

# Up-Sampling (5B)

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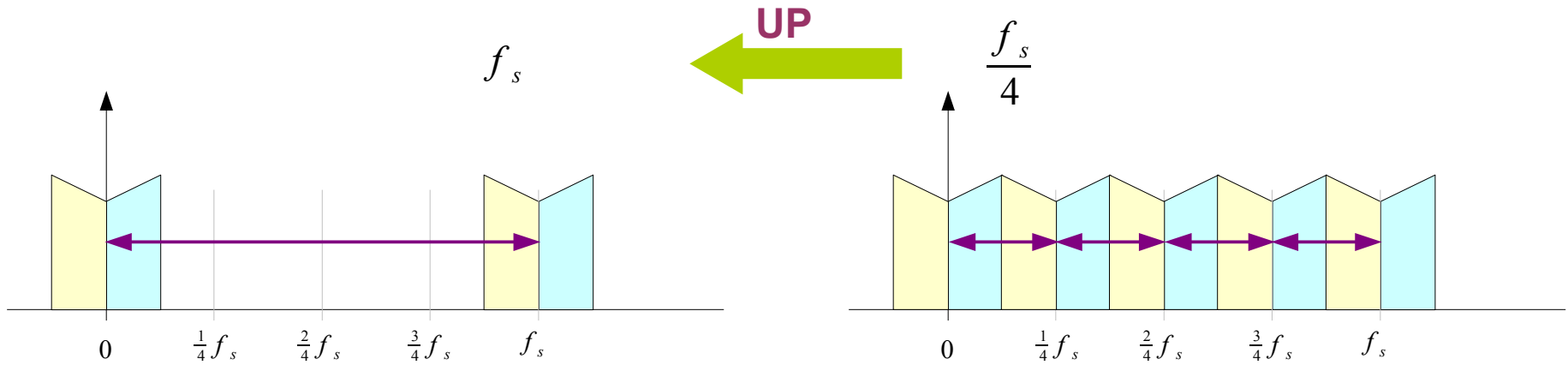
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# Increasing Sampling Frequency

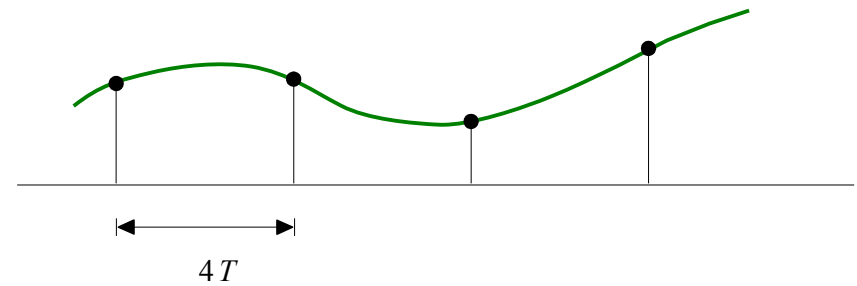
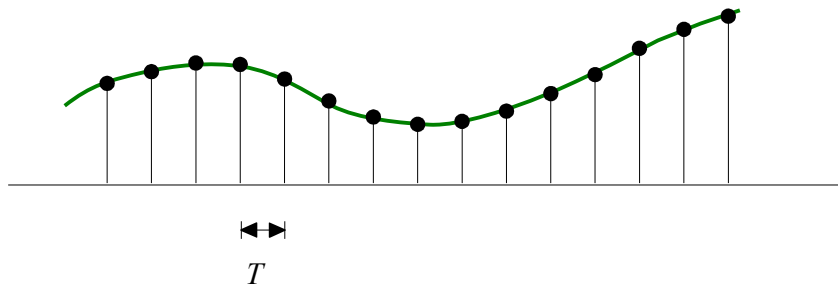


Sampling Frequency  $f_s$

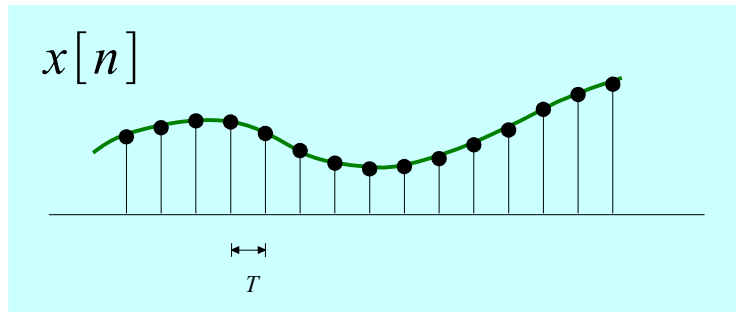
Sampling Time  $T = \frac{1}{f_s}$

Sampling Frequency  $f'_s = \frac{1}{4}f_s$

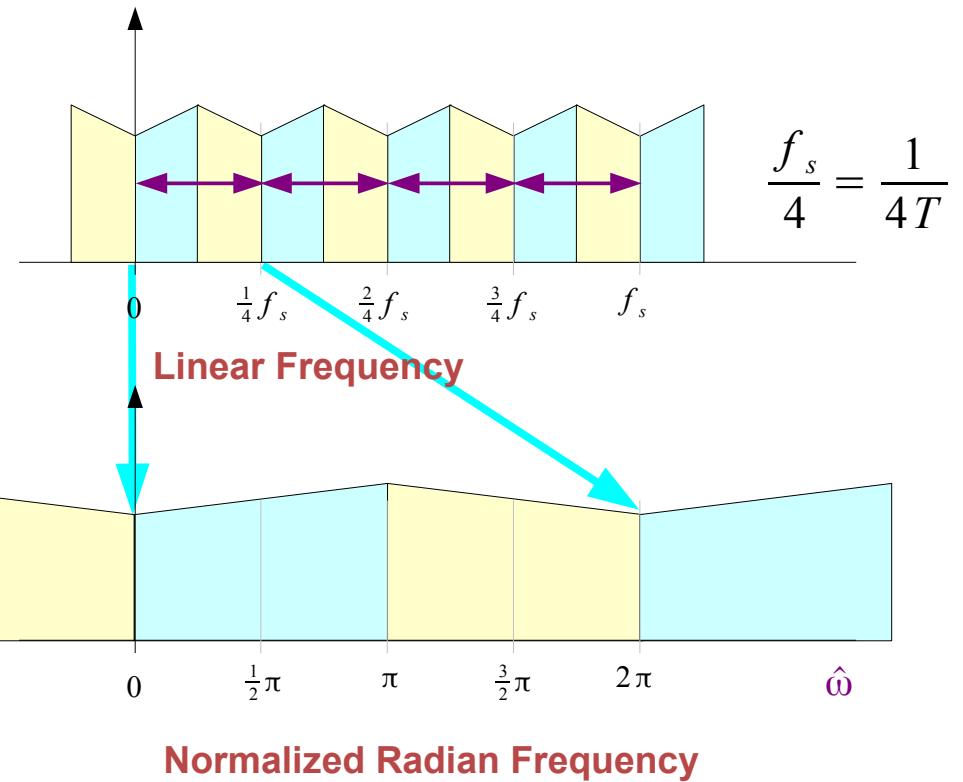
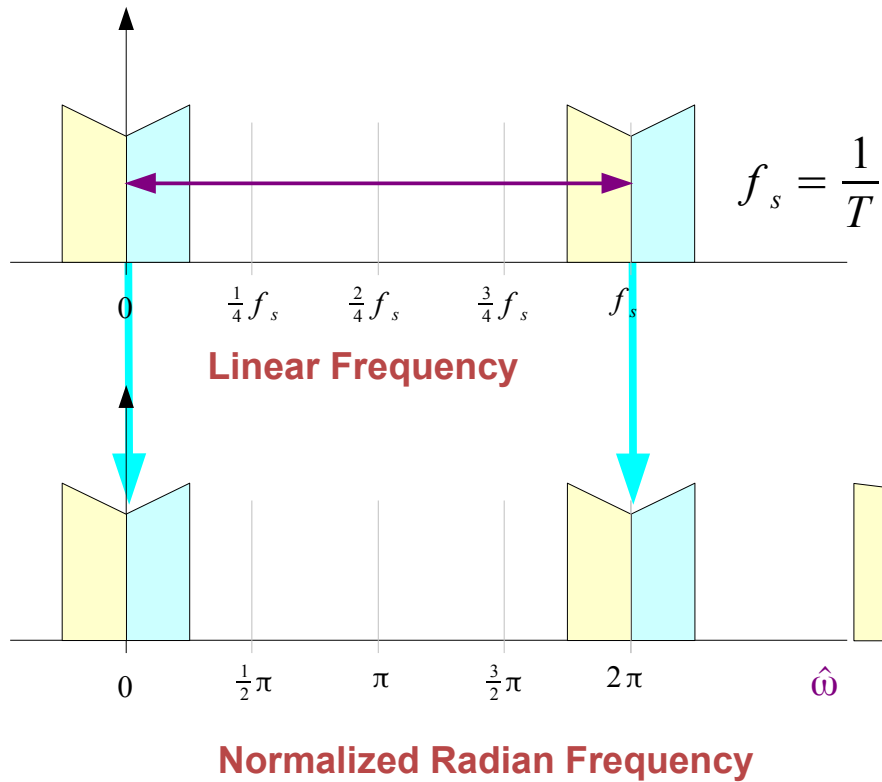
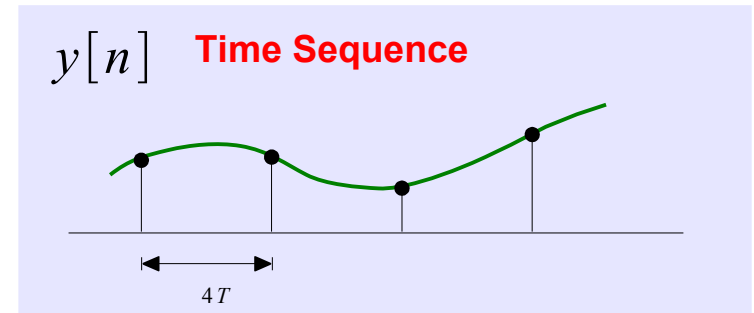
Sampling Time  $T' = \frac{4}{f_s}$



