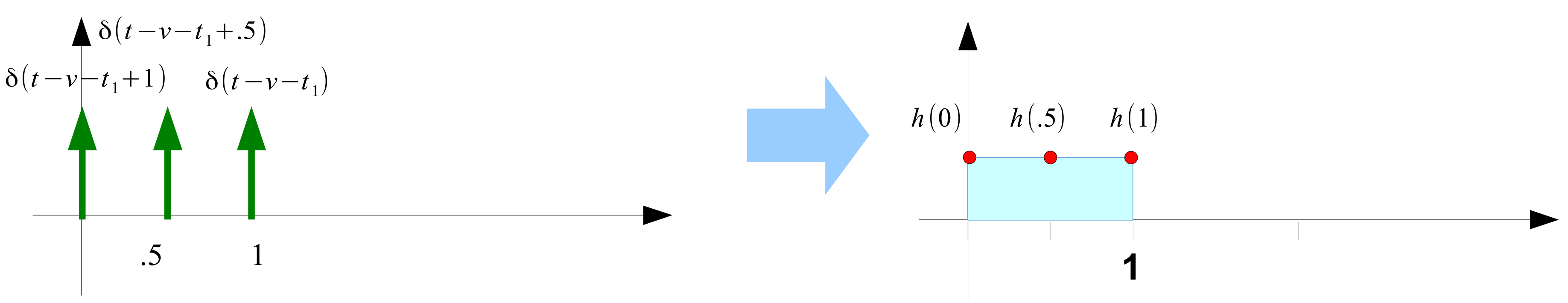
The Commutativity of Convolution (3)

