

CORDIC Background (3A)

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CORDIC Background

J. S. Walther, A Unified Algorithm for Elementary Functions

Unified CORDIC Iteration Eq

$$x_{i+1} = x_i - m \cdot y_i \cdot d_i \cdot 2^{-i}$$

$$y_{i+1} = y_i + x_i \cdot d_i \cdot 2^{-i}$$

$$z_{i+1} = z_i - d_i \cdot e_i$$

$$m = 1 \Rightarrow e_i = \tan^{-1}(2^{-i})$$

$$m = 0 \Rightarrow e_i = (2^{-i})$$

$$m = -1 \Rightarrow e_i = \tanh^{-1}(2^{-i})$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$\tanh \alpha = \frac{\sinh \alpha}{\cosh \alpha}$$

$$\exp \alpha = \sinh \alpha + \cosh \alpha$$

$$\ln \alpha = 2 \tanh^{-1}(y/x)$$

$$x = \alpha + 1$$

$$y = \alpha - 1$$

$$(\alpha)^{1/2} = (x^2 - y^2)^{1/2}$$

$$x = \alpha + 1/4$$

$$y = \alpha - 1/4$$

Unified CORDIC Iteration Eq

$$x_{i+1} = x_i - m \cdot y_i \cdot d_i \cdot 2^{-i}$$

$$y_{i+1} = y_i + x_i \cdot d_i \cdot 2^{-i}$$

$$z_{i+1} = z_i - d_i \cdot e_i$$

$$m = 1 \Rightarrow e_i = \tan^{-1}(2^{-i})$$

$$m = 0 \Rightarrow e_i = (2^{-i})$$

$$m = -1 \Rightarrow e_i = \mathbf{\tanh}^{-1}(2^{-i})$$

$$\mathbf{\cosh} i x = \frac{1}{2}(e^{ix} + e^{-ix}) = \cos x$$

$$\mathbf{\sinh} i x = \frac{1}{2}(e^{ix} - e^{-ix}) = i \sin x$$

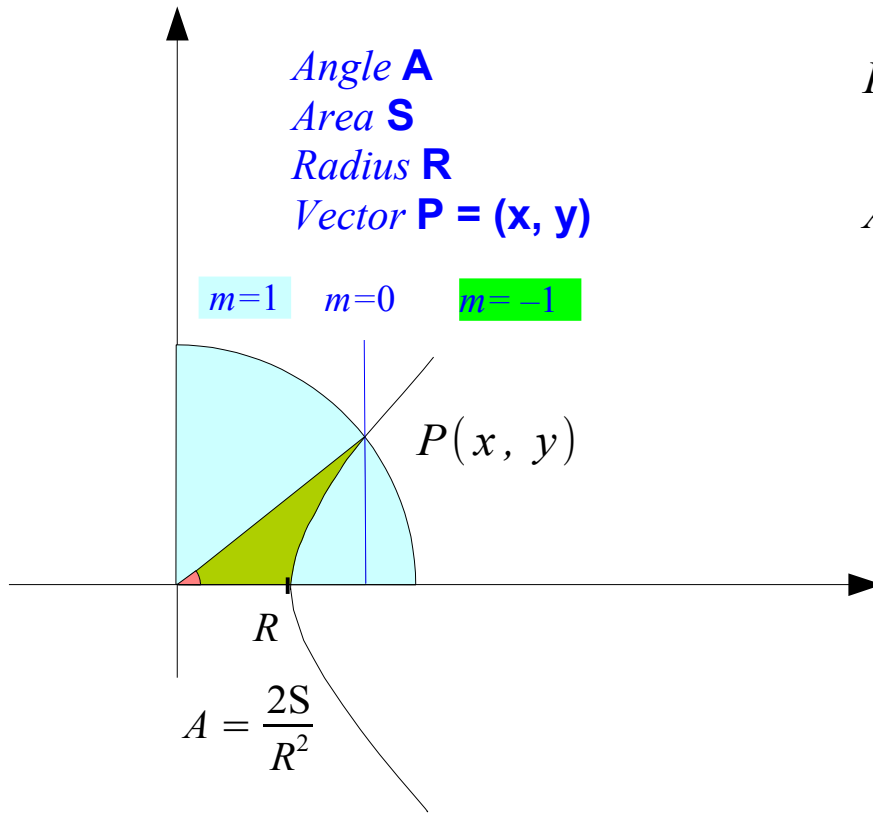
$$\mathbf{\tanh} i x = \frac{(e^{ix} + e^{-ix})}{(e^{ix} - e^{-ix})} = i \tan x$$