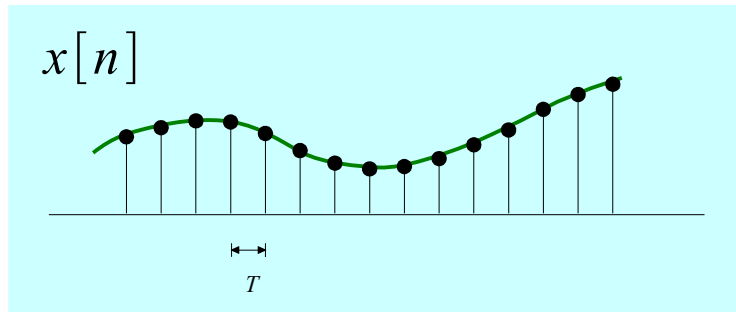
# Fine Sequence & Spectrum



← UP



# Normalized Radian Frequency

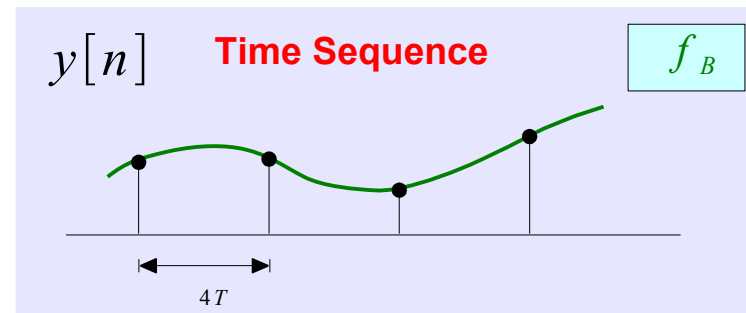


$$\hat{\omega} = \omega \cdot T_s = \frac{\omega}{1/T_s}$$

$$\hat{\omega} = \frac{\omega}{f_s} = 2\pi \frac{f}{f_s}$$

↑                      ↑  
Normalized to  $f_s$

Normalized Radian Frequency

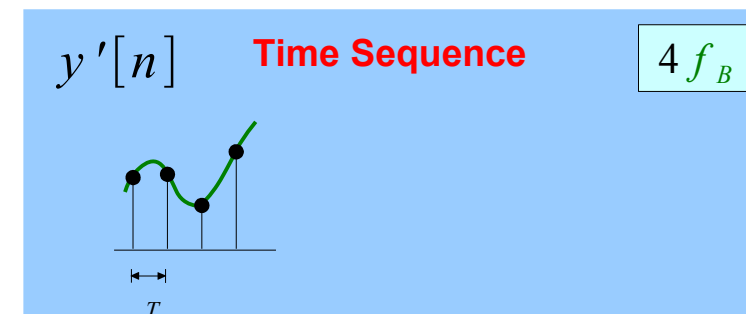


$$\frac{f}{f_s} = \frac{f_B}{1/4T} = f_B \cdot 4T$$

The Same

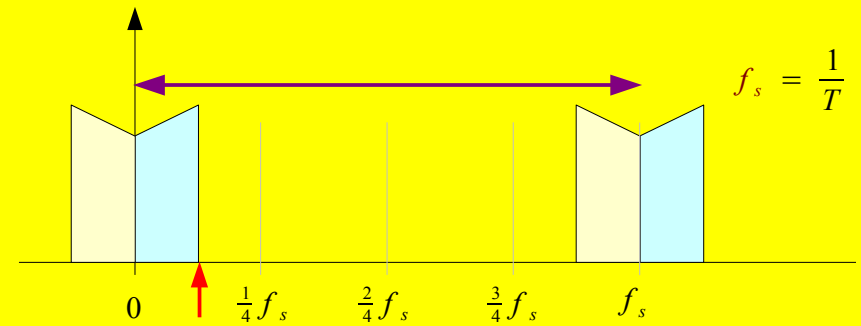
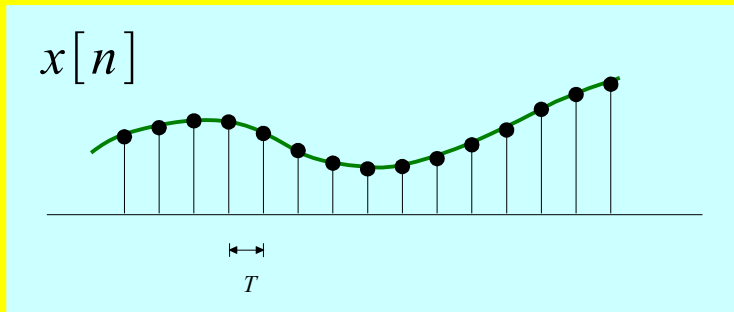
- Time Sequence
- Normalized Radian Frequency

$$\frac{f}{f_s} = \frac{4f_B}{1/T} = f_B \cdot 4T$$

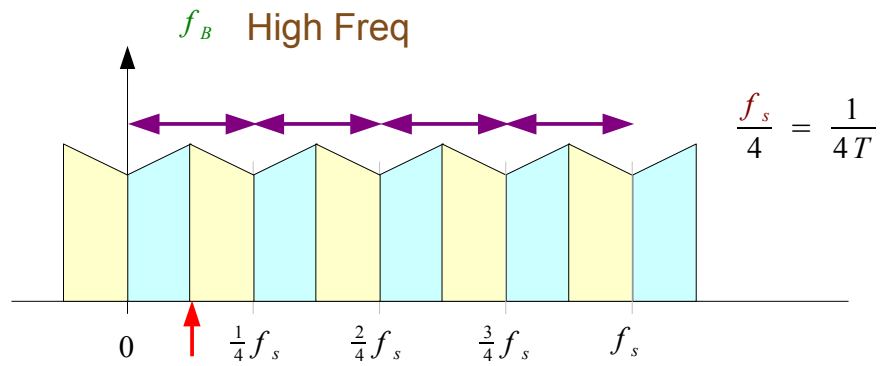
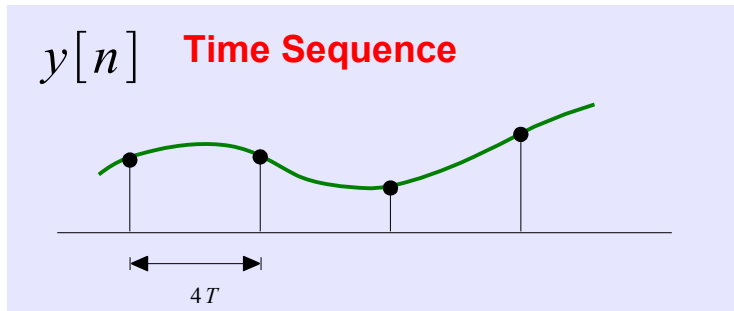


The Highest Frequency:

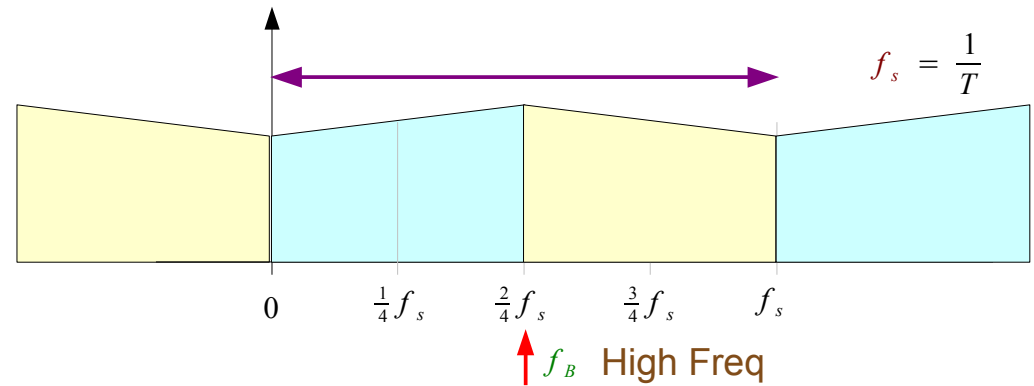
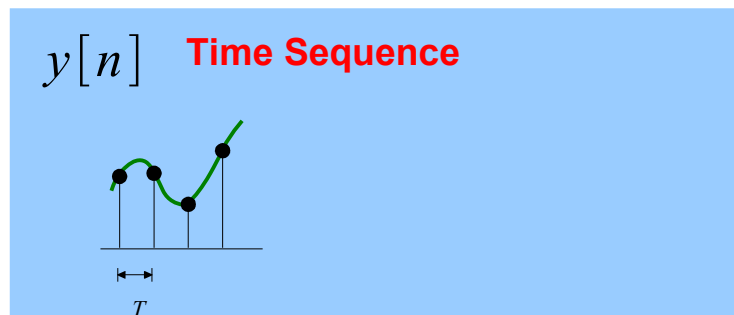
# Fine Sequence Spectrum – Linear Frequency