Flip around y-axis

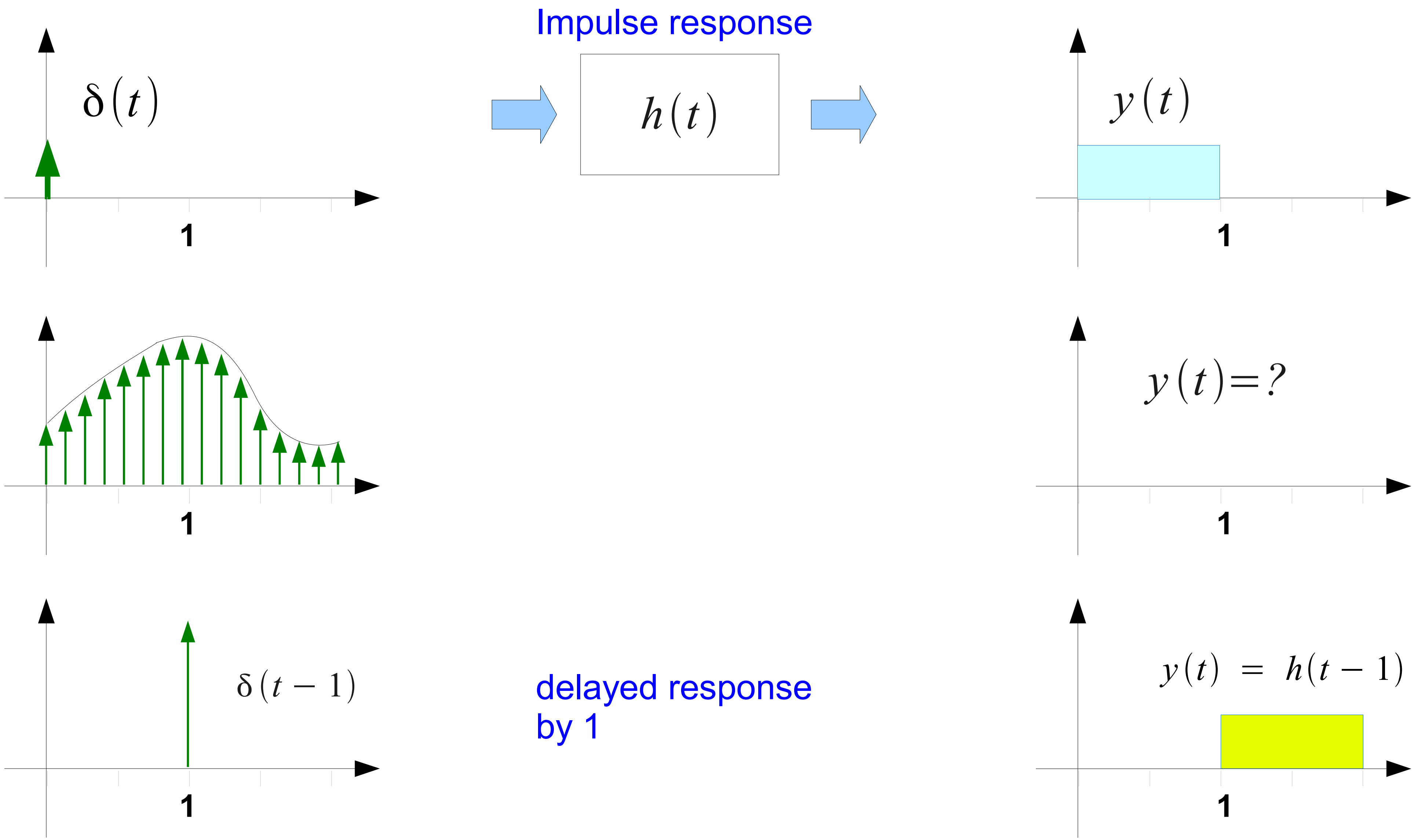


Shift to the right by t_1

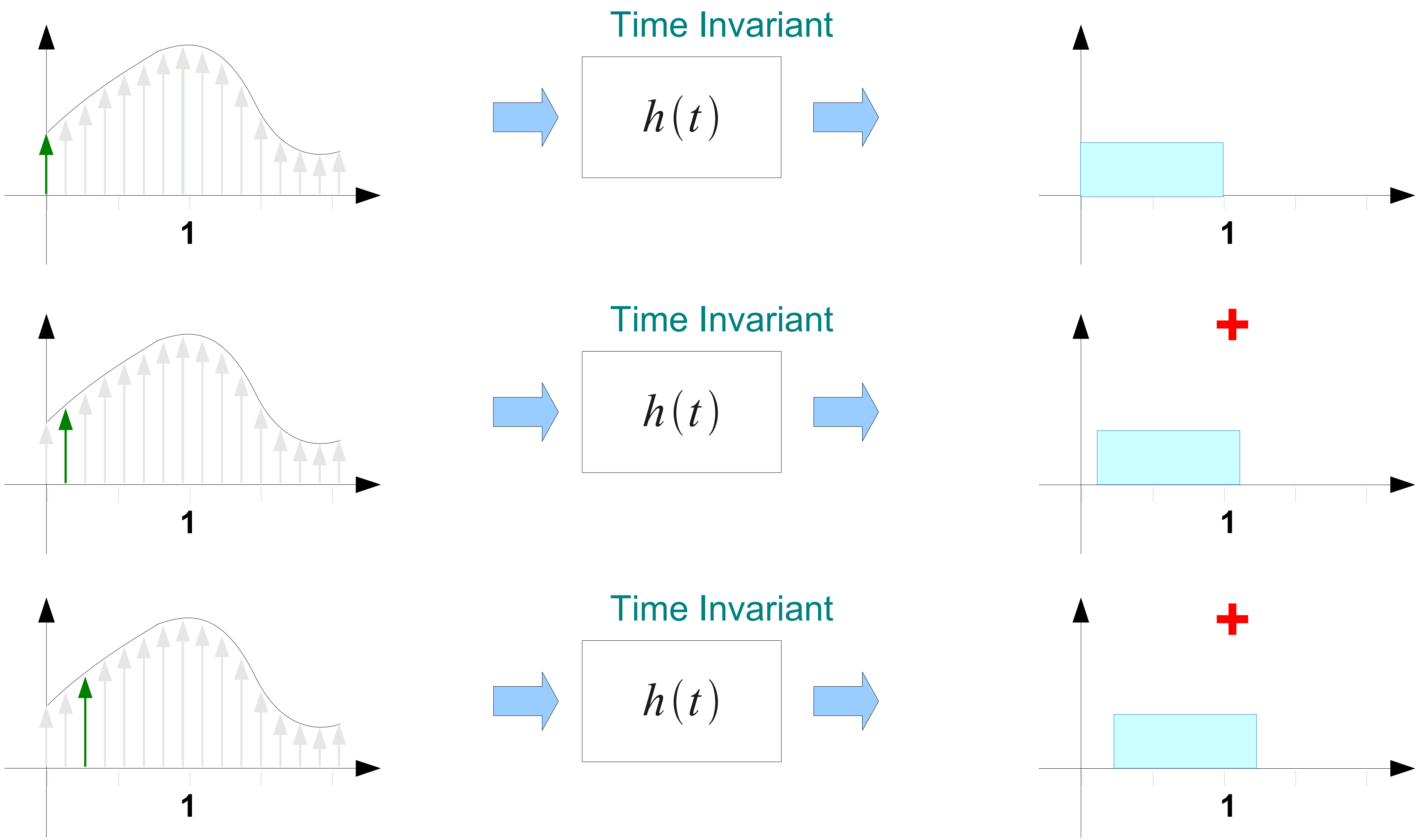




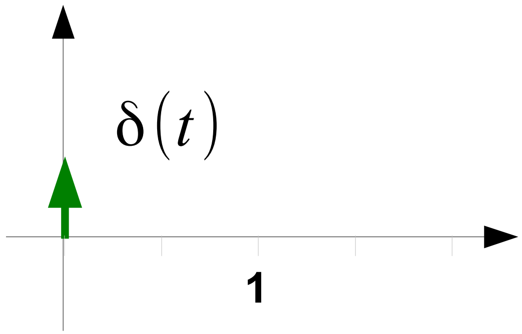
Convolution: delayed response of h(t) (1)



Convolution: delayed response of h(t) (2)

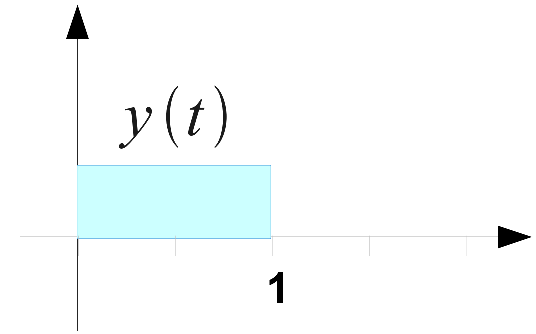


Convolution: delayed response of h(t) (3)



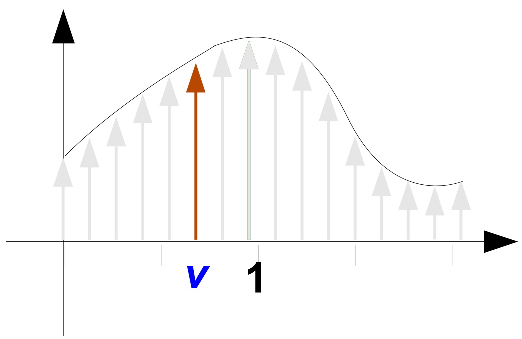
Linear System

$h(t)$



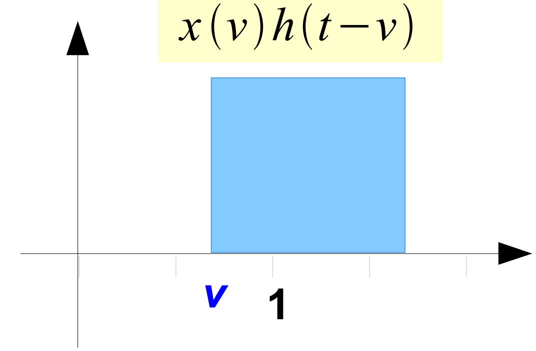
$\delta(t-v) \Rightarrow h(t-v)$

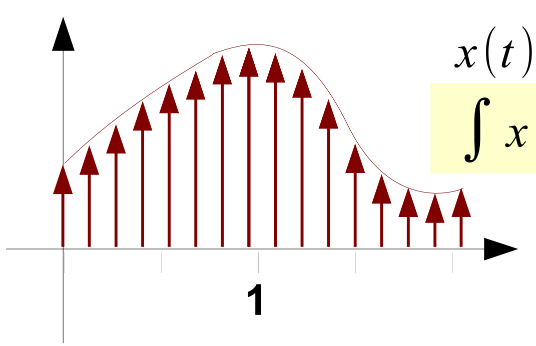
$x(v)\,\delta(t-v) \Rightarrow x(v)\,h(t-v)$



input value at time v
→ x(v)
delayed impulse response
→ x(v) h(t - v)