$$\mathbf{\cosh} x = \frac{1}{2}(e^x + e^{-x}) = \cos i x$$

$$\mathbf{\sinh} x = \frac{1}{2}(e^x - e^{-x}) = -i \sin i x$$

$$\mathbf{\tanh} x = \frac{(e^x + e^{-x})}{(e^x - e^{-x})} = -i \tan i x$$

2S = A (Area:S Angle:A)



$$R = (x^2 + y^2)^{1/2}$$

$$A = \tan^{-1}\left(\frac{y}{x}\right)$$

$$R = (x^2 - y^2)^{1/2}$$

$$A = -i \tan^{-1}\left(i \frac{y}{x}\right) = \mathbf{\tanh^{-1}}\left(\frac{y}{x}\right)$$

$$R = x$$

$$A =$$

Unified CORDIC Iteration Eq

$$P=(x, y)$$

$$R = \sqrt{x^2 + m y^2}$$

$$A = \frac{1}{\sqrt{m}} \tan^{-1} \left(\sqrt{m} \frac{y}{x} \right)$$

$$P_{i+1}=(x_{i+1}, y_{i+1}) \rightarrow P_i=(x_i, y_i)$$

$$x_{i+1} = x_i + m y_i \delta_i$$

$$y_{i+1} = y_i - x_i \delta_i$$

$$A_{i+1} = A_i - \alpha_i$$

$$R_{i+1} = R_i \cdot K_i$$

$$\alpha_i = \frac{1}{\sqrt{m}} \tan^{-1} (\sqrt{m} \delta_i)$$

$$K_i = \sqrt{1 + m \delta_i^2}$$

$$A_n = A_0 - \alpha$$

$$R_n = R_0 \cdot K$$

$$\alpha = \sum_{i=0}^{n-1} \alpha_i$$

$$K = \prod_{i=0}^{n-1} K_i$$

Unified CORDIC Iteration Eq

$$A_{i+1} = A_i - \alpha_i \quad \alpha_i = \frac{1}{\sqrt{m}} \tan^{-1}(\sqrt{m} \delta_i)$$

$$R_{i+1} = R_i \cdot K_i \quad K_i = \sqrt{1 + m \delta_i^2}$$

$$A_{i+1} = A_i - \frac{1}{\sqrt{m}} \tan^{-1}(\sqrt{m} \delta_i)$$

$$R_{i+1} = R_i \cdot \sqrt{1 + m \delta_i^2}$$

$$m = +1$$

$$x_{i+1} = x_i + y_i \delta_i$$

$$y_{i+1} = y_i - x_i \delta_i$$

$$A_{i+1} = A_i - \tan^{-1}(\delta_i)$$

$$R_{i+1} = R_i \cdot \sqrt{1 + \delta_i^2}$$

$$m = -1$$

$$x_{i+1} = x_i - y_i \delta_i$$

$$y_{i+1} = y_i - x_i \delta_i$$

$$A_{i+1} = A_i + i \tan^{-1}(i \delta_i)$$

$$R_{i+1} = R_i \cdot \sqrt{1 - \delta_i^2}$$

$$m = 0$$

$$x_{i+1} = x_i$$

$$y_{i+1} = y_i - x_i \delta_i$$

$$A_{i+1} = A_i - \delta_i$$

$$R_{i+1} = R_i$$

Unified CORDIC Iteration Eq

$$\mathbf{m} = -1 \Rightarrow \mathbf{e}_i = \mathbf{tanh}^{-1}(2^{-i})$$

$$\mathbf{cosh} \, i x = \frac{1}{2}(e^{ix} + e^{-ix}) = \cos x$$

$$\mathbf{sinh} \, i x = \frac{1}{2}(e^{ix} - e^{-ix}) = i \sin x$$

$$\mathbf{tanh} \, i x = \frac{(e^{ix} + e^{-ix})}{(e^{ix} - e^{-ix})} = i \tan x$$

$$\mathbf{cosh} \, x = \frac{1}{2}(e^x + e^{-x}) = \cos i x$$

$$\mathbf{sinh} \, x = \frac{1}{2}(e^x - e^{-x}) = -i \sin i x$$

$$\mathbf{tanh} \, x = \frac{(e^x + e^{-x})}{(e^x - e^{-x})} = -i \tan i x$$

Unified CORDIC Iteration Eq

References

- [1] <http://en.wikipedia.org/>
- [2] CORDIC FAQ, www.dspguru.com
- [3] R. Andraka, A survey of CORDIC algorithms for FPGA based computers
- [4] J. S. Walther, A Unified Algorithm for Elementary Functions