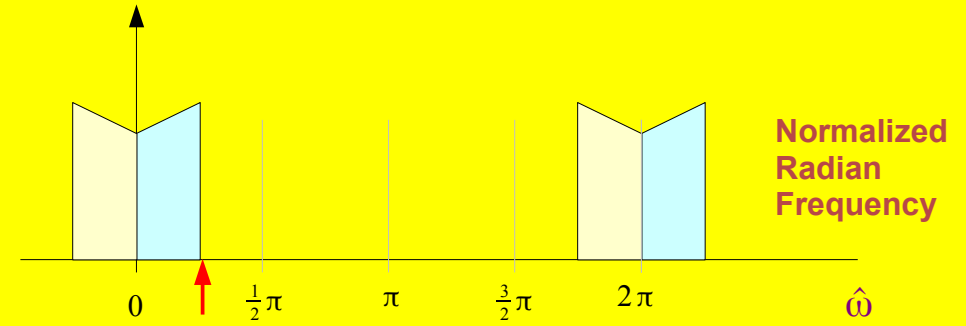
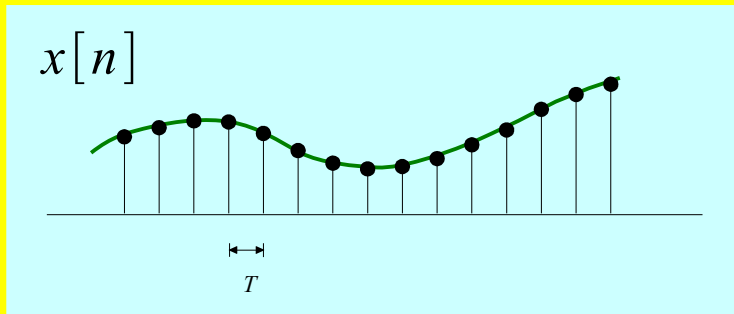
↑ UP



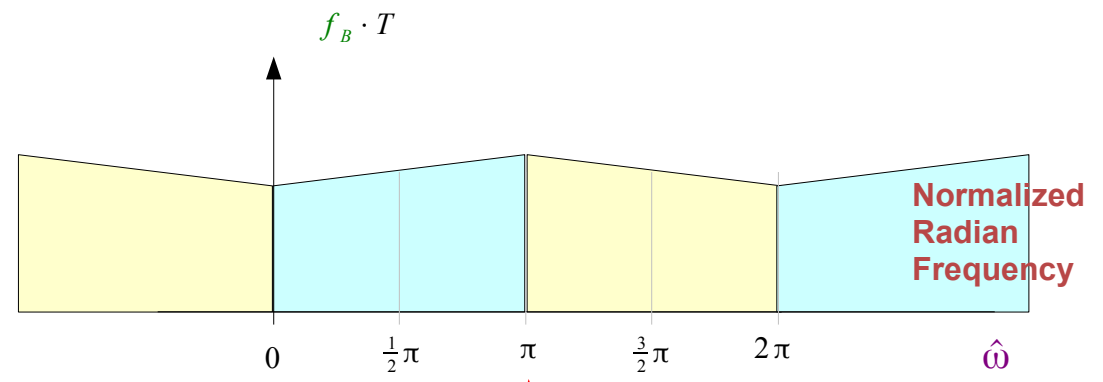
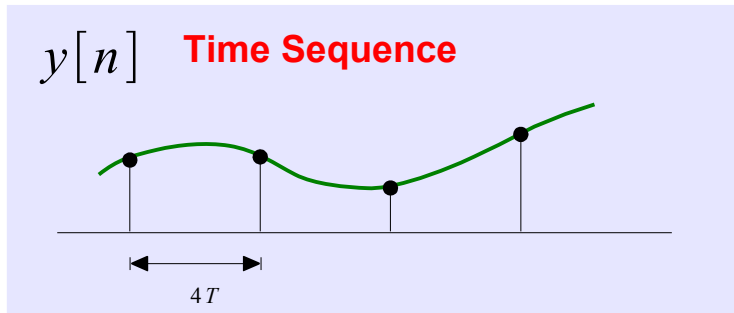
|| The Same Time Sequence



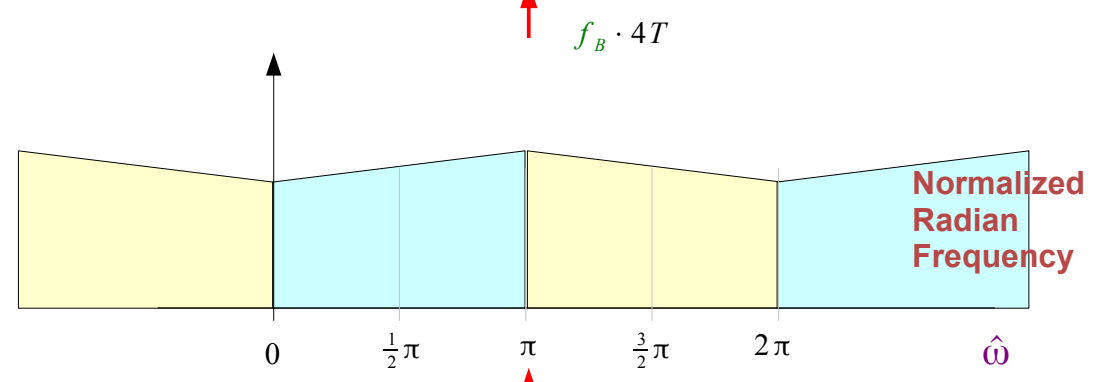
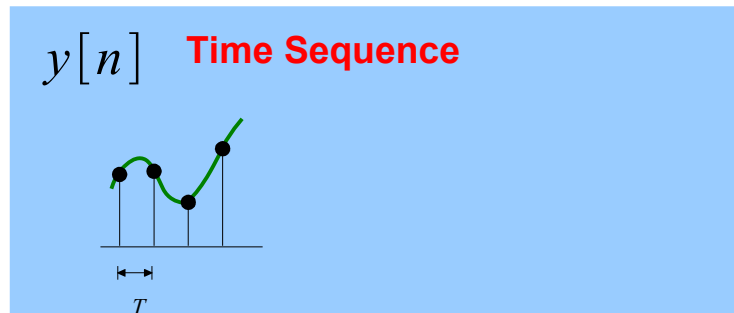
# Fine Sequence Spectrum – Normalized Frequency