$x(v)h(t-v)$

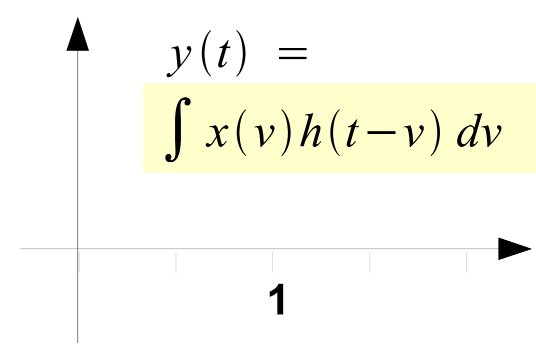




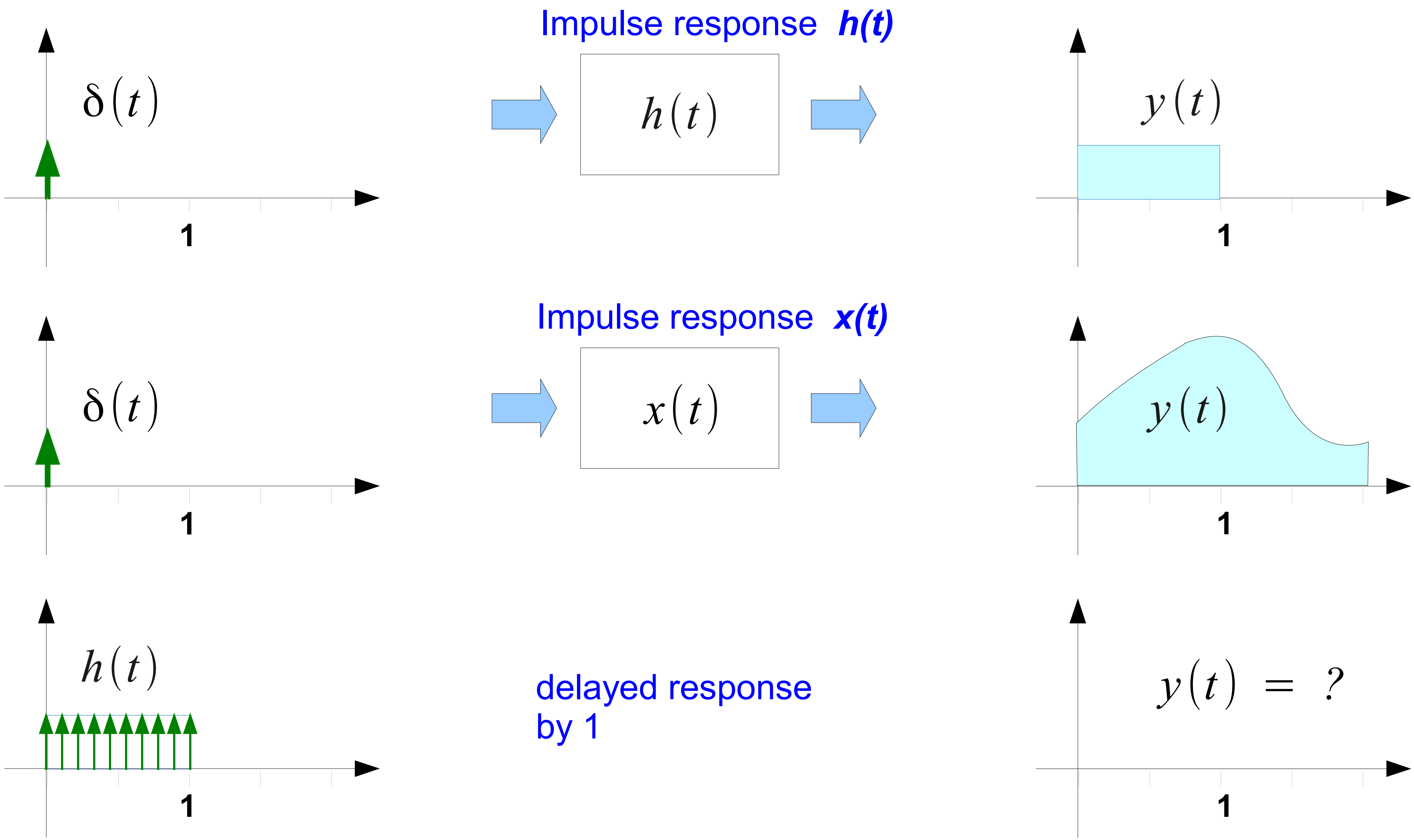
$x(t) = \int x(v)\delta(t-v)\,dv$

$\Rightarrow$

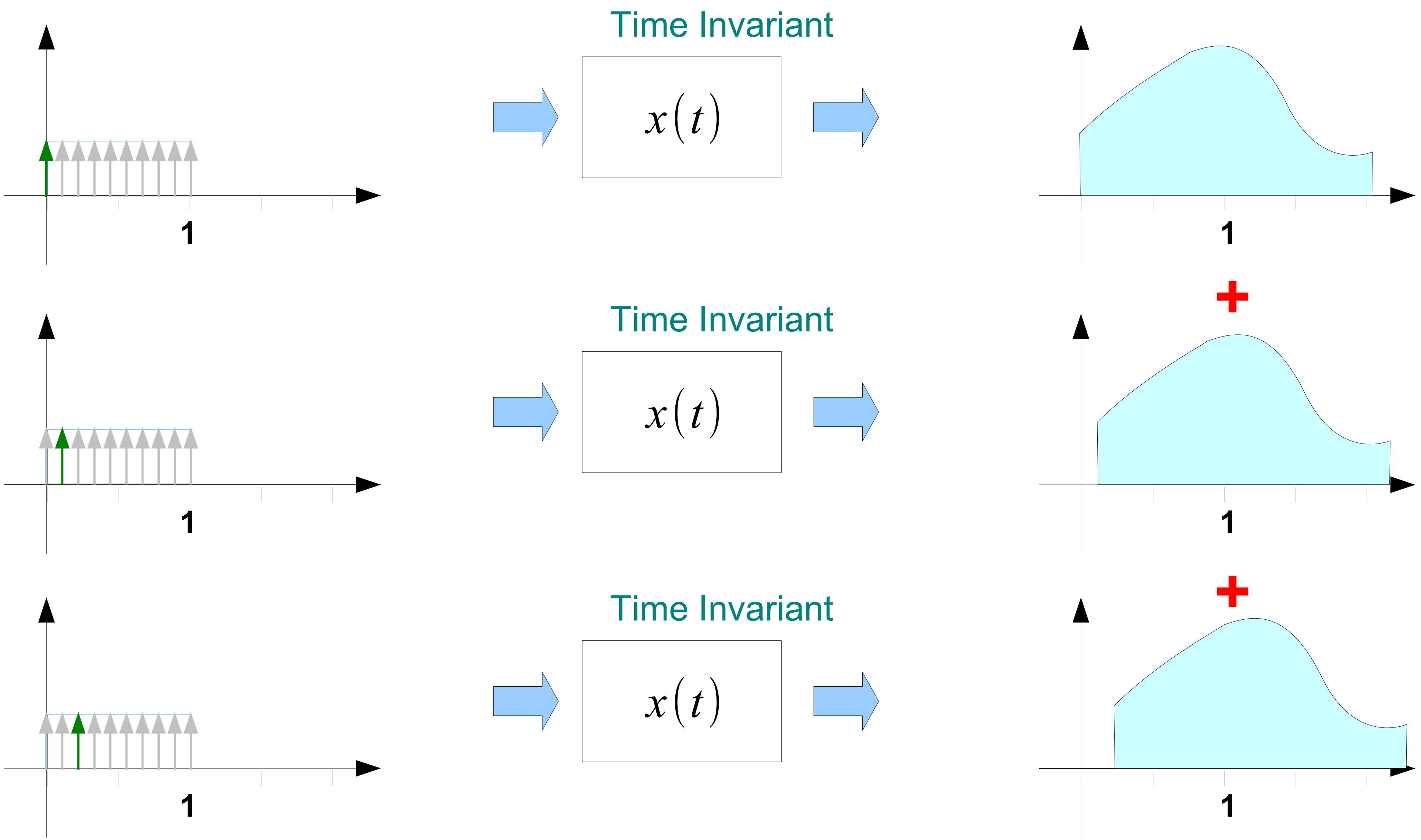
$y(t) = \int x(v)h(t-v)\,dv$



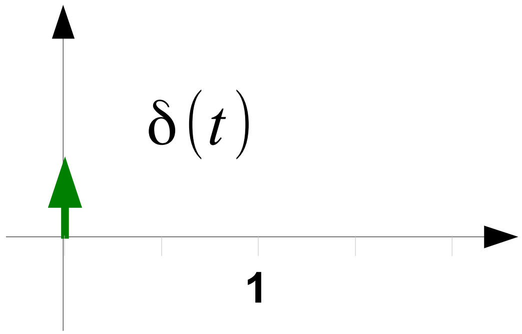
Convolution: delayed response of x(t) (1)



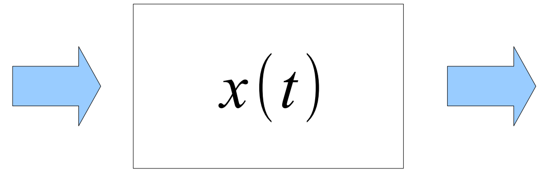
Convolution: delayed response of x(t) (2)

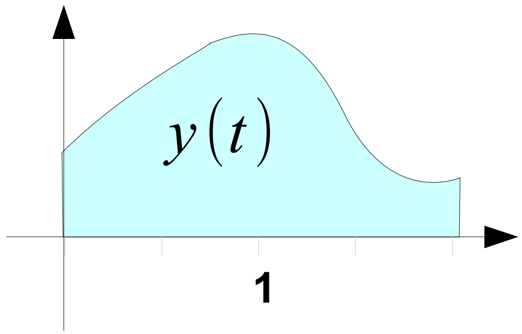


Convolution: delayed response of x(t) (3)



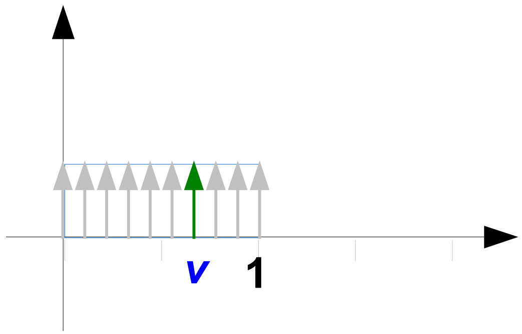
Linear System



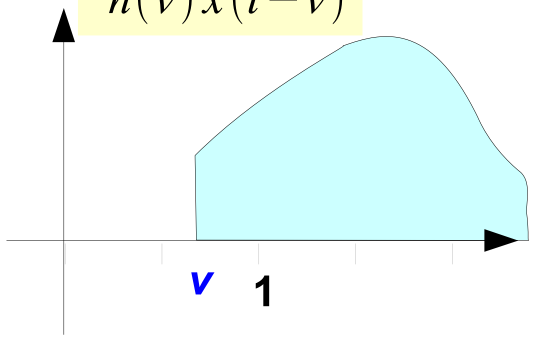


$\delta(t-v) \Rightarrow x(t-v)$

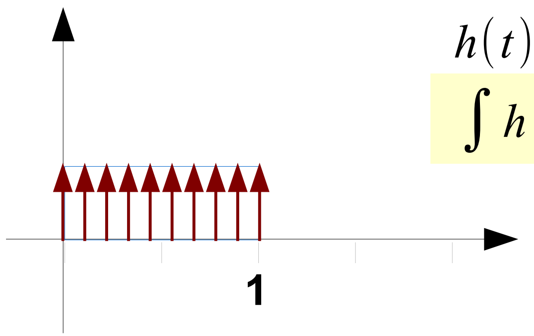
$h(v)\,\delta(t-v) \Rightarrow h(v)\,x(t-v)$