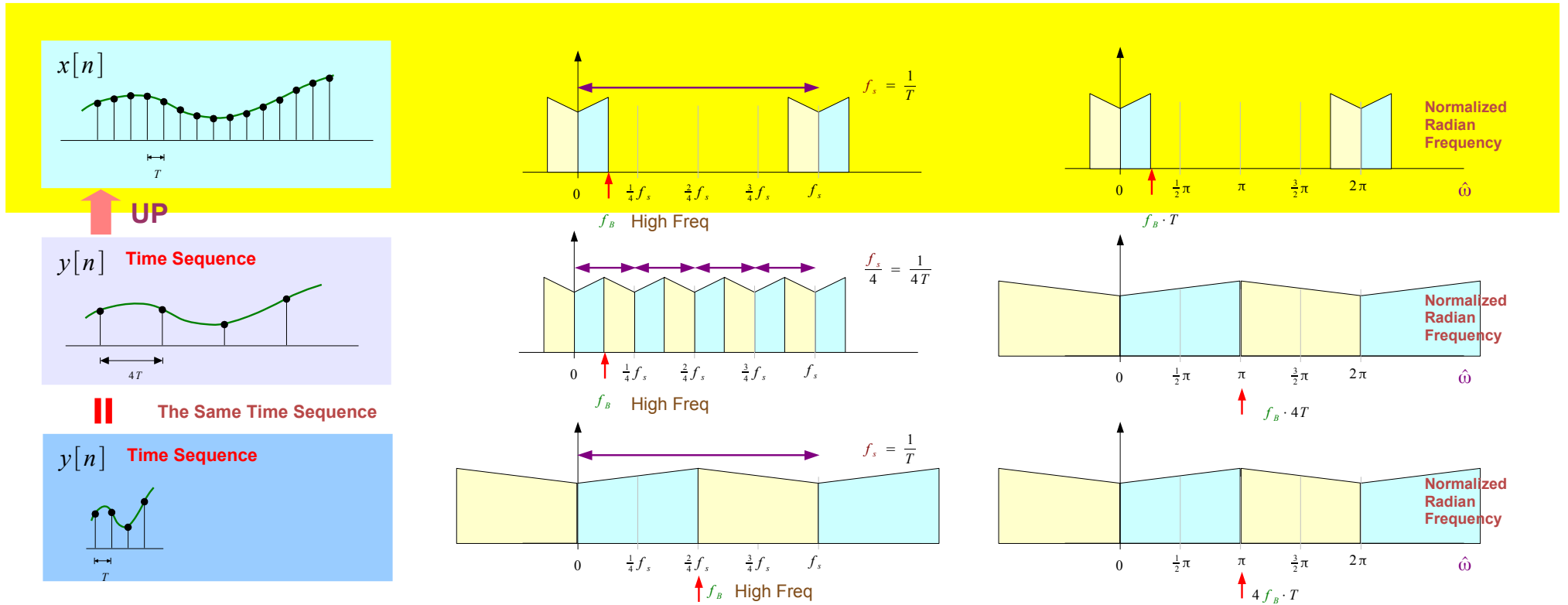
↑ UP



|| The Same Time Sequence

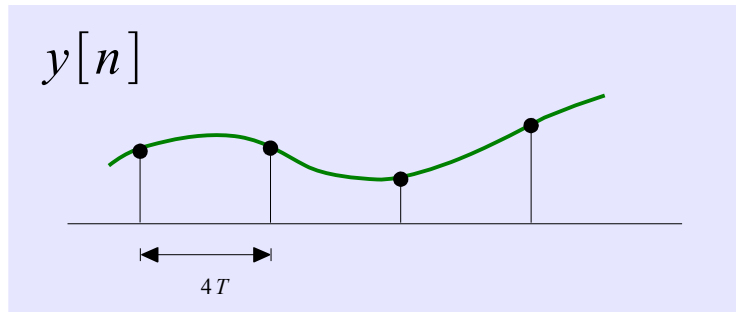


# Fine Sequence Spectrum

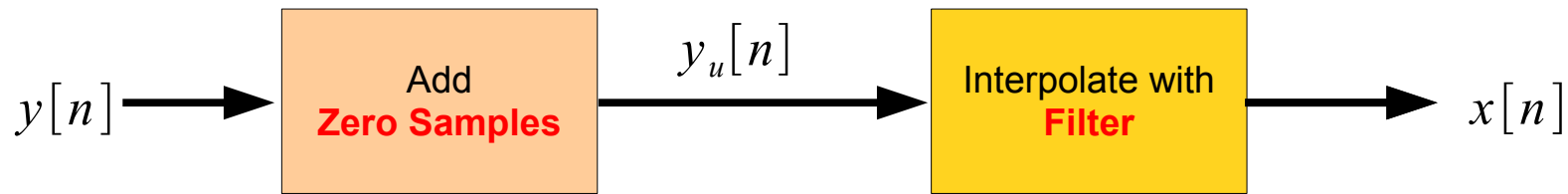
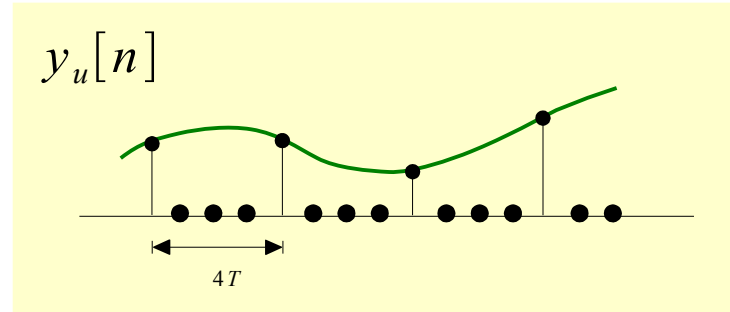




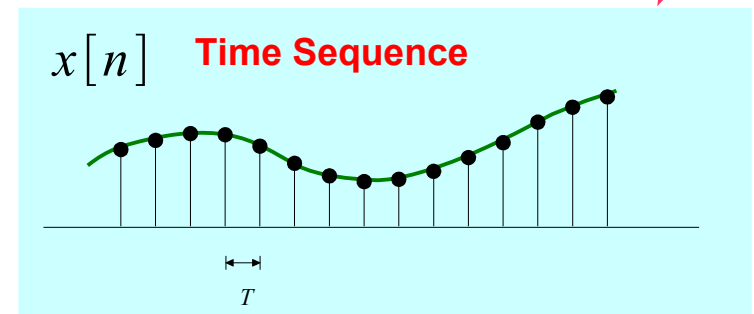
# Fine Sequence Generation



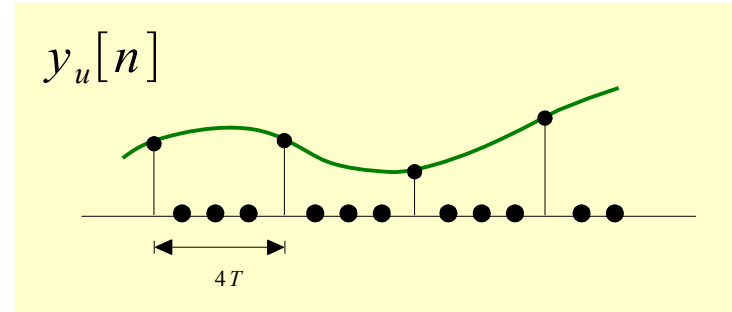
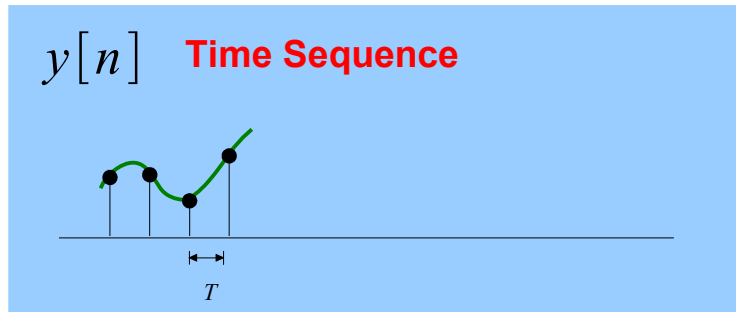
$4T$  Sampling Period



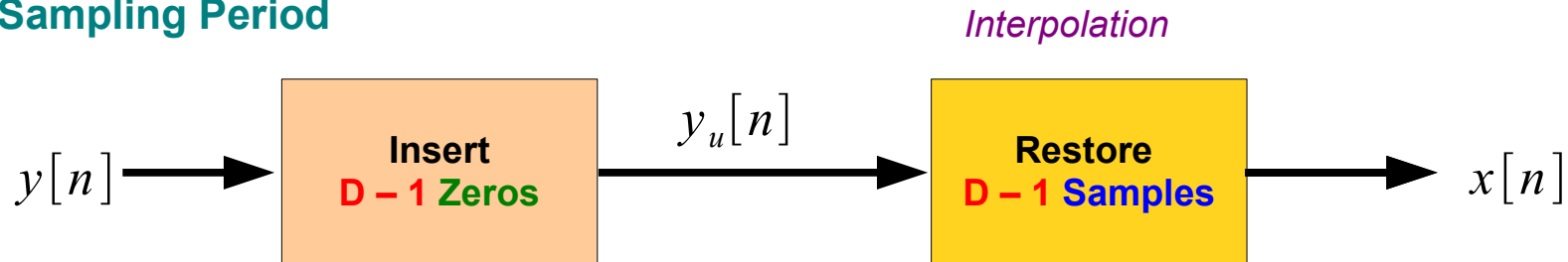
$T$  Sampling Period



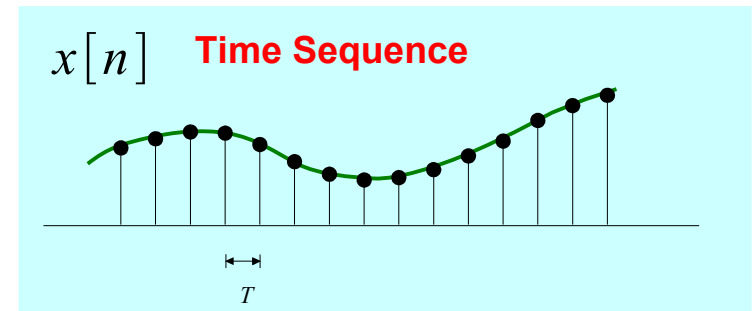
# Up Sampling in Two Steps



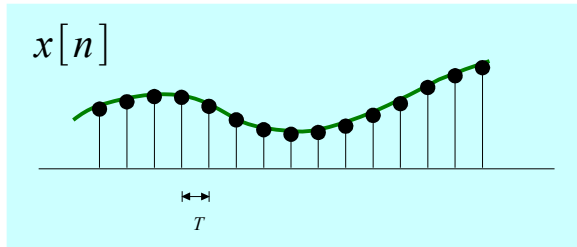
$4f_B$  Highest Frequency  
 $T$  Sampling Period



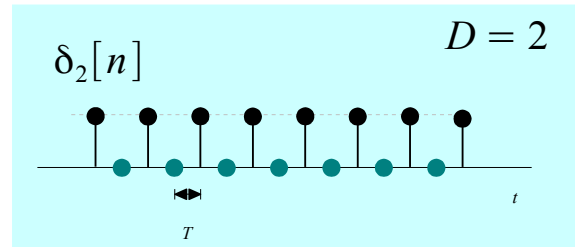
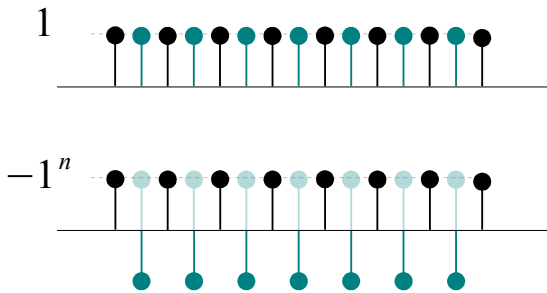
$f_B$  Highest Frequency  
 $T$  Sampling Period



# Example When D=2 (1)

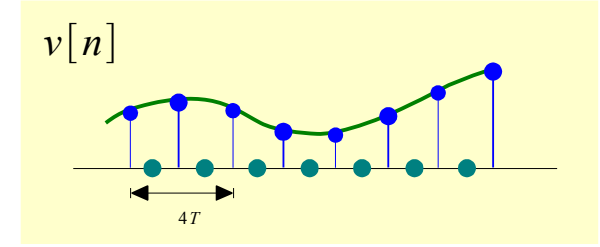


$$x[n] = e^{j\omega n}$$



$$\begin{aligned} \delta_2[n] &= \frac{1}{2}(1 + (-1)^n) \\ &= \frac{1}{2}(1 + e^{-j\pi n}) \\ &\quad (e^{-j\pi} = -1) \end{aligned}$$

$$V(z) = \frac{1}{2} \sum_{n=-\infty}^{+\infty} (x[n]z^{-n} + x[n](-z)^{-n}) = \frac{1}{2}X(z) + \frac{1}{2}X(-z)$$



$$\begin{aligned} v[n] &= \frac{1}{2}x[n] + \frac{1}{2}e^{-j\pi n}x[n] \\ &= \frac{1}{2}e^{j\omega n} + \frac{1}{2}e^{-j\pi n}e^{j\omega n} \\ &= \frac{1}{2}e^{j\omega n} + \frac{1}{2}e^{+j(\omega-\pi)n} \end{aligned}$$

$$V(e^{j\hat{\omega}}) = \frac{1}{2}X(e^{j\hat{\omega}}) + \frac{1}{2}X(e^{-j\pi}e^{j\hat{\omega}})$$

$$V(\hat{\omega}) = \frac{1}{2}X(\hat{\omega}) + \frac{1}{2}X(\hat{\omega} - \pi)$$

# Z-Transform Analysis

$$\delta_D[n] = \begin{cases} 1 & \text{if } n/D \text{ is an integer} \\ 0 & \text{otherwise} \end{cases}$$

$$v[n] = \delta_D[n]x[n]$$

$$V[z] = \cdots + v[0]z^0 + v[D]z^{-D} + v[2D]z^{-2D} + \cdots \quad y[n]$$

$$V[z] = \sum_{n=-\infty}^{+\infty} v[n]z^{-n} = \sum_{m=-\infty}^{+\infty} v[mD]z^{-mD} = F(z^D)$$

$T$  Sampling Period

# Z-Transform Analysis

$$\delta_2[n] = \frac{1}{2}(1 + (-1)^n) = \frac{1}{2}(1 + e^{-j\pi n}) = \begin{cases} 1 & \text{if } n/2 \text{ is an integer (even)} \\ 0 & \text{otherwise} \end{cases}$$

$$e^{-j\pi} = -1$$

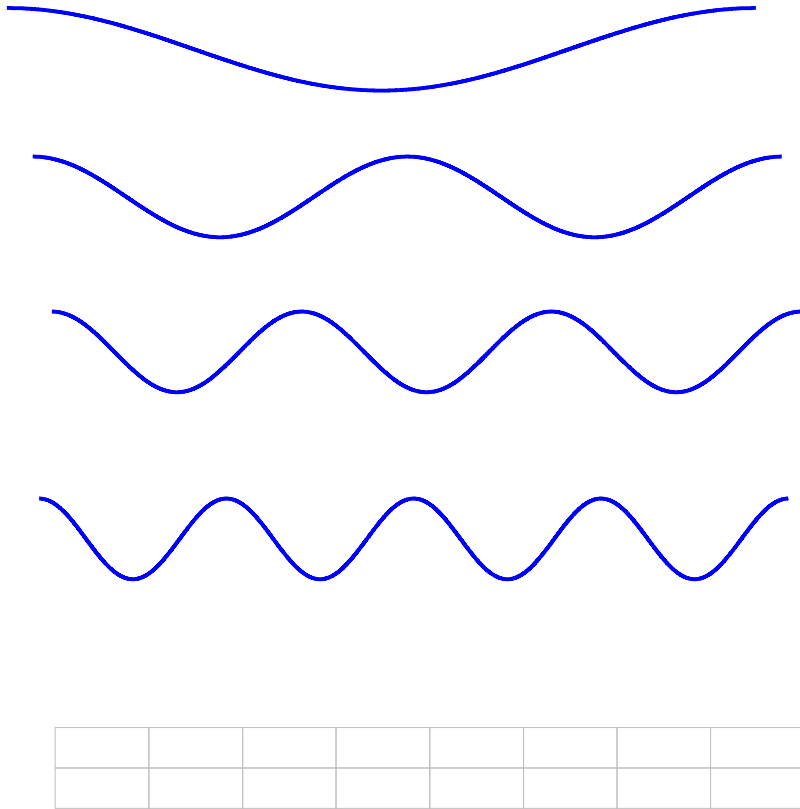
$$v[n] = \frac{1}{2}x[n] + \frac{1}{2}e^{-j\pi n}x[n] \quad x[n] = e^{j\omega n}$$

$$v[n] = \frac{1}{2}e^{j\omega n} + \frac{1}{2}e^{-j\pi n}e^{j\omega n} = \frac{1}{2}e^{j\omega n} + \frac{1}{2}e^{+j(\omega-\pi)n}$$

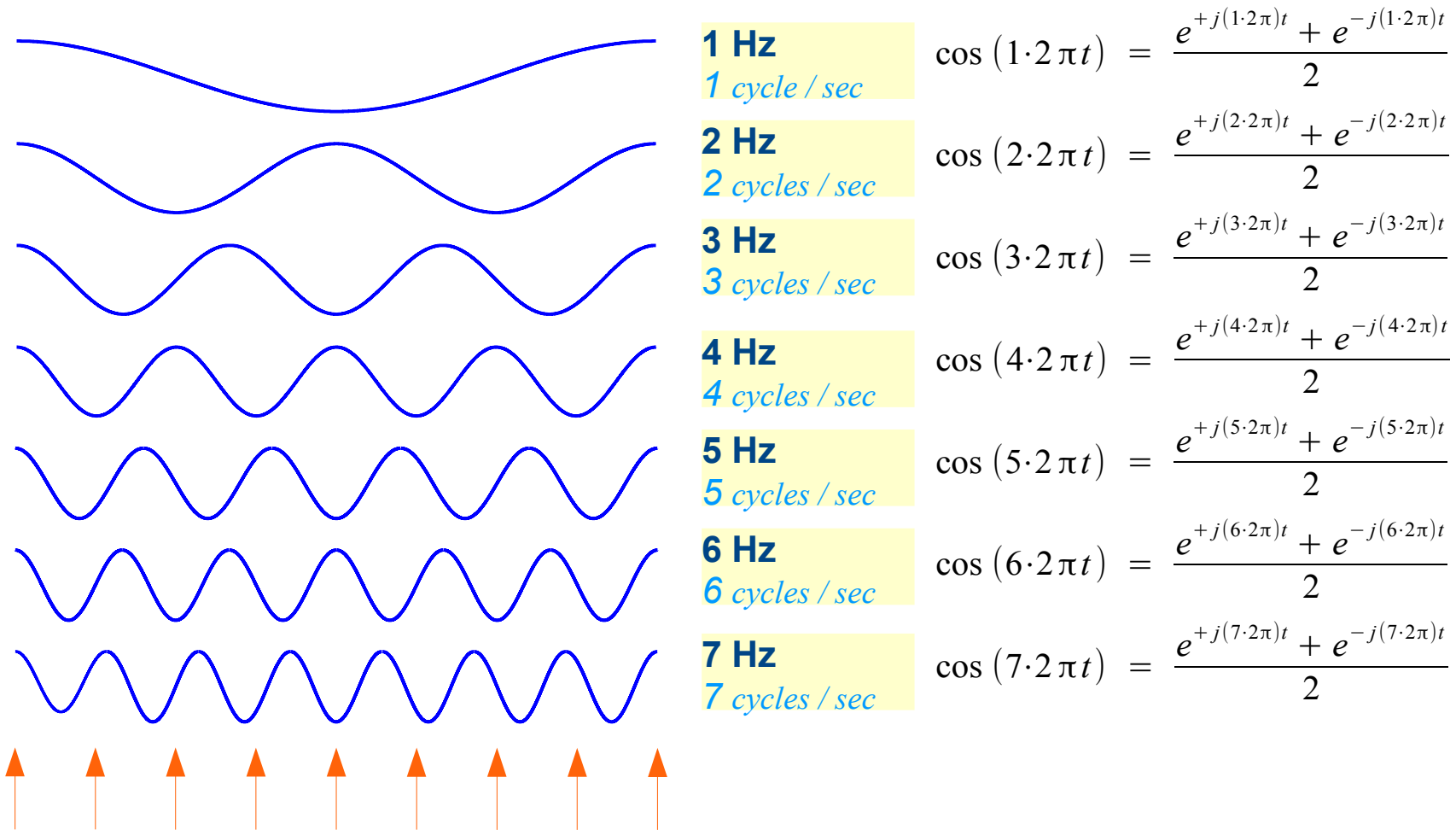
$$V(z) = \frac{1}{2} \sum_{n=-\infty}^{+\infty} (x[n]z^{-n} + x[n](-z)^{-n}) = \frac{1}{2}X(z) + \frac{1}{2}X(-z)$$

$$V(\omega) = V(e^{j\omega}) = \frac{1}{2}X(e^{j\omega}) + \frac{1}{2}X(e^{-j\pi}e^{j\omega}) = \frac{1}{2}X(\omega) + \frac{1}{2}X(\omega - \pi)$$