$h(v)\,x(t-v)$



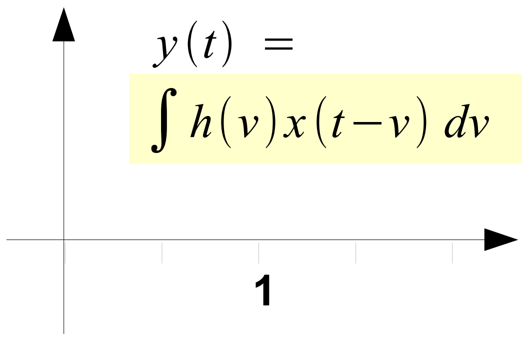
input value at time v
→ h(v)
delayed impulse response
→ h(v) x(t - v)



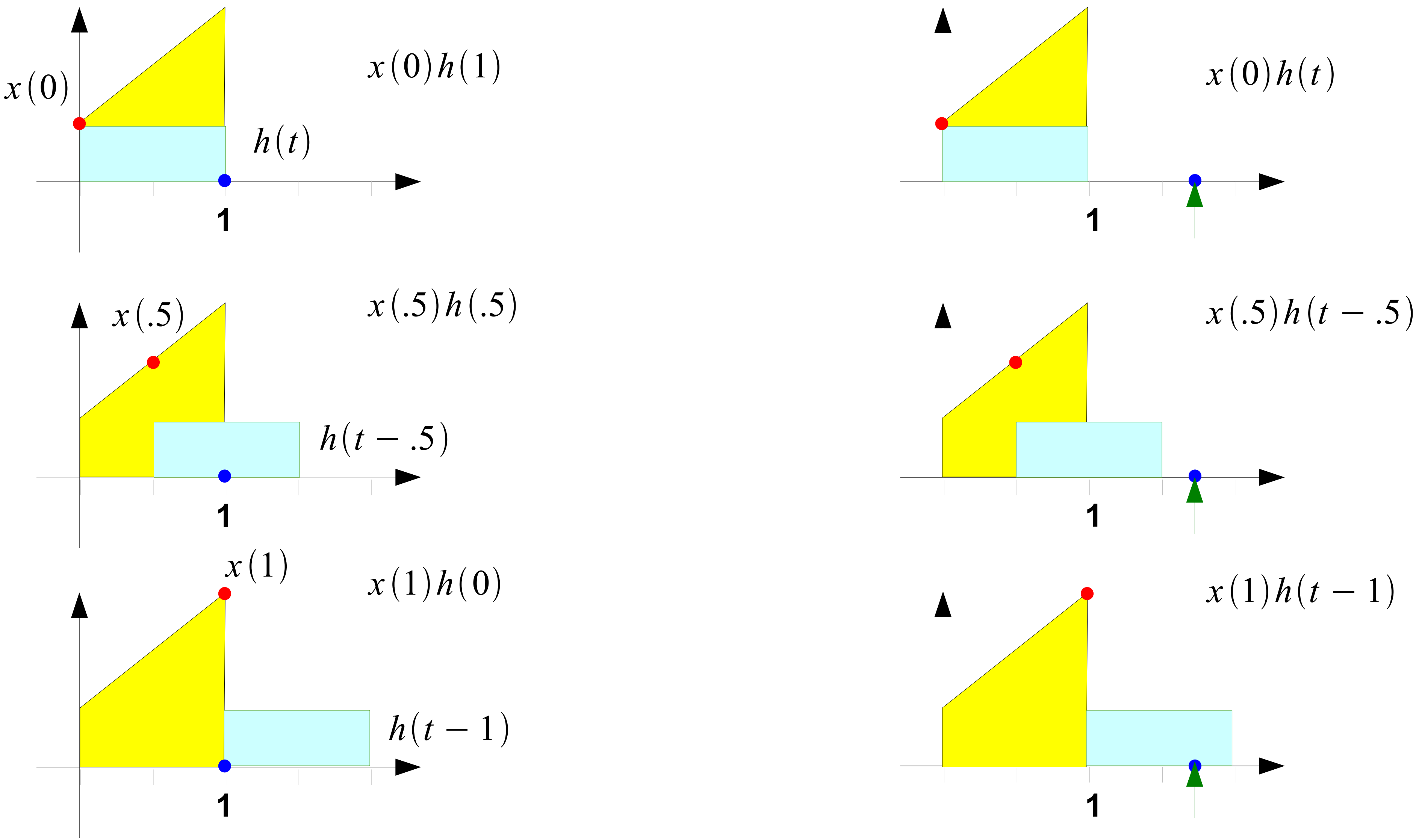
$$h(t) = \int h(v)\,\delta(t-v)\,dv$$

$\Rightarrow$

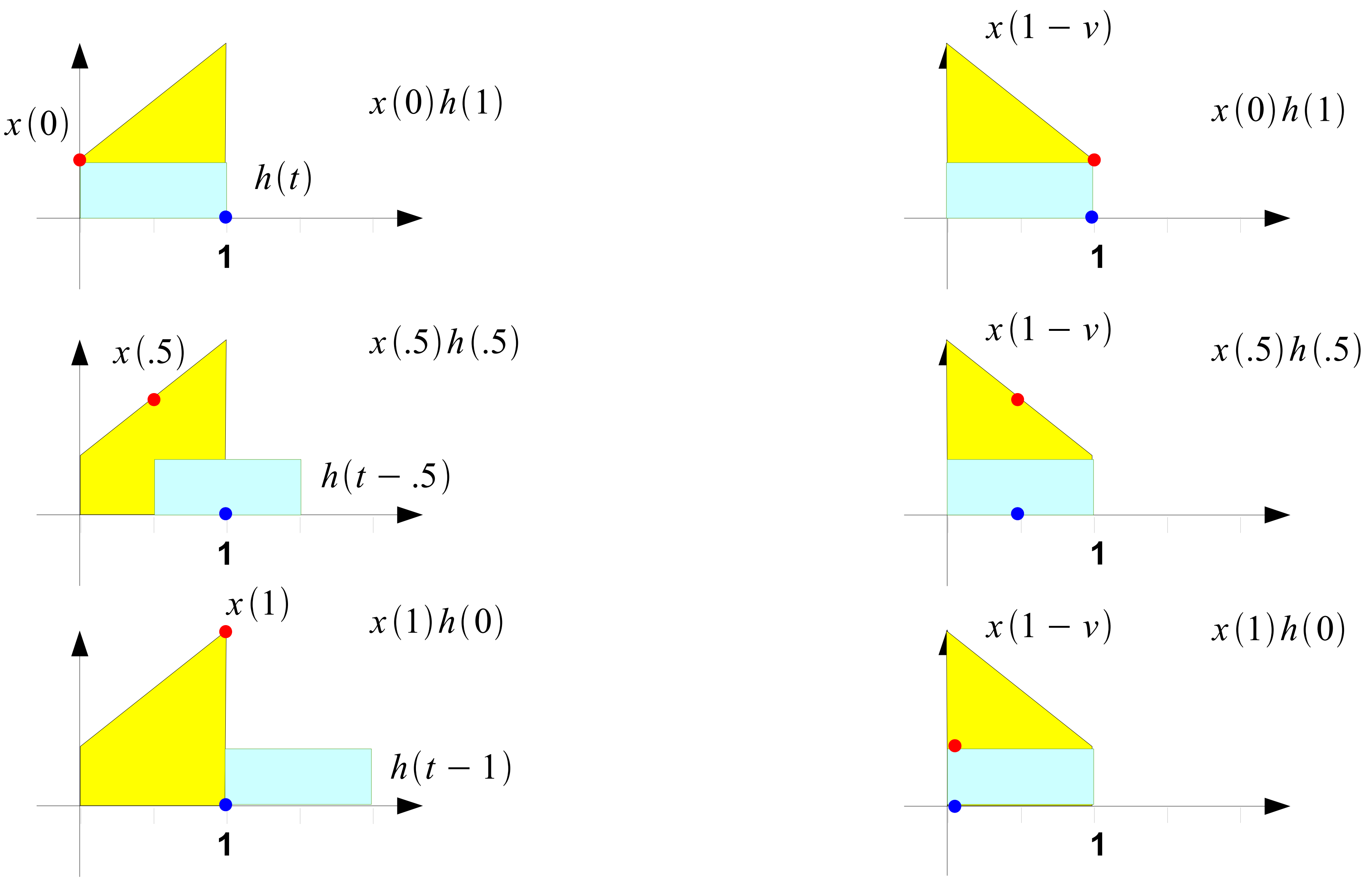
$$y(t) = \int h(v)\,x(t-v)\,dv$$



Impulse Response