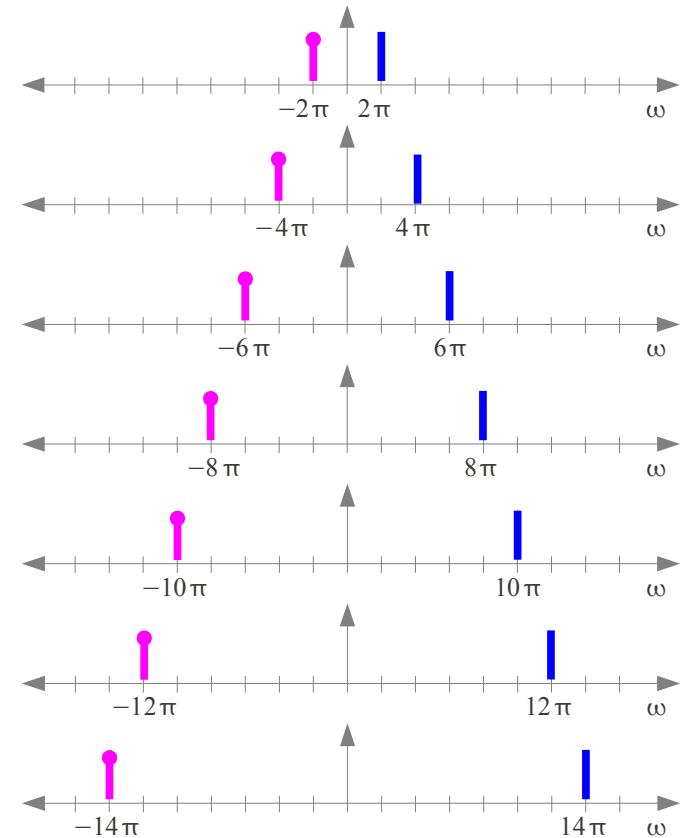
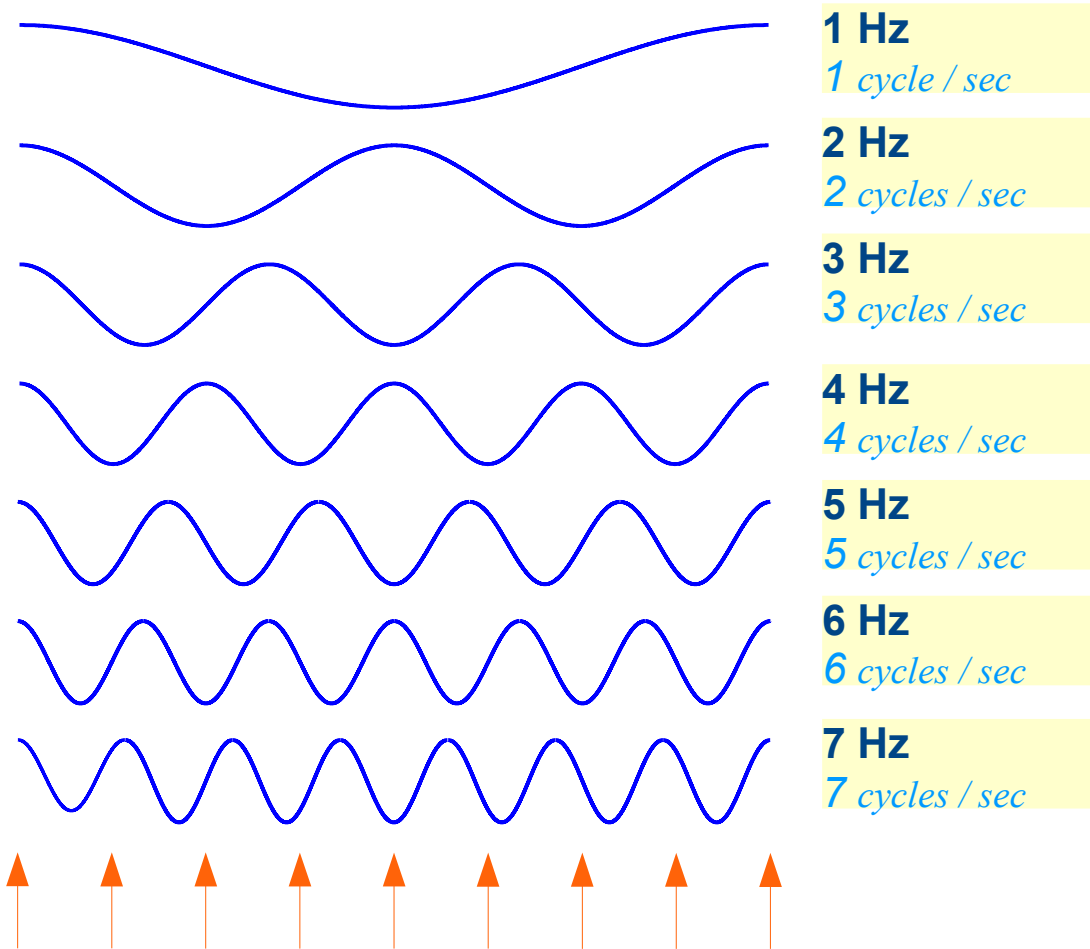
# Measuring Rotation Rate