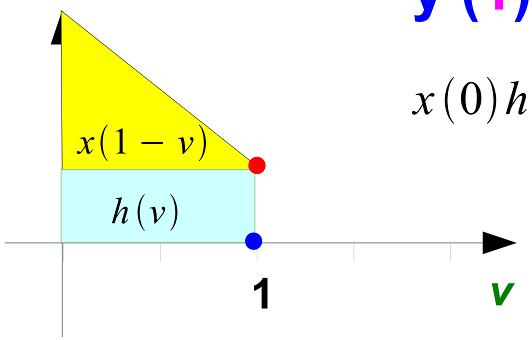


Impulse Response

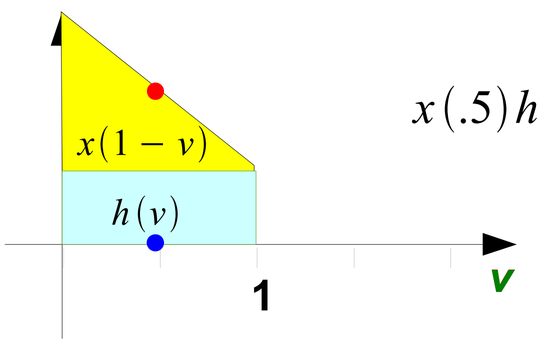


Impulse Response

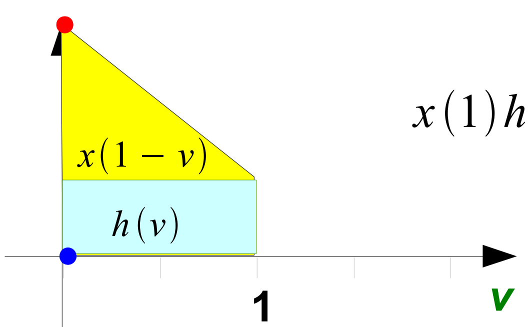
y (1)



$x(0)h(1)$

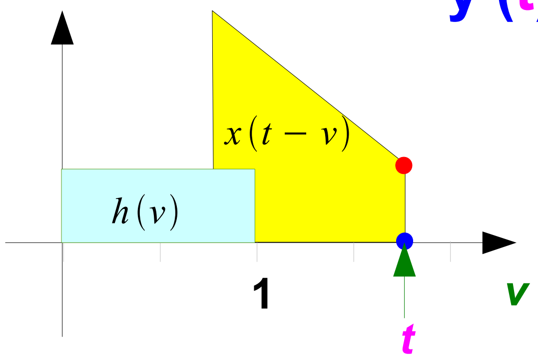


$x(.5)h(.5)$

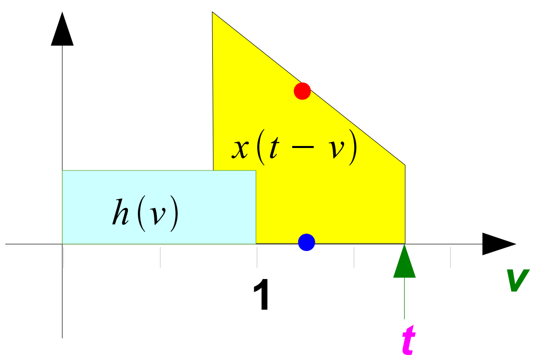


$x(1)h(0)$

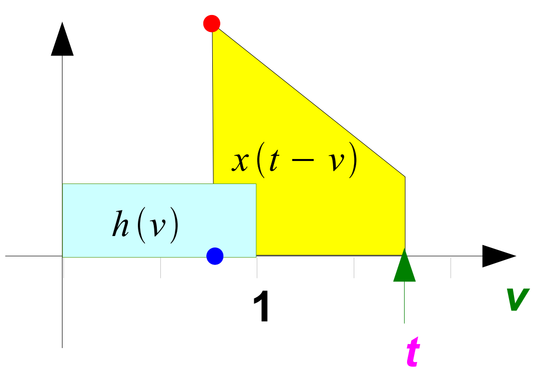
y (t)



$$x(t-t)h(t)$$
$$= x(0)h(t)$$

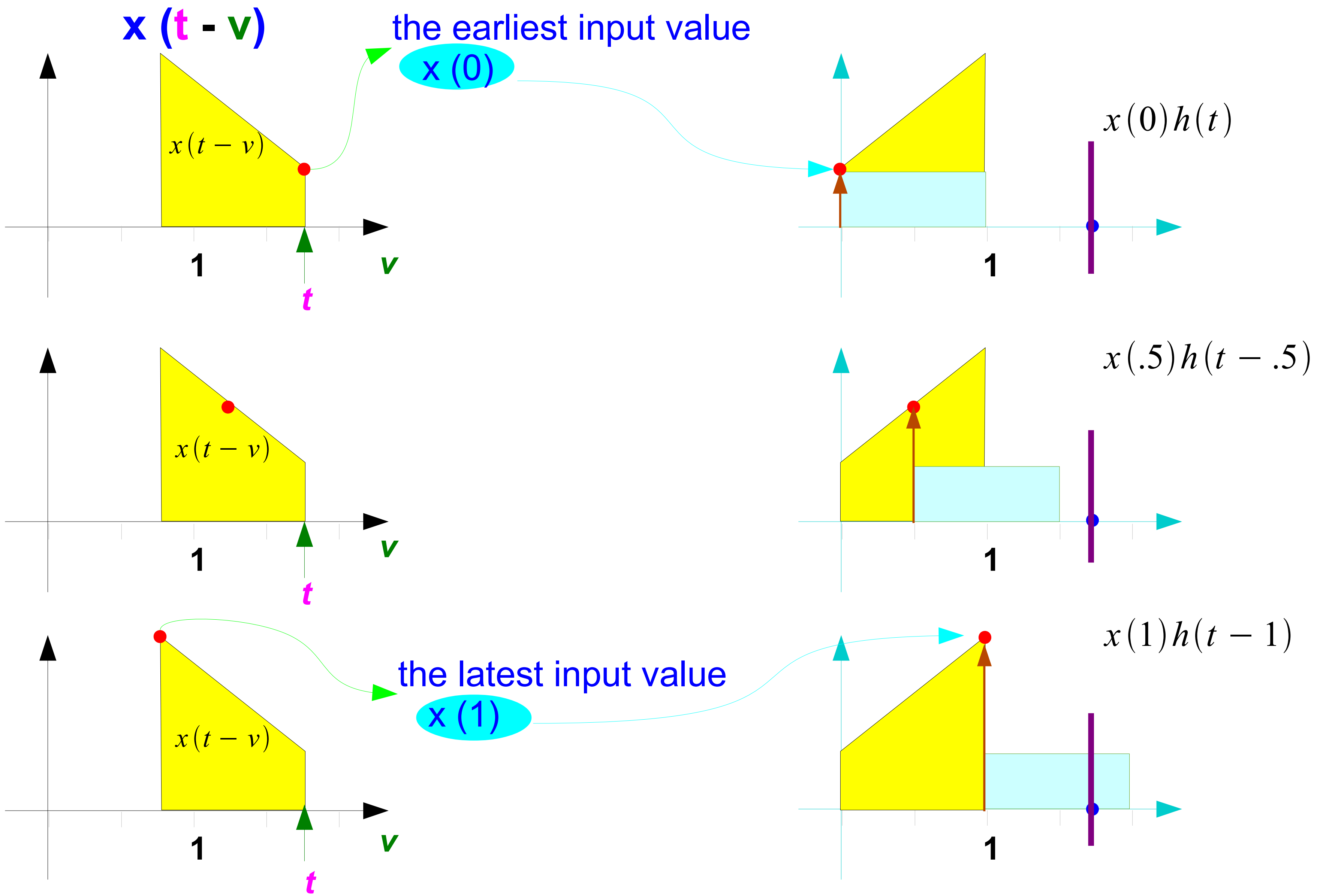


$$x(t-t+.5)h(t-.5)$$
$$= x(.5)h(t-.5)$$



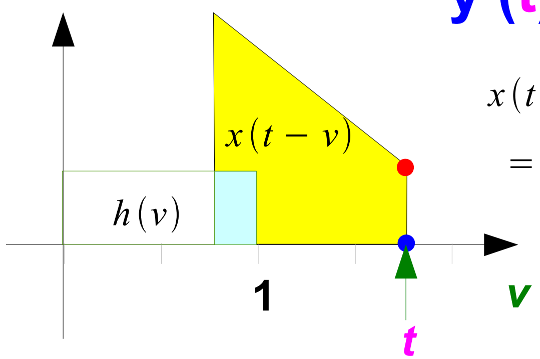
$$x(t-t+1)h(t-1)$$
$$x(1)h(t-1)$$

Impulse Response



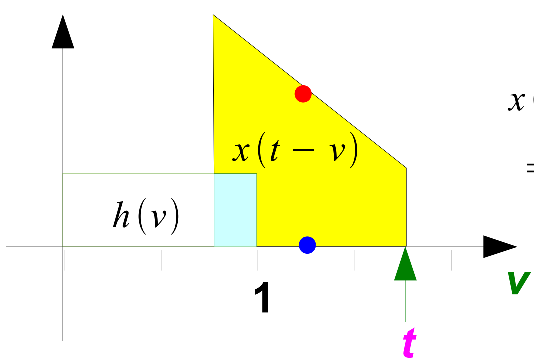
Impulse Response

y (t)



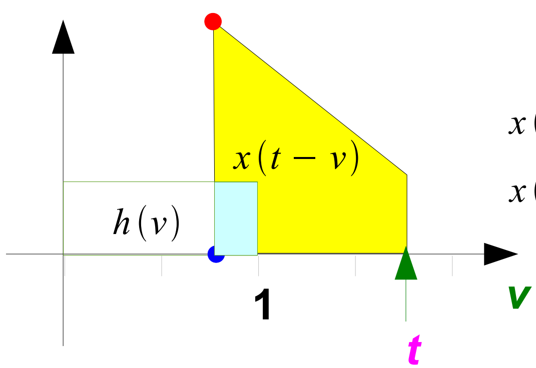
$x(t-t)h(t)$

$= x(0)h(t)$



$x(t-t+.5)h(t-.5)$

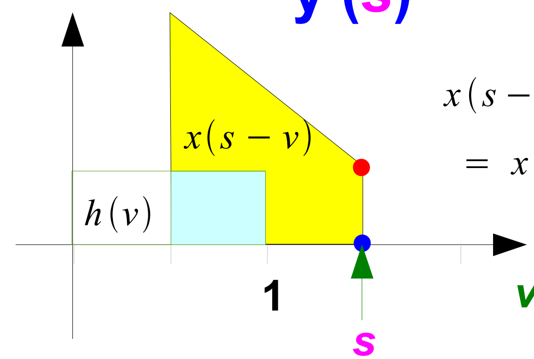
$= x(.5)h(t-.5)$



$x(t-t+1)h(t-1)$

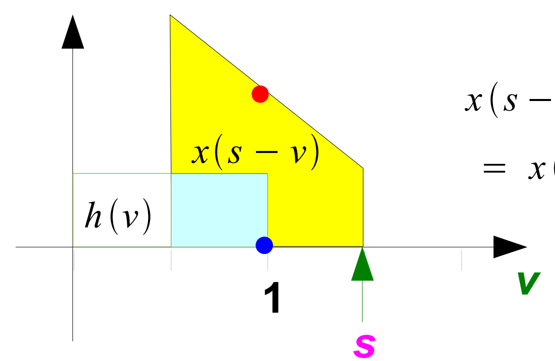
$x(1)h(t-1)$

y (s)