# Signals with Harmonic Frequencies (1)

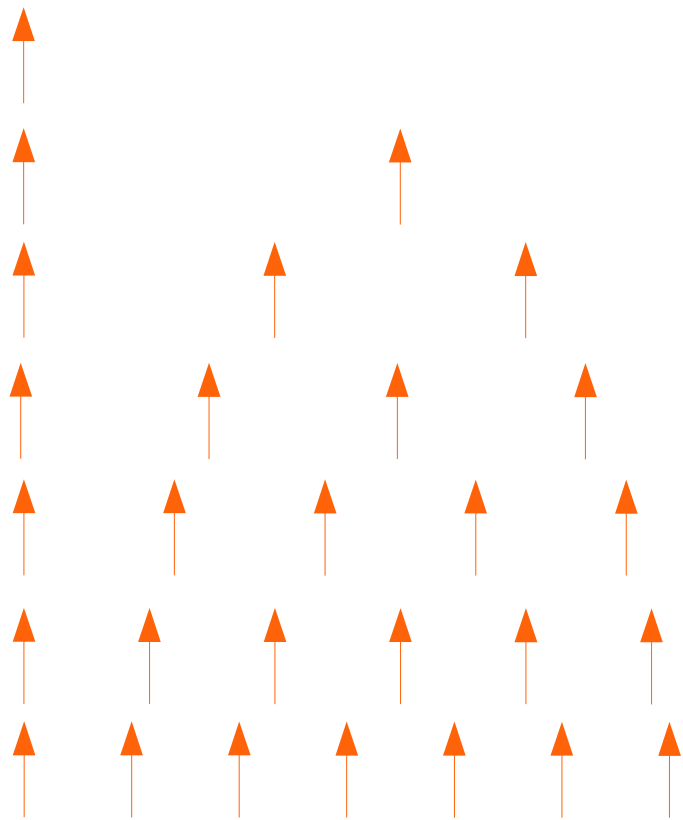


# Signals with Harmonic Frequencies (2)

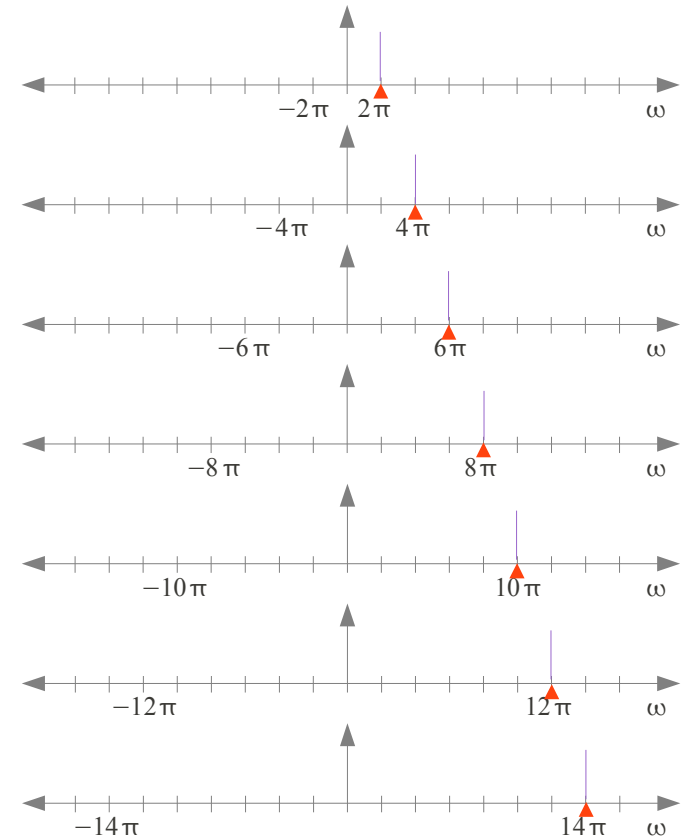




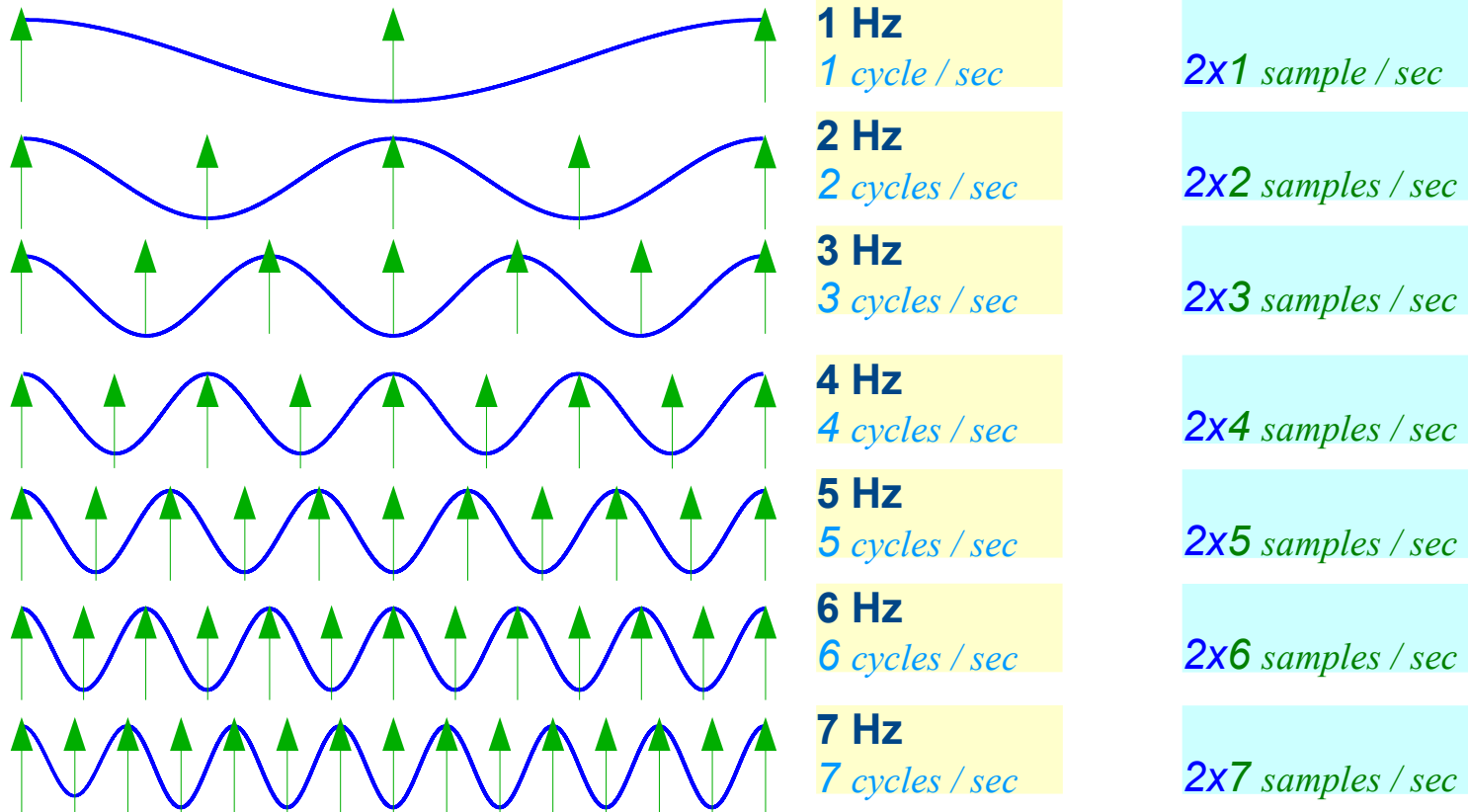
# Sampling Frequency



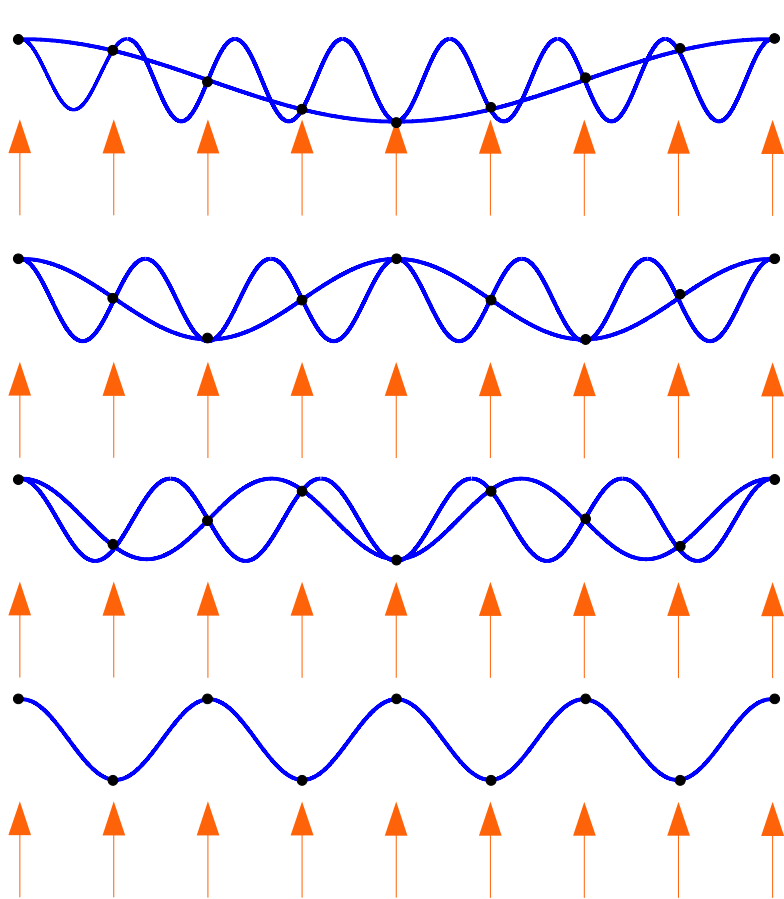
- 1 Hz  
*1 sample / sec*
- 2 Hz  
*2 samples / sec*
- 3 Hz  
*3 samples / sec*
- 4 Hz  
*4 samples / sec*
- 5 Hz  
*5 samples / sec*
- 6 Hz  
*6 samples / sec*
- 7 Hz  
*7 samples / sec*