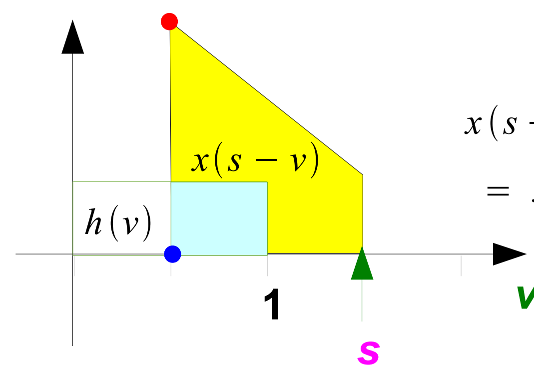
$x(s-s)h(s)$

$= x(0)h(s)$



$x(s-s+.5)h(s-.5)$

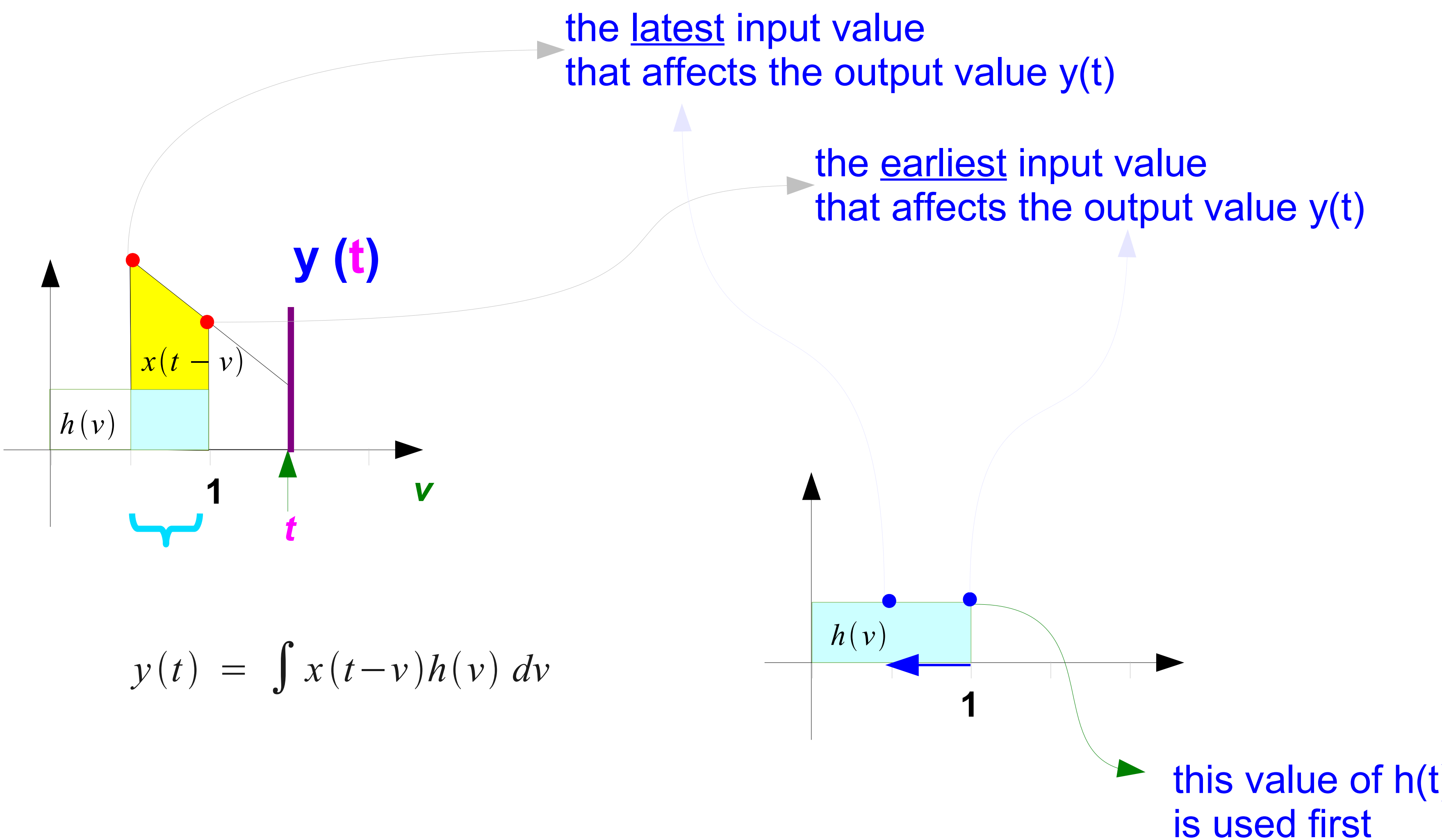
$= x(.5)h(s-.5)$



$x(s-s+1)h(s-1)$

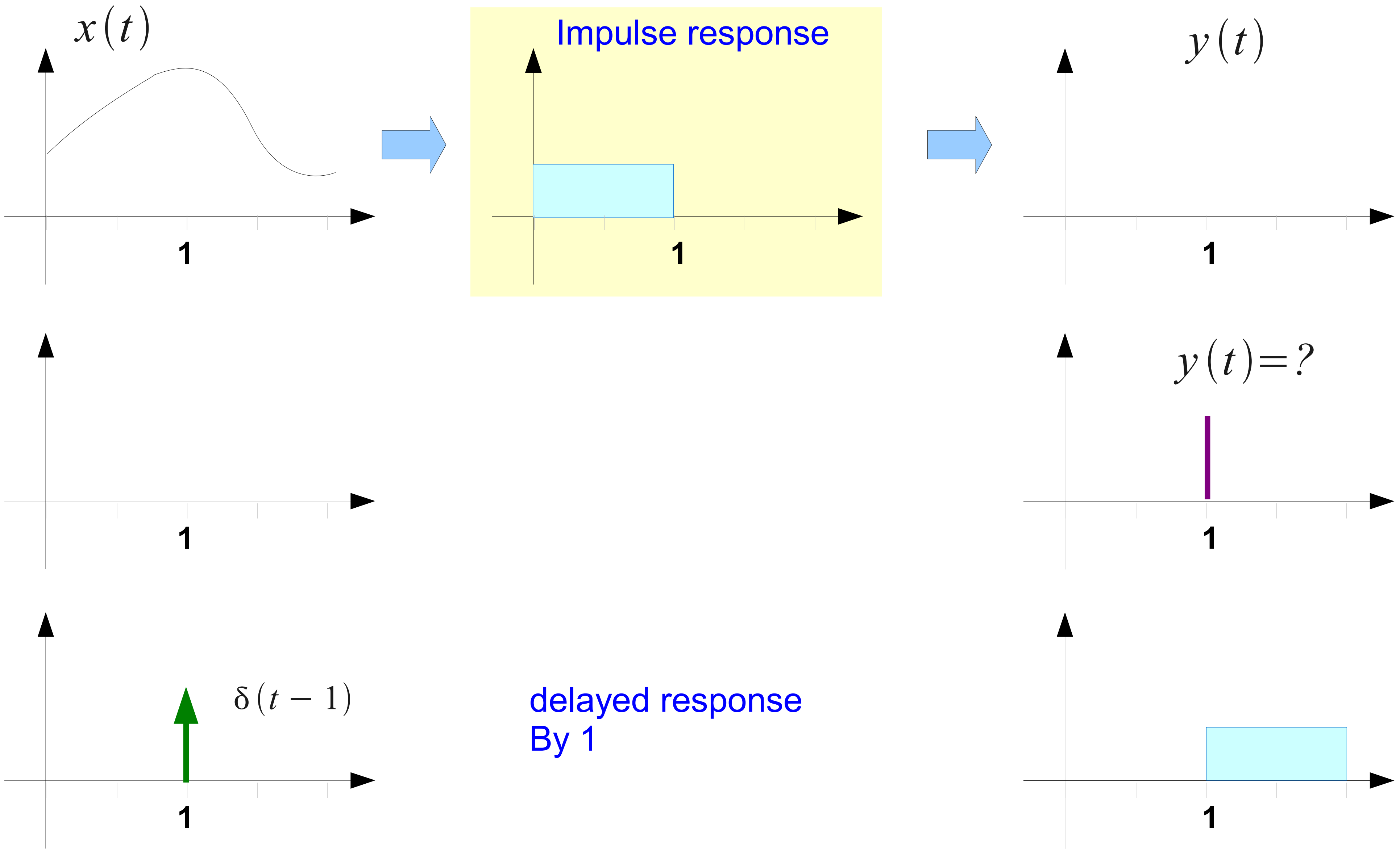
$= x(1)h(s-1)$

Impulse Response

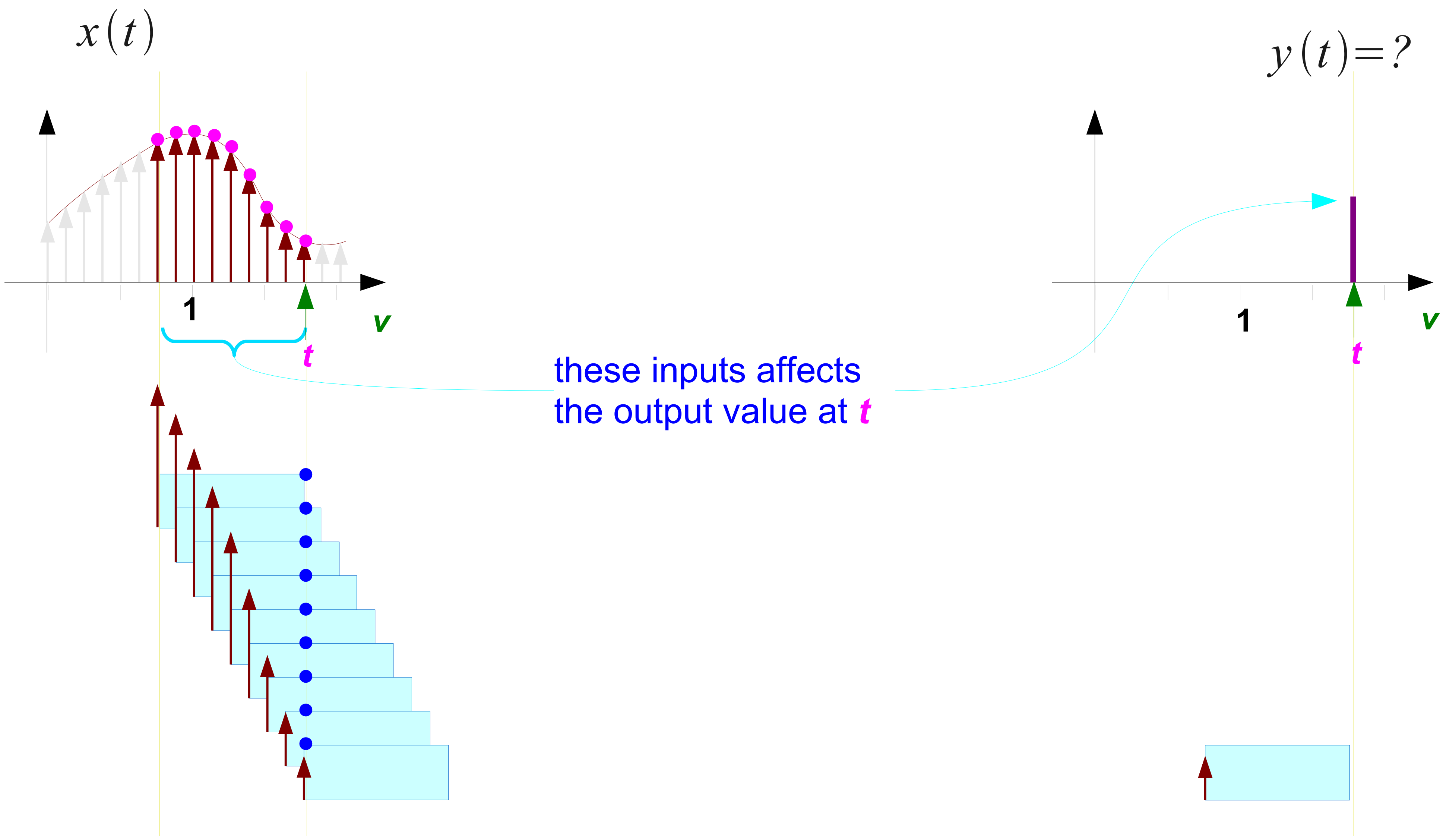


$$y(t) = \int x(t-v)h(v)\,dv$$

Impulse Response

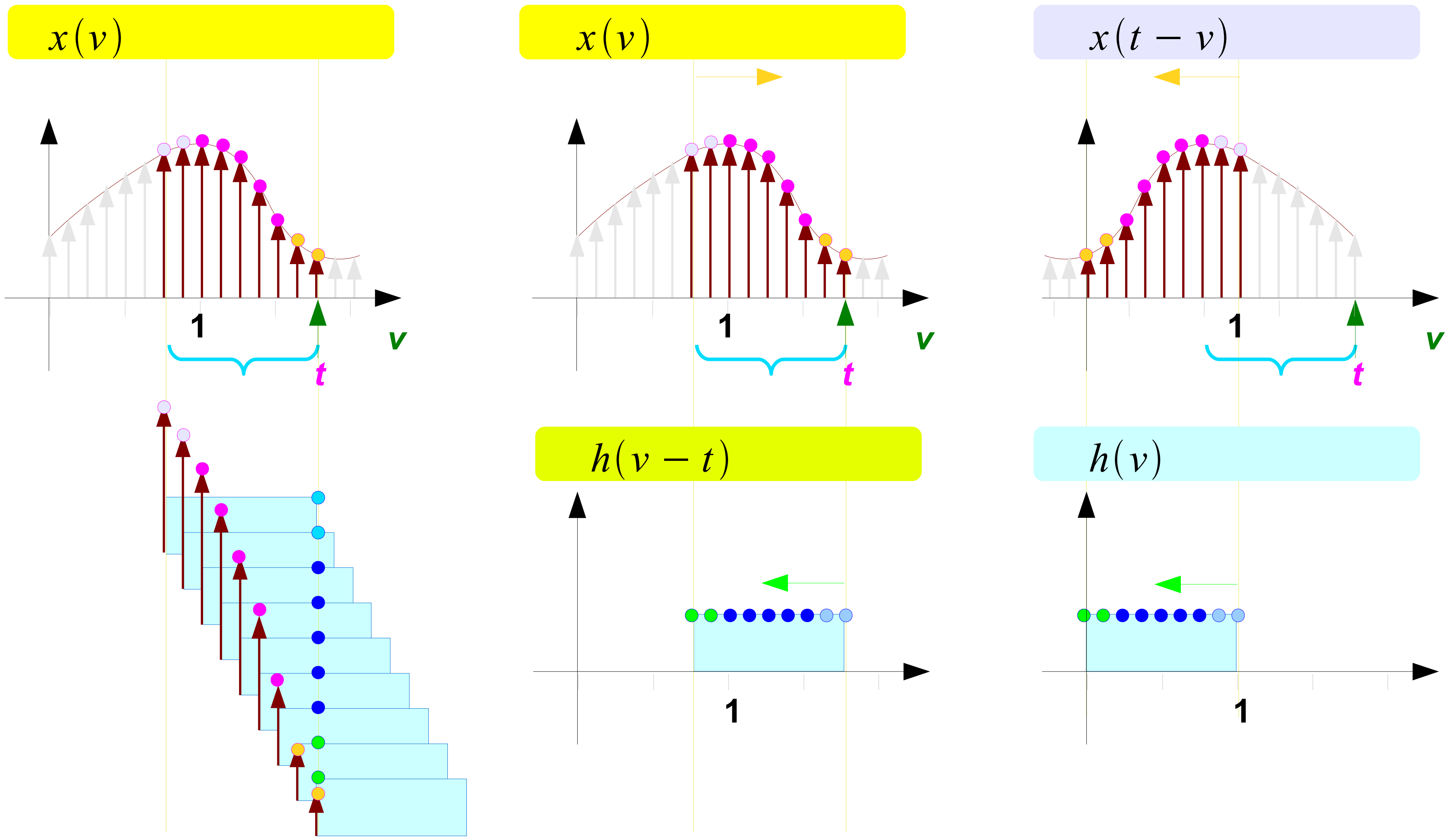


Impulse Response



these inputs affects
the output value at *t*

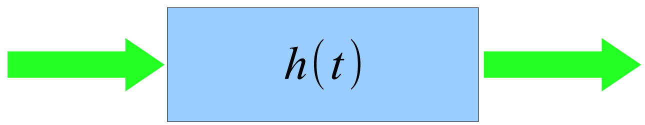
Impulse Response



Frequency Response

$$y(t) = h(t) * x(t) = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$x(t) = A\, e^{j\Phi}\, e^{j\omega t}$$



$$y(t) = H(jw) \cdot A\, e^{j\Phi}\, e^{j\omega t}$$

$$y(t) = \int_{-\infty}^{+\infty} h(\tau)\, A\, e^{j\Phi}\, e^{jw(t-\tau)}\, d\tau$$

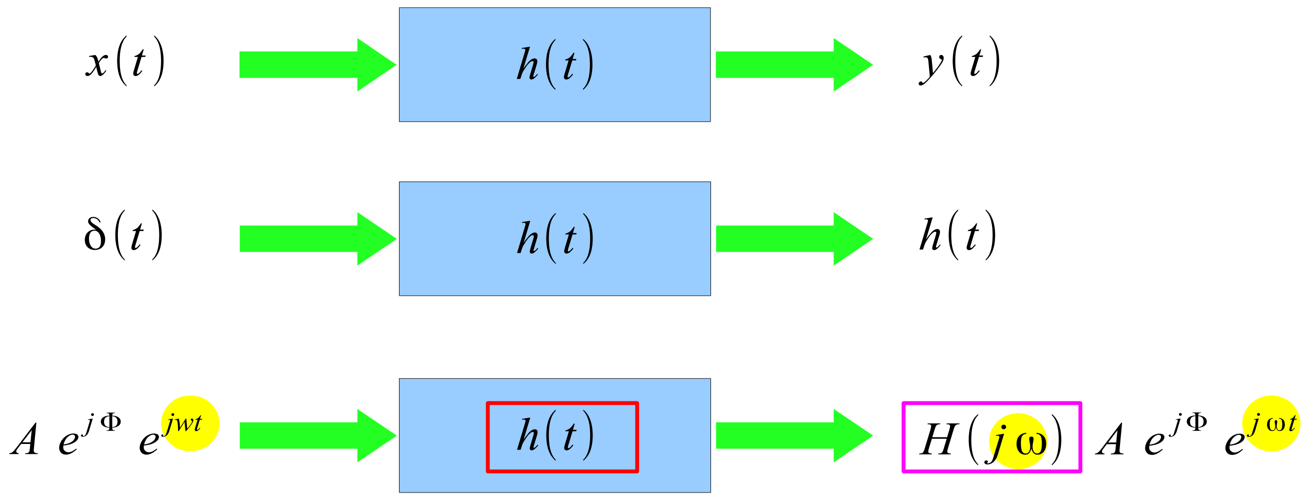
$$= \int_{-\infty}^{+\infty} h(\tau)\, A\, e^{j\Phi}\, e^{jwt}\, e^{-j\omega\tau}\, d\tau$$

$$= \underline{A\, e^{j\Phi}\, e^{jwt}} \cdot \underline{\int_{-\infty}^{+\infty} h(\tau)\, e^{-j\omega\tau}\, d\tau}$$

$$= \underline{x(t)} \cdot \underline{H(jw)}$$

Frequency Response

$$y(t) = h(t) * x(t) = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$$



single frequency component : ω

single frequency component : ω

$$H(j\omega) = \int_{-\infty}^{+\infty} h(\tau)\, e^{-j\omega\tau}\, d\tau$$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003