# Nyquist Frequency



# Aliasing



1 Hz  
7 Hz

$2 \times 4$  samples / sec

2 Hz  
6 Hz

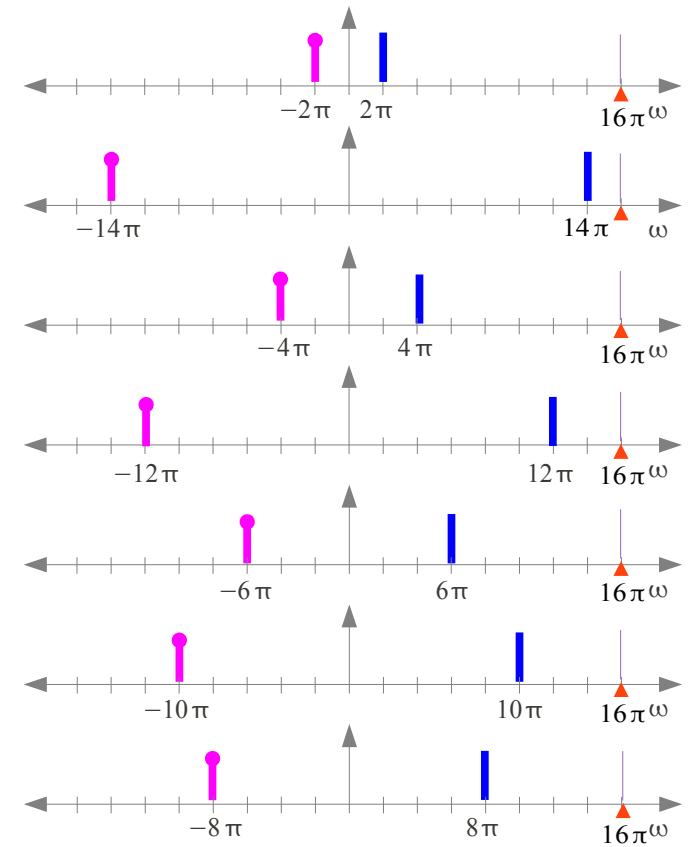
$2 \times 4$  samples / sec

3 Hz  
5 Hz

$2 \times 4$  samples / sec

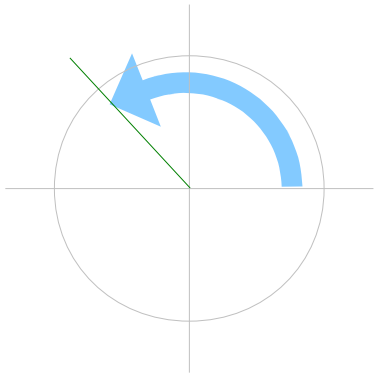
4 Hz

$2 \times 4$  samples / sec



# Sampling

$$\omega_s = 2\pi f_s \text{ (rad/sec)}$$



$$\omega_1 = 2\pi f_1$$

$$\omega_1 = \frac{\omega_s}{2} \text{ (rad/sec)}$$

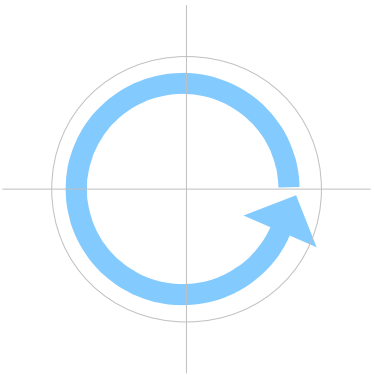
$$f_1 = \frac{f_s}{2} \text{ (rad/sec)}$$

$$\omega_2 = 2\pi f_2$$

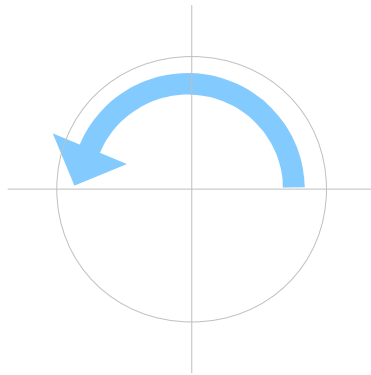
$$\omega_2 = -\frac{\omega_s}{2} \text{ (rad/sec)}$$

$$f_2 = -\frac{f_s}{2} \text{ (rad/sec)}$$

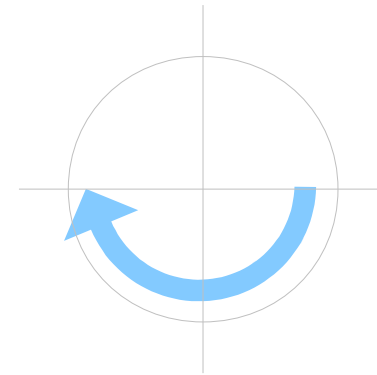
$$2\pi \text{ (rad)} / T_s \text{ (sec)}$$



$$\pi \text{ (rad)} / T_s \text{ (sec)}$$

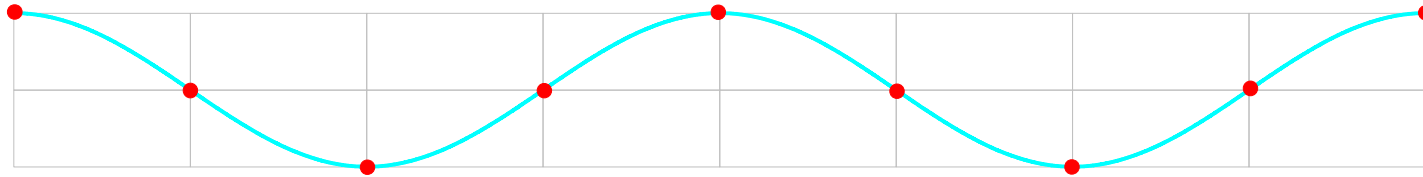


$$-\pi \text{ (rad)} / T_s \text{ (sec)}$$

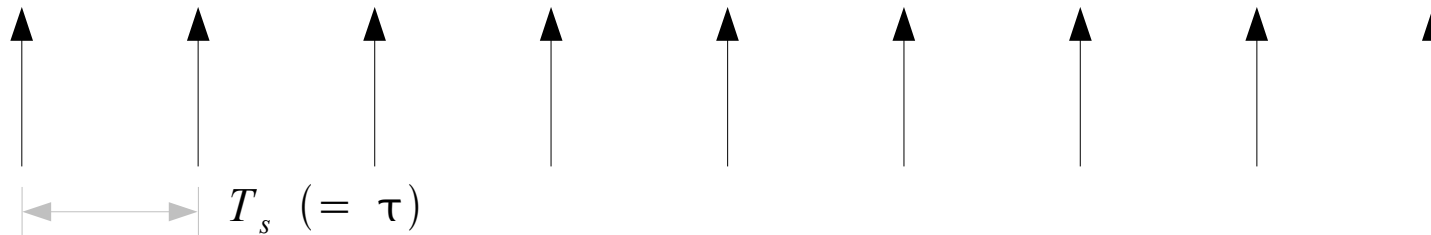


# Sampling

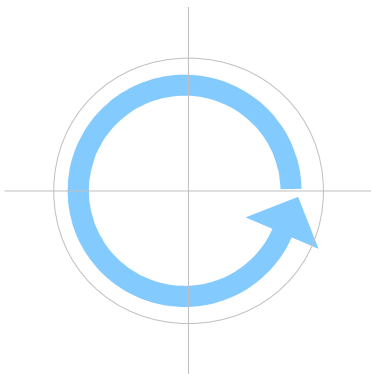
$$\omega_1 = 2\pi f_1 \text{ (rad/sec)}$$



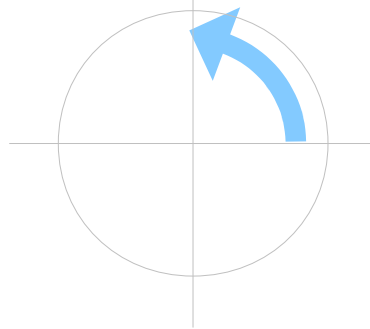
$$\omega_s = 2\pi f_s \text{ (rad/sec)}$$



$$2\pi \text{ (rad)} / T_s \text{ (sec)}$$



$$\frac{\pi}{2} \text{ (rad)} / T_s \text{ (sec)}$$



For the period of  $T_s$   
Angular displacement  $\frac{\pi}{2}$  (rad)

$$\begin{aligned} \hat{\omega} &= \omega \cdot T_s \text{ (rad)} \\ &= 2\pi f_1 \cdot T_s \text{ (rad)} \\ &= 2\pi \frac{f_s}{4} \cdot T_s \text{ (rad)} \\ &= \frac{\pi}{2} \text{ (rad)} \end{aligned}$$

# Angular Frequencies in Sampling

## continuous-time signals

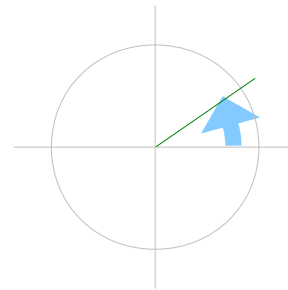
Signal Frequency

$$f_0 = \frac{1}{T_0}$$

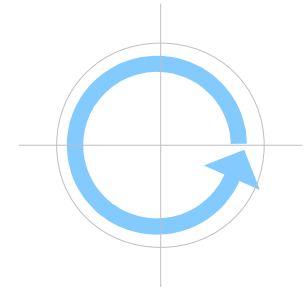
Signal Angular Frequency

$$\omega_0 = 2\pi f_0 \text{ (rad/sec)}$$

For 1 second  
 $2\pi f_0 \text{ (rad/sec)}$



For 1 revolution  
 $2\pi \text{ (rad)}$   
 $T_0 \text{ (sec)}$



## sampling sequence

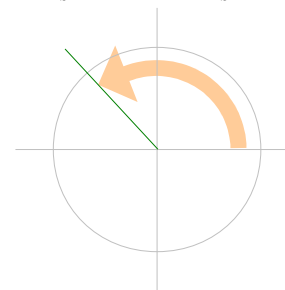
Sampling Frequency

$$f_s = \frac{1}{T_s}$$

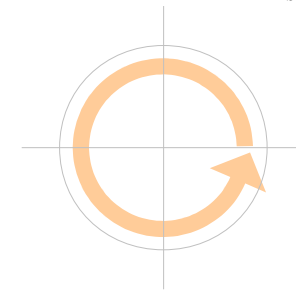
Sampling Angular Frequency

$$\omega_s = 2\pi f_s \text{ (rad/sec)}$$

For 1 second  
 $2\pi f_s \text{ (rad/sec)}$



For 1 revolution  
 $2\pi \text{ (rad)}$   
 $T_s \text{ (sec)}$











## References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] A "graphical interpretation" of the DFT and FFT, by Steve Mann
- [4] R. Cristi, "Modern Digital Signal Processing"